

Two-Sided Network Effects: A Theory of Information Product Design

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How can firms profitably give away free products? This paper provides a novel answer and articulates trade-offs in a space of information product design. We introduce a formal model of two-sided network externalities based in textbook economics—a mix of Katz and Shapiro network effects, price discrimination, and product differentiation. Externality-based complements, however, exploit a different mechanism than either tying or lock-in even as they help to explain many recent strategies such as those of firms selling operating systems, Internet browsers, games, music, and video.

The model presented here argues for three simple but useful results. First, even in the absence of competition, a firm can rationally invest in a product it intends to give away into perpetuity. Second, we identify distinct markets for content providers and end consumers and show that either can be a candidate for a free good. Third, product coupling across markets can increase consumer welfare even as it increases firm profits.

The model also generates testable hypotheses on the size and direction of network effects while offering insights to regulators seeking to apply antitrust law to network markets.

Key words: network effects; network externalities; two-sided markets; free information; business models; strategic complements; product design

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1. Introduction

This paper seeks to explain strategic pricing behavior and product design decisions in network markets. Given the seemingly anomalous practice of free-pricing, how is it possible that firms are willing to subsidize information and related products, apparently expecting neither future consumer exploitation nor tying? Why do firms give away applications development toolkits, portable document readers, and Internet browsers without metering tie-ins to those same consumers? What theory explains this economic logic?

In answer, we find that designing matched product pairs and discounting one relative to the independent goods case changes the shape of demand in markets joined by network effects. The insight is that characterizing network markets may require not only product standardization, essential to demand economies of scale (Katz and Shapiro 1985, Farrell and Saloner 1986), it may also require recognizing sharp distinctions between consumer types. This is essential for managing demand interdependence, implementing price discrimination, and raising barriers to entry.

Contributing to the recent two-sided network externality literature, we introduce a novel mechanism

(i) that explains firms' unbundled component sales and pricing strategies, (ii) that shows which half of a two-sided network market to discount, (iii) that is distinct from tying and traditional multimarket price discrimination, and (iv) whose formulation is robust in results to multiple specifications of demand. Of managerial interest is the proposition that discounting an unbundled component can increase profits to the point where negative prices become optimal. This helps not only the monopolist but also the oligopoly firm seeking barriers to entry. Profits increase conditionally, however, on promoting network effects through clever product design. They depend also on the correct choice of market to discount. Of legal and economic interest is the implied industry concentration and difficulty applying antitrust tests of predation. Pricing below marginal cost maximizes profits even in the absence of competition. Platform intermediaries that internalize these externalities can improve consumer welfare as well as profit.

The paper proceeds as follows. Section 2 provides a review of related literature in network externalities, multiproduct pricing, and bundling. In §3, we first develop a model of externality-based complements. We find the conditions for a subsidy market to exist

and identify which market receives the subsidy. We also demonstrate how the key insight is robust to several modeling generalizations. Section 4 offers further applications and extensions and §5 concludes.

2. Historical Context and Related Literature

Several branches of literature yield insight for this research, including network externalities, multi-product pricing, and to a lesser extent bundling. In the classic network externality story (Katz and Shapiro 1985, Farrell and Saloner 1986, Arthur 1989), demand economies of scale cause growth in an existing stock of consumer value as new consumers join the network. Various authors have used network effects to explain the popularity of QWERTY and VHS (Arthur 1994), ad subsidies of content in “circulation industries” (Chaudhri 1998), and the importance of standards and switching costs in network economies (Katz and Shapiro 1985; Shapiro and Varian 1998, 1999). In contrast, we consider a matched-market, chicken-and-egg problem, supporting an alternate interpretation of conventional wisdom. We distinguish between *intramarket* and *intermarket* network externalities. Producers want consumers and consumers want producers before either prefers a new format.

An incumbent firm producing content for one format probably does not welcome entry by a competing firm producing similar content if there is no profitable exchange between them. This mitigates a network effect *within* one side of the market. Buyers in the end-consumer market, however, welcome entry

because it increases the prospect of a viable format should the incumbent fail. It also increases variety while possibly lowering prices. This increases both the value to individuals and the number of individuals willing to switch formats. Thus, in the present example, the externality runs from content creators to end consumers.

Conversely, consider the end consumer’s original choice of VHS versus Beta. Initially, at least, consumers probably care less about adding another consumer to a new format—it could even bid up prices if supply is limited—than they care about the number and diversity of firms who provide content for that format. Producers, however, care immensely about the size of the consumer market. The existence of a larger consumer base makes production under any given format more attractive. Again, in the present articulation, the externality runs across markets from consumers to providers of content.

Importantly, the externality benefit can run across markets and back again. Both content creators and consumers do value growth in their own markets, but this may be mediated by the indirect effect of the internetwork externality. At issue is whether own-market entry expands participation on the other side of each transaction. Content creators may not object to other content-providing firms if effective consumer demand rises instead of falls.

Consider, finally, a third participant, the focus of our attention here, who produces tools to support both content creators and end consumers. These are “platform” intermediaries. Examples in Table 1 include Sun, Apple, and Microsoft, who support software developers as well as private and business

Table 1 Two-Sided Market Examples

Product category	Market 1	Intermediary	Market 2
Portable documents	Document reader [⊗]	Adobe	Document writer
Credit cards	Consumer credit [⊗]	Issuing bank	Merchant processing
Operating systems	Complementary applications	Microsoft, Apple, Sun	Systems developer toolkits [⊗]
Plug-ins	Applications software	Microsoft, Adobe	Systems developer toolkits [⊗]
Ladies’ nights	Men’s admission	Bars, restaurants	Women’s admission [⊗]
TV format	Color UHF, VHF, HDTV [⊗]	Sony, Phillips, RCA	Broadcast equipment
Broadcast & publishing	Content [⊗]	Magazine publishers, TV, radio broadcasters	Advertisements
Computer games	Game engine/player	Ubisoft, ID, valve, electronic arts	Level editors [⊗]
Auctions	Buyers [⊗]	E-Bay, Christie’s, Sotheby’s	Sellers
Academic journals	Articles	<i>Management Science</i>	Author submissions [⊗]
Recruiting	Applicants [⊗]	Monster.com	Employers
Reservation systems	Travelers [⊗]	Expedia, Travelocity, Orbitz	Hotels, airlines, rental cars
Shopping malls	Shoppers [⊗]	Mall of America	Stores
Streaming audio/video	Content [⊗]	Real audio, Microsoft, Apple	Servers
Paid search	Searchers [⊗]	Google.com	Marketers
Stock exchange	Equity purchasers [⊗]	NYSE, NASDAQ	Listed companies
Home real estate	Home buyers [⊗]	Real estate agents	Home sellers

