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"Tying in Two-Sided Markets with Multi-Homing: Corrigendum and Comment"

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Tying in Two-Sided Markets with Multi-Homing: Corrigendum and Comment*

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Abstract

We identify two issues in Choi's (2010) paper on tying in two-sided markets published in this Journal, and provide solutions to both of them. First, we point out that the equilibrium in the absence of tying requires more restrictive conditions and does not satisfy a natural equilibrium refinement criterion. We offer an alternative timing structure that validates the equilibrium derived in Choi (2010) under the conditions provided there. Second, we show that his equilibrium analysis with tying ignores a profitable deviation. We rectify this analysis under our alternative timing structure and derive the (mixed-strategy) equilibrium with tying. We also show by means of simulations that tying is welfare-enhancing whenever it is profitable, which is consistent with the main finding in Choi (2010).

1 Introduction

Choi (2010) - hereafter C10 - analyzes the effects of tying in a two-sided market model that allows multi-homing on both sides of the market. It is shown that multi-homing has important implications for market competition and social welfare. In particular, tying can be welfare-enhancing if multi-homing is allowed, even in cases where its welfare impacts are negative in the absence of multi-homing. We identify two issues in C10's equilibrium analysis: one for the equilibrium with multihoming on both sides of the platforms in the absence of tying and the other for the equilibrium with tying. More specifically, the analysis in C10 did not take

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into consideration particular deviations that could upset the equilibrium. To resolve these issues, we offer an alternative timing structure of the game that would preserve the equilibrium without tying derived in C10. We also provide the correct equilibrium analysis with tying under this alternative timing assumption. Despite the errors in equilibrium characterizations, our simulation results show that the main finding in C10 is still valid: tying is welfare-enhancing whenever it is profitable.

2 Setup and issues

2.1 Description of the setup in C10

We first provide a brief description of the setup in C10 to facilitate our discussion and introduce the model notation. There are two platforms indexed by $i = A, B$. Consumers and content providers constitute two sides of the platforms. Let p_i and q_i denote platform i's charge to content providers and consumers, respectively. The corresponding number of content providers and consumers who participate in platform i are denoted by m_i and N_i, respectively.

The consumers' choice of platform is according to the Hotelling model of product differentiation with two platforms, A and B, being located at the end points of a line with length equal to 1. Consumers can choose to either single-home or multi-home. If a consumer located at point x participates in platform A only, his utility is given by $u_A(q_A, x) = bm_A - q_A - tx$ while his utility from participating in platform B is given by $u_B(q_B, x) = bm_B - q_B - t(1-x)$, where t is a 'transportation' cost parameter. If the consumer decides to multi-home, his utility is given by $u_{AB}(q_A, q_B, x) = bm - q_A - tx - q_B - t(1-x)$, where m is the total amount of content available to consumers who multi-home. This amount is given by $m = m_A + m_B - \delta$ if the extent of duplicative content across the platforms is δ.

On the content side, the total measure of content potentially available for each format is normalized to 1. Among them, a proportion λ is of the 'exclusive' type and can be encoded only for a particular format whereas $(1-\lambda)$ is of the 'nonexclusive' type and can also be encoded in the other format. When the nonexclusive type of content is encoded for both formats, content providers are said to multi-home. Each content provider gains additional utility (profit) of π from each consumer who has access to its content. The profit for content providers who create content on platform i is thus given by $\pi n_i - pi$.

The existence of exclusive content available for each format creates incentives for consumers to multi-home. With the possibility of multi-homing on the consumer side, let n_i and n_M respectively denote the number of consumers who single-home on platform i and the number of consumers who multi-home, where $i = A, B$. Then, the total number of consumers who participate in platform i is given by $N_i = n_i + n_M$.

2.2 Issues

2.2.1 Equilibrium with multi-homing in the absence of tying

C10 implicitly assumes that platforms A and B set *simultaneously* (and independently) their prices. More precisely, he considers the following timing:

1- Platforms A and B set their price pairs (p_A, q_A) and (p_B, q_B) simultaneously.

2- After observing these prices, consumers and content providers decide simultaneously to join both platforms, one of them, or none.

In this section we show that this timing raises two problems: first, the set of parameters for which an equilibrium with multi-homing exists is smaller than what is stated in Proposition 1 in C10. Second, the existence of an equilibrium with multi-homing relies on an equilibrium allocation of consumers and content providers (in the second stage) that violates a natural refinement criterion.

