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Type IIB 7-brane solutions from nine-dimensional domain walls

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Abstract
We investigate half-supersymmetric domain wall solutions of four maximally supersymmetric $D = 9$ massive supergravity theories obtained by Scherk–Schwarz reduction of $D = 10$ IIA and IIB supergravity. One of the theories does not have a superpotential and does not allow domain wall solutions preserving any supersymmetry. The other three theories have superpotentials leading to half-supersymmetric domain wall solutions, one of which has zero potential but non-zero superpotential. The uplifting of these domain wall solutions to ten dimensions leads to three classes of half-supersymmetric type IIB 7-brane solutions. All solutions within each class are related by $SL(2, \mathbb{R})$ transformations. The three classes together contain solutions carrying all possible (quantized) 7-brane charges. One class contains the well-known D7-brane solution and its dual partners and we provide the explicit solutions for the other two classes. The domain wall solution with zero potential lifts up to a half-supersymmetric conical spacetime.

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1. Introduction

Recently, much attention has been given to the study of domain wall solutions in (matter-coupled) supergravity theories. This is due to several reasons. First of all, the possibility of a supersymmetric RS scenario [1, 2] relies on the existence of a special domain wall solution containing a warp factor with the correct asymptotic behaviour such that gravity is suppressed in the transverse direction. Secondly, domain wall solutions play an important role in the AdS/CFT correspondence [3, 4]. A domain wall in D dimensions may describe the renormalization group flow of the corresponding field theory in D-1 dimensions. The geometrical warp factor now plays the role of an energy scale. Finally, domain wall solutions have been applied to cosmology (for some recent papers see, e.g., [5, 6]). In all these cases the properties of the domain wall crucially depend on the detailed properties of the scalar potential.

The highest-dimensional supergravity theory that allows a domain wall solution is the maximally supersymmetric $D = 10$ massive IIA supergravity [7]. This theory is a massive deformation, characterized by a mass parameter m_R, of the massless IIA supergravity theory [8, 9]. The particular domain wall solution, the D8-brane, has been constructed in [10, 11]. In a supersymmetric theory both the scalar potential V as well as the massive deformations in the supersymmetry transformations are often characterized by a superpotential W. In the IIA case the superpotential depends on just one scalar $\hat{\phi}$, the dilaton, and is of a simple exponential form:

$$W(\hat{\phi}) = \tfrac{1}{4}\,\mathrm{e}^{5\hat{\phi}/4} m_R. \tag{1}$$

In general, the lower-dimensional supergravity theories contain more scalars and have, correspondingly, a more complicated superpotential which is difficult to investigate. In fact, the most general form of the superpotential is not always known explicitly. In view of this, it is instructive to consider maximally supersymmetric $D = 9$ massive supergravity theories. These theories on one hand share some of the complications of the lower-dimensional supergravity theories and on the other hand are simple enough to study in full detail.

The most general Scherk–Schwarz reduction [12] of $D = 10$ IIB supergravity has been considered in [13][1]. It leads to $SL(2,\mathbb{R})$-covariant[2] $D = 9$ massive supergravity theories with mass parameters m_1, m_2 and m_3. By $SL(2,\mathbb{R})$ transformations one can go to different mass parameters but the quantity ${m_1}^2 + {m_2}^2 - {m_3}^2$ is always invariant. One therefore has three different theories depending on whether this quantity is positive, negative or zero corresponding to the three different conjugacy classes of $SL(2,\mathbb{R})$ [15]. The supersymmetry transformations of these massive supergravities have been calculated recently [16]. The theory contains three scalars (ϕ, χ, φ) and we find that the superpotential is given by

$$W_{\mathrm{IIB}}(\phi, \chi, \varphi) = \tfrac{1}{4}\mathrm{e}^{2\varphi/\sqrt{7}}\left(m_2 \sinh(\phi) + m_3 \cosh(\phi) + m_1\,\mathrm{e}^{\phi}\chi - \tfrac{1}{2}(m_2 - m_3)\,\mathrm{e}^{\phi}\chi^2\right). \tag{2}$$

The scalar potential is given in terms of this superpotential via the expression that follows from the positive energy requirement [17]:

$$V = 4\left(\gamma^{AB}\frac{\delta W}{\delta\Phi^A}\frac{\delta W}{\delta\Phi^B} - \frac{D-1}{D-2}W^2\right), \tag{3}$$

with $D = 9$ and $\Phi^A = (\phi, \chi, \varphi)$ in this case. Here γ^{AB} is the inverse of the metric γ_{AB} occurring in the kinetic scalar term $-\gamma_{AB}\partial\Phi^A\partial\Phi^B$.

In the IIA case, the situation is more subtle. We find that there are two possibilities. Either one performs an (ordinary) Kaluza–Klein reduction of $D = 10$ massive IIA supergravity. This leads to a $D = 9$ massive supergravity theory which is covered by the above superpotential for the following choice of mass parameters (for more details, see the next section):

$$m_1 = 0, \qquad m_2 = m_3 = m_R. \tag{4}$$

This is the massive T-duality of [11]. The other possibility is to set $m_R = 0$ and perform a generalized Scherk–Schwarz reduction making use of the $SO(1,1)$ symmetry of the action [14]. Since the $SO(1,1)$ symmetry is only valid for $m_R = 0$, i.e. the massive Romans deformation breaks the $SO(1,1)$ symmetry, one cannot perform both reductions at the same time. The Scherk–Schwarz reduction leads to an $SO(1,1)$-covariant $D = 9$ massive

[1] A Scherk–Schwarz reduction leading to two mass parameters has been considered in [14].

[2] Since our analysis below is at the classical level we will work with $SL(2,\mathbb{R})$ instead of $SL(2,\mathbb{Z})$. It is only in section 5 that we will consider the breaking of $SL(2,\mathbb{R})$ to $SL(2,\mathbb{Z})$ at the quantum level.

supergravity containing a single mass parameter m_4. It turns out that in this case the massive deformations cannot be expressed in terms of a superpotential.

In this paper we study the domain wall solutions allowed by the $SL(2,\mathbb{R})$- and $SO(1,1)$-covariant $D=9$ massive supergravities[3]. We find that the $SO(1,1)$-covariant theory has no superpotential and does not allow domain wall solutions. The other three $SL(2,\mathbb{R})$-covariant theories have superpotentials that do allow half-supersymmetric domain wall solutions.

The uplifting of these domain walls to ten dimensions leads to three classes of half-supersymmetric type IIB 7-brane solutions. All solutions within each class are related by $SL(2,\mathbb{R})$ transformations and are characterized by two holomorphic functions. The two functions are restricted by the consistency requirement of yielding equal monodromy for the scalars and the Killing spinors. We have explicitly checked that the solutions of our three classes satisfy this requirement. These solutions give rise to all possible 7-brane charges. One class contains the well-known D7-brane solution and its $SL(2,\mathbb{R})$-related partners. We provide the previously unknown explicit solutions for the other two classes. For each class we show which solutions survive the quantization of $SL(2,\mathbb{R})$ to $SL(2,\mathbb{Z})$.

We find a special domain wall solution, corresponding to a zero potential but non-zero superpotential. This half-supersymmetric domain wall uplifts to either a fully supersymmetric Minkowski spacetime or to half-supersymmetric conical type IIB solutions with deficit angle $3\pi/2$ or $5\pi/3$ and without scalars. The conical solutions have non-trivial monodromy due to the fermionic sector.

This paper is organized as follows. In section 2, we review the IIA theory in ten and nine dimensions and give the domain wall, or D8-brane, solution of $D=10$ massive IIA supergravity. In section 3, we discuss the IIB theory in ten and nine dimensions and give the class of half-supersymmetric 7-brane solutions with two holomorphic functions. In section 4, we discuss the three classes of $D=9$ domain wall solutions and their uplifting to ten dimensions. The quantization conditions on the charges of the 7-branes and mass parameters of the domain walls are discussed in section 5. We will summarize and discuss our results in the conclusions. Our conventions are given in appendix A.

