Ann Oper Res
DOI 10.1007/s10479-015-2043-x

Two-stage security screening strategies in the face of strategic applicants, congestions and screening errors

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Abstract In a security screening system, a tighter screening policy not only increases the security level, but also causes congestion for normal people, which may deter their use and decrease the approver's payoff. Adapting to the screening policies, adversary and normal applicants choose whether to enter the screening system. Security managers could use screening policies to deter adversary applicants, but could also lose the benefits of admitting normal applicants when they are deterred, which generates a tradeoff. This paper analyzes the optimal screening policies in an imperfect two-stage screening system with potential screening errors at each stage, balancing security and congestion in the face of strategic normal and adversary applicants. We provide the optimal levels of screening strategies for the approver and the best-response application strategies for each type of applicant. This paper integrates game theory and queueing theory to study the optimal two-stage policies under discriminatory and non-discriminatory screening policies. We extend the basic model to the optimal allocation of total service rate to the assumed two types of applicants at the second stage and find that most of the total service rate are assigned to the service rate for the assumed "Bad" applicants. This paper provides some novel policy insights which may be useful for security screening practices.

Keywords Security screening policy · Two-stage queueing network · Waiting time · Game theory · Imperfect screening

This research was partially supported by the United States National Science Foundation (NSF) under award numbers 1200899 and 1334930. This research was also partially supported by the United States Department of Homeland Security (DHS) through the National Center for Risk and Economic Analysis of Terrorism Events (CREATE) under award number 2010-ST-061-RE0001. In addition, this research is partially supported by Science Foundation of China University of Petroleum (Beijing) under award number 2462014YJRC051. However, any opinions, findings, and conclusions or recommendations in this document are those of the authors and do not necessarily reflect views of the NSF, DHS, or CREATE.

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Published online: 23 October 2015

1 Introduction

Since September 11, 2001, the issue of homeland security has received much attention, and the government has taken tighter screening measures to increase the security level. The 9/11 Commission Act requires a 100 % of scanning of US bound containers by radiation detection and non-intrusive inspection equipment at foreign ports before being loaded on a vessel (U.S. Government Printing Office 2007). The Transportation Security Administration (TSA) developed a Certified Cargo Screening Program to reach 100 % of screening cargo transported on a passenger aircraft for explosives (Transportation Security Administration 2013). Strict security screening policies could identify and deter adversary applicants, which prevents damages. On the other hand, it can also cause congestion and delays, which may discourage normal application and bring in high economic losses. For example, the U.S. General Accounting Office (2004) estimates that the average waiting time for a visa security clearance is 67 days, which could result in the loss of technology and advanced knowledge due to excessive waiting times. Cudmore and Whalley (2005) observe that the border delay due to trade liberalization through tariff reduction decreases about 30 % of imports' value. Closing two US ports in Washington D.C. for 3 days would result in a major economic loss of up to $58 billion (Gerencser and Vincent 2003). One Federal Reserve Bank of New York capital report estimates that the travel delays due to heightened airport security carry a $12 billion cost in 2003 (Cordes et al. 2006). Similarly, Schneier (2012) estimates about $10 billion per year loss due to the post-9/11 airport security procedures made by the TSA. Such security-screening-related huge economic losses motivates this research to explore "better" screening policies, which would not only minimize risks but also consider the normal passengers' welfare and strategies.

The screening processes are generally not perfect. For example, Ding et al. (1998) illustrate two types of testing errors—nonconforming items could be tested as conforming or conforming items could be tested as nonconforming. McLay et al. (2009) formulate dynamic programming, knapsack and sequential assignment problems into Markov decision processes to maximize the number of true alarms, taking the capacity and assignment constraints and passengers' perceived risk levels into consideration. Nie et al. (2009b) apply a mixed integer linear program to minimize the overall false alarm probability and maintain the overall false clear probability, incorporating passenger risk levels into different risk classes. This paper considers two types of screening errors—the adversary applicant is erroneously screened as normal or the normal applicant is erroneously screened as an adversary.

Some literatures suggest that a multi-stage system is better than a single-stage one. For example, Kobza and Jacobson (1997) demonstrate that the multiple-device system is better than the single-device one under certain error probability measures for accessing security system architectures. Poole and Passantino (2003) suggest that multiple levels of security can pre-clear low-risk passengers and provide extra scrutiny for high-risk passengers.

Table 1 summarizes the scope of coverage by previous research on the aspects of game theory, queueing theory, or multi-stage inspection on security issues and establish the contribution of the paper.

Queueing theory is the mathematical theory of waiting lines, or queues, which is constructed to study queue lengths and waiting times (Allen 1990). It has been applied in many fields, such as traffic engineering (Menasce et al. 2004), the design of factories (Schlechter 2009), and telecommunications (Telecommunication Networks Group 2013). Queueing theory has been widely applied for security congestion issues caused by inspection. For example, Zhang (2009) proposes the congestion-based staffing policy to maintain average queue length with a Markovian benchmark model, balancing with the concerns of security. Bakshi et al.

Table 1 Comparison of literature on game theory, queueing theory, and multi-stage inspection

References	Queueing theory	Game theory	Multi-stage
Zhang (2009)	√		√
Bakshi et al. (2011)	√		√
Lee and Jacobson (2011)	√		
Wang and Zhuang (2011)	√	√	
Nie et al. (2012)	√		
Azaiez and Bier (2007)		√	
Zhuang et al. (2010)		√	
Golalikhani and Zhuang (2011)		√	
Haphuriwat and Bier (2011)		√	
Cavusoglu et al. (2013)		√	√
McLay et al. (2006)			√
Feng (2007)			√
Nie et al. (2009a)			√
Zhang et al. (2011)	√		√
Nikolaev et al. (2012)			√
This paper	√	√	√

(2011) analyze the relation between the fraction of inspected containers and the average delay time based on historical data and suggest a rapid primary test scan of all containers and then a more careful secondary scan of a few previous containers that failed the primary scan. Lee and Jacobson (2011) use queueing theory to study the passenger's expected screening time under a multi-level aviation security system. Wang and Zhuang (2011) apply game theory and an M/M/1 one-stage queueing system to analyze the strategic interaction and optimal security screening policies, balancing the system congestion and security issues. Nie et al. (2012) analyze how to assign passengers with different risk classes to the selectee queueing lane with a steady-state nonlinear binary integer model in an airport screening system.

Game theory is the study of mathematical models of conflict and cooperation among decision-makers (Myerson 1997). It is mainly used in political science (Downs 1957; Hausken and Zhuang forthcoming), logic (Smith and Price 1973), biology (Ben-David et al. 1994), psychology (Colman 2003; Xu and Zhuang, forthcoming), donation (Zhuang et al. 2014; Saxton and Zhuang 2013), and economics (Aparicio and Sanchez-Soriano 2008; Agarwal and Zeephongsekul 2011). Researchers apply game theory to study security problems, such as optimization of resource allocation (Xu et al. forthcoming; Xiang and Zhuang forthcoming). For example, Azaiez and Bier (2007) apply game theory to allocate security investments in series and parallel systems with a defense budget, assuming the cost of an attack against any given component increases linearly with the amount of defensive investment in that component. Zhuang et al. (2010) model a multiple-period signaling game with incomplete information between the defender and attacker, considering secrecy and deception strategies for the defender and balancing capital and expense for defense investments. Golalikhani and Zhuang (2011) consider a continuous-level optimal assignment of defensive resources based on functional similarity or geographical proximity in an attacker–defender game. Haphuriwat and Bier (2011) develop a game-theoretic model to optimally allocate resources between target hardening and overarching protection. Cavusoglu et al. (2013) analyze the profiling vulnerability under no-profiling and two profiling setups with one and two screening devices in the face of strategic attackers for aviation screening security problem, considering total expected security cost, inspection rate of normal passengers, and attacker detection rate.

A multi-stage screening process is common in product testing, security screening and inspection especially when the first-stage screening is not perfect. For example, when foreigners apply to US visa, they are interviewed by consular officers first, and then subject to additional administrative processing. Many researchers have considered multiple-stage inspection for security problems. McLay et al. (2006) study a multilevel allocation problem in an aviation security system, where passengers with different risk levels are assigned to different risk classes considering budget and assignment constraints. Feng (2007) point out a two-device systems is better than a single-device systems in terms of both cost effectiveness and accuracy for an airport checked-baggage security screening system. Nie et al. (2009a) study the impact of joint responses of device for airport inspection security problem in terms of expected cost of misclassification. Zhang et al. (2011) propose complete inspection at the first stage and further proportional inspection for US-Canadian border crossings, balancing security and customer service with a two-stage queueing model. Nikolaev et al. (2012) introduce a multi-stage sequential passenger screening problem to obtain an optimal screening policy that maximizes the total security, where the assessed passengers' threat value is dynamic and can be updated.

To our best knowledge, no previous work integrates queueing theory and game theory on multi-stage security problems, although congestion and strategic interaction are critical in multi-stage security systems. To fill this gap, based on a one-stage model from Wang and Zhuang (2011), this paper will analyze service allocation in a more realistic two-stage imperfect screening system on security and optimization problems to provide some screening policy insights for security decision makers.

The rest of this paper is structured as follows: Sect. 2 introduces a two-stage screening model. Section 3 shows the applicants' best responses with a numerical illustration in screening probabilities and the approver's optimal strategy; Sect. 4 provides some numerical sensitivity analysis as well as some extensions. Section 5 concludes this paper and provides some future research directions. The Appendix provides proofs of the propositions as well as additional numerical illustrations.

2 The model

This section introduces the basic model and preliminary results using the discriminatory and non-discriminatory policies in a two-stage game-theoretic model. In particular, Sect. 2.1 describes the screening system process; Sect. 2.2 introduces the notation and game tree with the strategic approver's and applicants' payoffs; Sects. 2.3–2.4 provides the optimization problems for the potential normal, adversary applicants and the approver.

2.1 System process

In the screening system, we consider that an approver (authority, manager, or screener) assigns screening policies to potential applicants based on the applicants' observable attributes, which are classified as normal (good) and adversary (bad) applicants. His purpose is to deter adversary applicants and at the same admit normal applicants. The potential applicants decide whether to submit their applications to the screening system based on the observable screen-

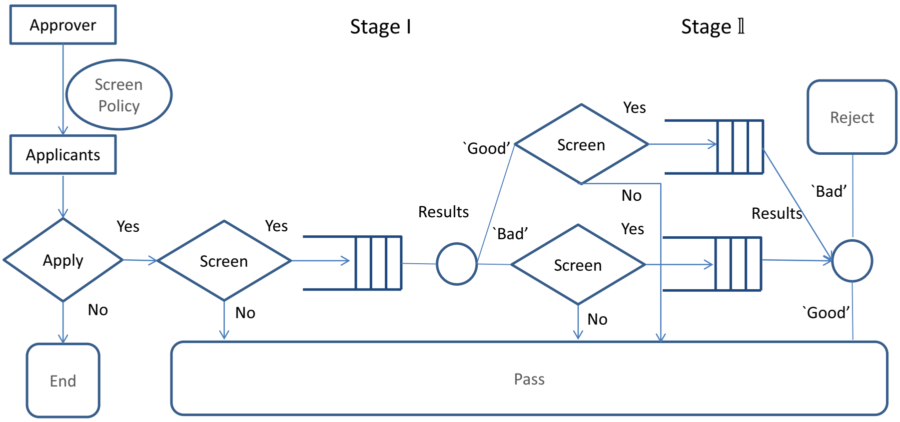

Fig. 1 A two-stage approval process with strategic applicants and congestion

ing policies and waiting line information. Figure 1 shows the screening process under an imperfect two-stage system.

