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Two proposals to improve freeway traffic flow ¹

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Abstract

Two proposals are presented. The first organizes vehicles in platoons in which the lead car is manually driven and the rest are under automatic spacing control. A plausible model of the resulting traffic flow indicates that for an average platoon size of 20, the freeway capacity increases by a factor of four. The second proposal begins with a macroscopic model of freeway congestion and then presents a control law for reducing congestion. Simulation of the resulting closed loop model indicates dramatic reduction in congestion.

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1 Introduction

Traffic flow studies specify an ‘equilibrium’ relation between the speed of cars on a freeway and the concentration or density, see eg. [1]. We propose an extension of one such relation [2] to the situation where traffic is organized in platoons of varying sizes from one to twenty cars. In each platoon the lead car is manually controlled, and the rest are under automatic spacing control [3,4]. The proposed extension is speculative since we lack empirical data. It is plausible, however, since we are essentially modeling the behavior of the lead car driver. Moreover, the final results seem not to be very sensitive to the details of the model. The extended model is presented in §2. Analysis of the model suggests that with platoons of size 20, the freeway capacity can be increased by as much as a factor of four.²

The equilibrium relation between speed and concentration is the basis of a model of driver response: If the concentration decreases (below its equilibrium value) the driver accelerates, if it increases the driver decelerates. Such a response characterization in turn is used to propose macroscopic³ models of traffic flow that exhibit congestion formation and propagation. In §3 we modify one such traffic model. In §4 we propose a feedback control law that dramatically reduces the propagation of congestion.

2 Flow and capacity when cars move in platoons

Suppose cars move in platoons as in Figure 1.

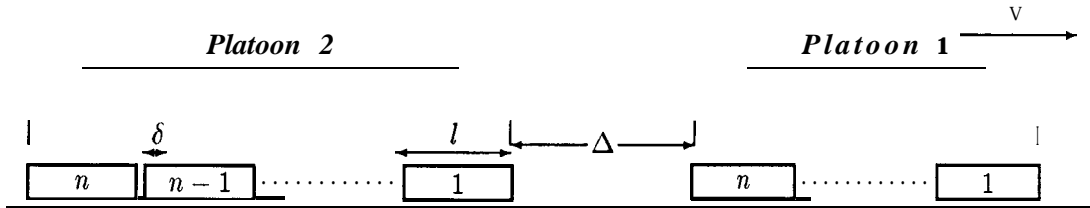


Figure 1: Platoons of vehicles on a freeway

The following notation will be used.

- A: Headway between platoons
- n : Number of cars in the platoon
- δ : Distance between vehicles in the platoon (in m)
- l : Average length of vehicles (in m)
- $k(\Delta, n)$: Average concentration (density) of vehicles (in *veh/m/lane*)
- $v(\Delta, n)$: Speed of platoons (in *m/sec*)

²It is this potentially large payoff that has stimulated considerable interest in platoon organization of traffic.

³By macroscopic we mean that traffic is modeled as a compressible fluid with state variables speed and concentration rather than as a collection of individual vehicles.

$\phi(\Delta, n)$: Flow (in *veh/sec/lane*)

Notice the identity

$$\phi(\Delta, n) = k(\Delta, n)v(\Delta, n) \quad (1)$$

2.1 Flow for $n = 1$

Following [2], the speed and concentration of individual cars are related in equilibrium as

$$v(k) = v_f \left(1 - \left(\frac{k}{k_j} \right)^{\frac{1}{2}} \right) \quad (2)$$

where v_f is the free speed and k_j is the jam concentration at which no movement is possible,

$$k(\Delta, 1) = \frac{1}{\Delta + l}, \quad k_j = \frac{1}{\Delta_j + l} \quad (3)$$

where Δ_j is the headway at the jam concentration. So the flow is given by

$$\phi(\Delta, 1) = \frac{1}{\Delta + l} v_f \left(1 - \left(\frac{\Delta_j + l}{\Delta + l} \right)^{\frac{1}{2}} \right) \quad (4)$$

2.2 Flow when $n > 1$

Assume all platoons have the same size n . For $n > 1$

$$k(\Delta, n) = \frac{n}{\Delta + n(l + \delta) - \delta} \quad (5)$$

It is reasonable to assume that $v(\Delta, n) \leq v(\Delta, 1)$, but the relation between $v(\Delta, n)$ and $v(\Delta, m)$ is not immediate, and we argue as follows. As shown in [3], a controller can be built such that if the lead car accelerates the deviations of the vehicles following it from their preassigned positions can be kept very small. A similar result is expected in the case of deceleration. These deviations increase with the number of cars in the platoon, which therefore has to be limited. This and other related considerations all suggest that $v(\Delta, n)$ must be smaller than $v(\Delta, m)$ if n is greater than m . One reasonable guess is to take a jam headway that increases with n . The following function is used for this purpose:

$$v = g(\Delta, n) = v_f \left(1 - \left(\frac{\Delta_{jn} + l}{\Delta + l} \right)^{\frac{1}{2}} \right) \quad \text{where } \Delta_{jn} = \Delta_{j1} \left(2 - e^{\frac{\sqrt{n-1}}{10}} \right) \quad (6a)$$

For future reference define the inverse function $h(n, v)$ of $g(\Delta, n)$:

$$\Delta = h(n, v) = g^{-1}(n, v) \quad (6b)$$

(6~) gives the following values:

$$\Delta_{j2} = 1.10 \Delta_{j1}$$

$$\Delta_{j5} = 1.18 \Delta_{j1}$$

$$\Delta_{j10} = 1.26 \Delta_{j1}$$

$$\Delta_{j20} = 1.35 \Delta_{j1}$$

It should be emphasized that this function is just a guess used to predict the platoon speed. However, the results below do not depend strongly on this function so long as $v(\Delta, n)$ is close to $v(\Delta, 1)$.

We focus on a single lane. The flow and capacity for k lanes will approximately be k times those of a single lane. The following average values are used: $\delta = 1 \text{ m}$, $l = 5 \text{ m}$, $\Delta_{j1} = 5.6 \text{ m}$ (obtained from a traffic data) [1]. We divide the flow by v_f because v_f is the characteristic of the particular road under consideration,

$$\frac{\phi(\Delta, n)}{v_f} = \frac{n}{\Delta + 6n - 1} \left[1 - \left(\frac{5.6(2 - e^{-*}) + \dots}{A + 5} \right)^{\frac{1}{2}} \right] \quad (7)$$