2. IIA supergravity in ten and nine dimensions

2.1. $D=10$ *massive IIA supergravity*

We first consider $D=10$ massive IIA supergravity. This theory contains one scalar, the dilaton $\hat{\phi}$. For our purposes, it is enough to consider only the kinetic terms for the graviton, dilaton and *R–R* vector plus the mass term. In the Einstein frame this part of the Lagrangian reads

$$\mathcal{L}_{\mathrm{mIIA}} = \tfrac{1}{2}\sqrt{-\hat{g}}\big[-\hat{R} - \tfrac{1}{2}(\partial\hat{\phi})^2 - \tfrac{1}{4}\,\mathrm{e}^{3\hat{\phi}/2}(\partial\hat{A})^2 - V\big], \tag{5}$$

with the potential V given by a superpotential:

$$V = 8\left(\frac{\delta W}{\delta\hat{\phi}}\right)^2 - \frac{9}{2}W^2 = \frac{1}{2}\,\mathrm{e}^{5\hat{\phi}/2}m_{\mathrm{R}}^2 \qquad \text{with} \quad W = \frac{1}{4}\,\mathrm{e}^{5\hat{\phi}/4}m_{\mathrm{R}}. \tag{6}$$

The corresponding supersymmetry transformations of the fermions are

$$\begin{aligned} \delta\hat{\psi}_{\hat{\mu}} &= \left(D_{\hat{\mu}} + \frac{1}{64}\,\mathrm{e}^{3\hat{\phi}/4}(\partial\hat{A})^{\hat{\nu}\hat{\rho}}(\hat{\Gamma}_{\hat{\mu}\hat{\nu}\hat{\rho}} - 14\hat{g}_{\hat{\mu}\hat{\nu}}\hat{\Gamma}_{\hat{\rho}})\Gamma_{11} - \frac{1}{8}W\hat{\Gamma}_{\hat{\mu}}\right)\hat{\epsilon}, \\ \delta\hat{\lambda} &= \left(\hat{\partial\!\!\!/}\hat{\phi} + \frac{3}{8}\,\mathrm{e}^{3\hat{\phi}/4}(\partial\hat{A})^{\hat{\mu}\hat{\nu}}\hat{\Gamma}_{\hat{\mu}\hat{\nu}}\Gamma_{11} + 4\frac{\delta W}{\delta\hat{\phi}}\right)\hat{\epsilon}, \end{aligned} \tag{7}$$

[3] Domain wall solutions of one of the $SL(2,\mathbb{R})$-covariant theories have been discussed in [18]. We will compare our results with those of [18] in section 4.

Table 1. The $SO(1,1)$ weights of the IIA supergravity fields.

Field	$\hat{g}_{\hat{\mu}\hat{\nu}}$	$\hat{B}_{\hat{\mu}\hat{\nu}}$	$e^{\hat{\phi}}$	$\hat{A}_{\hat{\mu}}$	$\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}$	$\hat{\psi}_{\hat{\mu}}$	$\hat{\lambda}$	m_R
$SO(1,1)$	0	$\frac{1}{2}$	1	$-\frac{3}{4}$	$-\frac{1}{4}$	0	0	$-\frac{5}{4}$

where $D_{\hat{\mu}}\hat{\epsilon} = (\partial_{\hat{\mu}} + \hat{\omega}_{\hat{\mu}})\hat{\epsilon}$ with the spin connection $\hat{\omega}_{\hat{\mu}} = \frac{1}{4}\hat{\omega}_{\hat{\mu}}{}^{\hat{a}\hat{b}}\Gamma_{\hat{a}\hat{b}}$. All spinors $\hat{\psi}_{\hat{\mu}}, \hat{\lambda}, \hat{\epsilon}$ are real Majorana spinors. The above transformation rules are the Einstein-frame version of [19] and coincide with those of [7] up to rescalings. Note that all m_R-dependent terms in both the Lagrangian and transformation rules can be expressed in terms of the superpotential (1).

For $m_R = 0$ the Lagrangian is invariant under $SO(1,1)$ transformations with weights as given in table 1 (we include all IIA fields and use the Einstein frame metric). For $m_R \neq 0$ the Lagrangian is invariant if one also scales the mass parameter m_R as indicated in table 1.

The massive IIA supergravity theory has the D8-brane solution [10, 11]

$$\text{D8:} \qquad \mathrm{d}\hat{s}^2 = H^{1/8}\,\mathrm{d}s_9{}^2 + H^{9/8}\,\mathrm{d}y^2, \qquad e^{\hat{\phi}} = H^{-5/4}, \qquad \text{with} \quad H = 1 + m_R y, \tag{8}$$

where we only consider y such that $1 + m_R y$ is strictly positive in order to have a well-behaved metric. By patching this solution at, e.g., $y = 0$ with another solution having $H = 1 - m_R y$, a two-sided domain wall positioned at $y = 0$ can be obtained. In this paper we will always restrict to one side. The D8-brane solution has the following non-zero spin connections ($\hat{\mu} = (\mu, y)$) (for our conventions on underlined indices, see the appendix):

$$\hat{\omega}_\mu = \tfrac{1}{32}H^{-25/16}\hat{\Gamma}_{\mu\underline{y}}m_R, \qquad \hat{\omega}_y = 0. \tag{9}$$

It satisfies the Killing spinor equations (7) for

$$(1 - \Gamma_{\underline{y}})\hat{\epsilon} = 0 \qquad \text{with} \quad \hat{\epsilon} = H^{1/32}\hat{\epsilon}_0, \quad \hat{\epsilon}_0 \text{ constant.} \tag{10}$$

Thus the D8-brane solution describes a $1/2$ BPS state.

2.2. IIA reduction to nine dimensions

We first consider the ordinary Kaluza–Klein reduction of the massive IIA theory, i.e. $m_R \neq 0$, to nine dimensions. We use the following reduction rules for the bosons (with $a = \frac{1}{8}, b = -\frac{3}{8\sqrt{7}}, c = \frac{3}{4}, d = \frac{\sqrt{7}}{4}$):

$$\begin{aligned} \hat{g}_{\mu\nu} &= e^{a\phi + b\varphi}g_{\mu\nu}, & \hat{\phi} &= c\phi + d\varphi, \\ \hat{g}_{xx} &= e^{-7(a\phi+b\varphi)}, & \hat{A}_x &= -2\chi. \end{aligned} \tag{11}$$

The reduction rules of the fermions are given by

$$\begin{aligned} \hat{\psi}_\mu &= e^{(a\phi+b\varphi)/4}\big(\psi_\mu + \tfrac{1}{4}\Gamma_\mu(a\lambda + b\tilde{\lambda})\big), & \hat{\lambda} &= e^{-(a\phi+b\varphi)/4}(c\lambda + d\tilde{\lambda}), \\ \hat{\psi}_{\underline{x}} &= -\tfrac{7}{4}\Gamma_{\underline{x}}\,e^{-(a\phi+b\varphi)/4}(a\lambda + b\tilde{\lambda}), & \hat{\epsilon} &= e^{(a\phi+b\varphi)/4}\epsilon. \end{aligned} \tag{12}$$

The scalar dependence is put in such a way that we obtain the conventional form of the $D = 9$ supersymmetry rules corresponding to a standard kinetic term for the $D = 9$ graviton and gravitini. This also explains the mixing between ψ_μ and $\tilde{\lambda}$ in the first line, which implies that only $\delta\tilde{\lambda}$ contains $\partial\varphi$ terms. Thus the massive IIA theory (5) reduces to the $D = 9$ massive Lagrangian

$$\mathcal{L} = \tfrac{1}{2}\sqrt{-g}\big[-R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}(\partial\varphi)^2 - \tfrac{1}{2}e^{2\phi}(\partial\chi)^2 - V\big], \tag{13}$$

with the potential V given by a superpotential:

$$V = 8\left(\frac{\delta W}{\delta\phi}\right)^2 + 8\left(\frac{\delta W}{\delta\varphi}\right)^2 - \frac{32}{7}W^2 = \frac{1}{2}\,\mathrm{e}^{2\phi+4\varphi/\sqrt{7}}m_{\mathrm{R}}^2 \qquad \text{with} \quad W = \frac{1}{4}\,\mathrm{e}^{\phi+2\varphi/\sqrt{7}}m_{\mathrm{R}}. \tag{14}$$

The supersymmetry rules (7) reduce to

$$\begin{aligned}
\delta\psi_\mu &= \left(D_\mu + \frac{1}{4}\,\mathrm{e}^{\phi}\partial_\mu\chi\,\Gamma_{\underline{x}}\Gamma_{11} - \frac{1}{7}W\Gamma_\mu\right)\epsilon,\\
\delta\lambda &= \left(\partial\!\!\!/\phi - \mathrm{e}^{\phi}\partial\!\!\!/\chi\,\Gamma_{\underline{x}}\Gamma_{11} + 4\frac{\delta W}{\delta\phi}\right)\epsilon,\\
\delta\tilde{\lambda} &= \left(\partial\!\!\!/\varphi + 4\frac{\delta W}{\delta\varphi}\right)\epsilon.
\end{aligned} \tag{15}$$

Note that the massive deformations of both the Lagrangian and supersymmetry rules can be given in terms of a superpotential, as in ten dimensions. Later, in section 3, we will see that the above Lagrangian and transformation rules can also be obtained via a particular Scherk–Schwarz reduction of $D = 10$ IIB supergravity.