There is a screening line at each stage where applicants are potentially checked. If good or bad applicants do not apply, the process ends. If applicants submit an application, the approver chooses whether to screen them at each stage. If the applicants are not selected to be screened, they will automatically pass through. We assume that there are screening errors at each stage (Wang and Zhuang 2011) due to the imperfect screening system functionality or the bad applicants' deception. Once normal and adversary applicants are checked at the first stage, they could be erroneously labeled as either 'Good' or 'Bad' with corresponding screening error probabilities. Based on the checking results at the first stage, the approver will determine whether to screen applicants at the second stage with corresponding screening probabilities and service rates in two screening lines. We assume that the second stage screening/service rate for applicants screened as 'Good' is larger than the one for applicants screened as 'Bad'. The applicants will be determined to be rejected or passed based on the screening results at the second stage.

2.2 Notation

Table 2 lists the notation that is used throughout this paper, including five decision variables (screening probability at the first stage Φ_1, probability of screening 'Good' applicants at the second stage Φ_{2G}, probability of screening 'Bad' applicants at the second stage Φ_{2B}, normal and adversary applicant's submission probabilities P_G and P_B, respectively), six utility functions (approver's objective function $J(\mathbf{\Phi}, P_B, P_G)$, normal applicants' objective function $u_G(\mathbf{\Phi}, P_G, P_B)$, adversary applicants' objective function $u_B(\mathbf{\Phi}, P_B)$, and expected waiting time W), and 22 parameters (approver's reward for admitting each normal applicant R, approver's penalty for admitting each adversary applicant C, normal and adversary applicants' rewards if passed r_G and r_B, respectively, adversary applicant's penalty if rejected c_B, waiting cost per unit time for normal applicants c_W, arrival rate of all potential normal and adversary applicants Λ_G and Λ_B, actual arrival rate of normal and adversary applicants $\lambda_G = \Lambda_G P_G$ and $\lambda_B = \Lambda_B P_B$, the maximum arrival rate of screened normal applicants $\hat{\Lambda}_G$, the first stage screening/service rates μ_1, the second stage screening/ service rate for applicants screened as 'Good' or 'Bad' μ_{2G} and μ_{2B}, respectively, the first stage available

Table 2 Notations and explanations for the two-stage model

Decision variables	
$\Phi_1 \in [0, 1]$	Screening probability at the first stage
$\Phi_{2G}, \Phi_{2B} \in [0, 1]$	Probability of screening 'Good' or 'Bad' applicants at the second stage
$\mathbf{\Phi} = (\Phi_1, \Phi_{2G}, \Phi_{2B})$	Vector for approver's screening strategy
$P_{G,B} \in [0, 1]$	Potential normal and adversary applicant's submission probability
$P_G(\mathbf{\Phi}), P_B(\mathbf{\Phi})$	Potential applicant's best response for given $\mathbf{\Phi}$
Utility functions	
$J(\mathbf{\Phi}, P_B, P_G)$	Approver's objective function
$u_G(\mathbf{\Phi}, P_B, P_G)$	Normal applicants' objective function
$u_B(\mathbf{\Phi}, P_B)$	Adversary applicants' objective function
$W_1(\mathbf{\Phi}, P(\mathbf{\Phi}))$	Expected waiting time at the first stage
$W_{2G}(\mathbf{\Phi}, P(\mathbf{\Phi}))$	Expected waiting time at the second stage once screened as 'Good'
$W_{2B}(\mathbf{\Phi}, P(\mathbf{\Phi}))$	Expected waiting time at the second stage once screened as 'Bad'
Parameters	
R	Approver's reward for admitting each normal applicant
C	Approver's penalty for admitting each adversary applicant
r_G, r_B	Normal and adversary applicants' reward if passed, respectively
c_G, c_B	Normal and adversary applicant's penalty if rejected, respectively
c_W	Waiting cost per unit time for normal applicants
Λ_G, Λ_B	Arrival rate of all potential normal and adversary applicants
λ_G, λ_B	Actual arrival rate of normal and adversary applicants
$\hat{\Lambda}_G$	The maximum arrival rate of screened normal applicants
μ_1	The first stage screening/service rate
μ_{2G}	The second stage screening/service rate for applicants screened as 'Good'
μ_{2B}	The second stage screening/service rate for applicants screened as 'Bad'
μ'_1	The first stage available service rate for normal applicants
μ'_{2G}, μ'_{2B}	The second stage available service rate for normal applicants screened as 'Good' or 'Bad', respectively
P_G^+	The first stage upper bound for normal application probability
P_{GG}^{++}, P_{GB}^{++}	The second stage upper bound for normal application probability once normal applicants screened as 'Good' or 'Bad', respectively
$e_{ib} \in [0, 1], i = 1, 2$	Probability that adversary applicant screened as 'Good' at stage $i = 1, 2$
$e_{ig} \in [0, 1], i = 1, 2$	Probability that normal applicant screened as 'Bad' at stage $i = 1, 2$
$r \in [0, 1]$	Power function coefficient

available service rates for normal applicants μ'_1, the second stage available service rate for normal applicants screened as 'Good' or 'Bad' μ'_{2G} and μ'_{2B}, respectively, the first stage upper bound for normal application probability P_G^+, The second stage upper bound for normal application probability once normal applicants screened as 'Good' or 'Bad' P_{GG}^{++} and P_{GB}^{++}, respectively, the probabilities that adversary applicant screened as normal e_{ib}, the probabilities that normal applicant screened as adversary e_{ig} at stage $i = 1, 2$, respectively, and the power function coefficient r).

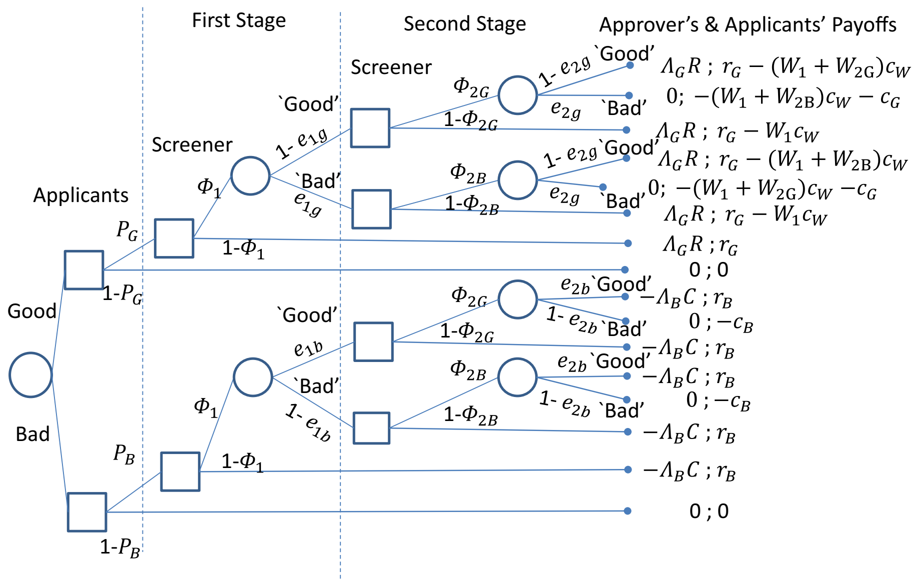

Fig. 2 Game tree and strategic approver's and applicants' payoffs in the two-stage model

Figure 2 shows the game tree of the screening system. At the beginning, the proportion of normal applicants who are *Good* and adversary applicants who are *Bad* decide the application probabilities P_G and P_B, respectively, based on the public screening policies determined by the approver. Based on the screening probability Φ_1 at the first stage, each applicant has the chance to get screened. The screening system is imperfect, and we model such screening errors using probabilities that the adversary applicant is incorrectly screened as 'Good' e_{ib}, and probabilities that the normal applicant is incorrectly screened as 'Bad' e_{ig} at stage $i = 1, 2$, respectively. The screened applicants are determined to be checked again based on the second-stage screening policies including the screening probabilities for 'Bad' ones Φ_{2B} and 'Good' ones Φ_{2G}.

The right side of Fig. 2 shows the approver's and applicants' payoffs. For the approver, without admitting any applicants, the approver's utility is 0. Once he passes one normal applicant as *Good*, he could get a reward R based on the normal application probability P_G, considering the potential normal arrival rate Λ_G. The approver passes normal applicants under one of the following five scenarios: go through both stages of screening and be screened as 'Good' at each stage with probability $\Phi_1(1-e_{1g})\Phi_{2G}(1-e_{2g})$, get screened as 'Good' at the first stage and be not screened at the second stage with probability $\Phi_1(1-e_{1g})(1-\Phi_{2G})$, incorrectly screened as 'Bad' at the first stage and screened as 'Good' at the second stage with probability $\Phi_1 e_{1g}\Phi_{2B}(1-e_{2g})$, incorrectly screened as 'Bad' at the first stage and not screened at the second stage with probability $\Phi_1 e_{1g}(1-\Phi_{2B})$, and not screened at all with probability $1-\Phi_1$.

On the other hand, once the approver passes an adversary applicant as *Bad*, he would receive a penalty C based on the adversary application probability P_B, while considering the potential adversary applicants arrival rate Λ_B. The approver passes each adversary applicant under one of the following five scenarios: go through both stages of screening and be incorrectly screened as 'Good' at each stage with probability $\Phi_1 e_{1b}\Phi_{2G}e_{2b}$, incorrectly screened as 'Good' at the first stage and not screened at the second stage with probability

$\Phi_1 e_{1b}(1-\Phi_{2G})$, screened as 'Bad' at the first stage but incorrectly screened as 'Good' at the second stage with probability $\Phi_1(1-e_{1b})\Phi_{2B}e_{2b}$, screened as 'Bad' at the first stage and not screened at the second stage with probability $\Phi_1(1-e_{1b})(1-\Phi_{2B})$, and not screened at all with probability $1-\Phi_1$.

If normal and adversary applicants do not apply to the system their utilities are 0 with probabilities $1-P_G$ and $1-P_B$, respectively. For the normal applicants as $Good$, once they are admitted, they will receive a reward r_G, otherwise they will receive a loss c_G. We assume that the normal applicants' application decision is affected by waiting cost, which equals the unit waiting cost c_W times the expected waiting time (W_1, W_{2G} and W_{2B} for stages 1 and 2, respectively) times the corresponding screening probability (Φ_1 for W_1, $\Phi_{2G}(1-e_{1g})$ for W_{2G}, and $\Phi_{2B}e_{1g}$ for W_{2B}). The normal applicants are screened at the second stage under one of the two following scenarios: screened as 'Good' at the first stage and also screened at the second stage with probability $\Phi_1(1-e_{1g})\Phi_{2G}$, and incorrectly screened as 'Bad' at the first stage and then screened at the second stage with probability $\Phi_1 e_{1g}\Phi_{2B}$. For the adversary applicants as Bad, if they are caught, they will receive a penalty c_B, otherwise they will receive a reward r_B. The adversary applicants can not pass the system under one of the two following scenarios: incorrectly screened as 'Good' at the first stage and screened as 'Bad' at the second stage with probability $\Phi_1 e_{1b}\Phi_{2G}(1-e_{2b})$, and screened as 'Bad' at both stages with probability $\Phi_1(1-e_{1b})\Phi_{2B}(1-e_{2b})$.