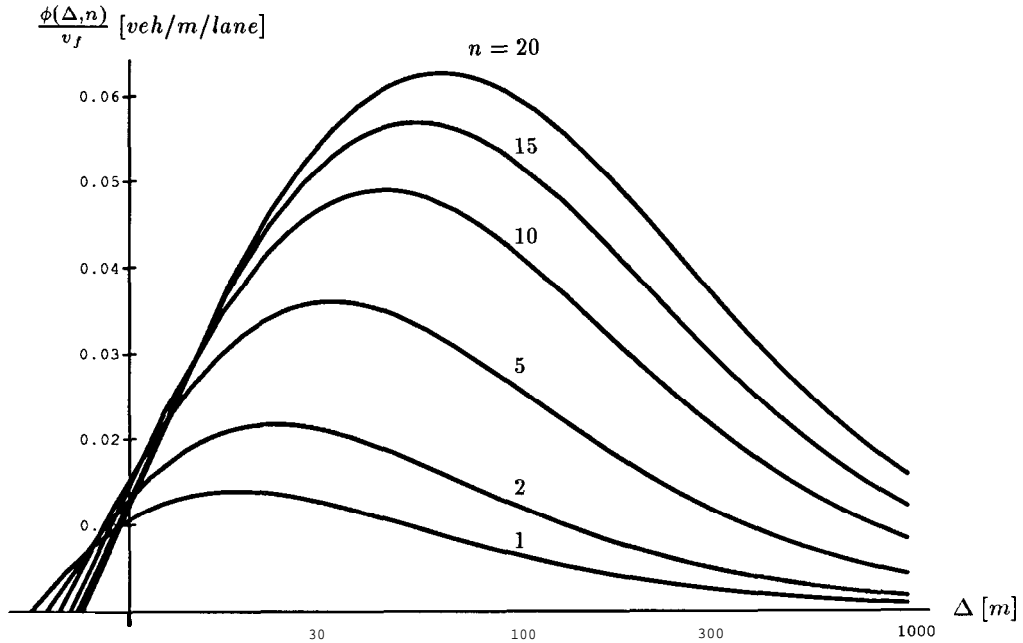


Figure 2: Change of flow as a function of headway and n .

In Figure 2, $\phi(\Delta, n)$ is plotted as a function of Δ for different values of n . It is seen that the maximum flow (capacity) increases significantly with n . For a given traffic volume ϕ there are multiple equilibria, each for a different platoon size n . But it is always better to increase platoon

size, so that platoon headway and speed increase, resulting in a shorter travel time. Some of the results are summarized in Table 1. In this table, v_o is the optimum (flow-maximizing) speed and v_f is the free speed.⁴

n	optimum headway [m]	v_o/v_f	capacity/ v_f [veh/m/lane]
1	18.85	0.333	0.0140
2	23.66	0.377	0.0217
5	33.21	0.449	0.0361
10	45.07	0.509	0.0489
15	54.78	0.545	0.0569
20	63.45	0.571	0.0626

Table 1: Capacity, optimum headway and speed for different n .

2.3 Random platoon size

Suppose the platoon size is random with a distribution given by p_i for $1 \leq i \leq n$, where n is the maximum allowable platoon size, p_i is the probability of a platoon of size i , and $\sum_{i=1}^n p_i = 1$. Then the average platoon size is

$$\mu = \sum_{i=1}^n i p_i \quad (8)$$

Since in the steady state all the platoons are moving with the same speed, the flow is given by

$$Flow = \frac{Ei}{E[h(i, v) + i(l + \delta) - \delta]} v = \frac{\mu}{Eh(i, v) + \mu(l + \delta) - \delta} v \quad (9)$$

where i is the random platoon size and $h(i, v)$ is the headway that a platoon of size i moving with speed v must keep in order to obey (6). But the $h(i, v)$ are very close to each other because Δ_{jn} changes only slightly with n . This allows us to write the steady state speed v in terms of the average platoon size and the average headway.

$$v \approx v_f \left(1 - \sqrt{\frac{\Delta_{j\mu} + l}{Eh(i, v) + l}} \right) \quad (10)$$

To verify this approximation we proceed as follows. From (6)

$$\frac{\Delta_{ji} + l}{h(i, v) + l} = \frac{\Delta_{j\mu} + l}{h(\mu, v) + l} = \left(1 - \frac{v}{v_f} \right)^2 = \text{constant} = k, \text{ for all } i \quad (11)$$

$\Delta_{j\mu}$ and $h(\mu, v)$ are defined by (6) with $n = \mu$. The error in (10) is due to replacing $h(\mu, v)$ with $Eh(i, v)$. $h(i, v)$ is a strictly increasing function of i for fixed v and its second derivative is

⁴The platoon headways in Table 1 meet certain safety considerations. If one requires that a platoon with a braking rate of 0.3g avoid colliding with the platoon in front that is braking at 1.0g, starting at an initial speed of 50 km/h, the headway should be 40 m, see [4].

negative. Therefore by Jensen's inequality $Eh(i, v)$ is always smaller than $h(\mu, v)$. Taking $n = 20$ we will find an upper bound on the error.

$$\begin{aligned}
\varepsilon &= \frac{\Delta_{j\mu} \text{ t } 5}{Eh(i, v) + 5} - \frac{\Delta_{j\mu} \text{ t } 5}{h(\mu, v) + 5} \\
&= \frac{(\Delta_{j\mu} + 5)(h(\mu, v) - Eh(i, v))}{(Eh(i, v) \text{ t } 5)(h(\mu, v) \text{ t } 5)} \\
&= k \frac{h(\mu, v) - Eh(i, v)}{Eh(i, v) + 5} \\
&\leq k \frac{h(\mu, v) - Eh(i, v)}{h(1, v) + 5} \\
&= \frac{k}{\frac{\Delta_{j1} + 5}{k}} (h(\mu, v) - Eh(i, v)) \\
&= \frac{k^2}{10.6} \varepsilon'
\end{aligned} \tag{12}$$

where

$$\varepsilon' = h(\mu, v) - Eh(i, v) \tag{13}$$

The maximum of ε' occurs at the distribution: $p_1 = 1 - p_{20}$ and $p_i = 0$ otherwise, and for an average platoon size μ determined as follows:

$$\begin{aligned}
\varepsilon' &= h(\mu, v) - Eh(i, v) \\
&= h(\mu, v) - \left(h(1, v) + \frac{h(20, v) - h(1, v)}{19} (\mu - 1) \right) \\
&= \frac{\Delta_{j\mu} \text{ t } 5}{k} - 5 - \left(\frac{\Delta_{j1} \text{ t } 5}{k} - 5 + \frac{\frac{\Delta_{j20} + 5}{k} - \frac{\Delta_{j1} + 5}{k}}{19} (\mu - 1) \right)
\end{aligned} \tag{14}$$

The solution of the equation $\frac{d\varepsilon'}{d\mu} = 0$ gives $\mu = 6.497$. For this μ , $\varepsilon' = 4.8 m$. For a typical speed of 55 mph and $v_f = 85 mph$, $k = 0.1246$ and $\varepsilon \leq 0.007$. So the maximum error in (10) is

$$\frac{55}{85} v_f - (1 - \sqrt{k + 0.007}) v_f = 0.01 v_f \tag{15}$$

which is 1.5 %.

$Eh(i, v)$ is the average headway between platoons in the steady state. Through an abuse of notation we call it A. Inserting (10) into (9) and using the same values for l, δ and Δ_{j1} as in §2.2 we get

$$\frac{\phi(\Delta, \mu)}{v_f} \approx \frac{\mu}{\Delta + 6\mu - 1} \left[1 - \left(\frac{5.6(2 - e^{-\frac{\sqrt{\mu-1}}{10}}) + 5}{\Delta + 5} \right)^{\frac{1}{2}} \right] \tag{16}$$

Notice the similarity between (16) and (7).