To see those issues, let us consider the symmetric equilibrium presented in C10, in which:

$$p_i^* = p^* = \pi\left(1 - \frac{\lambda b}{2t}\right);\ q_i^* = q^* = \frac{\lambda b}{2};\ n_i^* = n^* = 1 - \frac{\lambda b}{2t};\ N_i^* = N^* = \frac{\lambda b - q^*}{t} = \frac{\lambda b}{2t}$$

where $i \in \{A, B\}$. Note that non-exclusive content providers are then indifferent between single-homing and multi-homing because $p_i^* = \pi n_i^*$.

Suppose now that platform A deviates from (p^*, q^*) to $\left(p_A^d, q_A^d\right)$ with $q_A^d < q^*$. Then the mass of consumers N_A^d joining platform A is at least equal to $\frac{\lambda b - q_A^d}{t}$ (unless the price p_A^d is so high that platform A's captive content producers choose not to join it, which would never be optimal). As $q_A^d < q^*$, it must be the case that $N_A^d > N^*$. Therefore, the mass of consumers single-homing at platform B, $n_B^d = 1 - N_A^d$, is necessarily strictly less than n^*. This leads to

$$p_B^* > \pi n_B^d,$$

which implies that non-exclusive content providers do not multi-home.

The choice by non-exclusive content providers of the platform they single-home at depends on the prices charged by the two platforms, and their anticipations of what consumers will do. Consider the particular deviations with $\left(p^*, q_A^d\right)$ where q_A^d is smaller but close to q^*. Then, a natural anticipation of content providers is that more consumers will join platform A (which is undercutting platform B on the consumer side and offering the same price on the other side). With these anticipations, all non-exclusive content providers find it optimal to single-home at platform A. If consumers anticipate that, then there will be indeed more consumers joining platform A. More precisely, there will be $N_A^d = \min\left(1, \frac{b - q_A^d}{t}\right) > N^*$ on platform A. Thus, there exists an equilibrium allocation of content providers and consumers in the second stage such that the considered unilateral deviation in the first stage is strictly profitable if q_A^d is sufficiently close to q^*.

The above second-stage equilibrium allocation satisfies the *monotone allocation* refinement criterion defined in Caillaud and Jullien (2003) but it need not be the only second-stage equilibrium allocation following the first-stage deviation. More precisely, the outcome in which all non-exclusive content providers decide to single-home at platform B (because they anticipate that the mass of consumers joining platform A will decrease), and more consumers join platform B (because they anticipate that non-exclusive content providers single-home at platform B) can also be an equilibrium outcome of the second stage. A necessary condition for the candidate equilibrium constructed by C10 to be indeed an equilibrium is that the latter allocation, which does not satisfy the monotone allocation requirement, is selected whenever it is an equilibrium allocation of the second-stage subgame. When this is the case, a profitable deviation by platform A must ensure that this allocation (less favorable to A) is not an equilibrium allocation of the second-stage subgame, which requires reducing q_A by a large amount.

Denoting A3 the following condition:

$$\min\left(\frac{\lambda}{2}, 1-\lambda\right) < \frac{\pi}{b} < \left(3+2\sqrt{2}\right)(1-\lambda),$$

the reasoning above and Appendix A lead to the following statement:

Observation 1: In the absence of tying, the conditions stated for existence of an equilibrium with multi-homing on both sides in C10 are incorrect. If, in addition to the conditions stated there, condition A3 holds then there exists an equilibrium with multi-homing on both sides, but this equilibrium does not satisfy the monotone allocation refinement criterion defined in Caillaud and Jullien (2003).

2.2.2 Equilibrium with multi-homing under tying

We identified another issue in the section of C10 analyzing the equilibrium with tying. There, it is assumed that platform A is able to extract all content providers' surplus by setting a price π for them, while platform B focuses on its captive (i.e., exclusive) content providers and charges πn_B. This, however, ignores the fact that a non-exclusive content provider could decide to reduce his reach and join platform B if the price of B is attractive enough. In particular, if platform B deviates by charging a price slightly below πn_B on the content side, the non-exclusive content providers would join platform B as this would allow them to get a (strictly) positive surplus. Hence, platform B has a profitable deviation. This leads to the following statement:

Observation 2: The equilibrium analysis with tying in C10 is incorrect as it ignores a profitable deviation by platform B.