There is another massive 9D theory that can be obtained from reducing IIA supergravity. To obtain this 9D theory one has to use the $SO(1,1)$ scale symmetry of the 10D theory [14]. This symmetry implies the consistency of the generalized reduction rules with a specific x-dependence of the 10D fields, depending on their $SO(1,1)$ weights. This introduces a new mass parameter, which we call m_4, upon reduction to nine dimensions. Since the $SO(1,1)$ symmetry is broken by nonzero m_{R} (unless one scales it), the generalized reduction is only applicable to massless 10D IIA supergravity. The generalized reduction rules read

$$\begin{aligned}
\hat{g}_{\mu\nu} &= \mathrm{e}^{a\phi+b\varphi}g_{\mu\nu}, &\qquad \hat{\phi} &= c\phi + d\varphi + m_4 x,\\
\hat{g}_{xx} &= \mathrm{e}^{-7(a\phi+b\varphi)}, &\qquad \hat{A}_x &= -2\,\mathrm{e}^{-3m_4x/4}\chi,
\end{aligned} \tag{16}$$

with a, b, c, d as above and with the fermionic reduction rules as in the previous case, independent of x. Thus we find the following $SO(1,1)$-covariant $D = 9$ massive Lagrangian[4]:

$$\mathcal{L} = \tfrac{1}{2}\sqrt{-g}\left[-R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}(\partial\varphi)^2 - \tfrac{1}{2}\,\mathrm{e}^{2\phi}(\partial\chi)^2 - \tfrac{1}{2}\,\mathrm{e}^{\phi-3\varphi/\sqrt{7}}m_4^2\right] \tag{17}$$

with the supersymmetry rules

$$\begin{aligned}
\delta\psi_\mu &= \left(D_\mu + \tfrac{1}{4}\,\mathrm{e}^{\phi}\partial_\mu\chi\,\Gamma_{\underline{x}}\Gamma_{11}\right)\epsilon,\\
\delta\lambda &= \left(\partial\!\!\!/\phi - \mathrm{e}^{\phi}\partial\!\!\!/\chi\,\Gamma_{\underline{x}}\Gamma_{11} + \tfrac{3}{4}m_4\,\mathrm{e}^{\phi/2-3\varphi/2\sqrt{7}}\Gamma_{\underline{x}}\right)\epsilon,\\
\delta\tilde{\lambda} &= \left(\partial\!\!\!/\varphi + \tfrac{\sqrt{7}}{4}m_4\,\mathrm{e}^{\phi/2-3\varphi/2\sqrt{7}}\Gamma_{\underline{x}}\right)\epsilon.
\end{aligned} \tag{18}$$

A peculiar feature of this massive 9D theory is that the potential does not have a corresponding superpotential.

[4] Strictly speaking, the Lagrangian below is only $SO(1,1)$-invariant for $m_4 = 0$. To obtain manifest $SO(1,1)$ invariance one should replace m_4 by a scalar field via a Lagrange multiplier (see the conclusions). The same remark applies to the $SL(2,\mathbb{R})$-covariant massive supergravity theory of section 4.

3. IIB supergravity in ten and nine dimensions

3.1. $D = 10$ IIB supergravity

We next consider $D = 10$ IIB supergravity. This theory has two scalars, a dilaton $\hat{\phi}$ and an axion $\hat{\chi}$. We truncate to the gravity-scalar part. This part of the Lagrangian reads in the Einstein frame:

$$\begin{aligned}\mathcal{L}_{\mathrm{IIB}} &= \tfrac{1}{2}\sqrt{-\hat{g}}\big[-\hat{R} - \tfrac{1}{2}(\partial\hat{\phi})^2 - \tfrac{1}{2}\,\mathrm{e}^{2\hat{\phi}}(\partial\hat{\chi})^2\big]\\ &= \tfrac{1}{2}\sqrt{-\hat{g}}\big[-\hat{R} + \tfrac{1}{4}\mathrm{Tr}(\partial\hat{\mathcal{M}}\partial\hat{\mathcal{M}}^{-1})\big].\end{aligned} \tag{19}$$

The two scalars $\hat{\phi}$ and $\hat{\chi}$ parametrize an $SL(2,\mathbb{R})/SO(2)$ coset space as follows:

$$\hat{\mathcal{M}} = \mathrm{e}^{\hat{\phi}}\begin{pmatrix}|\hat{\tau}|^2 & \hat{\chi}\\ \hat{\chi} & 1\end{pmatrix} \qquad \text{with} \quad \hat{\tau} = \hat{\chi} + \mathrm{i}\mathrm{e}^{-\hat{\phi}}. \tag{20}$$

The $SL(2,\mathbb{R})$ duality acts in the following way:

$$\hat{\mathcal{M}} \to \Omega\hat{\mathcal{M}}\Omega^T, \qquad \text{or} \qquad \hat{\tau} \to \frac{a\hat{\tau}+b}{c\hat{\tau}+d}, \qquad \text{with} \quad \Omega = \begin{pmatrix}a & b\\ c & d\end{pmatrix} \in SL(2,\mathbb{R}). \tag{21}$$

For later use we give the two elements whose products span $SL(2,\mathbb{Z})$:

$$S = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}, \qquad T = \begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}. \tag{22}$$

The Einstein frame metric is $SL(2,\mathbb{R})$-invariant. The corresponding truncated supersymmetry variations of the fermions read

$$\delta\hat{\psi}_{\hat{\mu}} = \Big(D_{\hat{\mu}} + \tfrac{1}{4}\,\mathrm{i}\mathrm{e}^{\hat{\phi}}\partial_{\hat{\mu}}\hat{\chi}\Big)\,\hat{\epsilon}, \qquad \delta\hat{\lambda} = (\partial\!\!\!/\hat{\phi} + \mathrm{i}\mathrm{e}^{\hat{\phi}}\partial\!\!\!/\hat{\chi})\hat{\epsilon}^*, \tag{23}$$

where $D_{\hat{\mu}}\hat{\epsilon} = (\partial_{\hat{\mu}} + \hat{\omega}_{\hat{\mu}})\hat{\epsilon}$ with $\hat{\omega}_{\hat{\mu}} = \frac{1}{4}\hat{\omega}_{\hat{\mu}}{}^{\hat{a}\hat{b}}\Gamma_{\hat{a}\hat{b}}$ the spin connection. All spinors $\hat{\psi}_{\hat{\mu}}, \hat{\lambda}, \hat{\epsilon}$ are complex Weyl spinors. The fermions transform under the $SL(2,\mathbb{R})$ transformation (21) as [11][5]

$$\hat{\psi}_{\hat{\mu}} \to \left(\frac{c\hat{\tau}^*+d}{c\hat{\tau}+d}\right)^{1/4}\hat{\psi}_{\hat{\mu}}, \qquad \hat{\lambda} \to \left(\frac{c\hat{\tau}^*+d}{c\hat{\tau}+d}\right)^{3/4}\hat{\lambda}, \qquad \hat{\epsilon} \to \left(\frac{c\hat{\tau}^*+d}{c\hat{\tau}+d}\right)^{1/4}\hat{\epsilon}. \tag{24}$$

In particular, they are invariant under the shift symmetry $\hat{\chi} \to \hat{\chi} + b$ which has $a = d = 1$ and $c = 0$ and the scale symmetry $\hat{\tau} \to a^2\hat{\tau}$ which has $d = a^{-1}$ and $b = c = 0$.