Example 1

Suppose that $n = 20$ and the distribution of platoon sizes is uniform. Then $p_i = \frac{1}{20}, 1 \leq i \leq 20$. From (8)

$$\mu = \frac{1}{20} \sum_{i=1}^{20} i = 10.5$$

So in this case, the flow versus A curve will be slightly above the curve labeled 10 in Figure 2.

2.4 Conclusion

We can use Table 1 to evaluate the results of the last section by replacing n and A with their average values. The capacity for $\mu = 20$ is more than four times the capacity for $\mu = 1$. Another important result is that the speed at maximum flow, i.e., the optimum speed, increases with μ . For $\mu = 1$ the optimum speed is one third of the free speed whereas for $\mu = 20$ it is more than half the free speed. If the free speed is 80 *mph* then for $\mu = 20$ the maximum flow occurs at $v = 46$ *mph*.

For a maximum size n , μ is closer to n if larger platoons are favored. In 92.1 and in Example 1 the platoon size was constant regardless of concentration. But in practice the platoon size distribution is likely to be a function of the concentration. If the concentration is low, small platoons will be more likely, consequently μ and the capacity will be smaller. But this is not a disadvantage because at smaller concentrations we do not need high capacities. The traffic can still flow with a high speed. On the other hand, if the concentration is high, larger platoons are more likely which will result in a larger μ and a higher capacity. In this case even if the highway is used with full capacity, since the optimum speed is significantly higher than the one at current operating conditions, cars will reach their destinations in shorter times, which in turn will decrease the number of cars on the highways. This seemingly paradoxical conclusion can be traced to the fact that highway capacity is not a unique number but a function of the platooning policy, see Table 1.

3 A macroscopic model of traffic flow

Experience teaches us that small disturbances in the traffic flow can lead to long-lasting congestions. We will try to explain this phenomenon by deriving the differential equation governing the traffic flow. From now on we assume no platooning, $n = 1$.

Let $k(x, t)$ and $\phi(x, t)$ denote the concentration and flow of cars at location x and time t . Consider a length of road dx and an interval of time dt . The number of vehicles on dx at time t is $k(x, t)dx$ and the number of vehicles entering at x in the time interval dt is $\phi(x, t)dt$. Conservation of vehicles implies

$$[k(x, t + dt) - k(x, t)]dx = [\phi(x, t) - \phi(x + dx, t)]dt \quad (17)$$

$$\frac{\partial k(x, t)}{\partial t} = - \frac{\partial \phi(x, t)}{\partial x} \quad (18)$$

Using (2) and $\phi = kv$ gives

$$\frac{\partial k(x, t)}{\partial t} = -v_f \left(1 - \frac{3}{2} \sqrt{\frac{k(x, t)}{k_j}} \right) \frac{\partial k(x, t)}{\partial x} \quad (19)$$

Every $k(x, t) = k = \text{constant}$ is a solution of (19) where k is any number between 0 and k_j . If $\varepsilon(x, t)$ is a small perturbation around the equilibrium point such that $k(x, t) = k + \varepsilon(x, t)$ then (19) gives

$$\frac{\partial \varepsilon(x, t)}{\partial t} \approx -v_f \left(1 - \frac{3}{2} \sqrt{\frac{k}{k_j}} \right) \frac{\partial \varepsilon(x, t)}{\partial x} \quad (20)$$

The solution of (20) is any function $\varepsilon(x, t)$ such that $\varepsilon(x, t)$ is small and satisfies

$$\varepsilon(x, t) = \varepsilon(x - at, 0) \quad \text{where } a = v_f \left(1 - \frac{3}{2} \sqrt{\frac{k}{k_j}} \right) \quad (21)$$

Equation (21) shows that a small perturbation in the density will propagate with speed a along the freeway. If $a > 0$ the wave propagates upstream, and if $a < 0$ it propagates downstream; and $a > 0$ if

$$0 < k < \frac{4}{9} k_j \quad (22)$$

Note that $\frac{4}{9} k_j$ is the critical density at which the flow reaches its maximum value (capacity). The prediction of (21) in the overcritical region ($k > \frac{4}{9} k_j$) seems invalid since measurements show that traffic flow becomes unstable in this region due to the behavior of the individual drivers. The analysis above also assumed that a change in the traffic density results in a corresponding change in the traffic flow without any delay. We present a more sophisticated model which overcomes these two deficiencies.

3.1 The model

The first model was by Lighthill and Witham [5]. In this model traffic density was the only state variable, resulting in poor transient behavior. Payne overcame this by adding another differential equation representing the dynamics of the mean speed. His model was also used for example by Papageorgiou [6] and Cremer and May [7]. Payne's model which was derived on the basis of microscopic and empirical considerations accounts for the instability at critical density values as well as the occurrence of congestion.

Payne's earlier model used continuous space and time [8]; this was later discretized to obtain a discrete-space, discrete-time model of traffic flow [9]. In order to overcome some of the shortcomings of this model some modifications were proposed in [6,7]. We model in [7] for the simple case of a single lane freeway section with no on- or off-ramps. We will then propose some modifications. The freeway is subdivided into N sections with lengths L_i as in Figure 3.

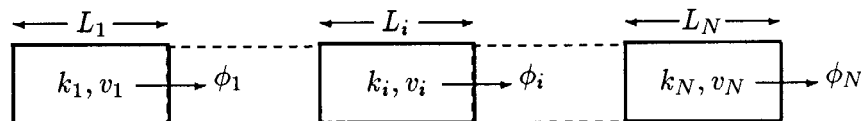


Figure 3: A freeway stretch divided into N sections.

The space-discretized traffic variables for a segment in Figure 3 are:

- $k_i(n)$: Density in section i at time nT (in $veh/km/Zone$)
- $u_i(n)$: Mean speed of vehicles in section i at time nT (in km/h)
- $\phi_i(n)$: Traffic volume leaving section i , entering section $i + 1$ at time nT (in veh/h)
- L_i : Length of the i th section (in km)
- T : Step size (in h)

Considering the vehicles entering and leaving section i the relation between the density at time $(n+1)T$ and time (nT) can easily be found,

$$k_i(n+1) = k_i(n) + \frac{T}{L_i}[\phi_{i-1}(n) - \phi_i(n)] \quad (23)$$

The traffic volume leaving section i is modelled as a weighted average of the volumes of section i and $i + 1$,

$$\phi_i(n) = \alpha k_i(n)v_i(n) + (1 - \alpha)k_{i+1}(n)v_{i+1}(n) \quad (24)$$

where α is between 0 and 1. One expects α to be close to 1 because the number of cars leaving section i is more dependent on the volume in section i than in section $i + 1$. For example, if the density in section i is zero there can be no flow into section $i + 1$, whereas if α is not close to 1 equation (24) may give an unrealistic result.