2.3 Adversary applicants' optimization problems

The applicants' expected utility payoffs are the summation of the weighted payoffs of the second column of the right side of Fig. 2. The adversary applicants choose the application probability P_B to maximize his expected utility payoff, which is shown in Eq. (1). We assume that the adversary applicants are patient and do not consider the waiting cost. We define the probability that adversary applicants are caught $\Phi^B \equiv \Phi_1\Phi_2^B(1-e_{2b})$. Particularly, the total probability of the adversary being caught across two stages equals the product of the screening probability at the first stage Φ_1, times the expected screening probability at the second stage $\Phi_2^B \equiv e_{1b}\Phi_{2G} + (1-e_{1b})\Phi_{2B}$, times the screening non-error probability at the second stage $1-e_{2b}$.

$$
\begin{aligned}
\max_{P_B} u_B(\mathbf{\Phi}, P_B) &= P_B r_B\Big(\Phi_1 e_{1b}\Phi_{2G}e_{2b} + \Phi_1 e_{1b}(1-\Phi_{2G}) \\
&\quad + \Phi_1(1-e_{1b})\Phi_{2B}e_{2b} + \Phi_1(1-e_{1b})(1-\Phi_{2B}) + (1-\Phi_1\Big) \\
&\quad - P_B c_B\Big(\Phi_1 e_{1b}\Phi_{2G}(1-e_{2b}) + \Phi_1(1-e_{1b})\Phi_{2B}(1-e_{2b})\Big) \\
&= P_B\Big(\underbrace{r_B(1-\Phi^B)}_{\text{Expected Reward}} - \underbrace{c_B\Phi^B}_{\text{Expected Penalty}}\Big)
\end{aligned} \tag{1}
$$

As shown in Eq. (1) above, the expected reward for adversary applicants by passing the system equals the reward r_B times the expected probability $1-\Phi^B$. The adversary are screened at the second stage under the scenarios that adversary applicants are erroneously labeled as 'Good' at the first stage but screened with probability $e_{1b}\Phi_{2G}$ and adversary applicants are correctly labeled as 'Bad' at the first stage but screened with probability $(1-e_{1b})\Phi_{2B}$. The expected penalty for adversary applicants by being caught equals the penalty for passing the system c_B times the expected probability Φ^B.

2.4 Normal applicants' optimization problems

The normal applicant chooses the application probability P_G to maximize his expected utility payoff, which is shown in Eq. (2). The normal applicants are affected by the waiting cost at each stage. Specifically, normal applicant's utility equals the application probability P_G times the net of the expected reward minus the expected waiting cost and the rejected loss. The expected reward equals the reward for passing the system r_G times the overall probability that normal applicants pass the system $1 - \Phi_1 \Phi_2^G e_{2g}$. Particularly, the total probability that normal applicants are caught across two stages equals the product of the screening probability at the first stage Φ_1, times the screening probability at the second stage $\Phi_2^G \equiv (1-e_{1g})\Phi_{2G} + e_{1g}\Phi_{2B}$, times the screening error probability at the second stage e_{2g}. Normal applicants are screened at the second stage under the scenarios that normal applicants are correctly labeled as normal but screened with probability $\Phi_{2G}(1-e_{1g})$ and normal applicants are erroneously labeled as adversary but screened with probability $\Phi_{2B}e_{1g}$. The expected waiting cost equals unit waiting time cost c_W times the product of the screening probability at the first stage Φ_1, and times the total waiting time across two stages $W_1 + W_{2G}(1 - e_{1g})\Phi_{2G} + W_{2B}e_{1g}\Phi_{2B}$. The total waiting time equals the summation of the expected waiting time at the first stage W_1 and the product of the expected waiting time at the second stage W_{2G} and W_{2B} with the corresponding screening probabilities at the second stage $(1 - e_{1g})\Phi_{2G}$ and $e_{1g}\Phi_{2B}$, respectively. The expected rejected loss equals the rejected loss c_G times the screening probability at the first stage Φ_1, the screening probability at the second stage Φ_2^G, and the screening error probability at the second stage e_{2g}.

$$
\begin{aligned}
\max_{P_G} u_G(\boldsymbol{\Phi}, P_G, P_B) = {} & P_G r_G \Big(\Phi_1(1-e_{1g})\Phi_{2G}(1-e_{2g}) + \Phi_1 e_{1g}(1-\Phi_{2G}) \\
& + \Phi_1 e_{1g}\Phi_{2B}(1-e_{2g}) + \Phi_1 e_{1g}(1-\Phi_{2B}) + 1 - \Phi_1\Big) \\
& - P_G c_W \Big(1-(1-\Phi_1)\Big)\Big(W_1(\Phi_1, P_G, P_B) \\
& + W_{2G}(\boldsymbol{\Phi}, P_G, P_B)(1-e_{1g})\Phi_{2G} + W_{2B}(\boldsymbol{\Phi}, P_G, P_B)e_{1g}\Phi_{2B}\Big) \\
& - P_G c_G \Big((1-e_{1g})\Phi_{2G} + e_{1g}\Phi_{2B}\Big)e_{2g} \\
= {} & P_G \Big(\underbrace{r_G(1-\Phi_1\Phi_2^G e_{2g})}_{\text{Expected Reward}} - \underbrace{c_W\Phi_1(W_1 + W_{2G}(1-e_{1g})\Phi_{2G} + W_{2B}e_{1g}\Phi_{2B})}_{\text{Expected Waiting Cost}} \\
& - \underbrace{c_G\Phi_1\Phi_2^G e_{2g}}_{\text{Expected Rejected Loss}} \Big)
\end{aligned}
\tag{2}
$$

Based on the $M/M/1$ queue theory, we have the expected waiting time $W = \frac{1}{\mu-\lambda}$, where μ is the service rate and λ is the arrival rate (Hines et al. 2003), the waiting time at the first stage W_1 and at the second stage W_{2G} and W_{2B} are expressed in Eqs. (3), (4), and (5) respectively. In particular, the expected waiting time at the first stage W_1 equals 1 over the net of the service rate μ_1 less the total arrival rate $\Phi_1(P_G\Lambda_G + P_B\Lambda_B)$, where the total arrival rate equals the screening probability at the first stage Φ_1 times the summation of the normal applicant's arrival rate $P_G\Lambda_G$ and the adversary arrival rate $P_B\Lambda_B$.

$$W_1(\Phi_1, P_G, P_B) = \frac{1}{\mu_1 - \Phi_1(P_G\Lambda_G + P_B\Lambda_B)} \tag{3}$$

The expected waiting time at the second stage once normal applicants are screened as 'Good' at the first stage W_{2G} equals 1 over the net of the service rate μ_{2G} less the total arrival rate, where the total arrival rate equals the screening probability at the first stage Φ_1 times the screening probability on 'Good' at the second stage Φ_{2G} times the summation of the normal applicants' arrival rate $(1 - e_{1g})P_G\Lambda_G$ and the adversary applicants arrival rate $e_{1b}P_B\Lambda_B$.

$$W_{2G}(\mathbf{\Phi}, P_G, P_B) = \frac{1}{\mu_{2G} - \Phi_1\Phi_{2G}\Big((1 - e_{1g})P_G\Lambda_G + e_{1b}P_B\Lambda_B\Big)} \tag{4}$$

The expected waiting time at the second stage once normal applicants are screened as 'Bad' at the first stage W_{2B} equals 1 over the net of the service rate μ_{2B} less the total arrival rate, where the total arrival rate equals the screening probability at the first stage Φ_1 times the screening probability on 'Bad' at the second stage Φ_{2B} times the summation of the normal applicants' arrival rate $e_{1g}P_G\Lambda_G$ and the adversary applicants arrival rate $(1 - e_{1b})P_B\Lambda_B$.

$$W_{2B}(\mathbf{\Phi}, P_G, P_B) = \frac{1}{\mu_{2B} - \Phi_1\Phi_{2B}\Big(e_{1g}P_G\Lambda_G + (1 - e_{1b})P_B\Lambda_B\Big)} \tag{5}$$

2.5 Approver's optimization problem and definition of equilibrium

The approver' expected utility payoff is the summation of the weighted payoff of the first column of the right side in Fig. 2. The approver chooses the screening probabilities Φ_1, Φ_{2B} and Φ_{2G} across the two stages to maximize his expected utility, which is summarized in Eq. (6). The utility consists of the expected benefit from passing normal applicants $\Lambda_G P_G R(1 - \Phi_1\Phi_2^G e_{2g})$, and the expected penalty from passing adversary applicants $\Lambda_B P_B C(1 - \Phi^B)$. Specifically, the expected benefit from passing normal applicants equals the normal applicant's arrival rate Λ_G times the normal application probability P_G, the reward for admitting each normal applicant R, and the probability for passing the normal applicants $(1 - \Phi_1\Phi_2^G e_{2g})$. The expected penalty from passing adversary applicants equals the adversary applicant's arrival rate Λ_B times adversary application probability P_B, the penalty for admitting each adversary applicant C, and the probability that passes each adversary applicant $1 - \Phi^B$.

$$\begin{aligned}
\max_{\Phi_1,\Phi_{2G},\Phi_{2B}} J(\mathbf{\Phi}, P_G, P_B) &= \Lambda_G P_G R\Big(\Phi_1(1 - e_{1g})\Phi_{2G}(1 - e_{2g}) + \Phi_1(1 - e_{1g})(1 - \Phi_{2G}) \\
&\quad + \Phi_1 e_{1g}\Phi_{2B}(1 - e_{2g}) + \Phi_1 e_{1g}(1 - \Phi_{2B}) + 1 - \Phi_1\Big) \\
&\quad - \Lambda_B P_B C\Big(\Phi_1 e_{1b}\Phi_{2G}e_{2b} + \Phi_1 e_{1b}(1 - \Phi_{2G}) \\
&\quad + \Phi_1(1 - e_{1b})\Phi_{2B}e_{2b} + \Phi_1(1 - e_{1b})(1 - \Phi_{2B}) + 1 - \Phi_1\Big) \\
&= \underbrace{\Lambda_G P_G R(1 - \Phi_1\Phi_2^G e_{2g})}_{\text{Expected Reward}} - \underbrace{\Lambda_B P_B C(1 - \Phi^B)}_{\text{Expected Cost}}
\end{aligned} \tag{6}$$

Definition 1 We call a collection of strategies $(P_B^*, P_G^*, \mathbf{\Phi}^*)$ a subgame perfect Nash equilibrium (SPNE), or 'equilibrium', if and only if Eqs. (7), (8) and (9) are satisfied, where none of the approver and two types of applicants have the incentives to change their move.

$$P_B^* = \hat{P}_B(\mathbf{\Phi}^*) = \underset{P_B \in [0,1]}{\text{argmax}}\ u_B(\mathbf{\Phi}^*, P_B) \tag{7}$$

$$P_G^* = \hat{P}_G(\mathbf{\Phi}^*, P_B^*) = \underset{P_G \in [0,1]}{\text{argmax}}\ u_G(\mathbf{\Phi}^*, P_G, P_B^*) \tag{8}$$

$$\mathbf{\Phi^*} = \hat{\mathbf{\Phi}}\Big(\hat{P}_B(\mathbf{\Phi}), \hat{P}_G(P_B, \mathbf{\Phi})\Big) = \underset{\mathbf{\Phi} \in [0,1]}{\text{argmax}}\ J\Big(\hat{P}_B(\mathbf{\Phi}), \hat{P}_G(\hat{P}_B, \mathbf{\Phi}), \mathbf{\Phi}\Big) \tag{9}$$

3 The analyses

3.1 Adversary applicants' best response

We assume that when the adversary applicant is indifferent between applying or not, he would not apply to the system as a tie breaker. Solving the optimization problem in Eq. (1), Proposition 1 provides the best response function for adversary applicants.