3 Solutions

3.1 Sequential timing

An alternative timing which is natural for the game considered by C10 is a sequential timing in which content prices are chosen after consumers make their decisionsabout which platform(s) to join (if any). Contrary to the simultaneous timing, the contination outcome is unique under the sequential timing. Moreover, a natural extension of this timing turns out to be useful to address the issue regarding the equilibrium analysis with tying.[1]

Consider the following timing for the game without tying:

0- Platform A sets the price q_M of good M.

1- Platforms set their prices q_A and q_B on the consumer side.

2- Consumers decide whether to join both platforms, one of them, or none.

3- Platforms set their prices p_A and p_B on the content side.

4- Content providers decide whether to join both platforms, one of them, or none.[2]

It can be easily shown that the symmetric equilibrium outcome with multi-homing that leads to Proposition 1 in C10 is an equilibrium outcome under this timing (and the same assumptions A1 and A2). To see why, note first that the reasoning in C10 rules out a profitable unilateral deviation in p_A (or p_B). Moreover, a deviation in q_A under the timing above is less profitable than in C10 (and is therefore unprofitable): if platform A undercuts platform B on the consumer side, this has no effect on platform A's choice of p_A but leads to a decrease in p_B because of the sequentiality in price setting, which makes the deviation profit smaller than that under the simultaneous price-setting environment in C10.

Thus, the equilibrim outcome with multi-homing exhibited in C10 is an equilibrium outcome under our sequential timing.

3.2 Analysis of Tying with Sequential Timing

To address this issue we consider a natural extension of the sequential timing above when the platform ties both products, which is the following:

0- Platform A sets a price $\tilde{q}_{M,A}$ for the bundle.

1- If platform A decided to tie the two products in stage 1, platform B sets a price $\tilde{q}_B$. Otherwise, platforms set simultaneously their prices q_A and q_B on the consumer side.

2- Consumers decide whether to join both platforms, one of them, or none.

3- Platforms set their prices on the content side.

4- Content providers decide whether to join both platforms, one of them, or none.

[1] Both the simultaneous timing in C10 and our sequential timing are reasonable. The reason why we favor the latter is that it is more convenient from a technical perspective.

[2] Each player is supposed to observe the actions of the agents who played in the previous stage(s) if any.

3.2.1 Pricing on the content side

Consider the last stage of pricing (stage 3) assuming that all consumers buy the bundle. The non-exclusive content providers never multi-home as platform A covers the whole population of consumers. They choose between platforms A and B with respective profits

$$\pi - p_A \ \text{ and } \pi \tilde{N}_B - p_B.$$

This leads to mixed strategies by platforms A and B. Let us consider a mixed strategy equilibrium candidate with the distribution of platform B's prices having possibly a mass point at $p_B = \pi \tilde{N}_B$. Let $F_i(.)$ denote platform i's cumulative distribution of prices, where $i = A, B$. It must hold that the upper bound of the distribution of the prices of platform A (resp. platform B) is less than or equal to π (resp. $\pi \tilde{N}_B$).

Then, the platforms' expected profits on the content side are given by

$$\tilde{\Pi}_A^c = p_A \{\lambda + (1-\lambda)) \left(1 - F_B \left(p_A + \pi \tilde{N}_B - \pi\right)\right)\}$$

and

$$\tilde{\Pi}_B^c = p_B \{1 - (1-\lambda) F_A \left(p_B - \pi \tilde{N}_B + \pi\right)\}$$

on their respective supports.

Platform B's expected profit on the content side is

$$\tilde{\Pi}_B^c = \lambda \pi \tilde{N}_B.$$

The property of mixed strategy equilibrium requires that

$$\left(p_A + \pi \tilde{N}_B - \pi\right) \{1 - (1-\lambda) F_A (p_A)\} = \lambda \pi \tilde{N}_B.$$

As a result, we have

$$F_A (p_A) = \frac{1}{1-\lambda} \left(\frac{p_A + \pi \tilde{N}_B - \pi - \lambda \pi \tilde{N}_B}{p_A + \pi \tilde{N}_B - \pi} \right)$$

on the support $[\pi - (1-\lambda) \pi \tilde{N}_B, \pi]$. Moreover, from

$$\tilde{\Pi}_A^c = \left(p_B - \pi \tilde{N}_B + \pi\right) \{1 - (1-\lambda) F_B (p_B)\}$$

it follows that

$$F_B (p_B) = \frac{1}{1-\lambda} \left(\frac{p_B - \pi \tilde{N}_B + \pi - \Pi_A^c}{p_B - \pi \tilde{N}_B + \pi} \right).$$

