Chapter 5

Two-Phase Network Data Envelopment Analysis: An Example of Bank Performance Assessment

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Additional information is available at the end of the chapter

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Abstract

Data envelopment analysis (DEA) models assess decision-making units (DMUs), which directly convert multiple inputs into multiple outputs. Network DEA models have been studied extensively. However, the performance indices that link the two stages are assumed to be fixed or non-discretionary; their values are not adjustable. These models only assumed that the reductions on the inputs and additions on the outputs would improve the overall efficiency. But in the real world, the link is always adjustable. "Free links" means that the intermediate items are adjustable or discretionary, and each DMU can be increased or decreased from the observed one. The current chapter introduces a two-phase procedure with free links to assess system performance, Phase-I is a proposed slack-based measurement (SBM) model to partition the links into two sets: as-input and as-output. Phase-II is a modified SBM model to determine the slack of each input, as-input link, output and as-output link. This proposed model counts the slacks associated with the intermediate items in the efficiency scores and determines the entire system performance by the directional distance function. It is validated using network procedure and assesses the performance of supply chain management system.

Keywords: data envelopment analysis, performance measure, directional distance function, network DEA, slack-based measure

1. Introduction

The data envelopment analysis (DEA) models assess a set of homogeneous decision-making units (DMUs) that convert inputs into outputs. Fewer input values and more output values are desired and DMUs may be classified as being either efficient or inefficient. Tone and Tsutsui [1, 2] introduce network and dynamic DEA and categorize links into two types—"fixed links" and "free links". The free links mean the links are adjustable; each DMU can be increased or decreased from the observed one and identifies the improvement target of each inefficient DMU on the frontier that is constructed by the efficient DMUs.

Seiford and Zhu [3] and Zhu [4] have introduced a two-stage process to measure the profitability and marketability of 55 US commercial banks and top Fortune 500 companies, respectively. They propose the effect of bank size on profitability and marketability through evaluating both technical and scale efficiencies. Sexton and Lewis [5] use a two-stage approach to evaluate the scores of American Major League Baseball teams. There are many other cases in which the whole operation is separated into more than two processes. These may have a series structure, a parallel structure, or a mixture of these. These structures are generally called network structures and the DEA technique to measure the efficiency of systems with a network structure is called network DEA (Färe & Grosskopf [6]). Färe and Whittaker [7] and Färe and Grosskopf [8] introduce models to compute the efficiency scores of sub-processes in network-structured DEA problems. Lewis and Sexton [9] introduce a network DEA model which focuses efficiency-enhancing strategies on individual stages of the production process. Kao and Hwang [10] introduce a framework for breaking down the efficiency of the entire process into the product of the efficiencies of the two-stage process. It assumes that the weights on the links are the same for the two stages, that is, the weights on the outputs in the first stage are assumed to be equal to the weights on the inputs in the second stage. In the real world, the relative weight of each stage is determined corresponding to its importance. Thus, the different weights in the entire system are mentioned in recent studies. Chen et al. [11] mentions that the overall efficiency scores resulting from Kao and Hwang [10] are not direct indicators of potential input reductions or output increases not realized by the inefficient DMUs. They develop an approach to determine the DEA frontier or DEA projections for inefficient DMUs. Chen et al. [12] note that the envelopment-based network DEA model should be used for determining the frontier projection for inefficient DMUs, whereas the multiplier-based network DEA model should be used for determining the divisional efficiency because it does not account for the intermediate links. Kao [13] proposes a dynamic DEA model to measure system and period efficiencies at the same time for multi-period systems. Chang et al. [14] take into account the ownership structure of networks in constructing effective network DEA models and accordingly develop three ownership-specified (centralized, distributed, and hybrid) network DEA models. Huang et al. [15] proposed a two-stage network model with bad outputs and supper efficiency (US-NSBM). Empirical comparisons show that the US-NSBM may be promising and practical for taking the nonperforming loans into account and being able to rank all samples.

However, these approaches do not count the slacks associated with the intermediate items in the efficiency scores. Consequently, the efficiency scores are greater than the actual efficiency. In addition, there is no DMU with an efficiency score equal to 1 because the properties of intermediate performance evaluation items would lead to conflicts. For instance, in the two-stage process problem, Stage-2 may have to reduce inputs (links) to achieve an efficient status. However, doing so would lead to a reduction in outputs in Stage-1, thereby reducing the efficiency of Stage-1. In other words, there are still two efficiency frontiers for the two sub-processes. One may desire a single frontier for the entire production system.

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"Link" cannot be adjusted freely in a radial model which adjusts the inputs and outputs by the efficiency scores in a two-stage process. For this model, the entire system efficiency cannot be improved by adjusting links, see Kao and Hwang [10] and Lewis and Sexton [9]. "Link" that applies in a non-radial model has been discussed in recent years. Tone and Tsutsui [1] introduce a network DEA and categorize links into two types—"fixed links" and "free links." "Free links" means the intermediate items are adjustable or discretionary; each DMU can be increased or decreased from the observed one and is free to assign each individual link to one of the three characteristics: as-input, as-output, or non-discretionary so that the entire system efficiency could be maximized. "Fixed links" means the intermediate products are beyond the control of DMUs. In the radial model, "links" cannot be adjusted freely, which adjust the inputs and outputs by the efficiency scores in a two-stage process. Tone and Tsutsui [2] introduce the dynamic slack-based measure (DSBM) model and the incorporation of slacks with *free* and *fixed* links into the efficiency score. They categorize the links into four types: desirable, undesirable, discretionary (free), and non-discretionary (fixed). The article incorporates the slacks of free links into the efficiency score in two ways: an ex-post approach (adjusted score) and incorporation through 0–1 MIP. The ex-post approach includes a two-phase procedure. Tone and Tsutsui [1] introduce the links are discretionary regarding their status, as-input or as-output. Liu and Liu [16, 17] adopt VGM and GBM models to assess the performance of supply chain management.