3.2. Half-supersymmetric 7-brane solutions

The $D = 10$ IIB supergravity theory allows for a family of 1/2-supersymmetric 7-brane solutions containing two functions f and g, which are seperately (anti-)holomorphic. For notational clarity we will always take both f and g to be holomorphic. In these solutions the scalar $\hat{\tau}$ is given by the function f [20, 21]. This function determines the monodromy of the scalars. The second function g appears only in the metric and in the Killing spinor. The monodromy of the Killing spinor is determined by f and g. The requirement that the monodromies of the scalars and the Killing spinor coincide puts restrictions on f and g. The function g can always be transformed by a holomorphic coordinate transformation[6] and only affects global issues such as monodromy and deficit angle. The occurrence of this function

[5] Note that the duality transformations of both the scalars and the fermions do not change if we replace Ω by $-\Omega$. Therefore these fields transform under $PSL(2,\mathbb{R})$. From now on we will only consider group elements Ω that are continuously connected to the unit element.

[6] This and related issues have been discussed independently by Tomás Ortín in unpublished notes.

in the metric was already considered in [13, 20]. The general solution with two holomorphic functions reads[7]

$$\hat{\mathrm{d}s}^2 = \mathrm{d}s_8^2 + \mathrm{Im}(f)\,\mathrm{e}^{-\mathrm{Re}(g)}\,\mathrm{d}z\,\mathrm{d}\overline{z}, \qquad \hat{\tau} = f, \tag{25}$$

with the holomorphicity conditions $\partial_{\overline{z}} f = \partial_{\overline{z}} g = 0$. The general 7-brane solution (25) has the spin connection ($\hat{\mu} = (\mu, z, \overline{z})$):

$$\hat{\omega}_\mu = 0, \qquad \hat{\omega}_i = \tfrac{1}{2}\Gamma_{\underline{ij}}(\mathrm{Im}(f)^{-1}\partial_i\,\mathrm{Im}(f) - \partial_i\,\mathrm{Re}(g)), \qquad i, j = (z, \overline{z}). \tag{26}$$

Solution (25) satisfies the Killing spinor equations (23) for

$$\Gamma_{\underline{z}}\hat{\epsilon} = 0 \qquad \text{with} \quad \hat{\epsilon} = \mathrm{e}^{\mathrm{i}\,\mathrm{Im}(g)/4}\hat{\epsilon}_0. \tag{27}$$

Thus the general 7-brane solution preserves $1/2$ of supersymmetry. The special case that $\partial_i \mathrm{Im}(f) = 0$ (thus implying that f is constant) can lead to an enhancement of supersymmetry. This case will be treated at the end of this section.

The holomorphic function $g(z)$ can be eliminated locally from the general 7-brane solution (25) via the holomorphic coordinate transformation

$$z' = \int_{z_0}^{z_0+z} \mathrm{d}\tilde{z}\,\mathrm{e}^{-\xi(\tilde{z})/2} \tag{28}$$

for $\xi(z) = g(z)$. This also transforms the Killing spinor (27) to a spacetime independent constant spinor $\hat{\epsilon}_0$. More generally, given a 7-brane solution with functions $f(z), g(z)$ the holomorphic coordinate transformation (28) gives us an equivalent 7-brane solution with $f'(z') = f(z)$ and $g'(z') = g(z) - \xi(z)$. Note that although the holomorphic function g can be transformed locally it may have implications on global issues such as the monodromy and the deficit angle. We note that the choice $g = -2B_1 \log(z)$ leads to the 7-brane solutions of [22].

Under an $SL(2,\mathbb{R})$ transformation a 7-brane solution (25) is transformed into another member of the same class (25). In particular, under the $SL(2,\mathbb{R})$ transformations (21) the holomorphic functions transform as

$$f \to \frac{af+b}{cf+d}, \qquad g \to g - 2\log(cf+d). \tag{29}$$

This relates for example the D7-brane solution to its S-dual partner, the Q7-brane [13] via an S transformation (22):

$$\text{D7:} \quad \begin{array}{l} f = \mathrm{i}m\log(-\mathrm{i}z) \\ g = 0 \end{array} \quad \xrightarrow{S} \quad \text{Q7:} \quad \begin{array}{l} f = (-\mathrm{i}m\log(-\mathrm{i}z))^{-1} \\ g = -2\log(-\mathrm{i}m\log(-\mathrm{i}z)). \end{array} \tag{30}$$

It is conventional to use polar coordinates for 7-branes: $z = r\,\mathrm{e}^{\mathrm{i}\theta}$ with $0 < r < \infty$ and $0 \leqslant \theta < 2\pi$. The D7-brane given above is an example of this. The 7-brane is located at $r = 0$ and therefore the monodromy is determined by going round in the θ direction [13]. The deficit angle can be determined by going to the Minkowski spacetime locally [23]. For the purpose of dimensional reduction we find it more convenient to use cylindrical coordinates: $z = x + \mathrm{i}y$ with $x \simeq x + 2\pi R$. The monodromy is then determined by the relation between the fields at x and $x + 2\pi R$. The cylindrical coordinates z are related to the polar coordinate z' by the holomorphic coordinate transformation (28) where $\xi(z) = 2\mathrm{i}z/R + 2\log(R)$. Thus the cylindrical D7-brane scalars $f'(z') = \mathrm{i}m'\log(-\mathrm{i}z')$ read in polar coordinates $f(z) = mz$ with $m = m'/R$ [11]. From now on we will use cylindrical coordinates unless explicitly indicated otherwise.

[7] The solutions we consider generically do not have finite energy. To obtain a globally well-defined, finite-energy solution one should use the so-called $j(\tau)$-function as explained in [20].

The $SL(2,\mathbb{R})$ monodromy of the scalars and the Killing spinors corresponding to the general solution (25) can be inferred from the relation between the fields at x and $x+2\pi R$. From the transformations (21) and (24) we can read off the relations

$$\hat{\tau}(x+2\pi R) = \frac{a\hat{\tau}(x)+b}{c\hat{\tau}(x)+d}, \qquad \hat{\epsilon}(x+2\pi R) = \left(\frac{c\hat{\tau}(x)^*+d}{c\hat{\tau}(x)+d}\right)^{1/4}\hat{\epsilon}(x). \quad (31)$$

It is convenient to parametrize the monodromy matrix Λ by

$$\Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \mathrm{e}^{2\pi RC} \qquad \text{with} \quad C = \tfrac{1}{2}\begin{pmatrix} m_1 & m_2+m_3 \\ m_2-m_3 & -m_1 \end{pmatrix}, \quad (32)$$

where $2\pi RC$ is a linear combination of the three generators of $SL(2,\mathbb{R})$. The constants $\vec{m} = (m_1, m_2, m_3)$ can be seen as the different charges of the 7-brane solution in some basis [13]. These charges are determined by the monodromy of the function $f(z)$. For example, the cylindrical D7-brane with $f(z) = mz$ leads to the monodromy relations

$$f(z+2\pi R) = f(z) + 2\pi mR \quad \Rightarrow \quad \Lambda = \begin{pmatrix} 1 & 2\pi mR \\ 0 & 1 \end{pmatrix} \quad \Rightarrow \quad \vec{m} = (0, m, m). \quad (33)$$

using (21) and (32).