The lower bound $\underline{p}_B$ of the support of platform B's prices must be such that

$$\pi \tilde{N}_B - \underline{p}_B = \pi - \underline{p}_A,$$

where $\underline{p}_A = \pi - (1-\lambda)\pi \tilde{N}_B$ is the lower bound of the support of platform A's prices. Therefore,

$$\underline{p}_B = \lambda \pi \tilde{N}_B$$

Then, from $F_B\left(\underline{p}_B\right) = 0$, it follows that

$$\tilde{\Pi}_A^c = \pi - (1-\lambda)\pi \tilde{N}_B.$$

Therefore,

$$F_B(p_B) = \frac{1}{1-\lambda}\left(\frac{p_B - \lambda\pi\tilde{N}_B}{p_B - \pi\tilde{N}_B + \pi}\right).$$

The value of this function at the upper bound of the support of platform B's prices, i.e. $\pi\tilde{N}_B$, is[3]

$$F_B\left(\pi\tilde{N}_B\right) = \frac{1}{1-\lambda}\left(\frac{\pi\tilde{N}_B - \lambda\pi\tilde{N}_B}{\pi}\right) = \tilde{N}_B < 1.$$

Hence, the distribution of prices of platform B has a mass of $1 - \tilde{N}_B$ at $\pi\tilde{N}_B$. This completes the characterization of the equilibrium of the pricing on the content side (stage 3).

Given $\tilde{N}_B$, the probability that the non-exclusive content providers join platform B is

$$\begin{aligned}
\kappa\left(\tilde{N}_B\right) &= \Pr(\pi\tilde{N}_B - p_B > \pi - p_A) \\
&= \int_{\lambda\pi\tilde{N}_B}^{\pi\tilde{N}_B}\left(1 - F_A\left(p_B - \pi\tilde{N}_B + \pi\right)\right) f_B(p_B)\, dp_B \\
&= \tilde{N}_B - \int_{\lambda\pi\tilde{N}_B}^{\pi\tilde{N}_B} F_A\left(p_B - \pi\tilde{N}_B + \pi\right) f_B(p_B)\, dp_B,
\end{aligned}$$

where

$$f_B(p_B) = F_B'(p_B) = \frac{1}{1-\lambda}\frac{\pi - \pi\tilde{N}_B + \lambda\pi\tilde{N}_B}{\left(p_B + \pi - \pi\tilde{N}_B\right)^2}.$$

Then,

$$\kappa\left(\tilde{N}_B\right) = \tilde{N}_B - \frac{\pi\left[1 - \left(1-\lambda\right)\tilde{N}_B\right)\right]}{(1-\lambda)^2}\int_{\lambda\pi\tilde{N}_B}^{\pi\tilde{N}_B}\left(\frac{p_B - \lambda\pi\tilde{N}_B}{p_B}\right)\frac{1}{\left(p_B + \pi - \pi\tilde{N}_B\right)^2} dp_B.$$

[3]More precisely, this is $\sup\left\{F_B(p_B) | \, p_B < \pi\tilde{N}_B\right\}$.

The computations in Appendix B show that this probability can be rewritten as

$$\kappa\left(\tilde{N}_B\right)=\tilde{N}_B-\frac{1}{(1-\lambda)^2}\left(1+\frac{\lambda\tilde{N}_B}{1-\tilde{N}_B}\right)\left\{1-\frac{\lambda\tilde{N}_B}{1-\tilde{N}_B}\ln\left(\frac{1-(1-\lambda)\tilde{N}_B}{\lambda}\right)\right\}.$$

3.2.2 Pricing on the consumer side

Consider now platform B's choice of $\tilde{q}_B$ at stage 1, which determines the mass of consumers $\tilde{N}_B$ joining platform B at stage 2. For a given $\tilde{N}_B$, the incremental value that a consumer located at x derives from joining platform B is $\lambda b+(1-\lambda)\,b\kappa\left(\tilde{N}_B\right)-t(1-x)$. Therefore, the price charged to consumers by platform B is related to $\tilde{N}_B$ as follows:

$$\tilde{q}_B=\lambda b+(1-\lambda)\,b\kappa\left(\tilde{N}_B\right)-t\tilde{N}_B.$$