The last equation describes the mean speed dynamics,

$$v_i(n+1) = v_i(n) + \frac{T}{\tau}[v_e(k_i(n)) - v_i(n)] + \frac{T}{L_i}v_i(n)[v_{i-1}(n) - v_i(n)] - \frac{\mu T}{\tau L_i} \frac{k_{i+1}(n) - k_i(n)}{k_i(n) + \chi} \quad (25)$$

Here τ is the relaxation time; $v_e(k)$ is the equilibrium speed for the density k ; and μ and χ are positive constants. As can be seen from (25) three terms influence the mean speed of a section:

- $\frac{T}{\tau}[v_e(k_i(n)) - v_i(n)]$ is the relaxation term describing the convergence of the mean speed to its equilibrium value at a rate determined by the time constant τ . For the equilibrium value, [7] uses

$$v_e(k) = v_f[1 - (\frac{k}{k_j})^l]^m \quad (26)$$

in which v_f, k_j, l, m are constants to be calibrated according to real traffic data. Note that (26) gives (2) with $l=0.5$ and $m=1$.

- $\frac{T}{L_i}v_i(n)[v_{i-1}(n) - v_i(n)]$ is the convection term. If vehicles enter a section at a different speed than the mean speed of that section, they affect its mean speed.
- $\frac{\mu T}{\tau L_i} \frac{k_{i+1}(n) - k_i(n)}{k_i(n) + \chi}$ is the anticipation term. It describes driver response to the downstream density. If the density downstream is lower, drivers tend to speed up and vice versa. The constant χ , absent in the Payne model, prevents the anticipation from becoming too large at low densities.

3.1.1 Boundary Conditions

The solution of the model requires specification of the boundary conditions at the entrance and exit, i.e., $k_0, v_0, k_{N+1}, v_{N+1}$. It is customary to choose a prescribed flow at the entrance and a stationary boundary condition at the exit of the freeway stretch. If $\lambda_0(n)$ is the flow at the entrance during time nT and $(n+1)T$ then these conditions are:

$$\begin{aligned} \text{entrance : } & \left. \begin{aligned} k_0(n) &= [\frac{\lambda_0(n)}{v_1(n)} - (1 - \alpha)k_1(n)]/\alpha \\ v_0(n) &= v_1(n) \end{aligned} \right\} \phi_0(n) = \lambda_0(n) \\ \text{exit : } & \begin{aligned} k_{N+1}(n) &= k_N(n) \\ v_{N+1}(n) &= v_N(n) \end{aligned} \end{aligned}$$

3.1.2 Parameter values

The different parameters in the model were calibrated to fit real observations from a Californian freeway [7].

v_f	k_j	l	m	α	X	μ	τ
93.1	110				9.5	23.9	20.4
<i>km/h</i>	<i>veh/km</i> <i>/lane</i>	1.86	4.05	0.95	<i>veh/km</i> <i>/lane</i>	<i>km²/h</i>	<i>sec</i>

Table 2: Calibrated model parameters of [7].

3.2 Modification of the model

The model described by (23), (24) and (25) is simulated with the parameters in Table 2 for a hypothetical freeway stretch of 12 sections, each 500 meters long. A step size of 15 seconds is used. The simulation is repeated for a set of different initial speed and density values in each section. The results show the need to change some of the terms in (25).

3.2.1 The convection term $\frac{T}{L_i} v_i(n)[v_{i-1}(n) - v_i(n)]$

This term is found to be stronger than it should be for most of the cases considered.

Example 2

Consider a case where section i is congested, with a density of 60 veh/km/lane and a mean speed of 20 km/h , but section $i - 1$ has a low density value of 20 veh/km/lane and traffic is flowing with 80 km/h .

In this case the convection term alone results in a 10 km/h increase in the mean speed of section i which, because section i has a high density value, causes a significant increase in the flow into section $i + 1$, and thus relieves the congestion in an unrealistic way. The problem arises because the model is spatially discrete and it is hard to predict the passing speeds at the boundaries of

the sections. It is reasonable to assume that the passing speed at the boundary of section $i - 1$ and section i is closer to the smaller one of the two section speeds since, in adapting to the speed of downstream traffic, the acceleration process is slower than the deceleration process. Thus we propose to model the passing speed as the geometric average of the section speeds,

$$v_{pi}(n) = \sqrt{v_{i-1}(n)v_i(n)} \quad (27)$$

Another shortcoming of the convection term in **(25)** is that it does not include the effects of the section densities. We will derive the convection term with these additions.

Consider a case where the relaxation and the anticipation terms are zero and the change in the mean speed in section i is due to the convection term alone. Since α is close to 1, the number of cars entering section i in a time interval T is approximately $Tv_{i-1}(n)k_{i-1}(n)$. The total number of cars in section i at the end of this period T is $L_i k_i(n + 1)$. Taking the weighted average of the speeds v_{pi} and v_i leads to:

$$v_i(n + 1) = \frac{Tv_{i-1}(n)k_{i-1}(n)v_{pi}(n) + [L_i k_i(n + 1) - Tv_{i-1}(n)k_{i-1}(n)]v_i(n)}{L_i k_i(n + 1)} \quad (28)$$

$$v_i(n + 1) = v_i(n) + \frac{T}{L_i} \frac{k_{i-1}(n)}{k_i(n + 1)} v_{i-1}(n) [v_{pi}(n) - v_i(n)] \quad (29)$$

From (29) the revised convection term is

$$\text{convection term} = \frac{T}{L_i} \frac{k_{i-1}(n)}{k_i(n + 1) + \chi'} v_{i-1}(n) [\sqrt{v_{i-1}(n)v_i(n)} - v_i(n)] \quad (30)$$

where we added a constant χ' to prevent the term from becoming too large when $k_i(n + 1)$ is close to zero. The factor $\frac{k_{i-1}(n)}{k_i(n + 1) + \chi'}$ in (30) is important and should not be omitted as was done in previous models. For example if the concentration in section $i - 1$ is much lower than the concentration in section i than the number of cars entering section i and their effect on the mean speed in that section will be small. With the revised convection term, in Example 2 the increase in the mean speed of section i due to this term is only around 4.1 km/h.

3.2.2 The anticipation term $\frac{\mu}{\tau} \frac{T}{L_i} \frac{k_{i+1}(n) - k_i(n)}{k_i(n) + \chi}$

The simulation of high density traffic with the model equations (23), (24) and (25) gave some unrealistic results. The traffic flow seemed to be stable even if initially there were highly congested sections. This is because the anticipation term is very strong and when the downstream traffic is less dense, congested sections can reach unrealistically high mean speeds relieving the congestion. If we weaken the term by decreasing the value of μ such that instability at the expected high density regions is demonstrated by the model then it is observed that, for some high initial density values, densities in some sections exceed the jam value and reach up to 170 veh/km/lane (compare with the jam value of 110 veh/km/lane in Table 2).