Proposition 1 *Adversary potential applicants' best responses are given by:*

$$P_B(\Phi) = \begin{cases} 1 & \textit{if } \Phi^B < s_b \equiv \frac{r_B}{r_B + c_B} \\ 0 & \textit{if } \Phi^B \geq s_b \equiv \frac{r_B}{r_B + c_B}. \end{cases} \tag{10}$$

where Φ^B is the probability that adversary applicants are caught as introduced in Sect. 2.3.

Remark Proposition 1 indicates that an adversary applicant is deterred if total effective screening probabilities for adversary applicants Φ^B is higher than or equal to the threshold value s_b. The adversary potential applicants' best responses P_B increases in adversary applicant's reward r_B and error probability that adversary applicant screened as 'Good' at the second stage e_{2b}, and decreases in penalty c_B.

3.2 Normal applicants' best response

We assume that when the normal applicant is indifferent between applying and not applying, he would apply to the system as a tie breaker. Normal potential applicants' utility depends on the decision of adversary applicants through congestion, as shown in Eqs. (2), (3), (4) and (5). Note that the traffic caused by the adversary applicants' arrival rate equals the summation of $\Phi_1 P_B \Lambda_B$ at stage one and $\Phi_1 \Phi_{2G} e_{1b} P_B \Lambda_B$ and $\Phi_1 \Phi_{2B}(1 - e_{1b}) P_B \Lambda_B$ at stage two, separately. Solving Eq. (2), Proposition 2 below provides the best response function for normal applicants, using some new notations: available service rates $\mu_1' \equiv \mu_1 - \Phi_1 P_B \Lambda_B$, $\mu_{2G}' \equiv \mu_{2G} - \Phi_1 \Phi_{2G} e_{1b} P_B \Lambda_B$ and $\mu_{2B}' \equiv \mu_{2B} - \Phi_1 \Phi_{2B}(1 - e_{1b}) P_B \Lambda_B$, and upper bounds for the normal application probability at the first stage $P_G^+ \equiv \frac{\mu_1'}{\Phi_1 \Lambda_G}$, the second stage $P_{GG}^{++} \equiv \frac{\mu_{2G}'}{\Phi_1 \Phi_{2G}(1-e_{1g})\Lambda_G}$ and $P_{GB}^{++} \equiv \frac{\mu_{2B}'}{\Phi_1 \Phi_{2B} e_{1g} \Lambda_G}$, and three conditions (d_1) $u_G(\mathbf{\Phi}, P_G = 1, P_B) < 0$, (d_2) $u_G(\mathbf{\Phi}, P_G = 1, P_B) \geq 0$, $P_G^+ \in [0, 1)$ or $P_{GG}^{++} \in [0, 1)$ or $P_{GB}^{++} \in [0, 1)$, and (d_3) $u_G(\mathbf{\Phi}, P_G = 1, P_B) \geq 0$ and $P_G^+ \in [0, 1)$.

Proposition 2 *Given $\mathbf{\Phi}$ and $P_B(\mathbf{\Phi})$, the normal potential applicant's best response $P_G(\mathbf{\Phi}, P_B)$ is given by:*

(i) *If the approver does not screen at the first stage; i.e., $\Phi_1 = 0$, then $P_G = 1$.*
(ii) *If the approver screens at both stages; i.e., $\Phi_1 \in (0, 1]$, $\Phi_2^G \in (0, 1]$, then*

(a) If $\mu_1' \le 0$ or $\mu_{2G}' \le 0$ or $\mu_{2B}' \le 0$, then $P_G = 0$.

(b) If $\mu_1' > 0$, $\mu_{2G}' > 0$, $\mu_{2B}' > 0$, neither (d_1) nor (d_2), then $P_G = 1$.

(c) If $\mu_1' > 0$, $\mu_{2G}' > 0$, $\mu_{2B}' > 0$, either (d_1) or (d_2) or both, then

$$P_G = \max\left(\min\left(\frac{\hat{\Lambda}_G}{\Phi_1\Phi_{2G}(1-e_{1g})\Lambda_G}, \frac{\hat{\Lambda}_G}{\Phi_1\Phi_{2B}e_{1g}\Lambda_G}, 1, P_G^+, P_{GG}^{++}, P_{GB}^{++}\right), 0\right) \in [0, 1).$$

where the maximum arrival rate for normal applicants is

$$\hat{\Lambda}_G = -\frac{b'}{3a'} + \sqrt[3]{\frac{b'c'}{6a'^2} - \frac{b'^3}{27a'^3} - \frac{d'}{2a'} + \sqrt{\left(\frac{b'c'}{6a'^2} - \frac{b'^3}{27a'^3} - \frac{d'}{2a'}\right)^2 + \left(\frac{c'}{3a'} - \frac{b'^2}{9a'^2}\right)^3}}$$
$$+ \sqrt[3]{\frac{b'c'}{6a'^2} - \frac{b'^3}{27a'^3} - \frac{d'}{2a'} - \sqrt{\left(\frac{b'c'}{6a'^2} - \frac{b'^3}{27a'^3} - \frac{d'}{2a'}\right)^2 + \left(\frac{c'}{3a'} - \frac{b'^2}{9a'^2}\right)^3}} \quad (a' \neq 0)$$

or $\hat{\Lambda}_G = \frac{-c' \pm \sqrt{c'^2 - 4b'd'}}{2b'}$ $(a' = 0 \& b' \neq 0)$; or $\hat{\Lambda}_G = -\frac{d'}{c'}$ $(a' = 0 \& b' = 0 \& c' \neq 0)$.
where $a \equiv \Phi_{2G}(1-e_{1g})$, $b \equiv \Phi_{2B}e_{1g}$, $c \equiv \frac{r_G - (r_G + c_G)\Phi_1\Phi_2^G e_{2g}}{\Phi_1 c_W}$, $a' = abc$, $b' = (3ab - \mu_1' abc - \mu_{2G}' bc - \mu_{2B}' ac$, $c' = \mu_1'\mu_{2G}' bc + \mu_1'\mu_{2B}' ac + \mu_{2G}'\mu_{2B}' c - 2a\mu_{2B}' - 2b\mu_{2G}' - 2ab\mu_1'$, $d' = \mu_{2G}'\mu_{2B}' + a\mu_1'\mu_{2B}' + b\mu_1'\mu_{2G}' - \mu_1'\mu_{2G}'\mu_{2B}' c$.

(iii) *If the approver screens at the first stage $\Phi_1 \in (0, 1]$ but does not screen at the second stage $\Phi_2^G = 0$, then*

(a) If $\mu_1' \le 0$, then $P_G = 0$.

(b) If $\mu_1' > 0$, neither (d_1) nor (d_3), then $P_G = 1$.

(c) If $\mu_1' > 0$, either (d_1) or (d_3) or both, then $P_G = \frac{\lambda_G}{\Phi_1\Lambda_G} = \max\left(\min\left(\frac{\hat{\Lambda}_G'}{\Phi_1\Lambda_G}, 1, P_G^+\right), 0\right) \in [0, 1)$, where $\hat{\Lambda}_G' = \mu_1 - \Phi_1 P_B \Lambda_B - \frac{c_W\Phi_1}{r_G - (r_G + c_G)\Phi_1 e_{1g}}$.

Remark Proposition 2(i) shows that normal applicants would apply to the system if there is no screening. Propositions 2(ii)(a) and (iii)(a) show that normal applicants would not apply to the system if there is no available service rate. Propositions 2(ii)(b) and (iii)(b) show that normal applicants would apply to the system if there are positive service rates and payoffs. Propositions 2(ii)(c) and (iii)(c) show that the interior normal potential applicants' submission probability P_G decreases in screening probabilities Φ_1, Φ_{2G}, Φ_{2B}. In addition, it shows that the normal potential applicants' submission probability P_G increases in the service rates at each stage μ_1, μ_{2G}, μ_{2B}, increases in the normal applicants' reward if passed r_G, and decreases in unit waiting cost c_W, error probability that normal applicant screened as 'Bad' at the second stage e_{2g}, and normal applicant's potential arrival rate Λ_G.

3.3 Numerical illustration for applicants' best response

Following Cavusoglu et al. (2013), we use a power function $1 - e_{ib} = e_{ig}^r$ to study the correlation between two screening errors. We refer the false alarm and false clear data from Aguirre et al. (2012) to let the error probabilities $e_{1g} = e_{2g} = 0.125$ and $r = 0.0247$, then we have $e_{1b} = e_{2b} = 0.05$. Some parameters' values are based on Wang and Zhuang (2011): $R = 1$ and $c_B = 1$; with other revisions as $c_G = 0$; $r_G = 10$; $c_W = 50$; $r_B = 1$; $\lambda_G = 120$; $\lambda_B = 10$; $\mu_1 = 50$; $\mu_{2G} = 40$; $\mu_{2B} = 30$; and $C = 10$.

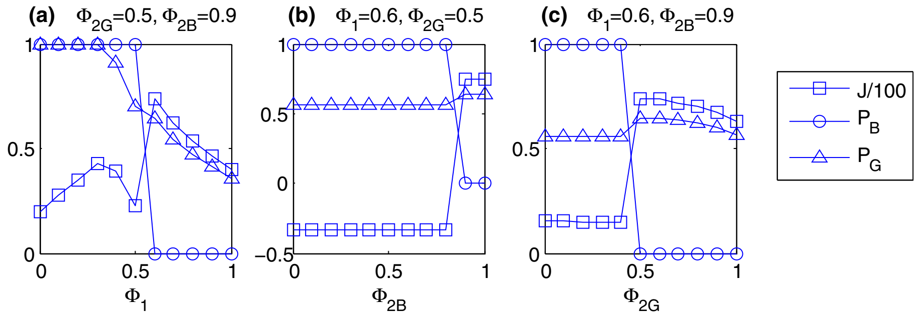

Fig. 3 The best response of adversary and normal applicants and approver's payoffs as a function of three individual screening probabilities

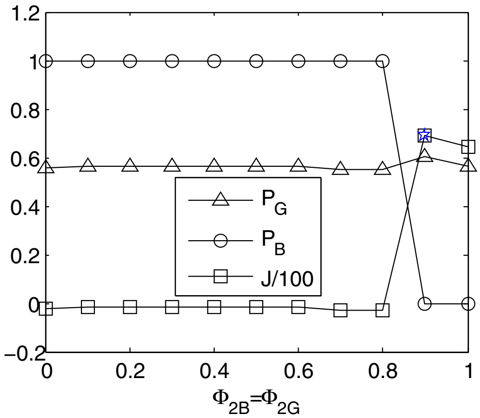

Fig. 4 Illustration of applicants' best response and approver's payoff affected by the non-discriminatory policy when $\Phi_1 = 0.6$

Figure 3 illustrates the best response of applicants to the individual screening probability. Figures 3a–c show the adversary application probability P_B is 1 if Φ_1, Φ_{2B} and Φ_{2G} are sufficiently small (such that $\Phi^B < \frac{r_B}{r_B + c_B}$ as predicted by Proposition 1) and zero otherwise. Figure 3a shows that the normal application probability P_G is 1 when Φ_1 is small. Figure 3a–c show the normal application probability P_G first decreases in Φ_1, Φ_{2B} and Φ_{2G}, respectively due to congestion, and then increases based on reduced congestion by adversary applicants when the adversary application is deterred (i.e. $P_B = 0$), and finally decreases again in Φ_1, Φ_{2B}, and Φ_{2G} due to additional congestion. The approver's payoff J is affected by the individual screening probability that affects applicants' application probabilities. The approver's payoff J first increases in Φ_1 due to more captures of adversary applicants, and then increases suddenly with the deterrence of adversary applicants, and finally decreases when the normal application probability decreases.