We can therefore write platform B's profit as a function of $\tilde{N}_B$:

$$\tilde{\Pi}_B=\tilde{q}_B\tilde{N}_B+\tilde{\Pi}_B^c=\left[\lambda b+(1-\lambda)\,b\kappa\left(\tilde{N}_B\right)-t\tilde{N}_B+\lambda\pi\right]\tilde{N}_B.$$

Differentiating the latter with respect to $\tilde{N}_B$ gives the equation defining the equilibrium mass $\tilde{N}_B^*$ of consumers joining platform B:

$$2t\tilde{N}_B^*=\lambda\,(b+\pi)+(1-\lambda)\,b\left[\kappa\left(\tilde{N}_B^*\right)+\kappa'\left(\tilde{N}_B^*\right)\tilde{N}_B^*\right].$$

The corresponding equilibrium price on the consumer side is

$$\tilde{q}_B^*=\frac{\lambda b+(1-\lambda)\,b\kappa\left(\tilde{N}_B^*\right)-\lambda\pi-(1-\lambda)\,b\kappa'\left(\tilde{N}_B^*\right)\tilde{N}_B^*}{2}.$$

Remark: The expressions in C10 can be derived from the expressions above by replacing $\kappa\left(\tilde{N}_B^*\right)$ by 0 (i.e. all non-exclusive content providers go to platform A) and $\kappa'\left(\tilde{N}_B^*\right)$ by 0 (because in Choi's timing consumers do not join platforms before platforms set their prices on the content side).

3.2.3 Pricing of the bundle

The lowest expected consumer utility is obtained by the marginal consumer located at $x=1-\tilde{N}_B^*$ and is equal to (when not including the price of the bundle):

$$\tilde{U}^*=v+b-(1-\lambda)\,b\kappa\left(\tilde{N}_B^*\right)-t\left(1-\tilde{N}_B^*\right)=v+b\,(1+\lambda)-t-\tilde{q}_B^*.$$

Thus, under tying, platform A sets the price of the bundle to

$$\tilde{q}_{M,A}^{*} = \tilde{U}^{*} = v + b\left(1+\lambda\right) - t - \tilde{q}_{B}^{*}$$

assuming that v is high enough for the tying platform to find it optimal to sell the bundle to all consumers.

3.2.4 Incentives for tying

Platform A's equilibrium (overall) profit under tying is

$$\tilde{\Pi}_{M}^{*} = v + b\left(1+\lambda\right) - t - \tilde{q}_{B}^{*} + \pi - \pi\tilde{N}_{B}^{*} + \lambda\pi\tilde{N}_{B}^{*} - c_{M}.$$

Since platform A's equilibrium profit without tying is

$$\Pi_{M}^{*} = v + b - t + \pi - \pi\frac{\lambda b}{2t} + \frac{\left(\lambda b\right)^{2}}{4t} - c_{M}$$

we have

$$\begin{aligned}
\tilde{\Pi}_{M}^{*} - \Pi_{M}^{*} &= \lambda b - \tilde{q}_{B}^{*} - \pi\tilde{N}_{B}^{*} + \lambda\pi\tilde{N}_{B}^{*} + \frac{\lambda b\pi}{2t} - \frac{\left(\lambda b\right)^{2}}{4t} \\
&= \frac{\lambda b}{2} + \frac{\lambda\pi}{2} + \frac{\lambda b\pi}{2t} - \frac{\left(\lambda b\right)^{2}}{4t} - \left(1-\lambda\right)\left(b\frac{\kappa\left(\tilde{N}_{B}^{*}\right) - \kappa'\left(\tilde{N}_{B}^{*}\right)\tilde{N}_{B}^{*}}{2} + \pi\tilde{N}_{B}^{*}\right)
\end{aligned}$$

while the corresponding difference between profits in C10 is

$$\frac{\lambda b}{2} + \frac{\lambda\pi}{2} + \frac{\lambda b\pi}{2t} - \frac{\left(\lambda b\right)^{2}}{4t}.$$

An analytical comparison of $\tilde{\Pi}_{M}^{*}$ and Π_{M}^{*} is much more complicated than in C10's analysis because $\tilde{N}_{B}^{*}$ does not have a closed-form expression in our setting. We therefore perform simulations at the end of our analysis to determine the set of parameters under which tying is profitable.