To eliminate this implausible behavior we use two different values for μ for the cases when the downstream traffic is more dense and when it is less dense. This is justified by our experience that drivers react differently in these two cases (stronger in the first case). To prevent the density from

exceeding the jam value we add a term in the first case which makes the anticipation stronger if the density in section $i + 1$ approaches the jam density. The revised anticipation term is

$$\mathbf{anticipation\ term} = \begin{cases} \frac{\mu_1}{\tau} \frac{T}{L_i} \frac{k_{i+1}(n) - k_i(n)}{k_i(n) + \chi} \frac{\rho}{k_j - k_{i+1}(n) + \sigma} & \text{if } k_{i+1}(n) \geq k_i(n) \\ \frac{\mu_2}{\tau} \frac{T}{L_i} \frac{k_{i+1}(n) - k_i(n)}{k_i(n) + \chi} & \text{otherwise} \end{cases} \quad (31)$$

where μ_1, μ_2, ρ and σ are constants.

Another change that improved results is a higher value for χ , because if χ is as small as 9.5 as in Table 2 then the anticipation term becomes unreasonably large at small densities. The values of the newly introduced parameters are listed in Table 3. Note, however, that these values are selected only because they give reasonable results in the simulation.

3.2.3 The revised model

The revised model is summarized below.

$$k_i(n + 1) = k_i(n) + \frac{T}{L_i} (\phi_{i-1}(n) - \phi_i(n)) \quad (32)$$

$$\phi_i(n) = \alpha k_i(n) v_i(n) + (1 - \alpha) k_{i+1}(n) v_{i+1}(n) \quad (33)$$

$$v_i(n + 1) = v_i(n) + \frac{T}{\tau} (v_e(k_i(n)) - v_i(n)) + \frac{T}{L_i} \frac{k_{i-1}(n)}{k_i(n) + \chi'} v_{i-1}(n) [\sqrt{v_{i-1}(n) v_i(n)} - v_i(n)] - \frac{\mu(n)}{\tau} \frac{T}{L_i} \frac{k_{i+1}(n) - k_i(n)}{k_i(n) + \chi} \quad (34)$$

where,

$$\mu(n) = \begin{cases} \mu_1 \frac{\rho}{k_j - k_{i+1}(n) + \sigma} & \text{if } k_{i+1}(n) \geq k_i(n) \\ \mu_2 & \text{otherwise} \end{cases} \quad (35)$$

v_f	k_j	l	m	a	χ	χ'	μ_1	μ_2	ρ	σ	τ	
93.1 km/h	110 veh/km /lane	1.8	6.4	0.5	0.95	0.95 veh/km /lane	4 veh/km /lane	12 km ² /h	6 km ² /h	120 veh/km /lane	35 veh/km /lane	20.4 see

Table 3: Parameter values for the revised model.

4 Control of freeway traffic flow

Several methods of controlling the traffic flow on freeways have been considered in the literature. These methods are based on controlling on-ramp traffic volumes and/or displaying variable advisory speeds on matrix signal boards above the road [6], [10], [11]. The input to the control algorithms are the on-ramp, off-ramp traffic volumes, the density and the mean speed in each section of the freeway. The state of the traffic, that is the section densities and mean speeds are

estimated using measurements of passing times and speeds of vehicles at specific locations along the freeway, [6], [12].

Many control algorithms try to minimize the total time spent by all drivers on the freeway. But because of the high order and the nonlinearities of the model the solution of the optimal control problem requires numerical search methods and therefore extensive computer time. To overcome these drawbacks multilayer control structures have been proposed, which try to approximate the ‘optimal’ solution [6].

The control strategy for the traffic flow must be fast enough so that it can adapt to rapidly changing traffic conditions. It must be robust and simple to be feasible. In the next section we propose a simple control strategy for a freeway section with no on- or off-ramps. We assume exact knowledge of the state of the traffic at all times. Then we give the results of the simulations of both the controlled and uncontrolled traffic for two different initial traffic conditions.

4.1 Control Law

Underlying the proposal is the observation that congestion occurs mainly because of the inhomogeneities in the traffic stream. Consider a freeway stretch along which there are regions with both high and low density values. If we can homogenize the density by moving some of the load in the congested regions to less dense regions, smoothing the density profile of the freeway gradually, we can eliminate congestion and reduce delays.

Since we consider a freeway with no on- or off-ramps the only variables that we may control are the mean speeds in the sections. Therefore the model equations for the controlled system will be those for the uncontrolled system with the speed equation altered to reflect control.

Since the convection term describes the effect of the speeds of the vehicles entering a section on the mean speed of that section, it is a term that we can not control. The relaxation term is also kept unchanged because when no control is applied mean speeds should converge to their equilibrium values. However, the anticipation term is replaced by a control term, f_c , which is a function of the section densities along the freeway.

In the initial design, we allow the control term for section i to depend on the densities of two sections downstream and two sections upstream of section i .

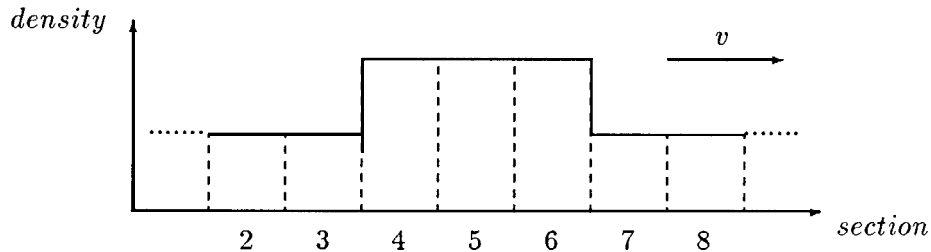


Figure 4: An imaginary density profile along a freeway.

In terms of the density profile of Figure 4, a control law which tries to smooth this profile should have the following effects:

1. Cars in section 6 should speed up.
2. Cars in section 7 and 8 should speed up but not as strongly as the cars in section 6.
3. Cars in section 5 should speed up but not as strongly as the cars in section 6. (See below for explanation.)
4. Cars in section 2 and 3 should slow down.
5. The decision for section 4 is not obvious and is discussed below.

The following control term is used to generate these effects:

$$f_c = \frac{\mu_c}{\tau} \frac{T}{L_i k_i(n) + \chi_c} \frac{1}{\chi_c} [c_1(c_2(k_i(n) - k_{i+1}(n)) + (1 - c_2)(k_{i-1}(n) - k_i(n))) + (1 - c_1)(c_2(k_i(n) - k_{i+2}(n)) + (1 - c_2)(k_{i-2}(n) - k_i(n)))] \quad (36)$$

In (36) c_1 and c_2 are constants in the range 0 to 1. It is obvious that the densities of the sections closer to section i are more important than the densities of the sections further away and also the density of the downstream traffic is more important than the density of the upstream traffic as far as the control law for section i is concerned. This is the reason why cars in section 5 should speed up in the above example although the density profile is symmetric with respect to section 5.

The simulation of the controlled system for different values of c_1 and c_2 gave some interesting results. A value of 0.7 for c_1 and a value of 1 for c_2 gave the best results. If c_2 is 1, then the density of the upstream traffic does not have any effect on the control term and the control term for section 4 is zero. But this means that the control terms for sections 7 and 8 are also zero, which conflicts with the second requirement in the above list. The reason for this conflict is that if c_2 is less than 1, then cars in section 4 will have to slow down, which has a negative effect on the flow. This negative effect dominates any positive effect we would get by choosing a smaller value for c_2 thus satisfying the second requirement in the list. We could improve the control policy by making the effect of the upstream density stronger when it is higher than when it is lower. However the improvement obtained using the information of the upstream density is not very significant and this fact encourages us to take $c_2 = 1$. Thus in the final design, the control depends only on the densities of two downstream sections.