Chambers et al. [18] introduced the directional distance function (DDF) based on the Luenberger benefit function to obtain the technical efficiency by increasing the outputs and reducing the inputs simultaneously. Later, Chambers et al. [19] introduced the DDF of DEA to measure the technical efficiency. This chapter develops a model for an improved efficiency measure through directional distance formulation of data envelopment analysis.

The contribution and innovative progress for this chapter are (1) creating a new SBM model and converting multi-efficiency frontiers for the separation processes to an aggregation efficiency frontier for the entire production system and (2) adopting free links application and introducing DDF with a virtual gap diagram to assess the performance of the entire system. The rest of this chapter is organized as follows. The proposed two-phase two-stage performance evaluation models and DDF are presented in Section 2. Because the uniqueness of the optimal solution is important, we report an experiment on this subject using a real-world bank performance assessment in Section 3. We conclude this chapter in the last section.

2. The proposed two-phase two-stage performance evaluation

J denotes the set of homogeneous decision-making units of a network process that are evaluated by a set of inputs, I, a set of free links, D^{free}, and a set of outputs R. DMU_o represents the DMU under evaluation. To maximize the system efficiency score of DMU_o, each link in set D^{free} is "free" to be assigned to one of the subsets – D_o^-, D_o^{free}, and D_o^+ if it is as-input, free link, and as-output, respectively. **Figure 1** depicts the two-phase procedure to evaluate the performance of DMUs using the two-stage and network processes. This two-phase procedure contains two

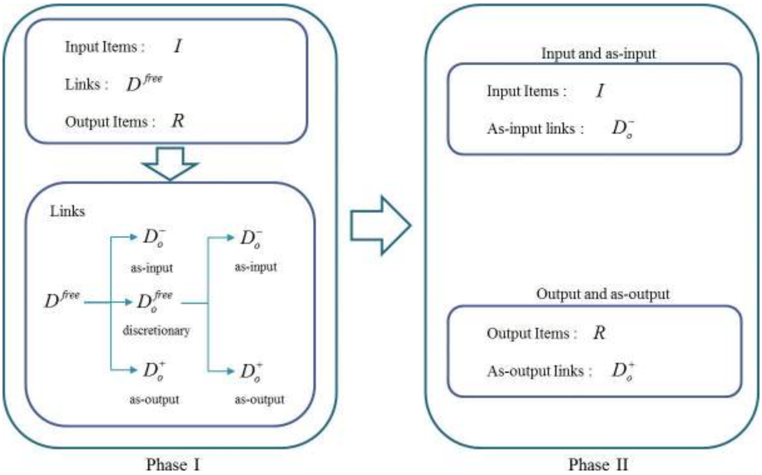

Figure 1. The flow of two-phase procedure.

slack-based DEA linear programming models. Phase-I sets all links in set D^{free} to discretionary and the objective is to determine the maximum slack values on each input and output so that the weights of each DMU in each stage can be assigned. The set D^{free} is partitioned into two subsets D_o^-, D_o^{free} and D_o^+. The output of Phase-I will indicate that several links in set D_o^{free} should be assigned to sets D_o^- and D_o^+. The target of Phase-II is thus to determine the maximum reduction value on each input and link in sets I and D_o^- and the addition value on each output and link in sets R and D_o^+ such that the weights of each DMU in the system can be assigned. The target of each input, free link, and output on the frontier is identified.

2.1. Two stages: Phase-I

Envelopment via the SBM fractional programming model [M1] is used to measure the relative performance of DMU_o. The decision variables of slack are the values to be subtracted at the ith input and the value to be added at the rth output, respectively. The decision variables denote the weights of DMU_j at Stage-1 and Stage-2, respectively. The right-hand side of constraints (1.2) and (1.3) expresses the targets of the inputs and links (outputs) at Stage-1. Each link d in set D^{free} is free to increase the slack s_{zd}^{free+} or decrease the slack s_{zd}^{free-}. These slacks are non-discretionary and not counted in the objective function. The decision variables on the left-hand side are $\lambda_{1j},\ j \in J$. Similarly, the right-hand side of constraints (1.4) and (1.5) expresses the targets of the links (inputs) and outputs at Stage-2. The decision variables on the left-hand side are $\lambda_{2j},\ j \in J$. Constraints (1.3) and (1.4) indicate that their right-hand sides are equal; each link d in set D^{free} can be freely adjusted to reach its single target at Stage-1 and Stage-2 simultaneously. The optimal slack value of link d in set D^{free}, $-s_{zd}^{free-*} + s_{zd}^{free+*}$, is >0, <0 or =0, in which case link d of DMU_o is assigned to sets D_o^+, D_o^-, and D_o^{free}, respectively. Two tasks remain for

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DMU_o. The first task is to assign all elements in the set to either D_o^+ or D_o^-. The second task is to place the slacks of link d in sets and add them to its aggregate performance score. Phase-II of our solving procedure addresses the first task.

