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Research paper

Two Warehouse Optimal Inventory Model for Non-Instantaneous Deteriorating Items

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Abstract

In this article, an optimal ordering policy for the stock model of items, whose deterioration starts after a certain period of time under the dispatching policy Last in First Out (i.e. LIFO) in owned and rented warehouses is presented with the demand of the product increases up to a certain point and that remains constant, deterioration rate which depends on time and inflation. The main aim of the present article is to find an optimal inventory model, which minimizes the total stock cost. Finally, this model is analyzed through by numerical examples .

Keywords: *Deterioration; Inflation; Ramp type demand; Two warehouse; LIFO*

1. Introduction

Various situations like price discounts given by the supplier, customer's high demand for the product, product's storage cost is low and seasonal products makes the retailer to buy more than the owned warehouse capacity in today's wholesale business market. Then the excess quantities are stored in additional storage place. Such place is called as rented warehouse.

Products start to deteriorate after a time lag, as the retailer received it from the supplier or from the factory without using any preservation technology. Those types of products are called as non-instantaneous deteriorating items. In that time lag, no deterioration occurs for the products.

Ghare and Schrader (1963) studied an inventory model for exponentially decaying inventories. Many researchers developed the two warehouse inventory models with various features like inflation, permissible delay in payments, allowing shortages or not and so on. Skouri, et al (2013) considered a two warehouse model with ramp type demand. Jaggi, et al (2017), Kumar et al (2015) and Sanni et al (2013) are developed the two warehouse inventory model for deteriorating items with various factors.

In this paper, an optimal ordering policy for the stock model for items, whose deterioration starts after a certain period of time under the dispatching policy Last in First Out (i.e. LIFO) in owned and rented warehouses is presented with the demand of the product increases up to a certain point and that remains constant, deterioration rate which depends on time and inflation. The main aim of the present article is to find an optimal inventory model, which minimizes the total stock cost. Finally, this model is analyzed through by numerical examples .

2. Assumptions

1. The single item is considered in this inventory system.
2. The ramp type demand rate is considered with shortages.
3. The time dependent deterioration rate is considered.
4. Holding cost is a linear function of time.
5. Lead time is zero.
6. A time during which no deterioration occurs is less than the time during the inventory in a rented warehouse becomes zero. i.e. $t_d < t_r$
7. Replenishment rate is infinite and instantaneous.

3. Notations

$I_r(t)$: Inventory in rented warehouse at any time *t*.
$I_o(t)$: Inventory in owned warehouse at any time *t*.
$I_s(t)$: Inventory at any time t, during shortage period.
t_d : A time gap in which the product exhibits no deterioration
t_r : Time for an inventory vanishes in RW.
t_w : Time for an inventory vanishes in OW.
Q : Total inventory level
Q_1 : Maximum positive inventory level at time $t = 0$
Q_2 : Maximum negative inventory level at time $t = T$
W : Capacity of owned warehouse
$Q_1 - W$: Capacity of rented warehouse
C_1 : Purchasing cost
C_2 : Shortage cost
C_3 : Lost sale cost
HC_r : Rented warehouse holding cost
HC_o : Owned warehouse holding cost

DC_r : Rented warehouse deterioration cost
DC_o : Owned warehouse deterioration cost
T : Length of time between two successive orders
A : Ordering cost
μ : Ramp type parameter function
TC : Total inventory cost

4. Formulation and solution of the inventory model:

4.1. Model:

$$\mathbf{0 < \mu < t_d < t_r < t_w < T}$$

The following differential equations presents inventory levels at any time 't' in the time period $[0, T]$

$$\frac{dI_r(t)}{dt} = -at, \quad 0 \le t \le \mu \qquad \text{with } I_r(0) = Q_1 - W \tag{1}$$

$$\frac{dI_r(t)}{dt} = -a\mu, \quad \mu \le t \le t_d \qquad \text{with } I_r(\mu -) = I_r(\mu +) \tag{2}$$

$$\frac{dI_r(t)}{dt} + \theta_1 t I_r(t) = -a\mu, \quad t_d \le t \le t_r \qquad \text{with } I_r(t_r) = 0 \tag{3}$$