Notes. This table shows how one side of a two-sided network market receives a discounted, free, or even subsidized good (indicated with [⊗]). In general though not always, Market 1 can be interpreted as the user/consumer market and Market 2 can be interpreted as the producer/developer market. We provide a test for which side receives the free good below.

buyers; Sony and Phillips, who support the entertainment industry as well as households; and firms like Adobe, who produce portable document writers as well as portable document readers. Applied to services, eBay coordinates buyers and sellers, while Visa coordinates merchants and card holders. For these firms, the chicken-and-egg profit-maximization problem is how to grow matched markets. A straightforward and widely observed solution is to discount one market in order to grow both, and to profit more from the other.¹ This model favors an intermediary who, straddling both markets, can set prices more efficiently by internalizing these two-sided externalities. Independent firms serving either market separately lose this advantage.

A key contribution of a two-sided network model is determining which side receives a discount. Different firms choose different beneficiaries. In streaming video, portable documents, and advertising, for example, the industry norm is to subsidize content consumers and charge content developers. The opposite, however, holds true for operating systems and multiplayer games in which content developers receive subsidies and consumers pay to join the network. We show how this depends on cross-price elasticities as well as the relative sizes of the two-sided network effects.

This analysis adds to recent literature on two-sided network effects (Armstrong 2002, Caillaud and Jullien 2003, Rochet and Tirole 2003) that make precise a form of indirect network effect (Katz and Shapiro 1985, Liebowitz and Margolis 1994). Indirect effects are consumption externalities from purchasing compatible products such as hardware and software. The key distinction for two-sidedness is that network effects must cross market populations. When they do not, the same consumer populations choose systems of compatible products only for themselves. These systems lead to pecuniary externalities, efficiently handled through the pricing system (Liebowitz and Margolis 1994). In contrast, two-sided networks yield true externalities in which one population chooses a good affecting another population's choice of a different good. In a recent working paper, Rochet and Tirole (2004, p. 3) state, "To use a celebrated example, the buyer of a razor internalizes in his purchase decision the net surplus that he will derive from buying razor blades. The starting point to the theory of two-sided markets by contrast is that an end-user does not internalize the welfare impact of his use of the platform on other end-users." As in Farrell and Saloner

(1985), coordinating purchases becomes essential to avoid inertia, but in two-sided networks, coordination across markets matters. Coordination within markets may have little effect.

Rochet and Tirole (2003), which focuses on competing credit card markets, parallels our work.² They model two-sided network effects using multiplicative demand and symmetric spillovers, and nicely capture "multihoming"—the decision to carry multiple credit cards from competing networks. Our framework, independently developed, differs in several important respects. (1) We show that key results generalize to alternative demand specifications. (2) We include models that correct for boundary condition problems. (3) We account for asymmetric and even negative network effects so that we can model markets with very different properties.

Caillaud and Jullien (2003) consider a matchmaking intermediary, as for dating services. Using linear demand and a Bertrand pricing model, they explain why agents register with more than one service, as in the case of multihoming credit card services. Under competition, two-sided network effects lead one firm to corner the market, or multiple firms to share the market with zero profits. They also show how transaction costs reduce total surplus. Our work focuses on information goods and product unbundling and generalizes to nonlinear demand. Our problem is also distinct from matching distinct agent pairs as in Mongell and Roth (1991), instead representing an effort to increase total matched transactions occurring on a common platform. A useful survey of our results and others' is provided in Armstrong (2002), examining which side of a market is subsidized and whether the outcome is socially efficient, and Rochet and Tirole (2004), which establishes general definitions for two-sided markets.

2.1. Multiproduct Pricing

The fact that lowering price on one half of a matched pair sells more of the other has precedent in multiproduct pricing. This parallels Cournot's (1838) observation that discounting zinc sells more copper in the brass market but without the novelty that sales accrue to different buyers. DeGraba (1996) also presents an example of firms willing to enter a zero-profit market, relying on declining average cost curves to increased sales. These mechanisms, however, differ in that analysis proceeds from a model of tying, like that in Whinston (1990).

¹ The consequences of mispricing internet markets can also be severe. In 2001, the number two auction site, Yahoo, raised seller fees and listings unravelled, dropping 90% (Hansell 2001). Sellers concluded that for such fees, they preferred the larger number of buyers on eBay.

² The two papers were independently and concurrently developed and were conceived to answer related but different questions. Rochet and Tirole cite an earlier version of this paper, titled "Information Complements, Substitutes, and Strategic Product Design," which was presented at the 1999 Workshop on Information Systems & Economics and was posted to *ssrn.com* in 2000.

Two-sided network externality coupling is not traditional second-degree price discrimination. It differs from tying razors and blades in that one market need never consume the complementary good. It also differs from penetration pricing in which a good is subsidized initially on expectation of future exploitation. In both cases, a single consumer internalizes his own value calculation such that price changes affecting one matched item are reflected in willingness to pay for the other. This property fails for two-sided network markets. Consumers of a portable document reader may, at their discretion, forgo the cost of the portable document creator now and forever.

Similarly, two-sided network externality coupling differs from traditional third-degree price discrimination. Firms offering nonlinear prices to mixed markets force heterogeneous consumers to self-select. Business travellers choose different options than tourists; heavy users of phones and electricity choose different tariffs than light users. Such mechanisms differentially extract consumer surplus and transfer it to the seller (Varian 1989). In contrast, platform intermediaries operating in two-sided markets seek to profit by transferring surplus from seller to consumer. Growth on one side of a matched market then induces growth on the other, creating exploitable surplus. This effect can prove unusually vexing to antitrust regulators because producer and consumer surplus can move together, as we show in Proposition 3.