3.2.5 Welfare analysis

Social welfare (net of v) under tying is

$$\begin{aligned}
\tilde{W} &= \left\{\left(1-\tilde{N}_{B}^{*}\right)\left[\lambda + \left(1-\lambda\right)\left(1-\kappa\left(\tilde{N}_{B}^{*}\right)\right)\right] + \tilde{N}_{B}^{*}\left(1+\lambda\right)\right\} b \\
&\quad - \left(\int_{0}^{1-\tilde{N}_{B}^{*}} tx dx + t\tilde{N}_{B}^{*}\right) + \left[\lambda + \lambda\tilde{N}_{B}^{*} + \left(1-\lambda\right)\left(1-\kappa\left(\tilde{N}_{B}^{*}\right) + \kappa'\left(\tilde{N}_{B}^{*}\right)\tilde{N}_{B}^{*}\right)\right]\pi
\end{aligned}$$

while in the absence of tying social welfare is given by

$$W = (1 + n_M^* \lambda) b - \left[\int_0^{1-N_A^*} tx dx + \int_0^{1-N_B^*} tx dx + t n_M^* \right] + \left[\lambda \left(N_A^* + N_B^* \right) + 1 - \lambda \right] \pi$$

where $N_A^* = N_B^* = \frac{\lambda b}{2t}$ and $n_M^* = \frac{\lambda b}{t} - 1$.

Again, the analytical comparison of social welfare with and without tying is very complicated, which requires us to run simulations.

3.2.6 Simulations

We now compute the effect of tying on profits $\tilde{\Pi}_M^* - \Pi_M^*$ and its effect on social welfare $\tilde{W} - W$ for a large number of discrete parameter values in the set defined by assumptions A1 and A2. More precisely, we normalize b to 1 (without any loss of generality) and consider values of λ, π and t such that

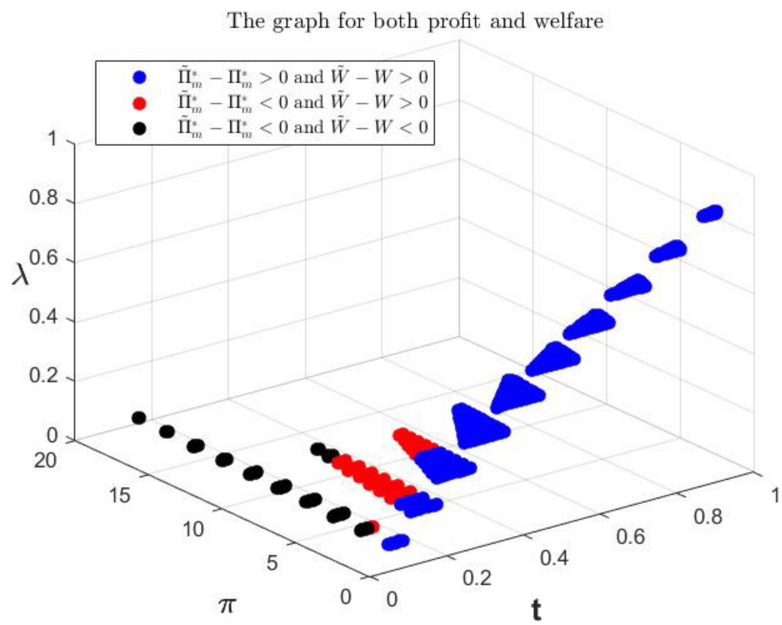
$$\begin{aligned} \lambda &\in (0, 1) \\ \pi &\in \left[0, \frac{2(1-\lambda)}{\lambda} \right) \\ t &\in \left[\frac{\lambda [\lambda \pi + 2(1+\lambda)]}{4}, \lambda \right). \end{aligned}$$

The graph below shows the effects of tying on profits and welfare. Blue (resp., black) depicts the combinations of parameter values such that the effects of tying on welfare and profits are positive (resp., negative), while red depicts the combinations of parameter values such that socially desirable tying does not occur in equilibrium because it is not profitable. The graph shows that there are parameter values for which tying is not profitable, while C10 finds that tying is always profitable under assumptions A1 and A2. However, we do not find any parameter values for which tying is both profitable and welfare-detrimental. In other words, if platform A engages in tying, this always increases social welfare, which is consistent with the main message of C10.[4]

4 Conclusion

We identify two issues in C10's analysis of tying in two-sided markets with multi-homing, and offer a solution to both of them based on an alternative timing. After correcting the two errors we find that, while tying is not always profitable, the main message in C10 remains correct under our alternative timing: with multi-homing on both sides, tying improves social welfare whenever it is profitable.

[4] The details of our simulation results are available upon request.