For μ_c we again use two values for the cases when the control term is positive and when it is negative.

$$\mu_c = \begin{cases} \mu_{c1} & \text{if } k_{i+1}(n) \geq k_i(n) \\ \mu_{c2} & \text{otherwise} \end{cases} \quad (37)$$

Because of safety considerations there must be an upper limit on the strength of the control factor when it is positive. We set this limit by requiring that the mean speeds do not exceed their equilibrium value by more than 10 km/h. Therefore one has to choose a smaller value for μ_{c2} than for μ_{c1} . The parameter values that we used in the simulation of the controlled system are listed in Table 4.

Thus, in the case of control, equation (34) is replaced by

$$v_i(n+1) = v_i(n) + \frac{T}{\tau}(v_e(k_i(n)) - v_i(n)) + \frac{T}{L_i} \frac{k_{i-1}(n)}{k_i(n+1) + \chi'} v_{i-1}(n) (\sqrt{v_{i-1}(n)v_i(n)} - v_i(n)) + \frac{\mu_c T}{\tau L_i} \frac{1}{k_i(n) + \chi_c} [c_1(k_i(n) - k_{i+1}(n)) + (1 - c_1)(k_i(n) - k_{i+2}(n))] \quad (38)$$

v_f	k_j	l	m	α	χ_c	χ'	μ_{c1}	μ_{c2}	c_1	τ
93.1 km/h	110 veh/km /lane	1.86	4.05	0.95	60 veh/km /lane	4 veh/km /lane	52.5 km ² /h	22.5 km ² /h	0.7	20.4 sec

Table 4: Parameter values for the controlled system.

Comparing (34) and (38) it is seen that the anticipation term in (34), which represents driver response, is replaced by a control term in (38). The speeds computed with (38) are the mean speeds, which are macroscopic variables. They may be regarded as target speeds in each section. These target speeds have to be reached using an appropriate control policy at the microscopic level to control individual vehicles. Such a two level control policy may be applied using communication and automatic cruise control methods. Alternatively, a similar effect may be achieved at least in part if the speeds computed using (38) are displayed as advisory speeds on signal boards above the road, [11].

4.2 Simulation Results

The model equations for the uncontrolled (34) and controlled (38) traffic were simulated for several different traffic conditions. The results of the simulations for two cases are given below both with and without control. It is seen that control improves the traffic flow considerably.

Case 1:

Number of sections: 12
Section length: 500m
Number of lanes: 1
Input volume ($\lambda_0(n)$): 1500veh/h
Initial conditions:

section number i	1	2	3	4	5	6	7	8	9	10	11	12
$k_i(0)$ veh/kmlane	18	18	18	18	18	52	52	52	18	18	18	18
$v_i(0)$ km/h	81	81	81	81	81	29	29	29	81	81	81	81

Table 5: Initial density and mean speed values.

The results of the simulation, the evolution of the density and mean speed for each section, have been drawn in Figures 5-8 for the cases with and without control. In the case of no-control

the density of the congested region grows and the congested region moves upstream due to the anticipation term. In some sections density reaches the jam value and speeds drop to zero. In the case of control the congested region disappears, section densities and mean speeds reach their equilibrium values after a short while.

Case 2:

Number of sections: 12
 Section length: 500m
 Number of lanes: 1
 Input volume ($\lambda_0(n)$): 1500veh/h
 Initial conditions:

section number i	1	2	3	4	5	6	7	8	9	10	11	12
$k_i(O)$ veh/km/lane	18	18	18	18	69	69	18	18	69	69	18	18
$v_i(O)$ km/h	81	81	81	81	10	10	81	81	10	10	81	81

Table 6: Initial density and mean speed values.

Initially there are two congested regions. The control is satisfactory as in case 1. The results are given in Figures 9-12.

4.3 Conclusion

The proposed control law is seen to be satisfactory using only the information about the densities of two downstream sections in order to determine the control to be applied. The control strategy may be improved using the knowledge of the densities of more sections and by optimizing the parameters in equation (38). Even without these refinements, the proposed control law seems very effective in reducing congestion.