Figure 4 demonstrates that the potential applicants' best response and approver's payoff under a non-discriminatory policy, where the screening probabilities at the second stage corresponding to each type of applicant are equal, $\Phi_{2G} = \Phi_{2B}$. The approver's payoff J decreases with decreasing normal application probability P_G based on increasing screening probability at the second stage. When the adversary application probability P_B is deterred due to high screening probability, the approver's payoff J suddenly increases with an increased normal application probability P_G.

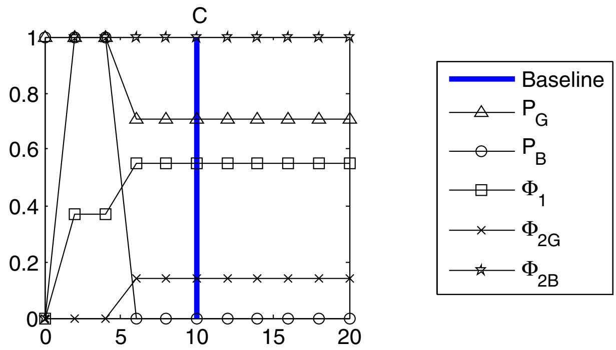

Fig. 5 Numerical sensitivity analysis under discriminatory policy for approver's penalty

3.4 Approver's best strategy

Following Wang and Zhuang (2011), for simplicity, we assume that when the approver is indifferent between different levels of screening probabilities, she will choose the lowest level. Solving Eq. (6), Proposition 3 below provides the optimal screening strategy for the approver.

Proposition 3 *The optimal approver's utility is given by:*

$$J(\mathbf{\Phi}) = \begin{cases} J_1(\mathbf{\Phi}), & \text{if } \Phi^B \geq s_b \\ J_2(\mathbf{\Phi}), & \text{if } \Phi^B \in [0, s_b). \end{cases} \tag{11}$$

where $J_1(\mathbf{\Phi}) = R(1 - \Phi_1\Phi_2^G e_{2g})P_G(\mathbf{\Phi}, P_B = 0)$, *and* $J_2(\mathbf{\Phi}) = R(1 - \Phi_1\Phi_2^G e_{2g})P_G(\mathbf{\Phi}, P_B = 1) - C(1 - \Phi_1\Phi_2^B(1 - e_{2b}))$

The optimal strategy for the approver is to solve:

$$J^* = \max_{\mathbf{\Phi}:\, 0 \leq \Phi^B \leq s_b} J(\mathbf{\Phi}) \tag{12}$$

Remark Equation (12) truncates the range of screening probability $\Phi_1\Phi_2^B$ from the range $[0, 1]$ to the range $[0, s_b]$ for maximizing the approver's utility, which provides an efficient way for computation.

4 Numerical illustrations and extensions

4.1 Numerical sensitivity analyses under discriminatory policy

Figure 5 illustrates the numerical sensitivity analysis under a discriminatory policy for penalty parameter C. It shows that as the approver's penalty of passing an adversary applicant C increases, the approver increases the screening probabilities Φ_1, Φ_{2G}, and Φ_{2B} to deter adversary applicants P_B, which decreases normal application probability P_G. Other results of numerical sensitivity analyses are shown in Appendix 4.

Figure 6 compares the model results between the non-discriminatory policy and the discriminatory policy: adversary application probabilities (P_{BN} and P_{BD}, respectively), normal application probabilities (P_{GN} and P_{GD}, respectively), and approver's payoffs (J_N and J_D,

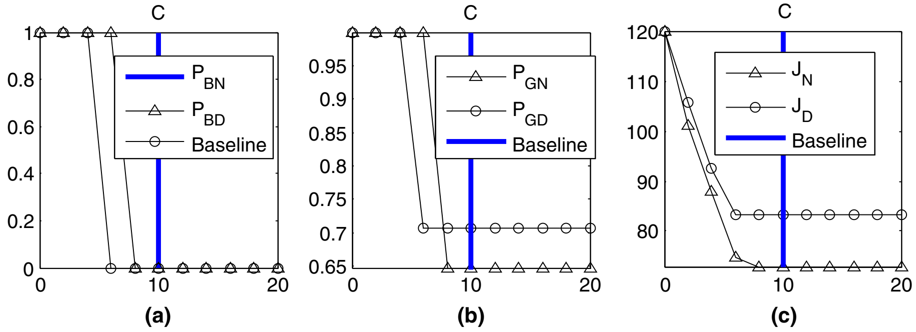

Fig. 6 Comparisons between the non-discriminatory and the discriminatory policies

respectively). Other results are shown in Appendix 5. In particular, Fig. 6a shows that the adversary application probability under a discriminatory policy is significantly lower than that under a non-discriminatory policy when C is low. Figure 6b shows that the normal application probability under a discriminatory policy is significantly higher than the one under a non-discriminatory policy when C is high. Figure 6c shows that the approver's payoffs under a discriminatory policy is significantly higher than the one under a non-discriminatory policy when C is high.

4.2 Extension: the allocation of total service rate in two-stage analysis

In this section, we extend our model to consider a new parameter: the total service rate μ, where the different service rates μ_{2G} and μ_{2B} at the second stage for the assumed 'Good' and 'Bad' applicants, respectively are new variables, which are constrained by μ. The total service rate is analyzed in terms of the optimal allocation to each type of assumed 'Good' and 'Bad' applicants at the second stage in order to maximize the approver's payoff. Based on Eq. (6), the approver's objective function can be rewritten as below:

$$\max_{\mathbf{\Phi},\mu_{2G},\mu_{2B}} J(\mathbf{\Phi}, \mu_{2G}, \mu_{2B}) \tag{13}$$

Equation (14) shows the summation of the service rates at the second stage meets a certain level (threshold) the total service rate μ.

$$\mu_{2G} + \mu_{2B} \leq \mu \tag{14}$$

Based on Propositions 1 and 2, we numerically find the optimal values of the service rates μ_{2G} and μ_{2B}. Figure 7 illustrates how to optimally allocate the total service resource μ at the second stage for assumed 'Good' and 'Bad' applicants under the discriminatory policy. Figure 7a shows that the service rate at the second stage for the assumed 'Good' applicants μ_{2G} increases dramatically in the total service rate μ. Figure 7b details the percentages of allocation of the total service resource μ to the assumed each type of applicants. It shows that the service rate at the second stage for the assumed 'Good' applicants takes at least 80 % of the total service rate.

4.3 Comparisons for one versus two-stage screening systems

This section introduces and compares one versus two-stage screening systems to find the best screening policy for the approver under certain situations. The utility for a one-stage

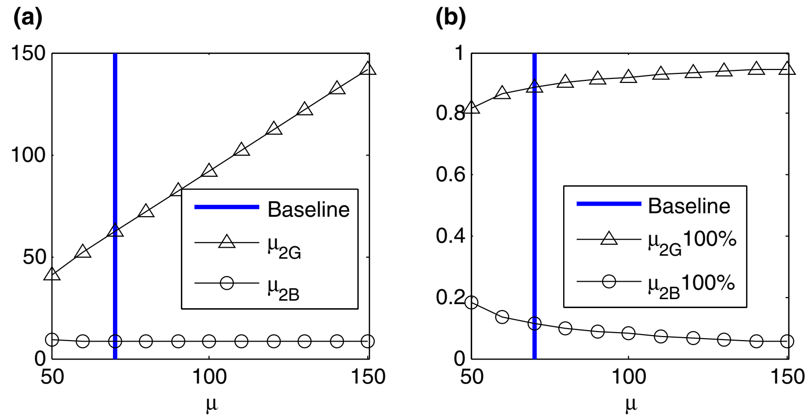

Fig. 7 The allocation of total service rate analysis in two-stage analysis. **a** Absolute values, **b** percentages

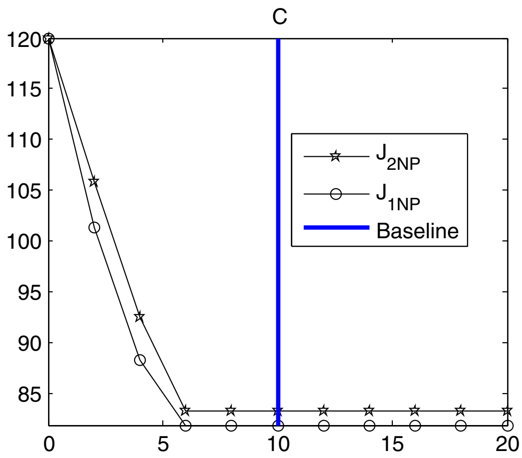

Fig. 8 Comparisons of approver's utilities in one-stage and two-stage systems

imperfect screening system is J_{1NP}. The utility for a two-stage imperfect screening system is J_{2NP}. Figure 8 compares the approver's utilities in one- and two-stage screening systems, which shows that the approver's payoff in a two-stage screening system J_{2NP} is significantly larger than the one in a one-stage system J_{1NP} when the penalty for approver once admitting each adversary applicant C is intermediate. Other results are shown in Appendix 6.

5 Conclusion and future research directions

Detecting and deterring adversary applicants during the security screening process is important but challenging for the approver, which could be affected by screening errors and service rates. Furthermore, appropriate screening policies are needed to control congestion and maintain safety for the welfare of normal applicants. In this paper, we model a security screening process in an imperfect and two-stage sequential game, where the approver, as the leader, determines the optimal screening strategies at each stage, and the normal and adversary applicants respond with whether to apply or not. We provide analytical solutions and numerical illustrations for the applicants' best responses and compare the equilibrium strategies for the approver and applicants between discriminatory and non-discriminatory screening policies. It shows that from the economic benefits perspective, the performance in discriminatory screening policy is better. This gives some insights for policy makers to determine which policy to apply in security screening context.

The adversary potential applicant's best response increases in reward and error probability that adversary applicant screened as "Good" at the second stage, and decreases in penalty. The normal potential applicant's submission probability increases in parameters that service rate at each stage, and reward if passed, and decreases in unit waiting cost, error probability that normal applicant screened as "Bad" at the second stage, and normal potential arrival rate. Security screening managers can thus better deter adversary applicants and attract normal applicants based on these insights.