Further, Rochet and Tirole (2004) note that demand creation—as distinct from surplus division—violates the Coase theorem (1960). This theorem states that, regardless of externalities, transactions volume will be efficient as long as property rights are clearly defined and there are no information asymmetries or transaction costs. Buyers and sellers will bargain their way to efficiency; pollution trading rights come to mind as an example.

The Coase theorem fails in the case of two-sided network effects. Property rights, symmetric information, and zero-cost transactions do not suffice for efficient trading volume when it is the presence of one consumer type that itself creates value for the other type. Ladies' nights, developer toolkits, ad-supported content, as well as cash-back bonuses and frequent flyer points on credit cards constitute continuing subsidies to one side of the market.

A product design strategy that discounts price to zero is aided by the unique properties of information. The key, however, relies less on nonrivalry than on low marginal costs. When these are negligible, a firm can subsidize an arbitrarily large market based solely on fixed initial costs. Providing each new sale or service then costs the clever product designer nearly nothing in incremental costs. Increased consumption of information may therefore have the potential to

increase the attractiveness of the proposed design strategy.

3. Markets

This section adapts a standard externality model to show why a firm spends resources creating a product it distributes for free when two-sided network externalities cross market pairs. It also shows which market receives the discount.

Let the first market represent the general consumer or end-user market, C , and let the second market represent the content creator, developer, or joint-producer market J . Index price p and quantity q terms by market such that p_c and p_j describe prices for consumers and content creators, respectively. Let marginal costs for information goods be negligible with profit given by $\pi = \pi_c + \pi_j = p_c q_c + p_j q_j$. To provide a standard concave profit function, let this be twice differentiable in both choice parameters, have $\partial^2 \pi / \partial p_i^2 < 0$, $i \in \{c, j\}$, and have a positive Hessian determinant. Accordingly, first-order conditions yield prices in both markets.

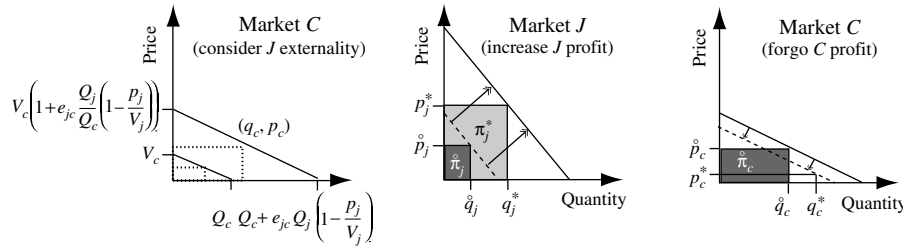
Demand is also standard but generalizes to allow a cross-market relationship. Each market has a continuum of consumers willing to buy one discrete unit of a good. If v is arbitrary willingness to pay, then $D(p)$, with upper bound \bar{V} , is simply $\int_{v \geq p}^{\bar{V}} dD dp$. We require the distribution of values to be bounded above and well defined over negative values to avoid corner solutions. In the linear demand case of Katz and Shapiro (1985), this is simply $v \in (-\infty, \bar{V}]$. In our model, this also has a natural interpretation of allowing price subsidies, and we generalize to nonlinear demand. For $i \in \{j, c\}$ we assume D_i is twice continuously differentiable and decreasing in p_i . We do not require concavity or convexity of demand but only restrict it sufficiently to ensure concavity of the profit function via the Hessian. Paralleling Katz and Shapiro, we also model total consumption as the sum of a good's intrinsic demand incremented by a function of network size in the other market. That is, we allow consumption q to rise with network effects as well as value v as specified next.

Our main point of departure is having externalities cross markets. The *internetwork* externality term e_{jc} measures how much effect purchases in the developer market have on the consumer market. More precisely, e_{jc} is the partial derivative of demand in the C market with respect to demand in the J market, or $\partial q_c / \partial q_j$. Conversely, e_{cj} determines how much effect purchases in the consumer market have on the joint-producer market. Together, these terms describe a pair of demand equations:

$$q_c = D_c(p_c) + e_{jc} D_j(p_j); \quad (1)$$

$$q_j = D_j(p_j) + e_{cj} D_c(p_c). \quad (2)$$

Figure 1 Markets Linked by Two-Sided Network Externalities



Notes. In the linear case, the left panel shows a network externality that shifts the demand curve out. The optimal independent pair (q_c, p_c) increases from the no-externality case $(Q_c/2, V_c/2)$. In the middle panel, a monopolist sets $p_j^* > \hat{p}_j$ and reaps profits π_j^* . In the right panel, price in the C market falls ($\hat{p}_c < \hat{p}_c$) in order to take profits in the J market. A higher price in market J then somewhat reduces the total externality impact in market C. Discounting the C market is rational when net profits rise.

Several notational conventions help develop useful intuition. We refer to the marginal cross-price contribution to sales $\partial q_j / \partial p_c = e_{cj} D'_c(p_c)$ as the two-sided network externality or simply “spillover” effect. The importance of network spillovers, as given by the ratio $r \equiv (\partial q_j / \partial p_c) / (\partial q_c / \partial p_j)$, plays an important role in subsequent analysis and will later draw attention to which market generates relatively more surplus. For convenience, we define p^* , \hat{p} , and \hat{p} as the optimal (coordinated across both markets) monopoly, uncoordinated (two independent firms making independent decisions), and no-externality prices, respectively. In general, $p^* \neq \hat{p} \neq \hat{p}$ unless $e_{jc} = e_{cj} = 0$, and $q_c \neq D_c(p_c)$ unless $e_{jc} = 0$. Where the context is clear, we simplify notation by suppressing parameters and write D_c for $D_c(p_c)$.

For illustration, the linear demand curves $D_c(p_c) = Q_c(1 - p_c/V_c)$ and $D_j(p_j) = Q_j(1 - p_j/V_j)$ yield simple and elegant results in terms of consumer surplus. Let exogenous parameters Q_i and V_i , $i \in \{c, j\}$ represent the maximum market size and maximum product value in the absence of externalities, respectively. A consumption externality from J creates an outward shift in C demand, raising value by an amount proportional to $D_c^{-1}(e_{jc} D_j)$; thus, \bar{V}_c rises from V_c to $V_c(1 + e_{jc}(Q_j/Q_c)(1 - p_j/V_j))$. In the C market, interpretation in terms of surplus space can then be found by integrating demand³ and defining substitutions.