Acting with an $SL(2,\mathbb{R})$ transformation (21) on the scalars amounts to the transformation of the monodromy matrix

$$\Lambda \to \Omega\Lambda\Omega^{-1}, \qquad \text{or} \qquad C \to \Omega C\Omega^{-1}. \quad (34)$$

Note that this leaves $\alpha^2 = -\det(C) = \frac{1}{4}\left({m_1}^2 + {m_2}^2 - {m_3}^2\right)$ invariant. Thus all $SL(2,\mathbb{R})$-related 7-brane solutions have the same value of α^2. Thus for the D7-brane and for all other 7-branes related to the D7-brane via an $SL(2,\mathbb{R})$ transformation we find $\alpha^2 = 0$. In section 4 we will see that the uplifting of certain $D = 9$ domain wall solutions will give us examples of 7-brane solutions with α^2 positive and negative as well.

Let us finally comment on the case of constant scalars, i.e. constant f. Solution (25) then becomes purely gravitational and has a second Killing spinor given by

$$\Gamma_{\underline{\bar{z}}}\hat{\epsilon} = 0 \qquad \text{with} \quad \hat{\epsilon} = \mathrm{e}^{-\mathrm{i}\,\mathrm{Im}(g)/4}\hat{\epsilon}_0. \quad (35)$$

The two Killing spinors build up a full $N = 2$ spinor. However, for the gravitational solution with constant f to have unbroken supersymmetry one must require equal monodromies for the two Killing spinors. The gravitational solution can be related locally to a Minkowski spacetime via the coordinate transformation (28) with $\xi(z) = g(z)$ but global issues may prevent the identification with the Minkowski spacetime. This depends on the boundary conditions on g. We will see an explicit example of this in section 6 where we will encounter a half-supersymmetric conical spacetime solution.

3.3. IIB reduction to nine dimensions

We now derive the relevant part of the $SL(2,\mathbb{R})$-covariant $N = 2$, $D = 9$ massive supergravity theories by performing a generalized Scherk–Schwarz reduction of the truncated IIB supergravity Lagrangian (19). For more details, see [13, 16]. To be specific, we make the following IIB reduction ansätze ($\hat{\mu} = (\mu, x)$):

$$\hat{g}_{\mu\nu} = \mathrm{e}^{\sqrt{7}\varphi/14} g_{\mu\nu}, \qquad \hat{g}_{xx} = \mathrm{e}^{-\sqrt{7}\varphi/2}, \qquad \hat{\mathcal{M}} = \Omega(x)\mathcal{M}\Omega(x)^T, \quad (36)$$

where we have given the $D = 10$ dilaton $\hat{\phi}$ and axion $\hat{\chi}$ an x-dependence via the $SL(2,\mathbb{R})$ element[8]

$$\Omega(x) = \mathrm{e}^{xC} = \begin{pmatrix} \cosh(\alpha x) + \frac{m_1}{2\alpha}\sinh(\alpha x) & \frac{m_2+m_3}{2\alpha}\sinh(\alpha x) \\ \frac{m_2-m_3}{2\alpha}\sinh(\alpha x) & \cosh(\alpha x) - \frac{m_1}{2\alpha}\sinh(\alpha x) \end{pmatrix} \tag{37}$$

with α and C defined in the previous subsection. Note that this reduction ansatz implies the identification of the monodromy matrix of 7-brane solutions in 10D with the mass matrix of domain walls in 9D. Thus the charges of the 7-branes provide the masses of the domain walls upon reduction [13].

These reduction ansätze lead to the following truncated $N = 2$, $D = 9$ $SL(2,\mathbb{R})$-covariant massive supergravity Lagrangian[9]:

$$\mathcal{L}_{9\mathrm{D}} = \tfrac{1}{2}\sqrt{-g}\big[-R + \tfrac{1}{4}\,\mathrm{Tr}(\partial\mathcal{M}\partial\mathcal{M}^{-1}) - \tfrac{1}{2}(\partial\varphi)^2 - V(\phi,\chi,\varphi)\big]. \tag{38}$$

The potential $V(\phi,\chi,\varphi)$ is given by

$$\begin{aligned} V(\phi,\chi,\varphi) &= \frac{1}{2}\,\mathrm{e}^{4\varphi/\sqrt{7}}\,\mathrm{Tr}(C^2 + C\mathcal{M}^{-1}C^T\mathcal{M}), \\ &= 8\left(\frac{\delta W}{\delta\phi}\right)^2 + 8\,\mathrm{e}^{-2\phi}\left(\frac{\delta W}{\delta\chi}\right)^2 + 8\left(\frac{\delta W}{\delta\varphi}\right)^2 - \frac{32}{7}W^2, \end{aligned} \tag{39}$$

with the superpotential $W(\phi,\chi,\varphi)$:

$$W(\phi,\chi,\varphi) = \tfrac{1}{4}\,\mathrm{e}^{2\varphi/\sqrt{7}}\left(m_2\sinh(\phi) + m_3\cosh(\phi) + m_1\,\mathrm{e}^{\phi}\chi - \tfrac{1}{2}(m_2 - m_3)\,\mathrm{e}^{\phi}\chi^2\right). \tag{40}$$

The supersymmetry transformations corresponding to the $D = 9$ massive action (38) follow from reducing the massless $D = 10$ supersymmetry rules (23) with the reduction ansätze

$$\begin{aligned} &\hat{\psi}_\mu = \mathrm{e}^{\sqrt{7}\varphi/56}\left(\frac{c\tau^* + d}{c\tau + d}\right)^{1/4}\left(\psi_\mu + \frac{1}{8\sqrt{7}}\Gamma_\mu\tilde{\lambda}^*\right), \qquad && \hat{\lambda} = \mathrm{e}^{-\sqrt{7}\varphi/56}\left(\frac{c\tau^* + d}{c\tau + d}\right)^{3/4}\lambda, \\ &\hat{\psi}_{\underline{x}} = -\frac{\sqrt{7}}{8}\Gamma_{\underline{x}}\,\mathrm{e}^{-\sqrt{7}\varphi/56}\left(\frac{c\tau^* + d}{c\tau + d}\right)^{1/4}\tilde{\lambda}^*, && \hat{\epsilon} = \mathrm{e}^{\sqrt{7}\varphi/56}\left(\frac{c\tau^* + d}{c\tau + d}\right)^{1/4}\epsilon. \end{aligned} \tag{41}$$

We have given the $D = 10$ fermions an x-dependence via the same $SL(2,\mathbb{R})$ element (37), i.e. the values of c and d in (41) are given by

$$c = \frac{m_2 - m_3}{2\alpha}\sinh(\alpha x), \qquad d = \cosh(\alpha x) - \frac{m_1}{2\alpha}\sinh(\alpha x). \tag{42}$$

The same considerations concerning the e^{φ}-dependence and mixing of ψ_μ and $\tilde{\lambda}^*$ apply as in the IIA case (12). The x-dependence via c and d is put in to ensure that the 9D theory is independent of x. With these reduction ansätze we obtain the following $D = 9$ massive supersymmetry rules:

$$\begin{aligned} \delta\psi_\mu &= \left(D_\mu + \frac{\mathrm{i}}{4}\,\mathrm{e}^{\phi}\partial_\mu\chi + \frac{\mathrm{i}}{7}\Gamma_{\mu\underline{x}}W\right)\epsilon, \\ \delta\lambda &= \left(\partial\!\!\!/\phi + 4\mathrm{i}\Gamma_{\underline{x}}\frac{\delta W}{\delta\phi} + \mathrm{i}\,\mathrm{e}^{\phi}\left(\partial\!\!\!/\chi + 4\,\mathrm{i}\Gamma_{\underline{x}}\,\mathrm{e}^{-2\phi}\frac{\delta W}{\delta\chi}\right)\right)\epsilon^*, \\ \delta\tilde{\lambda} &= \left(\partial\!\!\!/\varphi + 4\,\mathrm{i}\Gamma_{\underline{x}}\frac{\delta W}{\delta\varphi}\right)\epsilon^*, \end{aligned} \tag{43}$$

with the superpotential given by (40). These were also derived in [16].

[8] The precise rule for assigning the x-dependence is: (i) replace Ω by $\Omega(x)$ in the $D = 10$ $SL(2,\mathbb{R})$ transformation rule and (ii) replace the $D = 10$ fields occurring in the transformation rule by x-independent $D = 9$ fields.