The model is extended to analyze the optimal allocation of total service rates to assumed two types of applicants at the second stage. It demonstrates that when the total service rate is high, at least 80 % of the resources are assigned to the service rate for the assumed 'Good' applicants at the second stage. This result could be useful for decision makers to appropriately allocate limited resources among different targets and stages.

Finally, we compare two different screening systems and find that the two-stage screening system is significantly better than the one-stage system, especially when the adversary applicants' reward if passed, the service rate at the first stage, and the loss once normal applicants are rejected are high, or when the error probability that normal applicants are screened as "Bad" at the first stage, the penalty for approver once admitting each adversary applicant, the adversary applicant arrival rate, and error probabilities that normal applicants are screened as 'Bad' at the first and second stage are intermediate, or when the power function coefficient, the cost to adversary applicants being caught, the unit time cost, and the error probability that normal applicant screened as 'Bad' at the second stage are low. This means that in certain situations as long as the screening system is not perfect, no matters the screening system is in airport, subway stations, or borders, two-stage screening system would be more useful.

In the future, we could extend the model to a parallel queueing network, so that the applicants could be assigned into different lines with different service rates. We can classify applicants into more than two classes with different risk levels. In addition, with the real data, we could study dynamic screening policies and applicants' information for multi-period game analysis.

Appendix

Appendix 1: Proof of Proposition 1

The adversary potential applicant's application rate P_B depends on his utility function u_B, once the utility payoff is less than equal to 0, adversary applicants would have no interest in applying this system. His utility is $u_B = P_B \Lambda_B \left(r_B(1-\Phi^B) - c_B \Phi^B \right)$. The revised screening probability across the two stage is $\Phi^B \geq s_b \equiv \frac{r_B}{r_B + c_B}$, which is derived from $u_B \leq 0$, results in $P_B = 0$. The adversary potential applicants' best response P_B has the opposite change direction with Φ^B, which decreases in e_{2b}, where $\frac{\partial \Phi^B}{\partial e_{2b}} = -\Phi_1 \Phi_2^B \leq 0$. Therefore, adversary potential applicants' best response P_B increases in r_B and e_{2b}, and decreases in c_B.

Appendix 2: Proof of Proposition 2

We define $\mu_1' \equiv \mu_1 - \Phi_1 P_B \Lambda_B$, $\mu_{2G}' \equiv \mu_{2G} - \Phi_1 \Phi_{2G} e_{1b} P_B \Lambda_B$ and $\mu_{2G}' \equiv \mu_{2B} - \Phi_1 \Phi_{2B}(1-e_{1b}) P_B \Lambda_B$. There are upper bounds for the normal application probability at the first stage P_G^+ and at the second stage P_{GG}^{++} and P_{GB}^{++}, respectively, where $P_G^+ \equiv \frac{\mu_1'}{\Phi_1 \Lambda_G}$, and $P_{GG}^{++} \equiv \frac{\mu_{2G}'}{\Phi_1 \Phi_{2G}(1-e_{1g})\Lambda_G}$ and $P_{GB}^{++} \equiv \frac{\mu_{2B}'}{\Phi_1 \Phi_{2B} e_{1g} \Lambda_G}$. The normal application probability P_G must be at least smaller than or equal to them to satisfy the service rates.

(i) We substitute $\Phi_1 = 0$ into Eqs. (1) and (2), which results in $u_B = P_B r_B > 0$ and $u_G = P_G r_G > 0$. Thus potential applicants would apply with probability 1. Since there is no screening at the first stage, naturally there will be no further screening at the second stage, thus we have $\Phi_{2G} = \Phi_{2B} = \Phi_2^G = 0$.

(ii) If the approver screens at both stages; i.e., $\Phi_1 \in (0, 1]$, $\Phi_2^G \in (0, 1]$, then

(a) Once the service rates at the first and the second stage respectively can not satisfy normal applicants, they will drop the application. Thus when $\mu_1' \le 0$ or $\mu_{2G}' \le 0$ or $\mu_{2B}' \le 0$, we have $P_G = 0$.

(b) Once the normal application probability P_G satisfies the upper bounds and also makes the normal applicants' utility $\mu_G(\boldsymbol{\Phi}, P_G = 1, P_B) \ge 0$, the normal applicants would apply to the system $P_G = 1$.

(c) Once the normal application probability P_G makes the normal applicants' utility $\mu_G(\boldsymbol{\Phi}, P_G = 1, P_B) < 0$ or makes $\mu_G(\boldsymbol{\Phi}, P_G = 1, P_B) \ge 0$ with any upper bounds in the range of $[0, 1)$ that $P_G^+ \in [0, 1)$ or $P_{GG}^{++} \in [0, 1)$ or $P_{GB}^{++} \in [0, 1)$, it needs to decrease the probability value to the range of $[0, 1)$, considering P_G must be at least lower than or equal to the upper bounds P_G^+, P_{GG}^{++} and P_{GB}^{++}. For the screening policy, the maximum traffic of screened normal applicants $\hat{\Lambda}_G$ is derived from the normal applicant's zero utility [Eq. (2)] at the equilibrium:

$$r_G - (r_G + c_G)\Phi_1 \Phi_2^G e_{2g} - c_W \Phi_1 \Bigg(\frac{1}{\mu_1' - \hat{\Lambda}_G} + \frac{\Phi_{2G}(1-e_{1g})}{\mu_{2G}' - \Phi_{2G}(1-e_{1g})\hat{\Lambda}_G} + \frac{\Phi_{2B} e_{1g}}{\mu_{2B}' - \Phi_{2B} e_{1g} \hat{\Lambda}_G} \Bigg) = 0 \tag{15}$$

Then the maximum traffic of screened normal applicants is:

$$\hat{\Lambda}_G = -\frac{b'}{3a'} + \sqrt[3]{\frac{b'c'}{6a'^2} - \frac{b'^3}{27a'^3} - \frac{d'}{2a'} + \sqrt{\left(\frac{b'c'}{6a'^2} - \frac{b'^3}{27a'^3} - \frac{d'}{2a'}\right)^2 + \left(\frac{c'}{3a'} - \frac{b'^2}{9a'^2}\right)^3}} + \sqrt[3]{\frac{b'c'}{6a'^2} - \frac{b'^3}{27a'^3} - \frac{d'}{2a'} - \sqrt{\left(\frac{b'c'}{6a'^2} - \frac{b'^3}{27a'^3} - \frac{d'}{2a'}\right)^2 + \left(\frac{c'}{3a'} - \frac{b'^2}{9a'^2}\right)^3}} \quad (a' \ne 0)$$

$$\text{or } \hat{\Lambda}_G = \frac{-c' \pm \sqrt{c'^2 - 4b'd'}}{2b'} \quad (a' = 0 \& b' \ne 0)$$

$$\text{or } \hat{\Lambda}_G = -\frac{d'}{c'} \quad (a' = 0 \& b' = 0 \& c' \ne 0).$$

where $\Phi_2^G \equiv (1-e_{1g})\Phi_{2G} + e_{1g}\Phi_{2B}$ is introduced in Sect. 2.3, and we define $a \equiv \Phi_{2G}(1-e_{1g})$, $b \equiv \Phi_{2B} e_{1g}$, $c \equiv \frac{r_G - (r_G + c_G)\Phi_1 \Phi_2^G e_{2g}}{\Phi_1 c_W}$, $a' = abc$, $b' = (3ab - \mu_1' abc - \mu_{2G}' bc - \mu_{2B}' ac$, $c' = \mu_1' \mu_{2G}' bc + \mu_1' \mu_{2B}' ac + \mu_{2G}' \mu_{2B}' c - 2a\mu_{2B}' - 2b\mu_{2G}' - 2ab\mu_1'$, and $d' = \mu_{2G}' \mu_{2B}' +$

$a\mu_1'\mu_{2B}' + b\mu_1'\mu_{2G}' - \mu_1'\mu_{2G}'\mu_{2B}'c$. Using Theorem 1 in Balachandran and Schaefer (1980), there exists a unique equilibrium aggregate traffic rate: $\lambda_G = \max\big(\min(\hat{\Lambda}_G, \Phi_1\Phi_{2G}(1-e_{1g})\Lambda_G, \Phi_1\Phi_{2B}e_{1g}\Lambda_G), 0\big)$.

Since there are three maximum traffic of screened normal applicants, thus we have P_G^+, P_{GG}^{++} and P_{GB}^{++}. We choose the larger normal application probability that satisfies $P_G \in [0, 1]$. Thus, normal potential applicants' best response strategies satisfy:

$$P_G = \max\left(\min\left(\frac{\hat{\Lambda}_G}{\Phi_1\Phi_{2G}(1-e_{1g})\Lambda_G}, \frac{\hat{\Lambda}_G}{\Phi_1\Phi_{2B}e_{1g}\Lambda_G}, 1, P_G^+, P_{GG}^{++}, P_{GB}^{++}\right), 0\right)$$

We see that normal potential applicants' best response P_G has the opposite change direction with parameter Λ_G but same change direction with parameter $\hat{\Lambda}_G$. We have $\frac{\partial P_G}{\partial \Lambda_G} = -\frac{\hat{\Lambda}_G}{\Phi_1\Phi_{2G}(1-e_{1g})\Lambda_G^2} < 0$ or $-\frac{\hat{\Lambda}_G}{\Phi_1\Phi_{2B}e_{1g}\Lambda_G^2} < 0$, and $\frac{\partial P_G}{\partial \hat{\Lambda}_G} = \frac{1}{\Phi_1\Phi_{2G}(1-e_{1g})\Lambda_G} > 0$ or $\frac{1}{\Phi_1\Phi_{2B}e_{1g}\Lambda_G} > 0$. Equation (15) shows that parameter $\hat{\Lambda}_G$ has the opposite change direction with parameters c_W and e_{2g} but same change direction with parameters μ_1, μ_{2G}, μ_{2B}, and r_G since we have:

$$\frac{\partial c_W}{\partial \hat{\Lambda}_G} = -\frac{\frac{1}{(\mu_1'-\hat{\Lambda}_G)^2} + \frac{\Phi_{2G}^2(1-e_{1g})^2}{(\mu_{2G}'-\Phi_{2G}(1-e_{1g})\hat{\Lambda}_G)^2} + \frac{\Phi_{2B}^2 e_{1g}^2}{(\mu_{2B}'-\Phi_{2B}e_{1g}\hat{\Lambda}_G)^2}}{\frac{1}{\mu_1'-\hat{\Lambda}_G} + \frac{\Phi_{2G}(1-e_{1g})}{\mu_{2G}'-\Phi_{2G}(1-e_{1g})\hat{\Lambda}_G} + \frac{\Phi_{2B}e_{1g}}{\mu_{2B}'-\Phi_{2B}e_{1g}\hat{\Lambda}_G}} < 0$$

$$\frac{\partial e_{2g}}{\partial \hat{\Lambda}_G} = -\frac{c_W\Phi_1}{\Phi_1\Phi_2^G r_G}\left(\frac{1}{(\mu_1'-\hat{\Lambda}_G)^2} + \frac{\Phi_{2G}^2(1-e_{1g})^2}{(\mu_{2G}'-\Phi_{2G}(1-e_{1g})\hat{\Lambda}_G)^2} + \frac{\Phi_{2B}^2 e_{1g}^2}{(\mu_{2B}'-\Phi_{2B}e_{1g}\hat{\Lambda}_G)^2}\right) < 0$$