A Appendix: Existence of an equilibrium with multi-homing under simultaneous pricing

Assume that after a deviation by platform A, the continuation equilibrium where all non-exclusive content providers single-home at platform B prevails if it exists. In that case, the mass of consumers on platform A and B respectively would be

$$\hat{N}_A^d = \frac{\lambda b - q_A^d}{t} \text{ and } \hat{N}_B^d = \frac{b - q^*}{t}.$$

However, platform A can ensure (by charging content providers a price low enough) that this allocation is not an equilibrium allocation. For this to happen, it must be the case that

$$\pi \hat{N}_B^d - p^* \leq \pi \hat{N}_A^d - p_A^d$$

that is,

$$p_A^d \leq \pi \left(1 - \frac{b(1-\lambda)}{t} - \frac{q_A^d}{t} \right).$$

Platform A then finds it optimal to choose

$$p_A^d = \pi \left(1 - \frac{b(1-\lambda)}{t} - \frac{q_A^d}{t} \right)$$

and to choose the value of q_A^d that maximizes its profit

$$\begin{aligned} p_A^d + q_A^d N_A^d &= \pi\left(1-\frac{b(1-\lambda)}{t}-\frac{q_A^d}{t}\right)+q_A^d\frac{b-q_A^d}{t} \\ &= q_A^d\frac{\left(b-\pi-q_A^d\right)}{t}+\pi\left(1-\frac{b(1-\lambda)}{t}\right) \end{aligned}$$

subject to the constraint $q_A^d < q^*$. With a slight abuse of notation (due to an openness problem) that could be avoided at the cost of a longer exposition, the optimal deviation price on the consumer side is given by

$$q_A^d = \begin{cases} q^*-\epsilon & \text{if} \quad \frac{\pi}{b} \leq 1-\lambda \\ \frac{b-\pi}{2} & \text{otherwise} \end{cases}.$$

Then the optimal deviation profit (or, more rigorously the limit of the deviation profits when $\epsilon \longrightarrow 0$) is given by

$$\Pi_A^d = \begin{cases} \frac{\lambda b^2}{2t}\left(1-\frac{\lambda}{2}\right)+\pi\left(1-\frac{\lambda b}{2t}-\frac{(1-\lambda)b}{t}\right) & \text{if} \quad \frac{\pi}{b} \leq 1-\lambda \\ \frac{(b-\pi)^2}{4t}+\pi\left(1-\frac{(1-\lambda)b}{t}\right) & \text{otherwise} \end{cases}.$$

We need to compare this deviation profit to the equilibrium profit

$$\Pi_A^* = \frac{\lambda b^2}{4t}+\pi\left(1-\frac{\lambda b}{2t}\right).$$

Straightforward computations show that, for $\frac{\pi}{b} \leq 1-\lambda$,

$$\frac{\lambda b^2}{2t}\left(1-\frac{\lambda}{2}\right)+\pi\left(1-\frac{\lambda b}{2t}-\frac{(1-\lambda)b}{t}\right) > \Pi_A^* \Longleftrightarrow \frac{\pi}{b} < \frac{\lambda}{2}$$

and, for $\frac{\pi}{b} > 1-\lambda$,

$$\frac{(b-\pi)^2}{4t}+\pi\left(1-\frac{(1-\lambda)b}{t}\right) > \Pi_A^* \Longleftrightarrow \frac{\pi}{b} > \left(3+2\sqrt{2}\right)(1-\lambda).$$

Therefore, the considered deviation is not profitable if and only if

$$\min\left(\frac{\lambda}{2},1-\lambda\right) < \frac{\pi}{b} < \left(3+2\sqrt{2}\right)(1-\lambda).$$

For the candidate equilibrium with muti-homing derived by C10 to be indeed an equilibrium, it must hold that the above condition is satisfied, in addition to the conditions A1 and A2 defined in C10.[5]

[5] A1 stipulates that $\lambda b - d > t$ and A2 stipulates that $\frac{\lambda[\lambda\pi+2b(1-\lambda)]}{4} \leq t$.