[9] Strictly speaking the $D = 9$ Lagrangian is also covariant under an additional $SO(1,1)$ which acts on the scalars as $\varphi' = \varphi + c$ for constant c. Therefore the full symmetry group is $GL(2,\mathbb{R}) = SL(2,\mathbb{R}) \otimes SO(1,1)$.

The inclusion of the three mass parameters breaks the $SL(2,\mathbb{R})$ invariance. Rather, the duality transformation now maps between theories with different mass parameters:

$$C \to (\Omega^T)^{-1} C \Omega^T. \tag{44}$$

It is in this sense that the theory is covariant under $SL(2,\mathbb{R})$ transformations. Note that the relation is of the same form as (34): in fact the duality relations between 7-branes in 10D and domain walls in 9D is identical. Again this transformation preserves $\alpha^2 = \frac{1}{4}\big({m_1}^2 + {m_2}^2 - {m_3}^2\big)$. Thus one must distinguish three different theories depending on whether α^2 is positive, negative or zero corresponding to the three different conjugacy classes of $SL(2,\mathbb{R})$ [15]. For each class it is convenient to make a specific choice of basis for $\vec{m} = (m_1, m_2, m_3)$. For later use we give the explicit form of the potential in each class:

$$\begin{aligned}
&\text{class I:} \quad \alpha^2 = 0, \quad \vec{m} = (0, m, m): \quad V(\phi, \varphi, \chi) = \tfrac{1}{2}\, \mathrm{e}^{4\varphi/\sqrt{7}+2\phi} m^2, \\
&\text{class II:} \quad \alpha^2 > 0, \quad \vec{m} = (m, 0, 0): \quad V(\phi, \varphi, \chi) = \tfrac{1}{2}\, \mathrm{e}^{4\varphi/\sqrt{7}} (1 + \mathrm{e}^{2\phi}\chi^2) m^2, \\
&\text{class III:} \quad \alpha^2 < 0, \quad \vec{m} = (0, 0, m): \quad V(\phi, \varphi, \chi) = \tfrac{1}{2}\, \mathrm{e}^{4\varphi/\sqrt{7}} (\sinh^2(\phi) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + \chi^2(2 + \mathrm{e}^{2\phi}(2 + \chi^2))) m^2.
\end{aligned} \tag{45}$$

Comparing with the IIA results one finds that for the values $\vec{m} = (0, m_{\mathrm{R}}, m_{\mathrm{R}})$ (class I) the reduction of IIB supergravity equals the reduction of massive IIA supergravity. Also the superpotentials and hence the supersymmetry transformations are equal for these values of the mass parameters. This corresponds to the massive T-duality between the D8-brane solution (8) and the D7-brane solution [11]. The other massive deformation of IIA, coming from the $SO(1,1)$ scale symmetry, cannot be reproduced by the IIB reduction. This is obvious from the lack of a superpotential at the IIA side. Thus one can construct four different massive deformations of $D = 9$, $N = 2$ supergravity from considering both its IIA and IIB origin.

4. Domain wall solutions and their upliftings

We are now ready to investigate domain wall solutions for the three classes of nine-dimensional massive supergravity theories coming from the IIB side (classes I–III) and the massive supergravity theory coming from the IIA side (class IV). We do not consider seperately the theory obtained by reducing 10D massive IIA supergravity since, as mentioned above, this 9D theory coincides with class I if we set $m_2 = m_3 = m_R$ and $m_1 = 0$.

We will start by constructing half-supersymmetric solutions to the Killing spinor equations in nine dimensions that follow from the supersymmetry rules (43) and (18). Our only input will be a domain wall ansatz; i.e., we assume a diagonal $8+1$ split of the metric with all fields depending only on the single transverse y direction. These solutions automatically define half-supersymmetric domain wall solutions to the full equations of motion. After that we will uplift these solutions to ten dimensions. We find that all 10D 7-branes fall in the general class (25) with two holomorphic functions, as indicated in table 2.[10] The $D = 9$ domain walls correspond to the potential (39) with mass parameters $\vec{m} = (m_1, m_2, m_3)$. These mass parameters automatically define the charge of the $D = 10$ 7-brane solutions [13].

We find that there are three independent 7-brane solutions carried by scalars D7, R7 and T7. These cannot be related by $SL(2,\mathbb{R})$ transformations since their charges give rise to different $SL(2,\mathbb{R})$-invariant α^2. Unlike the well-studied D7-brane solution and its $SL(2,\mathbb{R})$-related partners, the R7- and T7-branes are new solutions which in the present context occur on the same footing as the D7-brane. We also find a G7 domain wall solution which has vanishing

[10] For clarity we have taken the constants C_1 and C_2 which appear later equal to $C_1 = 1$ and $C_2 = 0$.

Table 2. The table indicates the different solutions for the three classes. It gives the $\vec{m}$ charges and the functions $f(z)$ and $g(z)$ of the $D = 10$ 7-brane solutions that follow from uplifting of the $D = 9$ domain walls.

Class	α^2	$\vec{m}$		$f(z)$	$g(z)$
I	0	$(0, m, m)$	D7:	mz	0
II	$\frac{1}{2}m^2$	$(m, 0, 0)$	R7:	$\mathrm{i}\,\mathrm{e}^{mz}$	mz
III	$-\frac{1}{2}m^2$	$(0, 0, m)$	T7:	$\tan(\frac{1}{2}mz)$	$-2\log(\cos(\frac{1}{2}mz))$
			G7:	i	$\mathrm{i}mz$

potential but non-vanishing superpotential. It can be uplifted to a half-supersymmetic conical spacetime without scalars but with Killing spinors, giving rise to a non-trivial monodromy.

In this section we will present the explicit form of the solutions, both in $D = 9$ and $D = 10$, corresponding to the charges given in table 2.

4.1. Class I: $\alpha^2 = 0$

We find the following half-supersymmetric domain wall solution:

$$\mathrm{DW_I}: \quad \begin{cases} \mathrm{d}s^2 = (C_1 m y)^{1/7}\,\mathrm{d}s_8^2 + (C_1 m y)^{8/7}\,\mathrm{d}y^2, \\ \mathrm{e}^{\phi} = (my)^{-1}, \qquad \mathrm{e}^{\varphi} = (C_1 m y)^{-2/\sqrt{7}}, \qquad \chi = C_2, \end{cases} \tag{46}$$

where the constant C_2 is arbitrary while C_1 is strictly positive. The range of y is such that my is strictly positive in order to have a well-behaved metric. We have used here the freedom of making a reparametrization in the transverse direction in order to make the solution fall into the general class of 7-branes (25) after uplifting to ten dimensions. We can also solve for the Killing spinor giving

$$\epsilon = (my)^{1/28}\epsilon_0, \tag{47}$$

where ϵ_0 is a constant spinor satisfying[11] $(1 - \mathrm{i}\Gamma_{\underline{xy}})\epsilon_0 = 0$.

Uplifting the above domain wall solution to ten dimensions yields (with x being the reduction direction)

$$\mathrm{D7}: \quad \begin{cases} \mathrm{d}\hat{s}^2 = \mathrm{d}s_8^2 + C_1 m y\,(\mathrm{d}x^2 + \mathrm{d}y^2), \\ \mathrm{e}^{\hat{\phi}} = (my)^{-1}, \\ \hat{\chi} = mx + C_2. \end{cases} \tag{48}$$

The uplifted Killing spinor is constant and still satisfies $(1 - \mathrm{i}\Gamma_{\underline{xy}})\hat{\epsilon} = 0$. The scalars and spinors satisfy the monodromy requirement (31) with $a = 1$, $b = 2\pi m R$, $c = 0$ and $d = 1$. We find that this solution is a special case of the general 7-brane solution (25) with

$$f = mz + C_2, \qquad g = -\log(C_1), \tag{49}$$

We can thus identify the two free parameters C_1 and C_2 in the solution as coming from scalings (while keeping mz fixed) and shifts of the coordinates, respectively.