	Two-sided network externality surplus
Native surplus	
$S_c \equiv Q_c V_c$	$S_{cj} \equiv e_{cj} Q_c V_j$
$S_j \equiv Q_j V_j$	$S_{jc} \equiv e_{jc} Q_j V_c$

Native terms are proportional to consumer surplus in each market individually, whereas network terms

are proportional to the surplus created by consumption externalities in the complementary market.

3.1. Monopoly Choice

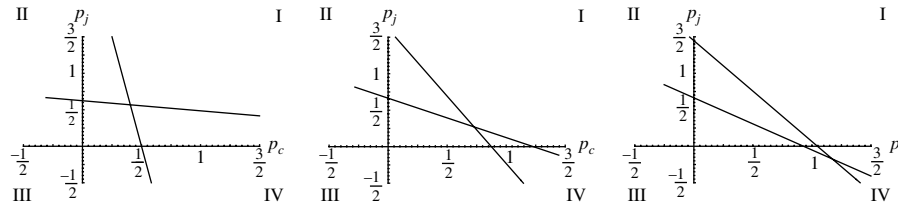
A simple linear example then illustrates how the model functions, although linearity is not required. Consider first a monopolist’s profit-maximizing decision in the absence of network externalities, $e_{cj} = e_{jc} = 0$. A straightforward maximization on $\pi_c = p_c Q_c(1 - p_c/V_c)$ yields $p_c^* = V_c/2$, $q_c^* = Q_c/2$, with profits of $\pi_c^* = V_c Q_c/4 = S_c/4$ and similarly for π_j as in the left-most panel of Figure 1. Optimal prices and quantities are simply half the consumer value and half the consumer market, respectively.

Allowing for positive spillover, the J market externality shifts the demand curve in the C market up and to the right relative to the no-externality case. Increasing p_j , however, reduces the total outward shift in the C market as shown in the right-most panel. Taking externality effects into account, the decision of how much to charge in each market is analyzed below. This graphic captures the idea that content providers stimulate demand among consumers, just as consumers stimulate interest from content providers.

The number of content providers who actually enter the market, however, governs the degree of shift. The externality or coupling between these two markets implies that the more a monopolist can coax consumers into adopting one of his products, the more it can charge for—and sell of—the other. If the increment to profit on one complementary good exceeds the lost profit on the other good, then a discount or even subsidy becomes profit maximizing. Free-goods markets can therefore exist whenever the profit-maximizing price of zero or less generates cross-market network externality benefits greater than intramarket losses. Firms do, in fact, offer subsidized goods when they offer free service and technical expertise. Historically, for example, Microsoft offered technical help on applications program interfaces (API) in addition to free systems development

³ $\int_0^{\bar{V}_c} [Q_c(1 - p_c/V_c) + e_{jc} Q_j(1 - p_j/V_j)] dp_c = \frac{1}{2} Q_c V_c + e_{jc} Q_j V_c (1 - p_j/V_j) + (1/(2Q_c V_c)) e_{jc}^2 Q_j^2 V_c^2 (1 - p_j/V_j)^2$, where $\bar{V}_c = V_c(1 + e_{jc}(Q_j/Q_c)(1 - p_j/V_j))$. The area of the triangles in Figure 1 gives a similar expression.

Figure 2 Intersecting Price Reaction Curves Show Consumer Price (p_c) Rising and Developer Price (p_j) Falling as the Developer-to-Consumer Externality (e_{jc}) Rises from 0 to $\frac{3}{4}$ to $\frac{11}{10}$ in the Linear Case with Parameters $\{Q_c = Q_j = V_c = V_j = 1\}$ and $e_{jc} = \frac{1}{3}$



toolkits, and this may have increased the number of user applications developed for its operating system. Apple did not and has since changed its strategy. This form of subsidy also addresses an interesting problem of adverse selection arising from a subsidy to a general audience. Only developers request API support, so the subsidy is not wasted on nondevelopers.

In general, the optimal price pair for markets linked by network externalities is not obvious. Figure 2 illustrates that a pricing decision falls into one of four quadrants. Regions II and IV represent a “free-goods” market because a firm charges nothing or spends money to place product in the hands of consumers or developers, respectively. In region I, both markets are charged positive prices while region III represents a subsidy to both markets and is never profit maximizing.

In principle, the markets are symmetric; in practice, computer games manufacturers charge consumers but give developers free toolkits, while streaming media companies give consumers free software players but charge developers to create content. Interestingly, banks selling credit card services charge both merchant and consumer fees, illustrating region I. The decision about which market to subsidize, if any, and which to charge, then rests on Corollary 2 below. Before analyzing this case, we first develop conditions for optimality in terms of elasticities. Definitions for own- and cross-price elasticities yield $\eta_c = p_c D'_c / (D_c + e_{jc} D_j)$ and $\eta_{cj} = e_{cj} p_j D'_j / (D_c + e_{jc} D_j)$. Recall also that $\pi = \pi_c + \pi_j = p_c q_c + p_j q_j$.

PROPOSITION 1. *Network effects on one side of the market render the matched side more inelastic. Optimal prices, however, also depend on cross-price elasticities and the ratio of surplus created on both sides. That is, p_c^* depends on cross-market elasticity of C sales with respect to J price times the spillover ratio. Equivalently, p_c^* depends on cross-market elasticity of J sales with respect to C price times the profit ratio:*

$$-1 = \eta_c + \eta_{cj} r \quad (3)$$

$$= \eta_c + \eta_{jc} \frac{\pi_j}{\pi_c}. \quad (4)$$

The condition for a free-goods market to exist is optimal subsidy price $p_c^ \leq 0$, which occurs at the point $-1 \geq \eta_{jc} r$.*