$$\frac{dI_o(t)}{dt} = 0, \quad 0 \le t \le \mu \qquad \text{with } I_o(\mu -) = I_o(\mu +) \tag{4}$$

$$\frac{dI_o(t)}{dt} = 0, \quad \mu \le t \le t_d \qquad \text{with } I_o(t_d) = W \tag{5}$$

$$\frac{dI_o(t)}{dt} + \theta_2 t I_o(t) = 0, \quad t_d \le t \le t_r \qquad \text{with } I_o(t_d) = W \tag{6}$$

$$\frac{dI_o(t)}{dt} + \theta_2 t I_o(t) = -a\mu, \quad t_r \le t \le t_w \qquad \text{with } I_o(t_w) = 0 \tag{7}$$

$$\frac{dI_s(t)}{dt} = -a\mu\delta, \quad t_w \le t \le T \qquad \text{with } I_s(t_w) = 0 \tag{8}$$

Now, solving all the equations (1) to (8) by using boundary conditions, we have

$$I_r(t) = -\frac{at^2}{2} + Q_1 - W, \quad 0 \le t \le \mu \tag{9}$$

$$I_r(t) = -a\mu t + \frac{a\mu^2}{2} + Q_1 - W, \quad \mu \le t \le t_d \tag{10}$$

$$I_r(t) = a\mu\left[t_r - t + \frac{\theta_1}{6}(t_r^3 - 3t^2 t_r + 2t^3)\right], \quad t_d \le t \le t_r \tag{11}$$

$$I_o(t) = W, \quad 0 \le t \le \mu \tag{12}$$

$$I_o(t) = W, \quad \mu \le t \le t_d \tag{13}$$

$$I_o(t) = W\left[1 + \frac{\theta_2}{2}(t_d^2 - t^2)\right], \quad t_d \le t \le t_r \tag{14}$$

$$I_o(t) = a\mu\left[t_w - t + \frac{\theta_2}{6}(t_w^3 - 3t^2 t_w + 2t^3)\right], \quad t_r \le t \le t_w \tag{15}$$

$$I_s(t) = a\mu\delta(t_w - t), \quad t_w \le t \le T \tag{16}$$

From equations (9) and (10), the stock level for rented warehouse is

$$Q_1 - W = a\mu\left[-\frac{\mu}{2} + t_r + \frac{\theta_1}{6}\left(t_r^3 - 3t_d^2 t_r + 2t_d^3\right)\right] \tag{17}$$

From equations (13) and (14), the stock level for owned warehouse is

$$W = a\mu\left[t_w - t_r + \frac{\theta_2}{6}\left(t_w^2 - t_r^3 - 3t_d^2 t_w + 3t_d^2 t_r\right)\right] \tag{18}$$

With $I_s(T) = -Q_2$, during shortage period the inventory level as

$$I_s(T) = -Q_2 = -a\mu\delta(t_w - T) \tag{19}$$

$$\text{Total Inventory}, Q = Q_1 + Q_2 \tag{20}$$

4.2. Total inventory cost:

Ordering cost: $OC = \frac{A}{T}$

Total holding cost:
For rented warehouse:

$$HC_r = \frac{1}{T}\int_0^{t_r}(x_1 + y_1 t)\, e^{-Rt} I_r(t)dt$$

$$HC_r = \begin{bmatrix} \frac{x_1}{T}\int_0^{\mu} e^{-Rt} I_r(t)dt + \frac{x_1}{T}\int_{\mu}^{t_d} e^{-Rt} I_r(t)dt \\ + \frac{x_1}{T}\int_{t_d}^{t_r} e^{-Rt} I_r(t)dt + \frac{y_1}{T}\int_0^{\mu} te^{-Rt} I_r(t)dt \\ + \frac{y_1}{T}\int_{\mu}^{t_d} te^{-Rt} I_r(t)dt + \frac{y_1}{T}\int_{t_d}^{t_r} te^{-Rt} I_r(t)dt \end{bmatrix}$$