[M1]

$$\rho_o^{(I)*} = \text{Min}\left[1 - \left(\sum_{i\in I}\frac{s_i^-}{x_{io}}\right)/|I|\right] \Big/ \left[1 + \left(\sum_{r\in R}\frac{s_r^+}{y_{ro}}\right)\Big/|R|\right]; \quad (1)$$

$$s.t. \quad \sum_{j\in J}\lambda_{1j}x_{ij} = x_{io} - s_i^-, \quad i\in I; \quad (2)$$

$$\sum_{j\in J}\lambda_{1j}z_{dj} = z_{do} + s_{zd}^{free+} - s_{zd}^{free-}, \quad d\in D^{free}; \quad (3)$$

$$\sum_{j\in J}\lambda_{2j}z_{dj} = z_{do} + s_{zd}^{free+} - s_{zd}^{free-}, \quad d\in D^{free}; \quad (4)$$

$$\sum_{j\in J}\lambda_{2j}y_{rj} = y_{ro} + s_r^+, \quad r\in R; \quad (5)$$

$$s_i^-, s_r^+, s_{zd}^{free+}, s_{zd}^{free-}, \lambda_{qj} \geq 0; \quad q = 1, 2; \quad j\in J; i\in I; \quad r\in R; \quad d\in D^{free}. \quad (6)$$

For the two-phase procedure which is depicted in **Figure 1**, Phase-I is to determine the maximum slack values on each input and output; [M1] presents this purpose and adopts Eq. (1.3) and (1.4) to distinguish links to be as-input, discretionary, and as-output, which express as three subsets, D_o^-, D_o^{free} and D_o^+, respectively. The aim of Phase-I is to assign each element in set D_o^{free} to either D_o^+ or D_o^-. [M2] is repeated until set D_o^{free} becomes empty. The fractional programming model [M2] measures the overall efficiency $\rho_o^{(II)*}$. The decision variables π_j^1 and π_j^2 denote the weights in Stage-1 and Stage-2, respectively, of DMU_j in evaluating DMU_o.

[M2]

$$\rho_o^{(II)*} = \text{Min}\frac{\left(\sum_{i\in I}\frac{x_{io}-s_i^-}{x_{io}} + \sum_{d\in D_o^-}\frac{z_{do}-s_{zd}^-}{z_{do}}\right)\Big/\left(|I| + |D_o^-|\right)}{\left(\sum_{r\in R}\frac{y_{ro}+s_r^+}{y_{ro}} + \sum_{d\in D_o^+}\frac{z_{do}+s_{zd}^+}{z_{do}}\right)\Big/\left(|R| + |D_o^+|\right)}; \quad (7)$$

$$s.t. \quad \sum_{j\in J}\pi_j^1 x_{ij} = x_{io} - s_i^-, \quad i\in I; \quad (8)$$

$$\sum_{j\in J}\pi_j^1 z_{dj} = z_{do} - s_{zd}^-, \quad d\in D_o^-; \quad (9)$$

$$\sum_{j \in J} \pi_j^1 z_{dj} = z_{do} - s_{zd}^{free-} + s_{zd}^{free+}, \quad d \in D_o^{free}; \tag{10}$$

$$\sum_{j \in J} \pi_j^2 z_{dj} = z_{do} - s_{zd}^{free-} + s_{zd}^{free+}, \quad d \in D_o^{free}; \tag{11}$$

$$\sum_{j \in J} \pi_j^2 z_{dj} = z_{do} + s_{zd}^{+}, \quad d \in D_o^{+}; \tag{12}$$

$$\sum_{j \in J} \pi_j^2 y_{rj} = y_{ro} + s_r^{+}, \quad r \in R; \tag{13}$$

$$\pi_j^1, \pi_j^2 \geq 0, \quad s_i^-, s_r^+ \geq 0, \quad j \in J, i \in I, \quad r \in R; \quad s_{zd}^+ \geq 0, \quad d \in D_o^+; \quad s_{zd}^- \geq 0, \quad d \in D_o^-. \tag{14}$$

$$s_{zd}^{free+}, s_{zd}^{free-} \geq 0, \quad d \in D_o^{free}. \tag{15}$$

The solution of [M2] for each link d in set D_o^{free} is one of the following cases, $-s_{zd}^{free-*} + s_{zd}^{free+*} < 0$, $-s_{zd}^{free\ -*} + s_{zd}^{free+*} > 0$, and $-s_{zd}^{free-*} + s_{zd}^{free+*} = 0$. Next, d is assigned to set D_o^-, D_o^+, and D_o^{free} accordingly.

2.2. Two stages: Phase-II

The results of Phase-I indicate that DMU_o already partitioned set D_o^{free} into D_o^- and D_o^+. The fractional programming model [M3] is an SBM model (Tone and Tsutsui [21]) that measures the efficiency of converting the sets of input and as-input indices $I \cup D_o^-$ into the sets of output and as-output indices $R \cup D_o^+$. The frontiers of π_j^1 and π_j^2 are converted into an entire system frontier π_j in this phase.

[M3]

$$E_o^* = \text{Min} \frac{\left(\sum_{i \in I} \frac{x_{io} - s_i^-}{x_{io}} + \sum_{d \in D_o^-} \frac{z_{do} - s_{zd}^-}{z_{do}} \right) \Big/ \left(|I| + |D_o^-| \right)}{\left(\sum_{r \in R} \frac{y_{ro} + s_r^+}{y_{ro}} + \sum_{d \in D_o^+} \frac{z_{do} + s_{zd}^+}{z_{do}} \right) \Big/ \left(|R| + |D_o^+| \right)}; \tag{16}$$

$$s.t. \quad \sum_{j \in J} \pi_j x_{ij} = x_{io} - s_i^-, \quad i \in I; \tag{17}$$

$$\sum_{j \in J} \pi_j z_{dj} = z_{do} - s_{zd}^-, \quad d \in D_o^-; \tag{18}$$

$$\sum_{j \in J} \pi_j z_{dj} = z_{do} + s_{zd}^+, \quad d \in D_o^+; \tag{19}$$

$$\sum_{j\in J}\pi_j y_{rj}=y_{ro}+s_r^+,\quad r\in R; \tag{20}$$

$$\pi_j\geq 0,\ \ s_i^-,s_r^+\geq 0,\ \ j\in J, i\in I,\ \ r\in R;\ \ s_{zd}^+\geq 0,\ \ d\in D_o^+;\ \ s_{zd}^-\geq 0,\ \ d\in D_o^-. \tag{21}$$

The dual form of [M3] is expressed as [M4]. The decision variables of [M4] possess properties $\boldsymbol{v_i}$ and $\boldsymbol{u_r}$, representing the weight assigned to the *i*th input and the *r*th output, respectively. The terms w_d^+ and w_d^- represent the weight assigned to link *d* in sets D_o^+ and D_o^-, respectively.