PROOF. To see that e_{jc} makes $\eta_c = p_c D'_c / (D_c + e_{jc} D_j)$ more inelastic, note that increases push η_c toward zero, decreasing sensitivity to changes in p_c . To establish the unit elasticity condition, use the first-order condition on total profit with respect to price in the C market to find

$$\frac{\partial \pi}{\partial p_c} = D_c + e_{jc} D_j + p_c D'_c + e_{cj} p_j D'_c = 0, \quad (5)$$

which rearranges to $-1 = \eta_c + e_{cj} p_j D'_c / (D_c + e_{jc} D_j)$. Recall that $r = (\partial q_j / \partial p_c) / (\partial q_c / \partial p_j) = e_{cj} D'_c / (e_{jc} D'_j)$. Multiplying the final term by $1 = e_{jc} D'_j / (e_{jc} D'_j)$ and substituting for η_{cj} provides Equation (3). Then, when price decreases through zero $p_c^* \leq 0$, we have $\eta_c \geq 0$, proving the condition on existence of a free-goods market. Note that although r is undefined when $e_{jc} D'_j \rightarrow 0$, $\eta_{cj} r$ is still defined because the multiplicative term, $e_{jc} D'_j$ cancels out. To produce the final expression, insert definitions for elasticities and simplify to confirm that $(\eta_{jc} / \eta_{cj}) / (\pi_j / \pi_c) = r$. Substituting this equality in Equation (3) produces Equation (4). Note that symmetry implies a parallel set of expressions in the J market where the optimality condition is $-1 = \eta_j + \eta_{jc} (1/r)$. \square

Intuitively, rising network effects from J to C allow price to rise in C. If this were the only effect, we might expect network effects to increase prices on both sides of the market in small neighborhoods around the no-externality prices \hat{p}_c and \hat{p}_j . A firm, however, must manage cross-price effects and the ratio of spillovers. These effects are so strong that at the point a subsidy becomes optimal, a firm is choosing prices opposite those implied by own-price elasticities considered separately.

To see this, let $p_c^* < 0$, and thus necessarily $p_j^* > 0$ with $\pi_c^* < 0$ and $\pi_j^* > 0$. Then signing terms in the definition of η_c implies $\eta_c^* > 0$. At the same time, Proposition 1 establishes the relation $\eta_j^* = -1 - \eta_{jc}^* r = -1 - (e_{cj} p_c D'_c / (D_j + e_{cj} D_c)) (e_{cj} D'_c / (e_{jc} D'_j)) < -1$. In short, $p_c^* < 0$ implies that one-sided elasticities by themselves are $\eta_j^* < -1 < 0 < \eta_c^*$. When subsidies are optimal in the C market, price hikes rather than subsidies would (locally) increase sales in the C market and price drops in the J market would (locally) increase sales in the J market.

Relative cross-price effects are dramatic. Further rearranging Equation (3) shows that it is not one-sided prices alone that must be unit elastic, $\eta_c^* \neq -1$, but rather the one-sided prices adjusted for an externality adjusted price ratio $\eta_c^*(1 + e_{cj}(p_j^*/p_c^*)) = -1$. Graphically, the intuition is again that of Figure 1. These results are surprisingly robust and are true more generally for each of the following.

COROLLARY 1. *The demand elasticity properties of Proposition 1 hold for any of the following alternative demand specifications (equations are symmetric for q_j):*

Additive	$q_c = D_c + e_{jc}D_j$	
Additive recursive	$q_c = D_c + e_{jc}q_j$	
Multiplicative	$q_c = D_c D_j^{e_{jc}}$	(6)
Shifted multiplicative	$q_c = D_c(1 + D_j)^{e_{jc}}$	
Multiplicative recursive	$q_c = D_c q_j^{e_{jc}}$	

PROOF. Define $M \equiv 1/(1 - e_{cj}e_{jc})$ as a “market multiplier,” noting that $e_{cj}e_{jc} < 1$ must hold to render the definition meaningful. Resolving the simultaneous pairs $\{q_c, q_j\}$ for the two recursive cases results in $q_c = M(D_c + e_{jc}D_j)$ and $q_c = (D_c D_j^{e_{jc}})^M$. These differ only by the constant term M from nonrecursive cases and the differences disappear in the optimality calculations. Applying the sequence of steps from Proposition 1 yields equations identical to Equations (3) and (4). Moreover, elasticities have the structure $e_{cj}e_{jc}\eta_c\eta_j = \eta_{cj}\eta_{jc}$. A minor exception occurs for the shifted case which also shifts, becoming $e_{cj}e_{jc}\eta_c\eta_j(D_c D_j / ((1 + D_c)(1 + D_j))) = \eta_{cj}\eta_{jc}$. \square

The recursive formulations allow reciprocal externalities on both sides of the market. An additive formulation has a natural interpretation in allowing demand shifts as in Figure 1 whereas a multiplicative formulation pivots around the point of maximum value V_c . The latter is used in Rochet and Tirole (2003), which makes the specialized assumption that $e_{cj} = e_{jc} = 1$. The multiplicative formulation has an advantage in forcing high price, zero demand in the native market to cancel externality-driven demand from the cross-market. Introducing the shifted multiplicative case corrects for the exotic condition where $0 < D_j < 1$ causes q_c to fall in a rising externality. Equivalence of these five functional forms shows that the main insight is robust to nonlinear demand as well as several alternative combinations of demand. For completeness, we will occasionally compare results using the notations p_{c+} and $p_{c\times}$ for the additive and shifted multiplicative formulations, respectively.

Proposition 1 implies that cross-price elasticities matter in both directions. The proposition constrains

price through η_{cj} times the spillover ratio. The bidirectional nature of spillovers demonstrated in Proposition 1 implies that pricing decisions depend heavily on their relative sizes and, in fact, no difference in size can imply no difference in price. In the case of additive demand, this leads to the unusual result that prices will be optimal, even if set under the myopic assumption of no network effects, when spillovers between markets are equal.