$$HC_r = HC_1 + HC_2 + HC_3 + HC_4 + HC_5 + HC_6$$

where

$$HC_1 = \frac{x_1}{T}\left[-\frac{a\mu^3}{6} + (Q_1 - w)\mu + \frac{Ra\mu^4}{8} - \frac{(Q_1 - w)R\mu^2}{2}\right]$$

$$HC_2 = \frac{x_1}{T}\begin{bmatrix} -a\mu\left\{\frac{t_d^2 - \mu^2}{2} - \frac{R(t_d^3 - \mu^3)}{3}\right\} \\ + \left(\frac{a\mu^2}{2} + Q_1 - W\right)\left\{t_d - \mu - \frac{R(t_d^2 - \mu^2)}{2}\right\} \end{bmatrix}$$

$$HC_3 = \frac{x_1 a\mu}{T}\begin{bmatrix} \frac{t_r^2}{2} - t_d t_r + \frac{t_d^2}{2} + \frac{\theta_1}{6}\left(\frac{t_r^4}{2} - t_d t_r^3 + t_d^3 t_r - \frac{t_d^4}{2}\right) \\ -R\begin{Bmatrix} \frac{t_r^3}{6} - \frac{t_d^2 t_r}{2} + \frac{t_d^3}{3} \\ + \frac{\theta_1}{6}\left(\frac{3t_r^5}{20} - \frac{t_d^2 t_r^3}{2} + \frac{3t_d^4 t_r}{4} - \frac{2t_d^5}{5}\right) \end{Bmatrix} \end{bmatrix}$$

$$HC_4 = \frac{y_1}{T}\left[-\frac{a\mu^4}{8} + \frac{(Q_1 - w)\mu^2}{2} + \frac{Ra\mu^5}{10} - \frac{(Q_1 - w)R\mu^3}{3}\right]$$

$$HC_5 = \frac{y_1}{T}\begin{bmatrix} -a\mu\left\{\frac{t_d^3 - \mu^3}{3} - \frac{R(t_d^4 - \mu^4)}{4}\right\} \\ + \left(\frac{a\mu^2}{2} + Q_1 - W\right)\left\{\frac{t_d^2 - \mu^2}{2} - \frac{R(t_d^3 - \mu^3)}{3}\right\} \end{bmatrix}$$

$$HC_6 = \frac{y_1 a\mu}{T}\begin{bmatrix} \frac{t_r^3}{6} - \frac{t_d^2 t_r}{2} + \frac{t_d^3}{3} + \frac{\theta_1}{6}\left(\frac{3t_r^5}{20} - \frac{t_d^2 t_r^3}{2} + \frac{3t_d^4 t_r}{4} - \frac{2t_d^5}{5}\right) \\ -R\begin{Bmatrix} \frac{t_r^4}{12} - \frac{t_d^3 t_r}{3} + \frac{t_d^4}{4} \\ + \frac{\theta_1}{6}\left(\frac{t_r^6}{15} - \frac{t_d^3 t_r^3}{3} + \frac{3t_d^5 t_r}{5} - \frac{t_d^6}{3}\right) \end{Bmatrix} \end{bmatrix}$$

For owned warehouse:

$$HC_o = \frac{1}{T}\int_0^{t_w}(x_2 + y_2 t)\, e^{-Rt} I_o(t)dt$$

$$HC_o = \begin{bmatrix} \frac{x_2}{T}\int_0^{t_d} e^{-Rt} I_o(t)dt + \frac{x_2}{T}\int_{t_d}^{t_r} e^{-Rt} I_o(t)dt \\ + \frac{x_2}{T}\int_{t_r}^{t_w} e^{-Rt} I_o(t)dt + \frac{y_2}{T}\int_0^{t_d} te^{-Rt} I_o(t)dt \\ + \frac{y_2}{T}\int_{t_d}^{t_r} te^{-Rt} I_o(t)dt + + \frac{y_2}{T}\int_{t_r}^{t_w} te^{-Rt} I_o(t)dt \end{bmatrix}$$