CASE 1

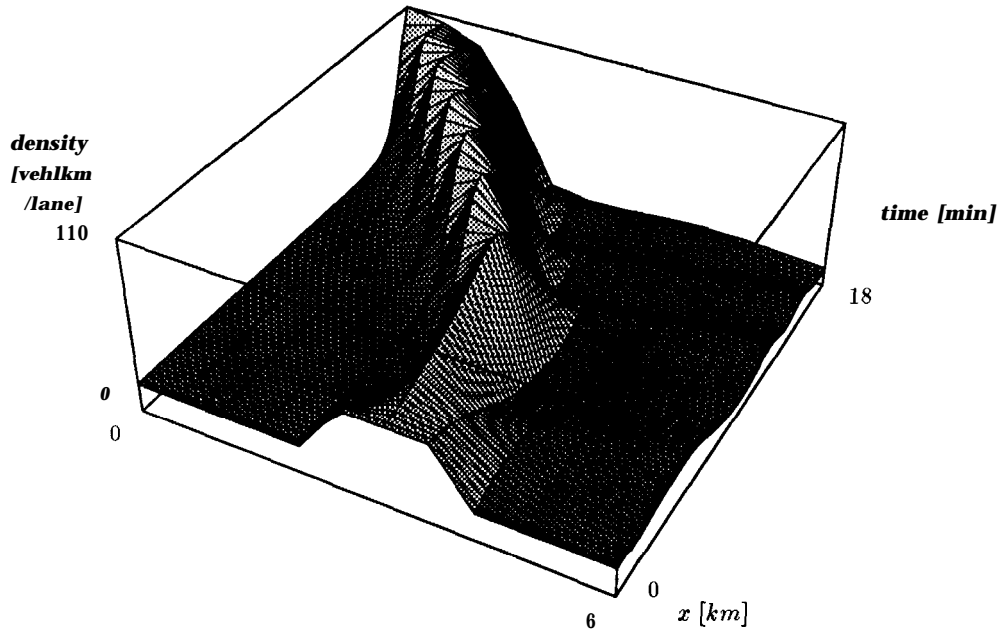


Figure 5: The evolution of density without control

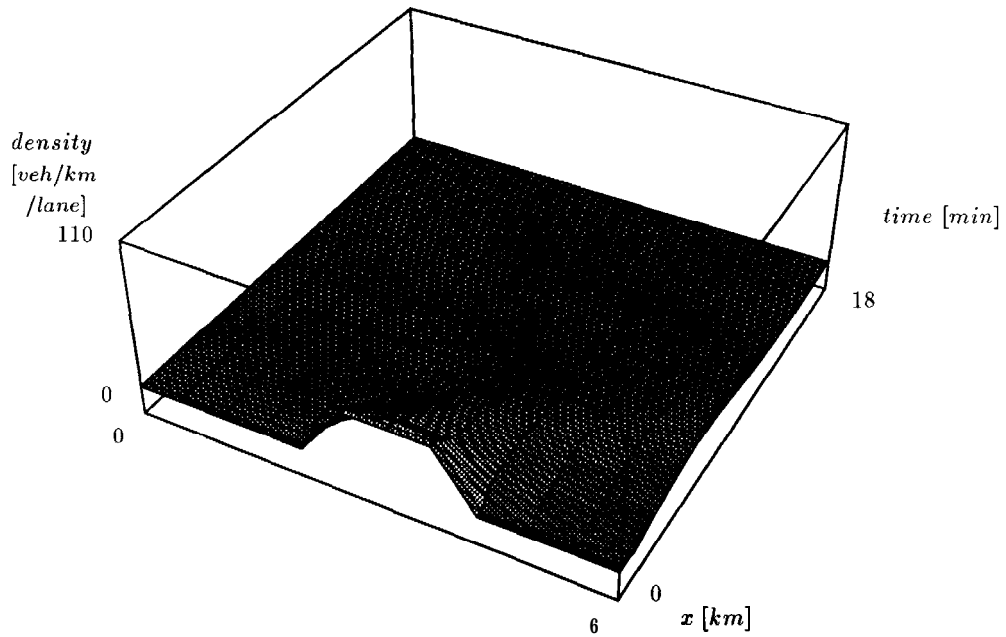


Figure 6: The evolution of density with control

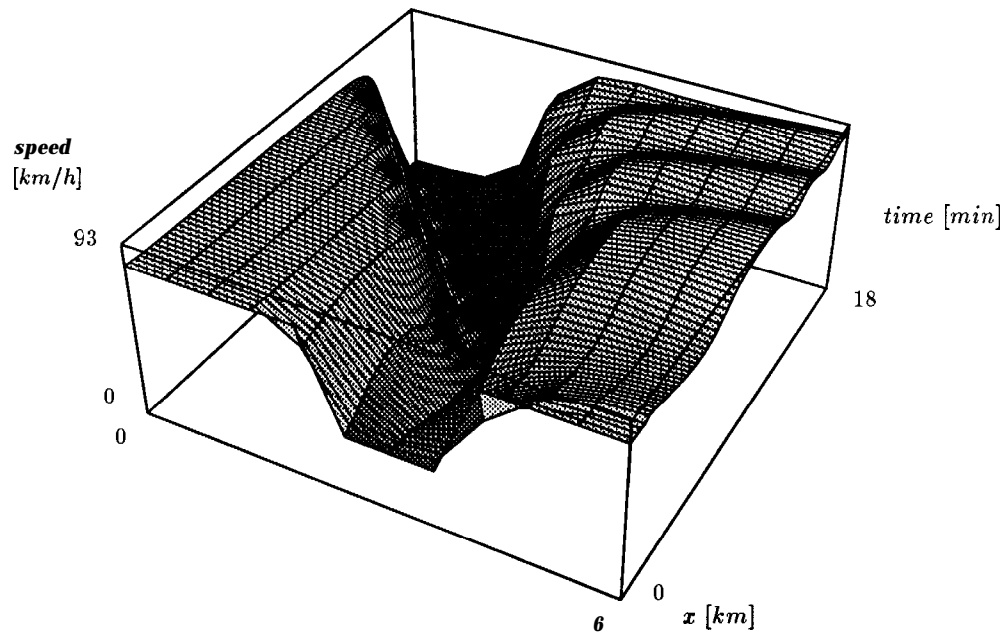


Figure 7: The evolution of speed without control

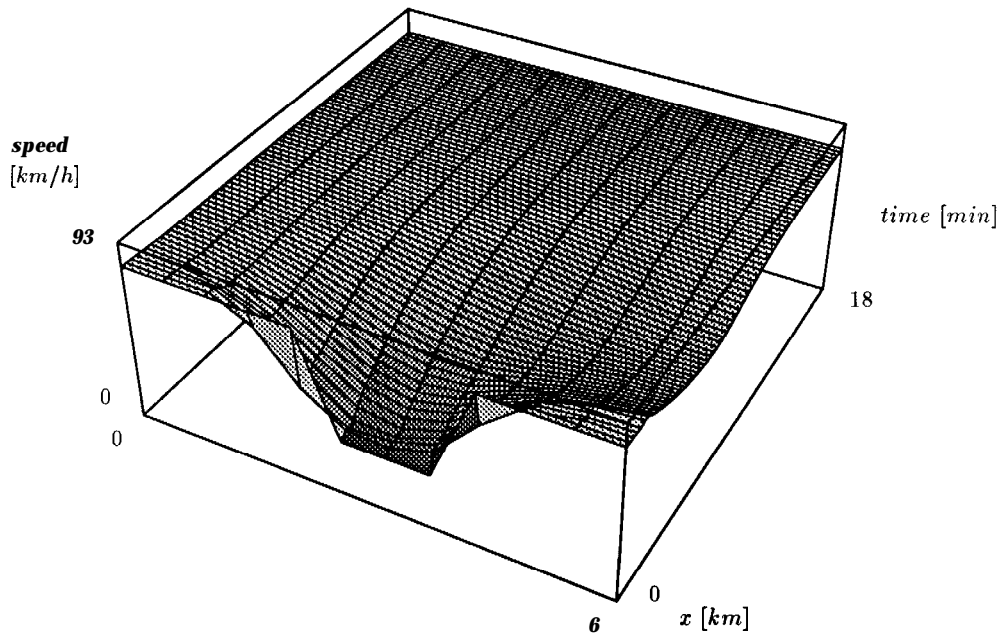


Figure 8: The evolution of speed with control

CASE 2