$$\frac{\partial \mu_1}{\partial \hat{\Lambda}_G} = (\mu_1'-\hat{\Lambda}_G)^2\left(\frac{\Phi_{2G}^2(1-e_{1g})^2}{(\mu_{2G}'-\Phi_{2G}(1-e_{1g})\hat{\Lambda}_G)^2} + \frac{\Phi_{2B}^2 e_{1g}^2}{(\mu_{2B}'-\Phi_{2B}e_{1g}\hat{\Lambda}_G)^2}\right) + 1 > 0$$

$$\frac{\partial \mu_{2G}}{\partial \hat{\Lambda}_G} = \frac{\left(\frac{1}{(\mu_1'-\hat{\Lambda}_G)^2} + \frac{\Phi_{2B}^2 e_{1g}^2}{(\mu_{2B}'-\Phi_{2B}e_{1g}\hat{\Lambda}_G)^2}\right)(\mu_{2G}'-\Phi_{2G}(1-e_{1g})\hat{\Lambda}_G)^2}{\Phi_{2G}(1-e_{1g})} + \Phi_{2G}(1-e_{1g}) > 0$$

$$\frac{\partial \mu_{2B}}{\partial \hat{\Lambda}_G} = \frac{\left(\frac{1}{(\mu_1'-\hat{\Lambda}_G)^2} + \frac{\Phi_{2G}^2(1-e_{1g})^2}{(\mu_{2G}'-\Phi_{2G}(1-e_{1g})\hat{\Lambda}_G)^2}\right)(\mu_{2B}'-\Phi_{2B}e_{1g}\hat{\Lambda}_G)^2}{\Phi_{2B}^2 e_{1g}^2} + \Phi_{2B}^2 e_{1g}^2 > 0$$

$$\frac{\partial r_G}{\partial \hat{\Lambda}_G} = \frac{c_W\Phi_1}{1-\Phi_1\Phi_2^G e_{2g}}\left(\frac{1}{(\mu_1'-\hat{\Lambda}_G)^2} + \frac{\Phi_{2G}^2(1-e_{1g})^2}{(\mu_{2G}'-\Phi_{2G}(1-e_{1g})\hat{\Lambda}_G)^2} + \frac{\Phi_{2B}^2 e_{1g}^2}{(\mu_{2B}'-\Phi_{2B}e_{1g}\hat{\Lambda}_G)^2}\right) > 0$$

(iii) If the approver screens at the first stage $\Phi_1 \in (0, 1]$ but does not at the second stage $\Phi_2^G = 0$, then

(a) Once the service rates at the first can not satisfy normal applicants, they will drop the application. Thus when $\mu_1' \leq 0$, we have $P_G = 0$.
(b) Once the normal application probability P_G satisfies the upper bound at the first stage P_G^+ and also makes the normal utility $\mu_G(\boldsymbol{\Phi}, P_G, P_B) \geq 0$, the normal applicants would apply to the system.
(c) Once the normal application probability P_G makes the normal utility $\mu_G(\boldsymbol{\Phi}, P_G = 1, P_B) < 0$ or makes $\mu_G(\boldsymbol{\Phi}, P_G = 1, P_B) \geq 0$ with upper bound $P_G^+ \in [0, 1)$, it needs to decrease the probability value to the range of $[0, 1)$, considering P_G must be at least lower than or equal to the upper bound P_G^+. For the screening policy, the maximum traffic of screened normal applicants $\hat{\Lambda}_G$ is derived from the normal applicant's

zero utility as follow: $r_G - (r_G + c_G)\Phi_1 e_{1g} - c_W \Phi_1 \left(\frac{1}{\mu_1 - \Phi_1 P_B \Lambda_B - \hat{\Lambda}'_G}\right) = 0$, and $\hat{\Lambda}'_G = \mu_1 - \Phi_1 P_B \Lambda_B - \frac{c_W \Phi_1}{r_G - (r_G + c_G)\Phi_1 e_{1g}}$. There exists a unique equilibrium aggregate traffic rate: $\lambda_G = \max\left(\min(\hat{\Lambda}'_G, \Phi_1 \Lambda_G), 0\right)$. Thus, normal potential applicants' best response strategies satisfy: $P_G = \frac{\lambda_G}{\Phi_1 \Lambda_G} = \max\left(\min(\frac{\hat{\Lambda}'_G}{\Phi_1 \Lambda_G}, 1, P_G^+), 0\right)$. We see that the normal potential applicants' best response P_G has the opposite change direction with parameter Λ_G, but same change direction with parameter $\hat{\Lambda}'_G$. We have $\frac{\partial P_G}{\partial \Lambda_G} = -\frac{\hat{\Lambda}'_G}{\Phi_1 \Lambda_G^2} < 0$, and $\frac{\partial P_G}{\partial \hat{\Lambda}'_G} = \frac{1}{\Phi_1 \Lambda_G} > 0$. The parameter $\hat{\Lambda}'_G$ has the opposite change direction with parameter c_W but same change direction with parameters μ_1 and r_G. We have $\frac{\partial \hat{\Lambda}'_G}{\partial c_W} = -\frac{\Phi_1}{r_G(1 - e_{1g}\Phi_1)} < 0$, $\frac{\partial \hat{\Lambda}'_G}{\partial \mu_1} = 1 > 0$, and $\frac{\partial \hat{\Lambda}'_G}{\partial r_G} = \frac{c_W \Phi_1 (1 - e_{1g}\Phi_1)}{(r_G - (r_G + c_G)\Phi_1 e_{1g})^2} > 0$.

Therefore, normal potential applicants' best response P_G decreases in parameters c_W and e_{2g} and increases in parameters μ_1, μ_{2G}, μ_{2B}, and r_G.

Appendix 3: Proof of Proposition 3 in discriminatory policy

According to the Proposition 1, when the total effective screening probability $\Phi_1 \Phi_2^B$ is larger than or equal to the threshold s_b, none of the adversary potential applicants submit their applications. We assume that when the approver is indifferent between different levels of screening probabilities, the lowest level will be chosen.

1. When $\Phi^B = s_b$, then $P_B = 0$, there are no bad applicants. Then the approver's objective value becomes: $J_1(\mathbf{\Phi}) = \Lambda_G P_G R(1 - \Phi_1 \Phi_2^G e_{2g}) = R(1 - \Phi_1 \Phi_{2G} e_{2g}) P_G(\mathbf{\Phi}, P_B = 0)$.
2. When $\Phi^B \in [0, s_b)$, then $P_B = 1$, all adversary potential applicants submit their applications. Then the approver's objective value becomes: $J_2(\mathbf{\Phi}) = R(1 - \Phi_1 \Phi_2^G e_{2g}) P_G(\mathbf{\Phi}, P_B = 1) - C(1 - \Phi^B)$.

$$J(\mathbf{\Phi}) = \begin{cases} J_1(\mathbf{\Phi}), \text{ if } \Phi^B = s_b \\ J_2(\mathbf{\Phi}), \text{ if } \Phi^B \in [0, s_b). \end{cases} \tag{16}$$

Thus, the optimal best strategy for the approver is to solve: $J^* = \max_{0 \le \Phi_1 \Phi_2^B \le s_b} J(\mathbf{\Phi})$.

Appendix 4: Numerical sensitivity analyses under discriminatory policy

Figure 9 illustrates the numerical sensitivity analysis under a discriminatory policy, which implies that the screening probabilities for 'Good' and 'Bad' applicants at stage 2, Φ_{2G} and Φ_{2B}, respectively, could be different. The screening probability Φ_1 increases in C, Λ_B, r_G, μ_1, μ_{2B}, and c_G (Fig. 9c, f, j, k, m, n), decreases in Λ_G, and R (Fig. 9g, h), and first increases and then decreases in e_g, r, c_B, R and r_B (Fig. 9a, b, d, h, i). The probability of screening 'Good' applicants at the second stage Φ_{2G} generally is low except when e_g is high (Fig. 9a), or when r is intermediate (Fig. 9b), or when R is low (Fig. 9h). The probability of screening 'Bad' applicants at the second stage Φ_{2B} generally remains high except when e_g and r are high (Fig. 9a, b), or when C, r_B, r_G, and μ_1 are low (Fig. 9c, i–k). The adversary application probability P_B stays at zero except when e_g, r, Λ_G, R and r_B are high (Fig. 9a, b, g–i), or when C, c_B, Λ_B, r_G, μ_1, and μ_{2B} are low (Fig. 9c, d, f, j, k, m). The normal application probability P_G generally increases in c_B, R, r_G, μ_1, and μ_{2B} (Fig. 9d, h, j, k, m), decreases in e_g, C, c_W, Λ_B, Λ_G, R, r_B, and c_G (Fig. 9a, c, e, f, g, i, n). The normal application probability

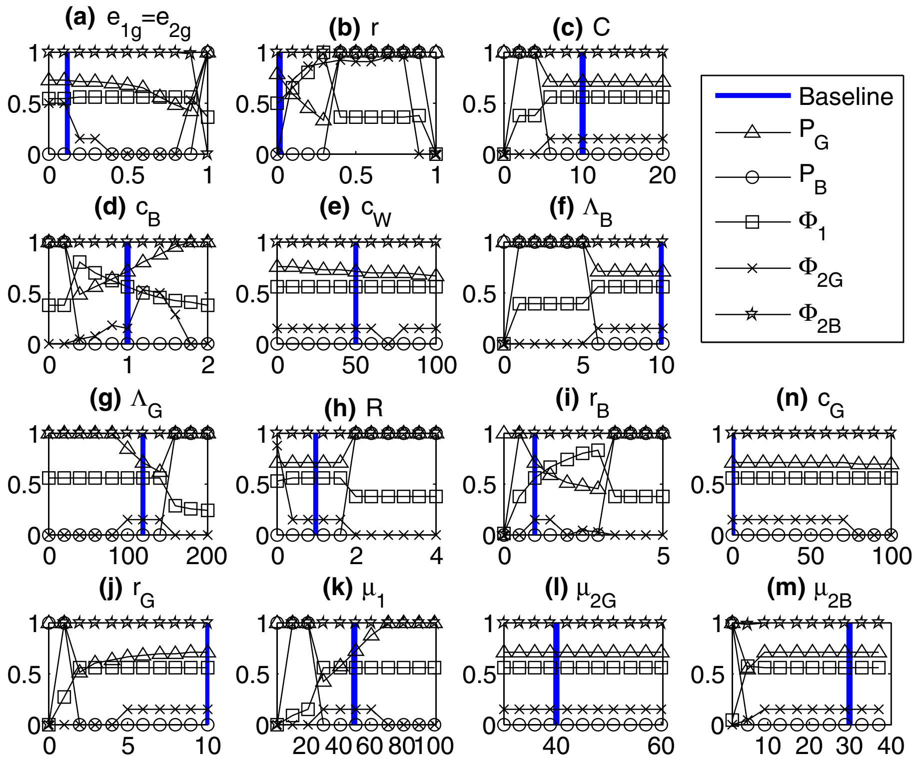

Fig. 9 Numerical sensitivity analysis under discriminatory policies

P_G keeps at 1 due to low screening probability Φ_1 when e_g, r, and R are high (Fig. 9a, b, h), or when C, c_B, Λ_B, Λ_G, r_B, r_G, and μ_1 are low (Fig. 9c, d, f, g, i, j, k).