4.2. Class II: $\alpha^2 > 0$

In this class we find the following half-supersymmetric domain wall solution:

$$\mathrm{DW_{II}}: \quad \begin{cases} \mathrm{d}s^2 = (C_1 \cos(my))^{1/7}\,\mathrm{d}s_8{}^2 + (C_1\cos(my))^{8/7}\,\mathrm{d}y^2, \\ \mathrm{e}^{\phi} = (\mathrm{e}^{C_2}\cos(my))^{-1}, \qquad \chi = -\mathrm{e}^{C_2}\sin(my), \\ \mathrm{e}^{\varphi} = (C_1\cos(my))^{-2/\sqrt{7}}, \end{cases} \tag{50}$$

[11] The chirality is determined by our convention that we choose the transverse vielbein to be positive.

where C_2 is arbitrary, C_1 is strictly positive and the range of y has to be restricted so that $\cos(my)$ is strictly positive. The Killing spinor corresponding to the present solution is given by

$$\epsilon = (C_1 \cos(my))^{1/28}\, \mathrm{e}^{\mathrm{i}my/4}\epsilon_0, \tag{51}$$

where $(1 - \mathrm{i}\Gamma_{\underline{xy}})\epsilon_0 = 0$.

Note that in this class there is no solution with constant axion. This is consistent with the fact that for zero axion the potential corresponding to class II reads

$$V(\varphi) = \tfrac{1}{2}m^2\, \mathrm{e}^{4\varphi/\sqrt{7}}, \tag{52}$$

which, using the terminology of [24, 25], is a $\Delta = 0$ potential for which the standard domain wall solution does not work.

The uplifting of this solution to ten dimensions is given by

$$\text{R7:}\quad \begin{cases} \mathrm{d}\hat{s}^2 = \mathrm{d}s_8^2 + C_1\cos(my)(\mathrm{d}x^2 + \mathrm{d}y^2),\\ \mathrm{e}^{\hat{\phi}} = \mathrm{e}^{-mx-C_2}(\cos(my))^{-1},\\ \hat{\chi} = -\mathrm{e}^{mx+C_2}\sin(my), \end{cases} \tag{53}$$

where the Killing spinor is now given by

$$\hat{\epsilon} = \mathrm{e}^{\mathrm{i}my/4}\hat{\epsilon}_0. \tag{54}$$

The scalars and spinors satisfy the monodromy requirement (31) with $a = \mathrm{e}^{m\pi R}$, $b = 0$, $c = 0$ and $d = \mathrm{e}^{-m\pi R}$. This solution falls in our general class of 7-branes (25) with

$$f = \mathrm{i}\,\mathrm{e}^{mz+C_2}, \qquad g = mz + C_2 - \log(C_1). \tag{55}$$

Thus the constants C_1 and C_2 have the same origin as in the class I solution: scalings and shifts of the coordinates.

4.3. *Class III:* $\alpha^2 < 0$

This class is divided into two subclasses depending on whether the dilaton ϕ is non-zero (class IIIa) or zero (class IIIb).

4.3.1. *Class IIIa:* $\alpha^2 < 0$ *and* $\phi \neq 0$.

For non-zero dilaton we find the following half-supersymmetric domain wall solution:

$$\mathrm{DW_{IIIa}}:\quad \begin{cases} \mathrm{d}s^2 = (C_1\sinh(my))^{1/7}\,\mathrm{d}s_8^2 + (C_1\sinh(my))^{8/7}\,\mathrm{d}y^2,\\ \mathrm{e}^{\phi} = \dfrac{\cos(C_2) + \cosh(my)}{\sinh(my)}, \qquad \chi = \dfrac{\sin(C_2)}{\cos(C_2) + \cosh(my)},\\ \mathrm{e}^{\varphi} = (C_1\sinh(my))^{-2/\sqrt{7}}, \end{cases} \tag{56}$$

where C_2 is an arbitrary angle between[12] $-\pi/2$ and $\pi/2$, C_1 is a strictly positive constant and the range of y is restricted by requiring $\sinh(my)$ to be strictly positive. The Killing spinor for this solution is given by

$$\epsilon = (C_1\sinh(my))^{1/28}\,\mathrm{e}^{\mathrm{i}\beta}\epsilon_0, \qquad \beta = \frac{1}{4}\mathrm{arccot}\left(\frac{1+\cos(C_2)\cosh(my)}{\sin(C_2)\sinh(my)}\right), \tag{57}$$

where $(1 - \mathrm{i}\Gamma_{\underline{xy}})\epsilon_0 = 0$.

[12] It is of course possible to extend the domain of C_2, but with the choice of $-\pi/2$ to $\pi/2$ no solutions are related via $SL(2,\mathbb{Z})$ and in this sense C_2 covers the space of solutions exactly once.

Lifting solution (56) to ten dimensions gives

$$\text{T7:}\quad \begin{cases} \hat{\mathrm{d}s}^2 = \mathrm{d}s_8^2 + C_1 \sinh(my)(\mathrm{d}x^2 + \mathrm{d}y^2), \\ \mathrm{e}^{\hat{\phi}} = \dfrac{\cos(mx + C_2) + \cosh(my)}{\sinh(my)}, \\ \hat{\chi} = \dfrac{\sin(mx + C_2)}{\cos(mx + C_2) + \cosh(my)}. \end{cases} \tag{58}$$

The Killing spinor can also be lifted using (41) yielding

$$\hat{\epsilon} = \mathrm{e}^{\mathrm{i}\beta}\hat{\epsilon}_0, \qquad \beta = \tfrac{1}{2}\arctan\left(\tan\left(\tfrac{1}{2}(mx + C_2)\right)\tan\left(\tfrac{1}{2}my\right)\right). \tag{59}$$

Note that here the Killing spinor acquires non-trivial x-dependence. We have explicitly checked that the monodromy requirement (31) is satisfied with $a = \cos(m\pi R)$, $b = \sin(m\pi R)$, $c = -\sin(m\pi R)$ and $d = \cos(m\pi R)$. We note that also this class falls into the general class of 7-brane solutions (25) with

$$f = \tan\left(\tfrac{1}{2}(mz + C_2)\right), \qquad g = -2\log\left(\cos\left(\tfrac{1}{2}(mz + C_2)\right)\right) - \log(C_1). \tag{60}$$

4.3.2. Class IIIb: $\alpha^2 < 0$ and $\phi = 0$. For the case with vanishing dilaton we find the following half-supersymmetric domain wall solution[13]:

$$\mathrm{DW}_{\mathrm{IIIb}}:\quad \begin{cases} \mathrm{d}s^2 = \mathrm{e}^{my/7}\,\mathrm{d}s_8^2 + \mathrm{e}^{8my/7}\,\mathrm{d}y^2, \\ \mathrm{e}^{\varphi} = \mathrm{e}^{-2my/\sqrt{7}}, \qquad \phi = \chi = 0, \end{cases} \tag{61}$$

where the range of y is unrestricted. The corresponding Killing spinor reads

$$\epsilon = \mathrm{e}^{my/28}\epsilon_0, \tag{62}$$

with $(1 - \mathrm{i}\Gamma_{\underline{xy}})\epsilon_0 = 0$.

We note that for this solution, since $\chi = 0$, the potential and superpotential read

$$V(\phi,\varphi) = \tfrac{1}{2}m^2\,\mathrm{e}^{4\varphi/\sqrt{7}}\sinh^2(\phi), \qquad W(\phi,\varphi) = \tfrac{1}{4}m\,\mathrm{e}^{2\varphi/\sqrt{7}}\cosh\phi. \tag{63}$$

The above potential has occurred recently, see equation (77) of [5], in the context of a possible inflation along flat directions. An interesting feature of this case is that the flat direction, $\phi = 0$, corresponds to a vanishing potential, $V(\varphi) = 0$, despite a non-vanishing superpotential, $W = \frac{1}{4}m\,\mathrm{e}^{2\varphi/\sqrt{7}}$. Such a situation has occurred recently in the context of quintessence in $N = 1$ supergravity (see section 3 of [6]).