PROPOSITION 2. *For additive demands, a monopolist selling to complementary markets with symmetric spillovers sets optimal price independent of the level of two-sided network effects. Mathematically, $r = 1$ if and only if $(p_{c+}^* = \hat{p}_{c+}$ and $p_{j+}^* = \hat{p}_{j+})$. In the (shifted) multiplicative case, a monopolist always sets p^* less than or equal to the independent and no-externality prices. That is, $p_{c\times}^* \leq \hat{p}_{c\times} = \hat{p}_{c\times}$.*

PROOF. Recall that $M = 1/(1 - e_{cj}e_{jc})$. First-order conditions (Equation (5)) indicate $p_c^* = (D_c + e_{jc}D_j + e_{cj}p_j^*D_c)/-D_c'$. For reference, we also calculate independent firm and no-externality prices \hat{p}_c and \hat{p}_c . In the case of independent firms the final term in the numerator of p_c^* is zero, so that $\hat{p}_c = (D_c + e_{jc}D_j)/-D_c'$, and in the case of no externalities of either sort, $\hat{p}_c = D_c/-D_c'$. For optimal profits, simultaneous solution of p_c^* and p_j^* yields implicit price

$$p_c^* = M \left(\frac{D_c + e_{jc}D_j}{-D_c'} - e_{cj} \frac{D_j + e_{cj}D_c}{-D_j'} \right), \quad (7)$$

with a symmetric expression for p_j^* . More intuitively, this can be interpreted as the market multiplier times the difference in independent prices $M(\hat{p}_c - e_{cj}\hat{p}_j)$, showing that own-price generally declines in own externality. To complete the proof, note that $r = 1$ requires $e_{jc}D_j' = e_{cj}D_c'$. Combining fractions and twice substituting $e_{jc}D_j'$ for $e_{cj}D_c'$ collapses the equation to $p_c^* = D_c/-D_c' = \hat{p}_c$. Symmetry forces $p_j^* = \hat{p}_j$, which is the required forward result. If both no-externality prices are optimal, then reversing each step of the equality requires $e_{jc}D_j' = e_{cj}D_c'$.

For the (shifted) multiplicative case, FOCs give:

$$\begin{aligned} \frac{\partial \pi}{\partial p_c} &= D_c(1 + D_j)^{e_{jc}} + p_c D_c'(1 + D_j)^{e_{jc}} \\ &+ e_{cj}p_j D_j(1 + D_c)^{e_{cj}-1} D_c' = 0. \end{aligned} \quad (8)$$

It is then straightforward to establish that $\hat{p}_c = \hat{p}_c = D_c/-D_c'$ and that $p_c^* = D_c/-D_c' - e_{cj}(p_j D_j(1 + D_c)^{e_{cj}})/((1 + D_j)^{e_{jc}}(1 + D_c)) = \hat{p}_c - e_{cj}\pi_j/((1 + D_j)^{e_{jc}}(1 + D_c))$. Because all terms in the rightmost expression are positive, p_c^* must fall relative to \hat{p}_c . \square

The proposition states that if the demand externality is additive and network effects across markets are approximately equal in size—the presence of developers is as attractive to consumers as the presence

of consumers is to developers—then a monopolist can safely ignore these effects in all pricing decisions. A firm can set prices *as if* the network terms e_{cj} and e_{jc} were both zero. This does *not* imply that network externalities have no effect on quantity sales. In general, output exhibits substantial increases due to the market multiplier $M = 1/(1 - e_{cj}e_{jc})$. This result is unusual in that one might have expected prices to rise with increased demand, but the monopolist takes the gain exclusively in terms of increased sales rather than higher prices. The point is that firms need to consider a joint pricing decision that accounts for network externalities only when the difference between the spillover effects on sales, $e_{cj}D'_c - e_{jc}D'_j$, is substantial. Proposition 2 shows that for additive demand externalities, optimal price equals no-externality price $p_c^* = \hat{p}_c$ whenever $r = 1$, while for the multiplicative case, independent price equals no-externality price $\hat{p}_c^* = \hat{p}_c$ always.

Building on this analysis, we can now answer the more general question of which market a profit-maximizing firm discounts when spillover levels are unequal.

COROLLARY 2. *A monopolist selling to complementary additive demand externality markets discounts the product with the greater spillover effect. That is, $r < 1 \Leftrightarrow p_c^* < \hat{p}_c$. The optimal C market price falls relative to the no-externality price if and only if market C contributes more network externality surplus to market J, or $p_c^* < \hat{p}_c \Leftrightarrow S_{cj} > S_{jc}$.*

PROOF. We have that $r < 1$ implies $e_{cj}D'_c = e_{jc}D'_j - \varepsilon$ for $\varepsilon > 0$. An identical sequence of substitutions as those above yields

$$p_c^* = \frac{D_c}{-D'_c} - M \left(\frac{\varepsilon}{-D'_c} \right) \left(\frac{D_j + e_{cj}D_c}{-D'_j} \right). \quad (9)$$

Only $D'_c, D'_j < 0$ so that p_c^* falls. Alternatively, this can be interpreted as $p_c^* = \hat{p}_c - (\varepsilon/-D'_c)M\hat{p}_j$, and we note that conclusions are similar for $e_{cj}D'_c = e_{jc}D'_j + \varepsilon$. To establish the condition on linear surplus, observe that when $D_c(p_c) = Q_c(1 - p_c/V_c)$, then $\partial q_j/\partial p_c$ is simply $-e_{cj}(Q_c/V_c)$, with a similar expression for $\partial q_c/\partial p_j$. Arrange both terms as an inequality. The result, $-e_{cj}(Q_c/V_c) < -e_{jc}(Q_j/V_j)$, then simplifies to $e_{cj}Q_cV_j > e_{jc}Q_jV_c$, which completes the proof. Discounting the C market must create more J market surplus than vice versa for C's price to fall. \square

A few additional points follow from this analysis. First, for additive demand externalities, a firm can rationally discount only one market at a time. This means that a discount in one market implies a price premium in the other, or $p_c^* < \hat{p}_c$ if and only if $p_j^* > \hat{p}_j$. The reason is the fact that $p_c^* < \hat{p}_c$ implies $e_{cj}D'_c < e_{jc}D'_j$ and the markets are symmetric. An immediate corollary shows that in the linear case, optimal prices fall

in the market that contributes more externality surplus to the other market.

While the preceding analysis compares no-externality and optimal prices, it is straightforward to compare independent and optimal prices. In an example of a classic result on the uncoordinated pricing of complements, we show below that factoring network externalities into the pricing decision, but failing to coordinate the prices across markets, leads to inefficient price setting. Where p^* is again optimal, let \hat{p} represent independent firm price as distinct from the no-externality price \hat{p} .