[M4]

$$\Delta_o^*=\text{Max}\left(-\sum_{i\in I}v_i x_{io}-\sum_{d\in D_o^-}w_d^- z_{do}+\sum_{d\in D_o^+}w_d^+ z_{do}+\sum_{r\in R}u_r y_{ro}\right); \tag{22}$$

$$s.t.\quad -\sum_{i\in I}v_i x_{ij}-\sum_{d\in D_o^-}w_d^- z_{dj}+\sum_{d\in D_o^+}w_d^+ z_{dj}+\sum_{r\in R}u_r y_{rj}\leq 0,\ \ j\in J; \tag{23}$$

$$v_i\geq(1/x_{io})/\left(|I|+\left|D_o^-\right|\right),\quad i\in I; \tag{24}$$

$$w_d^-\geq(1/z_{do})/\left(|I|+\left|D_o^-\right|\right),\quad d\in D_o^-; \tag{25}$$

$$w_d^+\geq\varsigma\times(1/z_{do})/\left(|R|+\left|D_o^+\right|\right),\quad d\in D_o^+; \tag{26}$$

$$u_r\geq\varsigma\times\left(1/y_{ro}\right)/\left(|R|+\left|D_o^+\right|\right),\quad r\in R; \tag{27}$$

$$\varsigma=\left(1-\sum_{i\in I}v_i x_{io}-\sum_{d\in D_o^-}z_{do}w_d^-+\sum_{d\in D_o^+}z_{do}w_d^++\sum_{r\in R}u_r y_{ro}\right); \tag{28}$$

$$v_i,\,u_r\ \text{free in sign},\ i\in I,\ \ r\in R; \tag{29}$$

$$w_d^-\ \ \text{free in sign},\ d\in D_o^-;\ w_d^+\ \ \text{free in sign},\ d\in D_o^+. \tag{30}$$

Inequality (4.2) may be revised such that $\left(\sum_{r\in R}u_r y_{rj}+\sum_{d\in D_o^+}w_d^+ z_{dj}\right)\Big/\left(\sum_{i\in I}v_i x_{ij}+\sum_{d\in D_o^-}w_d^- z_{dj}\right)\leq 1$, and the constraint ensures that the maximum performance value of each DMU_j is not greater than 1.

2.3. Proposed directional distance function approach

The directional distance function (DDF) measures the distance from a certain operation point (e.g., DMU_o) to the efficient frontier of the technology along the positive semi-ray defined by vector g. Given a directional vector $g=\left(-g_X^-,g_Y^+\right)$, $g_X^-\in\Re_+^I\bigcup\Re_+^{D^-}$ and $g_Y^+\in\Re_+^R\bigcup\Re_+^{D^+}$. The objective function (4.1) can be modeled by using the DDF. We denote virtual input by ($X=\sum_{i\in I}v_i x_{io}+\sum_{d\in D_o^-}w_d^- z_{do}$) and virtual output by ($Y=\sum_{d\in D_o^+}w_d^+ z_{do}+\sum_{r\in R}u_r y_{ro}$) which are identified by specifying a directional vector g. The objective function (4.1) can be converted to (4.9) which is to minimize the virtual input and maximize the virtual output to reach the efficient frontier.

$$\Delta_o^* = \text{Max} \left[\left(\sum_{i \in I} v_i x_{io} + \sum_{d \in D_o^-} w_d^- z_{do} \right) (-g_X^-) + \left(\sum_{d \in D_o^+} w_d^+ z_{do} + \sum_{r \in R} u_r y_{ro} \right) (-g_Y^+) \right]; \quad (31)$$

The graph technology can be represented by $T = \left\{ (X, Y); \ X \in \mathfrak{R}_+^I \bigcap \mathfrak{R}_+^{D_o^-}, Y \in \mathfrak{R}_+^R \bigcap \mathfrak{R}_+^{D_o^+} \right\}$. The optimal solution of virtual gap Δ_o^* expresses as DDF: $\vec{D}_g\left(X, Y; \ -g_X^-, g_Y^+\right) = \sup\{\sum_{i \in I} x_{io} v_i^* \times (-g_X^-) + \sum_{d \in D_o^-} z_{do} w_d^{-*} \times (-g_X^-), \sum_{r \in R} y_{ro} u_r^* \times (g_Y^+) + \sum_{d \in D_o^+} z_{do} w_d^{+*} \times (g_Y^+) \in T(X, Y)\}$. This chapter defines a virtual gap diagram; the summation of input and as-input is the x-axis $(\sum_{i \in I} v_i x_{io} + \sum_{d \in D_o^-} w_d^- z_{do})$ and the summation of output and as-output is the y-axis $(\sum_{d \in D_o^+} w_d^+ z_{do} + \sum_{r \in R} u_r y_{ro})$. The geometry on the virtual gap diagram is the slope of the line from DMU_o to origin. To evaluate different DMU_o, one may directly compare their vectors of weights; virtual gap, Δ_o; virtual input and virtual as-input, Δ_o^I; and virtual as-output and virtual output, Δ_o^O. It is obvious that the minimum virtual gap "$\boldsymbol{\Delta_o^*}$" is equivalent to the maximum efficiency score of the entire network. It ensures that the nearest improvement target is found. **Figure 2** depicts the virtual gap diagram; x-axis denotes the virtual input and y-axis denotes the virtual output.