$$HC_o = HC_1 + HC_2 + HC_3 + HC_4 + HC_5 + HC_6$$

where

$$HC_1 = \frac{x_2 W}{T}\left[t_d - \frac{Rt_d^2}{2}\right]$$

$$HC_2 = \frac{x_{2W}}{T}\begin{bmatrix} t_r - t_d + \frac{\theta_2}{2}\left(t_r t_d^2 - \frac{2t_d^3}{3} - \frac{t_r^3}{3}\right) \\ -R\left\{\frac{t_r^2 - t_d^2}{2} + \frac{\theta_2}{2}\left(\frac{t_r^2 t_d^2}{2} - \frac{t_d^4}{4} - \frac{t_r^4}{4}\right)\right\} \end{bmatrix}$$

$$HC_3 = \frac{x_2 a\mu}{T}\begin{bmatrix} \frac{t_w^2}{2} - t_w t_r + \frac{t_r^2}{2} + \frac{\theta_2}{6}\left(\frac{t_w^4}{2} - t_r t_w^3 - t_w t_r^3 - \frac{t_r^4}{2}\right) \\ -R\begin{Bmatrix} \frac{t_w^3}{6} - \frac{t_r^2 t_w}{2} + \frac{t_r^3}{3} \\ + \frac{\theta_2}{6}\left(\frac{3t_w^5}{20} - \frac{t_r^2 t_w^3}{2} + \frac{3t_r^4 t_w}{4} - \frac{2t_r^5}{5}\right) \end{Bmatrix} \end{bmatrix}$$

$$HC_4 = \frac{y_2 W}{T}\left[\frac{t_d^2}{2} - \frac{Rt_d^3}{3}\right]$$

$$HC_5 = \frac{y_2W}{T}\left[\begin{array}{c}\frac{t_r^2-t_d^2}{2}+\frac{\theta_2}{2}\left(\frac{t_r^2t_d^2}{2}-\frac{t_d^4}{4}-\frac{t_r^4}{4}\right)\\ -R\left\{\frac{t_r^3-t_d^3}{3}+\frac{\theta_2}{2}\left(\frac{t_r^3t_d^2}{2}-\frac{2t_d^5}{15}-\frac{t_r^5}{5}\right)\right\}\end{array}\right]$$

$$HC_6 = \frac{y_2a\mu}{T}\left[\begin{array}{c}\frac{t_w^3}{6}-\frac{t_r^2t_w}{2}+\frac{t_r^3}{3}+\frac{\theta_2}{6}\left(\frac{3t_w^5}{20}-\frac{t_r^2t_w^3}{2}+\frac{3t_r^4t_w}{4}-\frac{2t_r^5}{5}\right)\\ -R\left\{\begin{array}{c}\frac{t_w^4}{12}-\frac{t_r^3t_w}{3}+\frac{t_r^4}{4}\\ +\frac{\theta_2}{6}\left(\frac{t_w^6}{15}-\frac{t_r^3t_w^3}{3}+\frac{3t_r^5t_w}{5}-\frac{t_r^6}{3}\right)\end{array}\right\}\end{array}\right]$$

Total holding cost, $HC = HC_r + HC_o$

Total deterioration cost:

For rented warehouse:

$$DC_r = \frac{C_1}{T}\int_{t_d}^{t_r}\theta_1 te^{-Rt}I_r(t)dt$$

$$DC_r = \frac{C_1\theta_1a\mu}{T}\left[\frac{t_r^3}{6}-\frac{t_d^2t_r}{2}+\frac{t_d^3}{3}-R\left(\frac{t_r^4}{12}-\frac{t_d^3t_r}{3}+\frac{t_d^4}{4}\right)\right]$$