Appendix 5: Comparisons between discriminatory policy and non-discriminatory policy

This section compares the model results between the non-discriminatory policy and the discriminatory policy: adversary application probabilities (P_{BN} and P_{BD}, respectively), normal application probabilities (P_{GN} and P_{GD}, respectively), and approver's payoffs (J_N and J_D, respectively). In particular, Figure 10 shows the comparison of adversary application probabilities P_{BN} and P_{BD} between the non-discriminatory and the discriminatory policies. The adversary application probability under the non-discriminatory policy is significantly higher than that in the discriminatory policy when the adversary applicants' reward if passed r_B is high (Fig. 10i), or when the penalty for approver once admitting each adversary applicant C, and the approver's reward for admitting each normal applicant R are intermediate (Fig. 10c, h), or when the cost to adversary applicants being caught c_B, the adversary applicant arrival rate Λ_B, and the reward for normal applicant to pass the system r_G are low (Fig. 10d, f, j).

Figure 11 compares the normal application probabilities P_{GN} and P_{GD} between the non-discriminatory and the discriminatory policies. The normal application probability in a discriminatory policy is significantly higher than the one in a non-discriminatory policy when the error probabilities that normal applicants are screened as 'Bad' at the first and

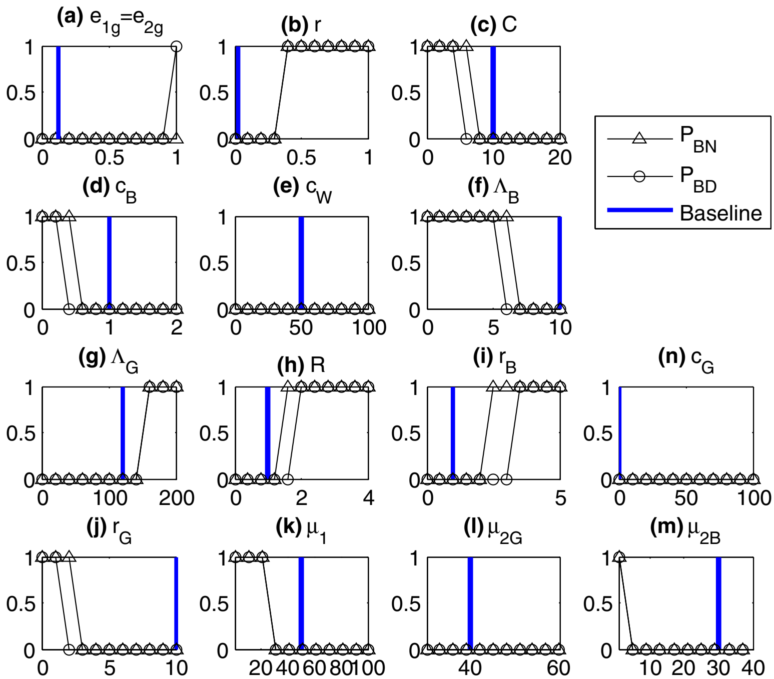

Fig. 10 Comparing adversary application rates P_{BN} and P_{BD} between the non-discriminatory and the discriminatory policies

second stage $e_{1g} = e_{2g}$, the unit time cost c_W, the service rate at the first stage μ_1, and the loss once normal applicants are rejected c_G are high (Fig. 11a, e, k, n), or when the the cost to adversary applicants being caught c_B, the adversary applicants' reward if passed r_B, and the second stage screening/service rate for applicants screened as 'Bad' μ_{2B} are intermediate (Fig. 11d, i, m), or when the power function coefficient r, the benefit of the approver for passing each normal applicant R, the reward for normal applicant to pass the system r_G, and the second stage screening/service rate for applicants screened as 'Good' μ_{2G} are low (Fig. 11b, h, j, l).

Figure 12 shows the comparison of the approver's payoffs J_N and J_D between the non-discriminatory and the discriminatory policies. The approver's payoffs under discriminatory policy is significantly higher than the one in a non-discriminatory policy, especially when the error probabilities that normal applicants are screened as 'Bad' at the first and second stage $e_{1g} = e_{2g}$, the penalty for the approver once admitting each adversary applicant C, the unit time cost c_W, the service rate at first stage μ_1, and the loss once normal applicants are rejected c_G are high (Fig. 12a, c, e, k, n), or when the cost to adversary applicants being caught c_B, the adversary applicant arrival rate Λ_B, the normal applicant arrival rate Λ_G, the reward for adversary to pass the system r_B, and the second stage screening/service rate for applicants screened as 'Bad' μ_{2B} are intermediate (Fig. 11d, f, g, i, m), or when the power function coefficient r, the the reward for normal applicant to pass the system r_G, and the second stage screening/service rate for applicants screened as 'Good' μ_{2G} are low (Fig. 11b, j, l).

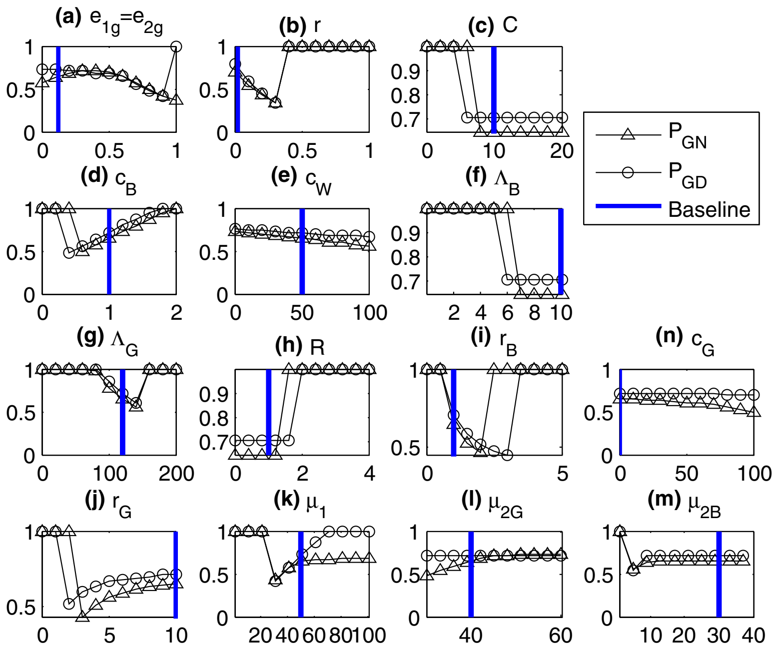

Fig. 11 Comparing normal application probabilities P_{GN} and P_{GD} between the non-discriminatory and the discriminatory policies

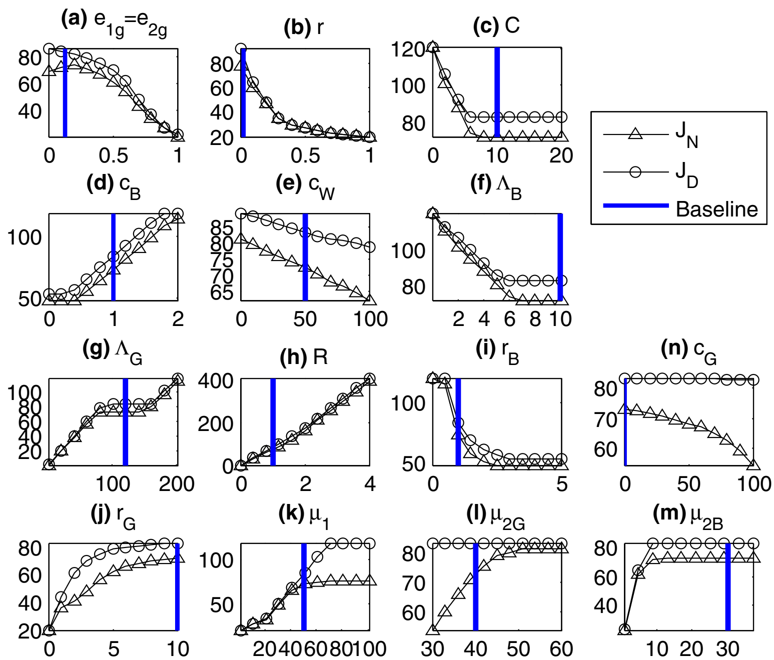

Fig. 12 Comparing approver's payoffs J_N and J_D between the non-discriminatory and the discriminatory policies

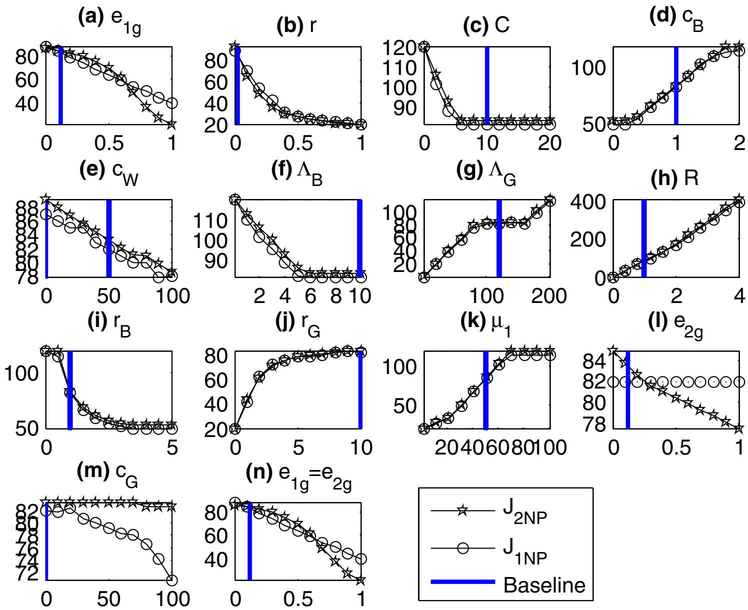

Fig. 13 Comparisons of approver's utilities in one-stage and two-stage systems

Appendix 6: Comparisons for one versus two-stage screening systems

This section introduces and compares one versus two-stage screening systems to find the best screening policy for the approver under certain situations. The utility for a one-stage imperfect screening system is J_{1NP}. The utility for a two-stage imperfect screening system is J_{2NP}. Figure 13 shows the comparison of the approver's utilities in one- and two-stage screening systems. It shows that the approver's payoff in a two-stage screening system J_{2NP} is significantly larger than the one in a one-stage system J_{1NP} when the adversary applicants' reward if passed r_B, the service rate at the first stage μ_1, and the loss once normal applicants are rejected c_G are high (Fig. 13i, k, m), or when the error probability that normal applicants are screened as 'Bad' at the first stage e_{1g}, the penalty for approver once admitting each adversary applicant C, the adversary applicant arrival rate Λ_B, and error probabilities that normal applicants are screened as 'Bad' at the first and second stage $e_{1g} = e_{2g}$ are intermediate (Fig. 13a, c, f, n), or when power function coefficient r, the cost to adversary applicants being caught c_B, the unit time cost c_W, and the error probability at the second stage e_{2g} are low (Fig. 13b, d, e, l).

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