2.4. Overall stage efficiencies

Similar to the SBM non-oriented models of Tone and Tsutsui [20], the solutions of Phase-II provide a reference set of DMUs for DMU_o. The target for the performance items in sets I, D_o^+,

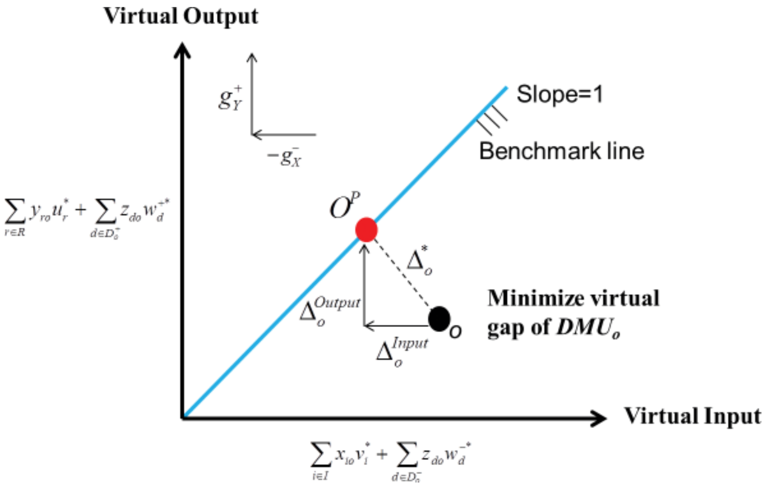

Figure 2. Virtual gap diagram.

D_o^-, and R can be obtained using [E1]. The measured performance value E_o^* is the best practice for DMU_o in the overall two-stage process which is expressed as [E2].

$$\begin{aligned}\widehat{x}_{io} &= x_{io} - s_i^{-*}, \quad i \in I; \\ \widehat{y}_{ro} &= y_{ro} + s_r^{+*}, \quad r \in R; \\ \widehat{z}_{do}^+ &= z_{do} + s_{zd}^{+*}, \quad d \in D_o^+; \\ \widehat{z}_{do}^- &= z_{do} - s_{zd}^{-*}, \quad d \in D_o^-.\end{aligned} \tag{32}$$

These points are the projection of DMU_o on the frontier.

$$E_o^* = \frac{\left(\sum_{i \in I} \frac{\hat{x}_{io}}{x_{io}} + \sum_{d \in D_o^-} \frac{\hat{z}_{do}^-}{z_{do}}\right) \Big/ \left(|I| + |D_o^-|\right)}{\left(\sum_{r \in R} \frac{\hat{y}_{ro}}{y_{ro}} + \sum_{d \in D_o^+} \frac{\hat{z}_{do}^+}{z_{do}}\right) \Big/ \left(|R| + |D_o^+|\right)} \tag{33}$$

The results of Phase-II, $s_i^{-*}, s_{zd}^{-*}, s_{zd}^{+*}$ and s_r^{+*}, are used to compute the efficiencies of Stage-1, E_1^*, and Stage-2; E_2^* is shown in the following two Eqs. (E3 and E4). For the efficiencies of Stage-1, the numerator is the summation of inputs ($x_{io}, \ i \in I$) and as-input ($z_{do}, \ d \in D_o^-$). The denominator is the as-output items ($z_{do}, \ d \in D_o^+$). Likewise, for the efficiencies of Stage-2, the numerator is the as-input item ($z_{do}, \ d \in D_o^-$). The denominator is the summation of the as-output ($z_{do}, \ d \in D_o^+$) and outputs ($y_{ro}, \ r \in R$).

$$E_o^{*1} = \frac{\left(\sum_{i \in I} \frac{\hat{x}_{io}}{x_{io}} + \sum_{d \in D_o^-} \frac{\hat{z}_{do}^-}{z_{do}}\right) \Big/ \left(|I| + |D_o^-|\right)}{\sum_{d \in D_o^+} \frac{\hat{z}_{do}^+}{z_{do}} \Big/ |D_o^+|} \tag{34}$$

If set D_o^+ is empty, the denominator is equal to 1.

$$E_o^{*2} = \frac{\sum_{d \in D_o^-} \frac{\hat{z}_{do}^-}{z_{do}} \Big/ |D_o^-|}{\left(\sum_{r \in R} \frac{\hat{y}_{ro}}{y_{ro}} + \sum_{d \in D_o^+} \frac{\hat{z}_{do}^+}{z_{do}}\right) \Big/ \left(|R| + |D_o^+|\right)} \tag{35}$$

If set D_o^+ is empty, the numerator is equal to 1.

From Eqs. [E3] and [E4], we obtain the performance scores of Stage-1 and Stage-2, respectively, which identify the performance of each stage.

2.5. To extend two-stage to network process

Liu and Liu [16] extend the two-stage to network process. The network contains a set of sub-processes (nodes), H. The nodes are assigned ordinal numbers 1, 2, 3,…, n. Let A denote the set of network links. There are n homogeneous DMUs in set J, named DMU_1, DMU_2,…, and DMU_n, which are randomly processed by the sub-processes in set H. The network structure is depicted in **Figure 3**.

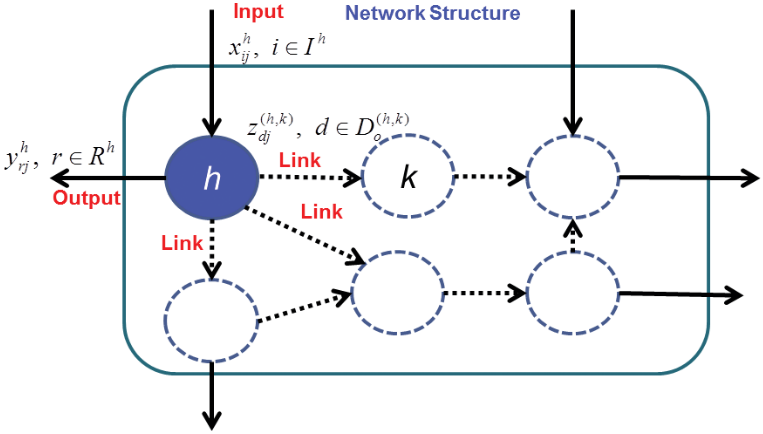

Figure 3. Network structure.