B Appendix: Computation of $\kappa\left(\tilde{N}_B\right)$

We have

$$\kappa\left(\tilde{N}_B\right) = \tilde{N}_B - \frac{\pi\left[1-\left(1-\lambda\right)\tilde{N}_B\right)\right]}{\left(1-\lambda\right)^2} \underbrace{\int_{\lambda\pi\tilde{N}_B}^{\pi\tilde{N}_B}\left(\frac{p_B-\lambda\pi\tilde{N}_B}{p_B}\right)\frac{1}{\left(p_B+\pi-\pi\tilde{N}_B\right)^2}dp_B}_{\equiv I}.$$

We can rewrite I as

$$I = \int_{\lambda\pi\tilde{N}_B}^{\pi\tilde{N}_B}\left(\frac{1}{\left(p_B+\pi-\pi\tilde{N}_B\right)^2} - \frac{\lambda\pi\tilde{N}_B}{p_B\left(p_B+\pi-\pi\tilde{N}_B\right)^2}\right)dp_B.$$

Using the following decomposition in irreducible rational fractions

$$\frac{\lambda\pi\tilde{N}_B}{p_B\left(p_B+\pi-\pi\tilde{N}_B\right)^2} = \frac{\lambda\pi\tilde{N}_B}{\left(\pi-\pi\tilde{N}_B\right)^2}\left[\frac{1}{p_B} - \frac{1}{p_B+\pi-\pi\tilde{N}_B} - \frac{\pi-\pi\tilde{N}_B}{\left(p_B+\pi-\pi\tilde{N}_B\right)^2}\right],$$

we can derive

$$\begin{aligned}
I &= \int_{\lambda\pi\tilde{N}_B}^{\pi\tilde{N}_B}\left\{\left(1+\frac{\lambda\pi\tilde{N}_B}{\pi-\pi\tilde{N}_B}\right)\frac{1}{\left(p_B+\pi-\pi\tilde{N}_B\right)^2} + \frac{\lambda\pi\tilde{N}_B}{\left(\pi-\pi\tilde{N}_B\right)^2}\left(\frac{1}{p_B+\pi-\pi\tilde{N}_B}-\frac{1}{p_B}\right)\right\}dp_B \\
&= \frac{1}{\pi-\pi\tilde{N}_B}\int_{\lambda\pi\tilde{N}_B}^{\pi\tilde{N}_B}\left\{\frac{\pi\left[1-\left(1-\lambda\right)\tilde{N}_B\right)\right]}{\left(p_B+\pi-\pi\tilde{N}_B\right)^2} + \frac{\lambda\tilde{N}_B}{1-\tilde{N}_B}\left(\frac{1}{p_B+\pi-\pi\tilde{N}_B}-\frac{1}{p_B}\right)\right\}dp_B.
\end{aligned}$$

From

$$\int_{\lambda\pi\tilde{N}_B}^{\pi\tilde{N}_B}\frac{1}{\left(p_B+\pi-\pi\tilde{N}_B\right)^2}dp_B = \frac{1}{\pi}\left(\frac{1}{1-\left(1-\lambda\right)n_B}\right)$$

$$\int_{\lambda\pi\tilde{N}_B}^{\pi\tilde{N}_B}\frac{1}{\left(p_B+\pi-\pi\tilde{N}_B\right)}dp_B = \ln\pi - \ln\left(\lambda\pi\tilde{N}_B+\pi-\pi\tilde{N}_B\right) = -\ln\left[1-\left(1-\lambda\right)\tilde{N}_B\right]$$

$$\int_{\lambda\pi\tilde{N}_B}^{\pi\tilde{N}_B}\frac{1}{p_B}dp_B = -\ln\lambda$$

it follows that

$$I = \frac{1}{\pi\left(1-\tilde{N}_B\right)}\left\{1-\frac{\lambda\tilde{N}_B}{1-\tilde{N}_B}\ln\left(\frac{1-\left(1-\lambda\right)\tilde{N}_B}{\lambda}\right)\right\}$$

We thus have

$$\begin{aligned}
\kappa\left(\tilde{N}_B\right) &= \tilde{N}_B - \frac{\left[1-\left(1-\lambda\right)\tilde{N}_B\right)\right]}{(1-\lambda)^2\left(1-\tilde{N}_B\right)}\left\{1-\frac{\lambda\tilde{N}_B}{1-\tilde{N}_B}\ln\left(\frac{1-(1-\lambda)\,\tilde{N}_B}{\lambda}\right)\right\} \\
&= \tilde{N}_B - \frac{1}{(1-\lambda)^2}\left(1+\frac{\lambda\tilde{N}_B}{1-\tilde{N}_B}\right)\left\{1-\frac{\lambda\tilde{N}_B}{1-\tilde{N}_B}\ln\left(\frac{1-(1-\lambda)\,\tilde{N}_B}{\lambda}\right)\right\}.
\end{aligned}$$

References

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