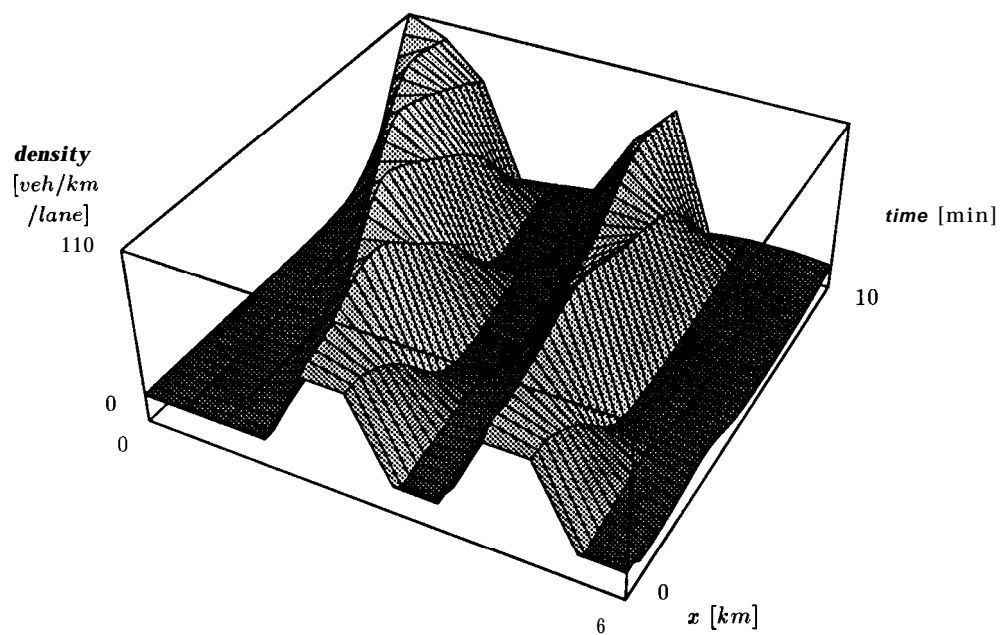


Figure 9: The evolution of density without control

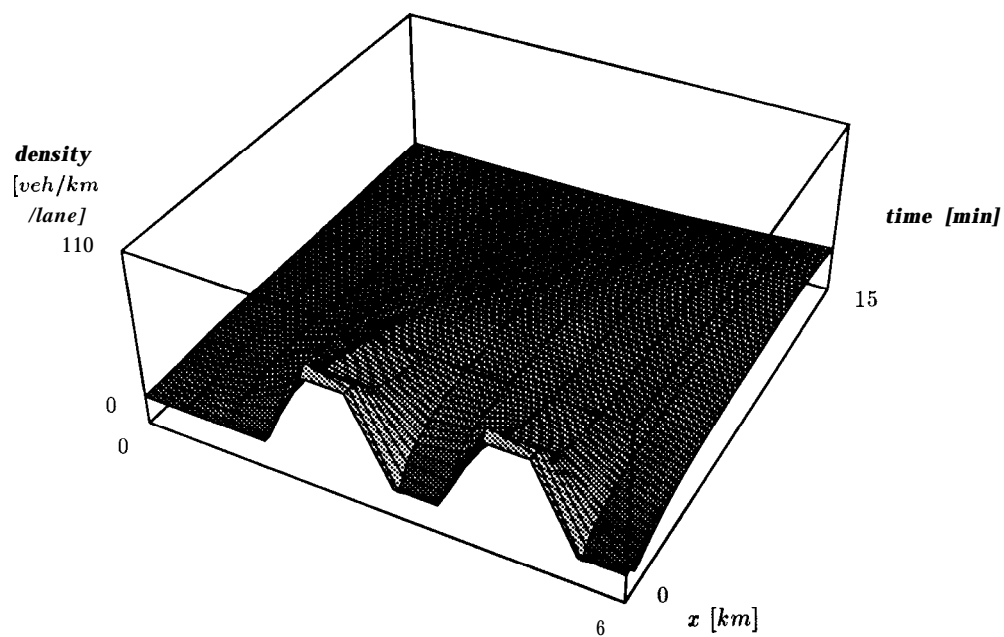


Figure 10: The evolution of density with control

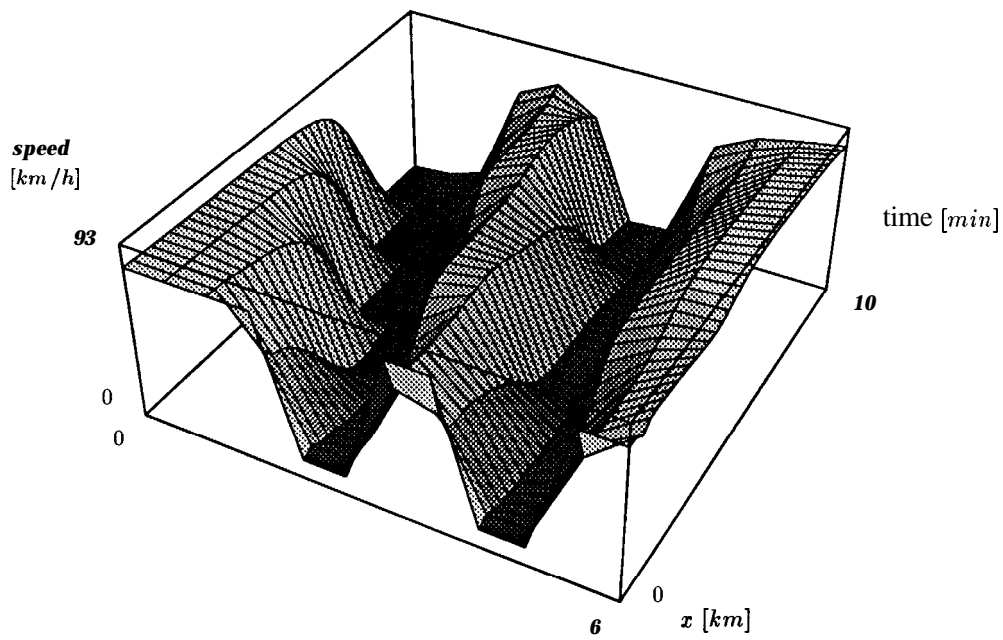


Figure 11: The evolution of speed without control

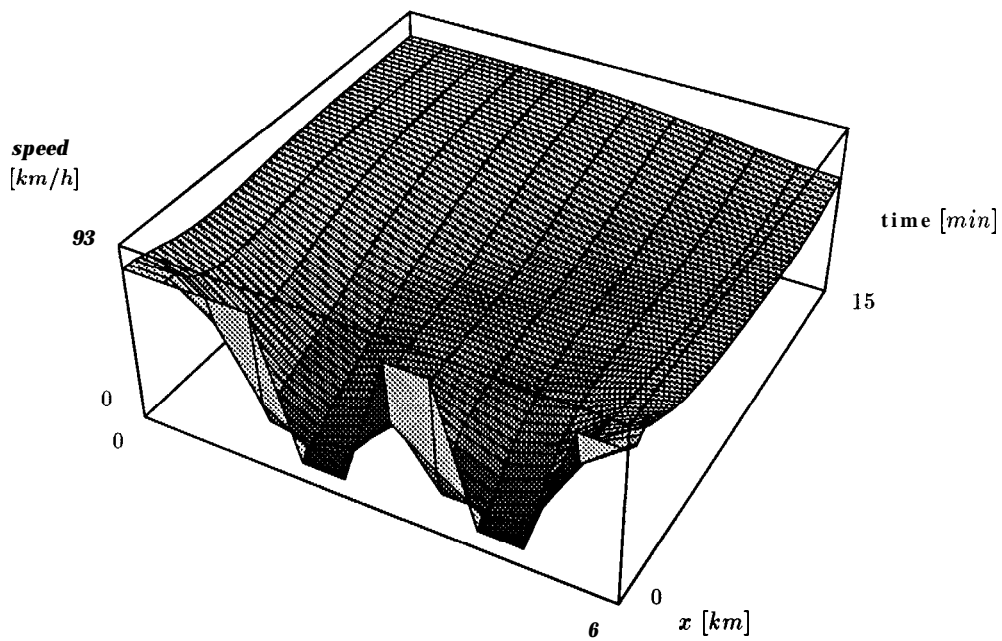


Figure 12: The evolution of speed with control

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