2.5.1. Inputs and outputs

At each sub-process h, there is a set of input measures I^h that flow into the network and a set of output measures R^h that flow out of the network. For DMU$_j$ in set J, let $x_{ij}^{h} \in \mathfrak{R}_{+}^{I^h}$ and $y_{rj}^{h} \in \mathfrak{R}_{+}^{R^h}$ denote the volumes of the ith input measure and the rth output measure at the sub-process h, respectively. Let s_i^{h-} and s_r^{h+} be the slack of the ith input and the rth output at sub-process h, respectively.

2.5.2. Links

Each sub-process may have links to other sub-processes. Let (h, k) denote the link between sub-processes h and k, $h > k$. Let $D^{(h,k)}$ denote the set of link measures on link (h, k).$z_{dj}^{(h,k)} \in \mathfrak{R}_{+}^{D^{(h,k)+}} \bigcup \mathfrak{R}_{+}^{D^{(h,k)-}}$ denotes the volume of the dth link in set $D^{(h,k)}$. Each DMU alternatively acts as the DMU_o that is under evaluation. The volume of link d on link (h, k) could be increased or decreased with a slack to improve the efficiency of DMU_o as well.

3. Illustrative examples

This study adopts a dataset covering 24 non-life insurance companies in Taiwan from Kao and Hwang [10] to illustrate the proposed two-phase procedure. **Table 1** summarizes the performance datasheet of 24 non-life insurance companies in Taiwan.

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Banks	DMU_j	Operation expenses	Insurance expenses	Direct written premiums	Reinsurance premiums	Underwriting profit	Investment profit
		(x_{1j})	(x_{2j})	(z_{1j})	(z_{2j})	(y_{1j})	(y_{2j})
Taiwan Fire	1	1,178,744	673,512	7,451,757	856,735	984,143	681,687
Chung Kuo	2	1,381,822	1,352,755	10,020,274	1,812,894	1,228,502	834,754
Tai Ping	3	1,177,494	592,790	4,776,548	560,244	293,613	658,428
China Mariners	4	601,320	594,259	3,174,851	371,863	248,709	177,331
Fubon	5	6,699,063	3,531,614	37,392,862	1,753,794	7,851,229	3,925,272
Zurich	6	2,627,707	668,363	9,747,908	952,326	1,713,598	415,058
Taian	7	1,942,833	1,443,100	10,685,457	643,412	2,239,593	439,039
Ming Tai	8	3,789,001	1,873,530	17,267,266	1,134,600	3,899,530	622,868
Central	9	1,567,746	950,432	11,473,162	546,337	1,043,778	264,098
The First	10	1,303,249	1,298,470	8,210,389	504,528	1,697,941	554,806
KuoHua	11	1,962,448	672,414	7,222,378	643,178	1,486,014	18,259
Union	12	2,592,790	650,952	9,434,406	1,118,489	1,574,191	909,295
Shingkong	13	2,609,941	1,368,802	13,921,464	811,343	3,609,236	223,047
South China	14	1,396,002	988,888	7,396,396	465,509	1,401,200	332,283
Cathay Century	15	2,184,944	651,063	10,422,297	749,893	3,355,197	555,482
Allianz President	16	1,211,716	415,071	5,606,013	402,881	854,054	197,947
Newa	17	1,453,797	1,085,019	7,695,461	342,489	3,144,484	371,984
AIU	18	757,515	547,997	3,631,484	995,620	692,731	163,927
North America	19	159,422	182,338	1,141,951	483,291	519,121	46,857
Federal	20	145,442	53,518	316,829	131,920	355,624	26,537
Royal & Sunalliance	21	84,171	26,224	225,888	40,542	51,950	6491
Aisa	22	15,993	10,502	52,063	14,574	82,141	4181
AXA	23	54,693	28,408	245,910	49,864	0.10	18,980
Mitsui Sumitomo	24	163,297	235,094	476,419	644,816	142,370	16,976

Table 1. Performance of 24 non-life insurance companies in Taiwan.

The inputs of the system:

Operation expenses (x_1): salaries of the employees and various types of costs incurred in daily operation and.

Insurance expenses (x_2): expenses paid to agencies, brokers, and solicitors and other expenses associated with marketing the service of insurance.

The links of the system:

Direct written premiums (z_1): premiums received from insured clients.

Reinsurance premiums (z_2): premiums received from ceding companies.

The outputs of the system:

Under-writing profit (y_1): profit earned from the insurance business.

Investment profit (y_2): profit earned from the investment portfolio.

3.1. Phase-I

Table 2 summarizes the results of Phase-I and Phase-II. In the Phase-I column, for example, when DMU_1 is being evaluated, DMU_o = DMU_1 and the optimal solution of [M1] is $s_{z1}^{+*} = 0$, $s_{z2}^{+*} = 549,067$, $s_{z1}^{-*} = 877,494$, and $s_{z2}^{-*} = 0$. Therefore, D_o^+ = {2}, D_o^- = {1}, and D_o^{free} = {}. The first row includes 2(549,067) and 1(877,494).

When DMU_4 is being evaluated, DMU_o = DMU_4 and the optimal solution of [M1] is $s_{z1}^{+*} = 0$, $s_{z2}^{+*} = 516,873$, $s_{z1}^{-*} = 0$, and $s_{z2}^{-*} = 0$. Therefore, D_o^+ = {2}, D_o^- = {}, andD_o^{free}={1}. The fourth row includes 2(516,873) and 1(0) in the Phase-I column. The solution of Phase-I indicates that D_o^+ = {2}, D_o^- = {}, and D_o^{free}={1}; the optimal solutions of [M4] are s_{z1}^{free-*} = 3,174,850 and s_{z1}^{free+*} =0. This calculation indicates that the natural link, d = 1, acts as an "as-input" item and may have a better solution. Therefore, 1(3,174,850) is recorded under the D_o^- column of Phase-I. DMU_7 and DMU_{18} have solution processes that are similar to DMU_4.