COROLLARY 3. *In the case of symmetric spillovers, if independent firms set prices in additive demand externality markets without coordination, then prices vary with the level of spillover and are inefficiently high, $\hat{p} \geq p^*$.*

PROOF. Proposition 2 established that for $r = 1$, prices are independent of network effects, but for independent prices, we have $\hat{p}_c = (D_c + e_{jc}D_j)/-D'_c$. The difference $\hat{p}_c - p_c^*$ is then $e_{jc}D_j/-D'_c$. For independent firms, prices are always positive and increasing in the size of cross-market network effect. \square

Without externalities, prices \hat{p}_c and p_c^* are identical and the inefficiency disappears. With externalities, however, independent firms err in direct proportion to their own level of network effect on their market complement. One consequence of Corollary 3 is that independent firm prices are strategic substitutes and a price increase by one firm causes a price decrease by the other.

3.2. Consumer Surplus

A firm's ability to manipulate two-sided network externalities to increase profits raises a concern about whether its gains represent consumer losses. Does the ability to link markets, particularly when two products are combined under monopoly ownership, hurt consumers?

In the case of symmetric spillovers, Proposition 2 and Corollary 3 imply that the answer is clearly no. Consumer surplus improves under monopoly ownership. Because two independent firms both price too high, integrated ownership reduces prices even as it increases profits.

Asymmetric spillovers may leave consumer welfare ambiguous because $p_c^* = 1/(1 - e_{cj}e_{jc})(\hat{p}_c - e_{cj}\hat{p}_j)$ and a careful choice of e_{cj}, e_{jc} can force $p_c^* > \hat{p}_c$. For linear demand, however, welfare analysis of asymmetric spillovers provides a particularly strong result. Welfare exhibits a strict Pareto improvement in the sense that even the market paying the increased premium-goods price prefers monopoly control. This result is stronger still in the case of multiplicative demand externalities. Let superscripts on consumer surplus CS^* and CS^\wedge represent the independent and joint optimization values from the firm's perspective.

PROPOSITION 3. *Consumers and producers benefit when firms merge or coordinate prices in markets with linear demand and two-sided network externalities. That is, $CS_c^* \geq CS_c^\wedge$ and $CS_j^* \geq CS_j^\wedge$ and thus necessarily $CS_c^* + CS_j^* \geq CS_c^\wedge + CS_j^\wedge$. $\pi^* \geq \hat{\pi}$, so total welfare improves. This is always true for multiplicative demands.*

PROOF. The upper bound on $\int_{p_c}^{\bar{V}_c} q_c(r) dr$ is chosen to avoid negative quantities and is given by $\bar{V}_c = D_c^{-1}(q_c - e_{jc}D_j)$. For $q_c = 0$, this becomes $V_c(1 + e_{jc}(Q_j/Q_c)(1 - p_j/V_j))$. Integrating Equation (1) and substituting for \bar{V}_c gives consumer surplus in the C market as

$$CS_c = \frac{(e_{jc}Q_jV_c(p_j - V_j) + Q_c(p_c - V_c)V_j)^2}{2Q_cV_cV_j^2}. \quad (10)$$

In computing optimal (p_c^*, p_j^*) and independent (\hat{p}_c, \hat{p}_j) prices, we condense several steps. We also simplify these expressions by converting to surplus notation. This collapses six apparent degrees of freedom to four true degrees. Substituting for (p_c^*, p_j^*) in (10) yields consumer surplus at optimal prices.

$$CS_c^* = \frac{(2S_c + S_{cj} + S_{jc})^2(S_cS_j - S_{cj}S_{jc})^2}{2S_c((S_{cj} + S_{jc})^2 - 4S_cS_j)^2}. \quad (11)$$

A similar substitution for (\hat{p}_c, \hat{p}_j) yields consumer surplus at independent prices.

$$CS_c^\wedge = \frac{S_c(S_{cj}S_{jc} - S_j(2S_c + S_{jc}))^2}{2(S_{cj}S_{jc} - 4S_cS_j)^2}. \quad (12)$$

Note that squares imply these are always positive. Imposing the Hessian condition $(4S_cS_j - (S_{cj} + S_{jc})^2 > 0)$ and simplifying the difference in total surplus $CS_c^* - CS_c^\wedge$ always proves nonnegative. Equality is achieved only when the markets are independent ($e_{cj} = e_{jc} = 0$ implying $S_{cj} = S_{jc} = 0$), in which case, $CS_c^* = CS_c^\wedge = \frac{1}{8}S_c$. The case for CS_j is symmetric. In the case of multiplicative demand, Proposition 2 proves $p_x^* \leq \hat{p}_x = \hat{p}_x$ always, thus CS improves. Finally, from the producer's perspective, a monopolist can always choose to set prices independently, thus $\pi^* \geq \hat{\pi}$ and total welfare Pareto improves. \square

The underlying intuition is that linking the markets allows a firm to manipulate the markets' total size via the cross-market network externality. Consumers then benefit to the extent that a self-interested firm sets prices more efficiently but cannot capture all consumer surplus from the efficiency gains. A firm internalizes the market externality but wins only a fraction of this effect.

4. Applications and Extensions

Two-sided markets coupled by network externalities are distinguished from other market types by heterogeneous matched consumers with interdependent demands. This has a number of implications beyond those of the present model.

One implication is that firms can deploy network effects to stimulate new surplus in contrast to price discrimination strategies that extract static surplus (Parker and Van Alstyne 2001). This provides counterpoint to key theories of bundling (Adams and Yellen 1976, Salinger 1995, Bakos and Brynjolfsson 1999), the advantage of which is reduced demand heterogeneity. As long as buyer values are not perfectly correlated, bundled demands converge to a point mass based on the central limit theorem. With zero marginal cost goods, firms can costlessly smooth idiosyncratic demands simply by combining items in a bundle. This allows a firm to predict and therefore extract consumer surplus. Rather than seeking to reduce market heterogeneity, two-sided product design seeks to leverage it. Unbundling together with a one-sided subsidy sacrifices the ability to predict transactions value in favor of growing transactions volume.