For owned warehouse:

$$DC_w = \frac{C_1}{T}\int_{t_d}^{t_w}\theta_2 te^{-Rt}I_o(t)dt$$

$$DC_w = \frac{C_1}{T}\left[\int_{t_d}^{t_r}\theta_2 te^{-Rt}I_o(t)dt + \int_{t_r}^{t_w}\theta_2 te^{-Rt}I_0(t)dt\right]$$

$$DC_w = DC_1 + DC_2$$

where

$$DC_1 = \frac{C_1\theta_2W}{T}\left[\frac{t_r^2-t_d^2}{2}-R\left(\frac{t_r^3-t_d^3}{3}\right)\right]$$

$$DC_2 = \frac{C_1\theta_2a\mu}{T}\left[\frac{t_w^3}{6}-\frac{t_r^2t_w}{2}+\frac{t_r^3}{3}-R\left(\frac{t_w^4}{12}-\frac{t_r^3t_w}{3}+\frac{t_r^4}{4}\right)\right]$$

Total deterioration cost, $DC = DC_r + DC_w$

Shortage cost:

$$SC = \frac{C_2}{T}\int_{t_w}^{T}[I_s(t)]e^{-Rt}\,dt$$

$$SC = -\frac{C_2a\mu\delta}{T}\left[Tt_w-\frac{t_w^2}{2}-\frac{T^2}{2}-R\left(\frac{T^2t_w}{2}-\frac{t_w^3}{6}-\frac{T^3}{3}\right)\right]$$

Cost due to lost sales:

$$CLS = \frac{C_3}{T}\int_{t_w}^{T}[(1-\delta)e^{-Rt}D(t)dt]dt$$

$$CLS = \frac{C_3a\mu(\delta-1)}{T}\left[T-t_w-R\left(\frac{T^2-t_w^2}{2}\right)\right]$$

Total inventory cost is given by

$$TC(t_w) = OC + HC + DC + SC + CLS \tag{21}$$

Using following optimal conditions, we minimize the total stock cost

(i) $\frac{d(TC)}{dt_w} = 0$ and (ii) $\frac{d^2(TC)}{dt_w^2} > 0$

5. Numerical illustrations:

Using MATLAB software, the following examples are solved and the optimal solutions are found.

Model I: Partial backlogging model

Example 1:

Let $A = 300$, $x_1 = 3$, $y_1 = 0.3$, $x_2 = 1$, $y_2 = 0.1$, $a = 150$, $W = 100$, $\mu = 1.5$ weeks, $R = 0.1$, $t_d = 2$, $t_r = 6$ weeks, $T = 10$ weeks, $\theta_1 = 0.5$, $\theta_2 = 0.5$, $\delta = 0.4$, $C_1 = 5$, $C_2 = 7$, and $C_3 = 3$

Optimal solutions are $t_w^* = 7.0701$ weeks and $TC_1{}^* = Rs.1{,}562$

Model II: Complete backlogging model

Example 2:

Let $A = 300$, $x_1 = 3$, $y_1 = 0.3$, $x_2 = 1$, $y_2 = 0.1$, $a = 150$, $W = 100$, $\mu = 1.5$ weeks, $R = 0.1$, $t_d = 2$, $t_r = 6$ weeks, $T = 10$ weeks, $\theta_1 = 0.5$, $\theta_2 = 0.5$, $\delta = 1$, $C_1 = 5$, $C_2 = 7$, and $C_3 = 3$.

Optimal solutions are $t_w^* = 7.9152$ weeks and $TC_1{}^* = Rs.1{,}862$

5.1. Observations

1. The total optimal stock cost (TC^*) in Model I is less than the total optimal stock cost (TC^*) in Model II.
2. The optimal time (t_w^*) in Model I is less than the total optimal time (t_w^*) in Model II.

6. Conclusion

In this article, a stock model is developed for non-instantaneous deteriorating items under a LIFO policy in owned and rented warehouse. Also, inflation and shortages in inventory are considered. An optimal policy which minimizes the total inventory cost is developed. A numerical illustration of each model is given to explain the developed model. This advanced model is very useful to retailers in wholesale business to minimize the stock cost for maintaining the inventory in various situations like price discounts given by supplier, customer's high demand for the product, product's storage cost is low and some new brand of cosmetic products, electronic items, seasonal products etc. are entered in the business market, the demand for those products are increasing at the beginning up to a particular time and then remain constant for the remaining period.

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