3.2. Phase-II

Because each link may be "as-input" or "as-output", the two links may have four possible combinations of D_o^+ and D_o^-. **Table 4** shows the four categories A, B, C, and D and their link settings. As indicated in the first column of **Table 3**, 15, 3, 3, and 3 DMUs belong to Categories A, B, C, and D, respectively. For instance, DMU_4 in Category A treats the first links as "as-input" (slack = 1,072,937) and is an undesirable output with respect to Stage-1 and a desirable input with respect to Stage-2. Meanwhile, the second set of links is "as-output" (slack = 135,818) and represents a desirable output with respect to Stage-1 but an undesirable input with respect to Stage-2.

Proceeding to Phase-II, which employs [M5], the optimal solutions for the evaluated DMU are listed in **Table 4**. The second column presents four efficiency scores obtained from (M5), Eqs. [E3], [E4], and [E2], which identify the Stage-1 efficiency (E_o^{1*}), Stage-2 efficiency (E_o^{2*}), and overall efficiency (E_o^*), respectively. The third column presents the reference DMU_j of each evaluated DMU_o. The projection points of DMU_o on the frontier which obtain from E_o^* presents at right sides.

The nine efficient DMUs, 1, 2, 3, 5, 11, 12, 20, 22, and 23, are consistent, with all of their performance scores in Stage-1 and Stage-2 being equal to one. The efficiency scores for the inefficient DMUs for both Stage-1 and Stage-2 are less than 1. For instance, DMU_4 has scores of 0.565 and 0.144 in Stage-1 and Stage-2. An obvious means of improving overall efficiency is to focus on Stage-2.

DMU_o	Phase-I					
	$\rho_o^{(I)*}$	D_o^+	D_o^-	D_o^{free}	$\rho_o^{(II)*}$	D_o^-
1	0.269	2(549,067)	1(877,494)			
2	0.149	2(888,783)	1(36,160)			
3	0.106	1(1,524,659)				
		2(612,849)				
4	0.066	2(516,873)		1(0)	0.183	1(3,174,850)
5	0.414	2(5,425,877)	1(1,093,578)			
6	0.219	2(494,470)	1(4,579,487)			
7	0.182	2(2,347,768)		1(0)	0.338	1(10,685,455)
8	0.199	2(2,888,495)	1(2,895,479)			
9	0.124	2(374,247)	1(8,184,538)			
10	0.192	2(1,429,397)	1(1,301,787)			
11	0.029	1(1,362,146)				
		2(466,500)				
12	0.413	2(8837)	1(78,018)			
13	0.186	1(11,144,019)	2(33,853)			
14	0.169	2(692,753)	1(3,258,713)			
15	0.465	2(689,745)	1(4,484,999)			
16	0.190	2(287,116)	1(3,141,120)			
17	0.300	2(954,161)	1(3,063,410)			
18	0.152		1(74,810)	2(0)	0.186	2(995,620)
19	0.333		1(558,474)			
			2(319,958)			
20	0.454	1(13,617)	2(39,418)			
21	0.154		1(145,060)			
			2(17,915)			
22	0.540	1(38,801)				
		2(4295)				
23	0.000	2(16,295)	1(9566)			
24	0.089	1(265,029)	2(585,641)			

$2^*(549,067)^+$:*, the name of the link; +, the slack of the item.

Table 2. Phase-I solutions.

The virtual weight is expressed as $v_i^* x_{ij} / \sum Virtual \quad Input, w_d^{-*} z_{dj} / \sum Virtual\ Input \quad w_d^{+*} z_{dj} / \sum Virtual\ Output$ and $u_r^* y_{rj} / \sum Virtual\ Output$ where $\sum Virtual\ Input = \sum_{i \in I} v_i x_{ij} + \sum_{d \in D_o^-} w_d^- z_{dj}$ and $\sum Virtual\ Output = \sum_{r \in R} u_r y_{rj} + \sum_{d \in D_o^+} w_d^+ z_{dj}$. These equations represent the percentage of each

Categories	A	B	C	D
D_o^+, D_o^-	$D_o^+ = \{2\}$, $D_o^- = \{1\}$	$D_o^+ = \{1,2\}$, $D_o^- = \{\}$	$D_o^+ = \{\}$, $D_o^- = \{1,2\}$	$D_o^+ = \{1\}$, $D_o^- = \{2\}$

Table 3. Sets D_o^+ and D_o^- of each DMU in Phase-II.