Introducing competition, analysis of two-sided network effects can also answer the question of why a duopolist might voluntarily enter into Bertrand price competition, forcing profits to zero in one market segment (Parker and Van Alstyne 1999). Even with an undifferentiated product, an entering firm can use a two-sided product complement to generate profits on the far side of a market. A firm can soften an incumbent's first-mover advantage and survive a market that becomes completely competitive post-entry. The product complement makes an entry threat credible. As in the case of the Internet browser wars, consumer surplus rises from the arrival of a zero price substitute and entrants can be unusually fierce competitors (Jackson 1999).

Empirically, a two-sided model has the advantage of suggesting new approaches for estimating network effects. Models of intramarket externalities suffer from endogeneity of demand estimation in that instruments can barely distinguish demand shocks from the network externality ripple effects they create. If, on the other hand, network effects cross markets, then instrumenting demand shocks in one market can proceed while holding demand constant in the coupled market, thus tracing a demand relationship. The proposed framework might therefore simplify estimation of network effects relative to earlier models. Testable propositions include which internetworked market to discount in markets with pricing power, which of several products firms will favor discounting in markets without pricing power, and whether discounting lasts longer than predicted by models of

penetration pricing with lock-in. An empirical test of Parker and Van Alstyne (1999) and Rochet and Tirole (2000, 2003) appears in Gallaugh and Wang (2002). They find significant positive externalities between the Web browser and Web server markets.

Failing to account for externality-based complements might therefore lead to pricing and product design errors. Using a product tying model, for example, firms might meter sales to both consumer types, missing the opportunity to manage network effects between them. Using a lock-in model, they might injudiciously discontinue discounts for the market stimulating cross-market sales, choking network effects. Using uncoupled multimarket price discrimination, they might fail to offer discounts at all. Each model suggests subtle differences in the locus and timing of pricing decisions. Suitable market divisions fall along several different lines based on demand attributes, product features, or time. Some practical applications of these divisions are given below.

(1) *Tangible goods and services with externalities between tangible and intangible goods*, $e_{it} > 0$ and $e_{ii} > 0$. Example: Historically, the first color TV broadcast occurred in 1951 but no firm realized a profit on color TV sets for almost a decade. Consumers delayed buying color TVs until the arrival of substantial content. RCA solved the chicken-and-egg problem in 1961 by subsidizing Disney's *Wonderful World of Color* (Shapiro and Varian 1998). In 2002, the Federal Communications Commission resolved an identical problem for HDTV by mandating the provision of HDTV sets by electronics firms.

(2) *Temporal complements with externalities between time 1 and time 2*. Example: Vendors of precedent and case law databases commonly subsidize student lawyers in time 1 while charging premium prices to these same people once they become professional lawyers in time 2. The time 1 version is fully functional, so there is no need to upgrade, nor is access ephemeral because it runs the full three years of law school. The spillover benefit is to save law firms the (re)training costs of having new lawyers learn the command structure and capabilities of a principal research tool. The benefit in the other direction is that, *ceteris paribus*, law students prefer to adopt the package adopted by more law firms.

(3) *Upgrades with externalities between novice and pro users*, $e_{np} > 0$. Example: Many firms give away novice versions of simulation or CAD software but charge for a professional version. The less functional or time-limited version encourages experimentation and purchase of the full-featured version. In contrast, conventional price discrimination uses a menu of different prices to force consumers with intrinsically different demand profiles to self-select on the basis of willingness to pay. The standard price dis-

crimination model, however, leaves open the question of why a firm should incur development costs on a good for which it never anticipates revenues.

(4) *Advertising market with externalities between consumers and advertisers*, $e_{ca} > 0$ and $e_{ac} < 0$ or $e_{ca} > 0$ and $e_{ac} > 0$. Example: Magazine publishers, TV and radio stations, software vendors, and Web portals offer free or discounted consumer content in "sponsored" mode in which consumer market C enjoys low prices while advertiser market A buys ad placement from the vendor in order to reach the C market. This has also long been an economic model supporting the free information content found in "circulation industries" (Chaudhri 1998). A larger consumer market increases the attractiveness of buying ads such that $e_{ca} > 0$, but the presence of ads may or may not increase the attractiveness of consuming content. For fashion magazines, the network externality term may be positive, $e_{ac} > 0$. But for many forms of advertising, consumers may experience clutter costs such that $e_{ac} < 0$.

In effect, these points illustrate different dimensions along which to cleave the halves of a two-sided market. These dimensions include tangibility, time, skill level, and content. As noted in Table 1, the force of two-sided network effects can also extend to near zero marginal cost services such as credit cards and auctions. Future research will likely enumerate various platforms across which providers meet consumers, and transactions volume is governed by tipping the platform to favor one or the other.

5. Conclusions

The model presented here argues for several simple and intuitive results. First, a firm can rationally invest in a product it intends to give away into perpetuity even in the absence of competition. The reason is that increased demand in a complementary premium-goods market more than covers the cost of investment in the free-goods market. In this case, market complementarity arises from an internetwork externality. This strategy also takes advantage of information's near zero marginal cost property as it allows a firm to subsidize an arbitrarily large market at a modest fixed cost.

Second, we identified distinct markets for content providers and end consumers and showed that either market can be a candidate for discounting or free distribution. Deciding which market to subsidize depends on the relative network externality benefits. At a high level of externality benefit, the market that contributes more to demand for its complement is the market to provide with a free good. At lower levels, firms may charge positive prices in both markets but keep one price artificially low.

Third, we show that, in general, consumer welfare is not harmed when firms set prices across markets

with positive complementarities. Firms can manipulate total market size through choice of price in each market. Consumers then benefit to the extent that a self-interested firm sets prices more efficiently but cannot capture all consumer surplus from the efficiency gains. A firm internalizes the market externality but wins only a fraction of this effect.

The modeling contribution is distinct from tying or second-degree price discrimination in the sense that consumers need never purchase both goods. Unlike razors and blades, the products are stand-alone goods. It also differs from multimarket or third-degree price discrimination in the sense that the firm may extract no consumer surplus from one of the two market segments, implying that this market would have traditionally gone unserved.

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