DMU_j	E_o^{1*}	E_o^{2*}	E_o^*	Reference DMU	Projected Points					
					(x_{1j})	(x_{2j})	(z_{1j})	(z_{2j})	(y_{1j})	(y_{2j})
1	1.000	1.000	1.000	1	1,178,744	673,512	7,451,757	856,735	984,143	681,687
2	1.000	1.000	1.000	2	1,381,822	1,352,755	10,020,274	1,812,894	1,228,502	834,754
3	1.000	1.000	1.000	3	1,177,494	592,790	4,776,548	560,244	293,613	658,428
4	0.565	0.144	0.168	22	601,320	386,832	2,101,914	507,681	2,842,519	177,331
5	1.000	1.000	1.000	5	6,699,063	3,531,614	37,392,862	1,753,794	7,851,229	3,925,272
6	0.654	0.316	0.437	12,22	1,384,733	668,363	4,734,713	976,808	4,422,004	415,058
7	0.300	0.212	0.296	22	1,679,395	1,102,796	5,467,038	1,530,389	8,625,473	439,039
8	0.333	0.223	0.316	22	2,382,571	1,564,544	7,756,129	2,171,174	12,237,025	622,868
9	0.322	0.112	0.213	22	1,010,217	663,372	3,288,623	920,585	5,188,537	264,098
10	0.308	0.395	0.423	2,22	1,303,249	1,057,171	6,739,349	1,437,967	4,040,010	554,806
11	1.000	1.000	1.000	11	1,962,448	672,414	7,222,378	643,178	1,486,014	18,259
12	1.000	1.000	1.000	12	2,592,790	650,952	9,434,406	1,118,489	1,574,191	909,295
13	0.976	0.020	0.345	5,22	2,558,630	1,368,802	13,923,383	769,865	3,609,236	1,449,235
14	0.310	0.206	0.284	22	1,396,002	916,702	4,544,491	1,272,140	7,169,973	364,952
15	0.591	0.516	0.702	12,22	1,721,308	651,063	6,056,436	1,001,547	3,489,394	555,482
16	0.460	0.210	0.332	12,22	712,644	415,071	2,369,909	586,830	3,069,552	197,947
17	0.215	0.254	0.343	22	1,422,899	934,363	4,632,050	1,296,650	7,308,093	371,984
18	0.820	0.201	0.240	22	757,515	497,432	2,465,985	690,305	3,890,642	198,035
19	0.582	0.312	0.474	5,22	159,422	102,620	556,110	134,910	755,528	46,857
20	1.000	1.000	1.000	20	145,442	53,518	316,829	131,920	355,624	26,537
21	0.458	0.265	0.286	22	24,829	16,304	80,827	22,626	127,524	6491
22	1.000	1.000	1.000	22	15,993	10,502	52,063	14,574	82,141	4181
23	1.000	1.000	1.000	23	54,693	28,408	245,910	49,864	0	18,980
24	0.504	0.073	0.177	22	163,297	107,231	531,590	148,806	838,704	42,690

Table 4. Final solutions' summary.

input or "as-input" link in the overall virtual input weight and the percentage of each output or "as-output" link in the overall virtual output weight. As shown at **Table 5**, it indicates the improving ratio of each inefficient DMU. For instance, DMU_4 has scores of 0.565 and 0.144 in Stage-1 and

DMU_j	E_o^*	Input Items		Intermediate Items				Output Items	
		$v_1x_1^*$	$v_2x_2^*$	$w_1z_1^{-*}$	$w_2z_2^{-*}$	$w_1z_1^{+*}$	$w_2z_2^{+*}$	$u_1y_1^*$	$u_2y_2^*$
1	1.000	76%	15%	9%	—	—	10%	7%	83%
2	1.000	84%	8%	8%	—	—	8%	8%	84%
3	1.000	4%	96%	—	—	19%	2%	0%	78%
4	0.168	53%	24%	24%	—	—	8%	8%	85%
5	1.000	78%	14%	8%	—	—	4%	10%	86%
6	0.437	26%	48%	26%	—	—	20%	20%	60%
7	0.296	33%	33%	33%	—	—	0%	28%	72%
8	0.316	33%	33%	33%	—	—	73%	10%	17%
9	0.213	33%	33%	33%	—	—	33%	33%	33%
10	0.423	59%	21%	21%	—	—	2%	2%	97%
11	1.000	1%	99%	—	—	91%	1%	8%	1%
12	1.000	7%	73%	20%	—	—	48%	3%	49%
13	0.345	92%	4%	—	4%	88%	—	8%	4%
14	0.284	33%	33%	33%	—	—	70%	9%	21%
15	0.702	21%	58%	21%	—	—	17%	26%	57%
16	0.332	28%	44%	28%	—	—	20%	20%	61%
17	0.343	33%	33%	33%	—	—	33%	11%	55%
18	0.240	25%	25%	25%	25%	—	—	50%	50%
19	0.474	25%	25%	25%	25%	—	—	44%	56%
20	1.000	2%	96%	—	2%	45%	—	40%	15%
21	0.286	25%	25%	25%	25%	—	—	40%	60%
22	1.000	55%	45%	—	—	23%	23%	32%	23%
23	1.000	4%	96%	—	—	—	100%	0%	0%
24	0.177	33%	33%	—	33%	44%	—	23%	33%

Table 5. Virtual weight percentage.

Stage-2. An obvious means of improving overall efficiency is to focus on Stage-2 output item $u_2y_2^*$; it is 85% of output and as-output items.

4. Discussion and conclusions

The objective of efficiency assessment is to identify weaknesses such that the appropriate steps to improve the entire system's performance. This chapter introduces a two-phase procedure to evaluate the two-stage and network models with "free" links. This new model adopts SBM and considers not only the input and output slacks in the objective function but also the slacks of links. The resultant DEA scores provide completely information on how to project inefficient DMUs onto the DEA frontier for specific two-stage processes. Instead of the two conflicting roles that each link plays in existing models, each link plays a single role in the proposed two-phase process system in that it is either desirable or undesirable. The SBM model in this chapter counts the slacks associated with links in the efficiency scores, overcoming the hurdle. The bank case study takes the example on adjustment in the slacks and defines the best practice performance that the DMU under evaluation will need to attain to achieve the best efficiency. To achieve the best-practice efficiency, each DMU determines a set of weights for input, output, and link, where the links are designated as either "as-input" or "as-output". Input and as-input measures reduce slacks, while output and as-output measures increase slacks to reach their targets on the production frontier. This study only introduces a two-stage procedure to assess the entire system. It also can be extended to more complex network processes, applied in series multistage, share resource (Chen et al. [21] and Liang et al. [22]), dynamic network DEA (Tone & Tsutsui [2] and Kao [13]), assurance region (Thompson et al. [23]), cone ratio model (Charnes et al. [24]), and virtual weight analysis models (Sarrico & Dyson [25]) in future research.

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