Lifting this solution to ten dimensions leads to the following purely gravitational solution[14]:

$$\text{G7:}\quad \begin{cases} \hat{\mathrm{d}s}^2 = \mathrm{d}s_8^2 + \mathrm{e}^{my}(\mathrm{d}x^2 + \mathrm{d}y^2), \\ \hat{\phi} = \hat{\chi} = 0. \end{cases} \tag{64}$$

For the lifted Killing spinor we find

$$\hat{\epsilon} = \mathrm{e}^{\mathrm{i}mx/4}\epsilon_0. \tag{65}$$

Again, this solution falls in the class of purely gravitational solutions discussed in section 3.1 with the identifications

$$f = \mathrm{i}, \qquad g = \mathrm{i}mz. \tag{66}$$

[13] This solution is related by a coordinate transformation to that of [18], where, contrary to our result, it was claimed that in order to preserve a fraction of the supersymmetry m should be zero, reducing the solution to the Minkowski spacetime with arbitrary constant scalars.

[14] Other examples of domain walls that lift up to purely gravitational solutions have been given in [26].

As discussed in section 3.2, the holomorphic function g can be transformed: the coordinate transformation (28) takes the form $r = \frac{2}{m}\,\mathrm{e}^{my/2}$ and $\theta = \frac{1}{2}(\pi - mx)$. The compactness of x translates into $\theta \sim \theta + m\pi R$.

We can now impose three different quantization conditions (to be discussed in section 5). The condition $m = 1/(2R)$ implies that this solution describes a conical spacetime with deficit angle $3\pi/2$. In other words, this is a half-supersymmetric $\mathrm{Mink}_8 \times \mathbb{C}/\mathbb{Z}_4$ spacetime with non-trivial monodromy, the bosonic part of which was also mentioned in [20]. The second quantization condition $\tilde{m} = 1/(3\sqrt{3}R)$ can only be applied to an $SL(2,\mathbb{R})$-related partner of the G7-brane and gives rise to a deficit angle of $5\pi/3$. This is a half-supersymmetric $\mathrm{Mink}_8 \times \mathbb{C}/\mathbb{Z}_6$ spacetime with non-trivial monodromy. The third quantization condition $m = 2/R$ yields the identification $\theta \sim \theta + 2\pi$ and indeed this is fully supersymmetric Mink_{10} spacetime. The monodromy is trivial and there is a second Killing spinor $\hat{\epsilon} = \mathrm{e}^{-\mathrm{i}mx/4}\epsilon_0$ with opposite chirality. For the previous two quantization conditions this second Killing spinor had a different monodromy and was therefore not consistent.

4.4. Class IV: m_4

We first substitute the domain wall ansatz in the Killing spinor equations, which are in this class given by (18). We find that we cannot construct a projector, yielding a 1/2 supersymmetric domain wall, out of the Γ-matrices appearing in the supersymmetry rules unless the scalars have a time dependence, i.e. the transverse direction has to be the time direction. In this respect class IV is fundamentally different from classes I–III. For time-dependent solutions we cannot assume that a solution to the Killing spinor equations is automatically a solution to the full equations of motion. A counter-example is provided by considering a scalar Φ that does only occur in the transformations of the spin-1/2 fermions as $(\partial\!\!\!/\Phi)\epsilon$. Clearly the Killing spinor equations can be solved for a flat metric and $\Phi = \Phi(u)$, $\gamma_v\epsilon = 0$ where we use lightcone coordinates $u = x + t$, $v = x - t$. The non-zero scalar leads to a nonzero uu-component of the energy–momentum tensor and the Einstein equations are not solved.

On the other hand, examples of time-dependent 1/2 supersymmetric BPS solutions are known. An example is the gravitational wave solution. For the present case, however, we find that it is not possible to construct a domain wall solution, time-dependent or not, preserving any fraction of the supersymmetry.

5. Quantization conditions

It is well known that at the quantum level the classical $SL(2,\mathbb{R})$ symmetry of IIB supergravity is broken to $SL(2,\mathbb{Z})$[15]. We would like to consider the effect of this on the solutions discussed in the previous sections. In particular, it implies that the monodromy matrix must be an element of the arithmetic subgroup of $SL(2,\mathbb{R})$:

$$\hat{\mathcal{M}}(x + 2\pi R) = \Lambda\hat{\mathcal{M}}(x)\Lambda^T \qquad \text{with} \quad \Lambda = \mathrm{e}^{2\pi RC} \in SL(2,\mathbb{Z}). \tag{67}$$

This will imply a charge quantization of the 7-brane solutions in 10D. Since these charges give rise to the mass parameters upon reduction, at the same time this requirement therefore implies a mass quantization.

We will apply the following procedure. The mass parameters will be parametrized by $\vec{m} = \tilde{m}(p, q, r)$. Then, given the radius of compactification R and the relative coefficients (p, q, r) of the mass parameters, one should choose the overall coefficient $\tilde{m}$ such that the monodromy lies in $SL(2,\mathbb{Z})$. This is not always possible; a necessary requirement in all but

[15] A similar quantization condition does not apply to the $SO(1,1)$ symmetry of IIA supergravity.

Table 3. The table summarizes the different $SL(2,\mathbb{Z})$ monodromies. It is organized according to the trace of the monodromy and gives the diophantic equation for (p,q,r). Explicit examples are given with the corresponding monodromies. For cases I and III all diophantic solutions are related by $SL(2,\mathbb{Z})$ to the examples given. In case II there are other conjugacy classes [27, 28].

Class	α^2	Tr (Λ)	$p^2+q^2-r^2$	(p,q,r)	Λ	
I	$=0$	2	0	$(0,n,n)$	$T^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$	$n \in \mathbb{Z}$
II	>0	n	n^2-4	$(\pm n,0,\pm 2)$	$(ST^{-n})^{\pm 1} = \begin{pmatrix} 0 & 1 \\ -1 & n \end{pmatrix}^{\pm 1}$	$3 \leqslant n \in \mathbb{Z}$
III	<0	0	-4	$(0,0,\pm 2)$	$S^{\pm 1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}^{\pm 1}$	
		1	-3	$(\pm 1,0,\pm 2)$	$(T^{-1}S)^{\pm 1} = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}^{\pm 1}$	
		2	-4		$1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	

one case will be that (p,q,r) are integers and satisfy a diophantic equation. Furthermore, we must require q and r to be either both even or both odd. Only in class III will it be possible to quantize for non-integers (p,q,r). Thus we get all $SL(2,\mathbb{Z})$ monodromies that can be expressed as the products of S and T (and their inverses) as defined in (22). These $SL(2,\mathbb{Z})$ conjugacy classes have been classified in [27, 28]. The ones corresponding to classes I and III have also been discussed in [29]. The situation is summarized in table 3. We consider each of the three classes separately.

- For class I with $\alpha^2 = 0$ the monodromy matrix reads

$$\Lambda = \begin{pmatrix} 1 + m_1\pi R & (m_2+m_3)\pi R \\ (m_2-m_3)\pi R & 1 - m_1\pi R \end{pmatrix}. \tag{68}$$

  We find that Λ is an element of $SL(2,\mathbb{Z})$ provided we have

$$\text{class I:} \quad \tilde{m} = \frac{1}{2\pi R} \qquad \text{and} \qquad p^2+q^2-r^2 = 0. \tag{69}$$

  All the solutions of the diophantic equation are related via $SL(2,\mathbb{Z})$ to the D7-brane solutions with $(p,q,r)=(0,n,n)$ with n an arbitrary integer [27–29], which is the explicit choice we have used for class I. This gives rise to the monodromy $\Lambda = T^n$. The quantization on $\tilde{m}$ is the same charge quantization condition as found in [11].

- For class II with $\alpha^2 > 0$ the monodromy matrix reads

$$\Lambda = \begin{pmatrix} \cosh(\alpha 2\pi R) + \frac{m_1}{2\alpha}\sinh(\alpha 2\pi R) & \frac{1}{2\alpha}(m_2+m_3)\sinh(\alpha 2\pi R) \\ \frac{1}{2\alpha}(m_2-m_3)\sinh(\alpha 2\pi R) & \cosh(\alpha 2\pi R) - \frac{m_1}{2\alpha}\sinh(\alpha 2\pi R) \end{pmatrix}. \tag{70}$$

  We find that Λ is an element of $SL(2,\mathbb{Z})$ provided we have

$$\text{class II:} \quad \tilde{m} = \frac{\operatorname{arccosh}(n/2)}{\pi R\sqrt{n^2-4}} \qquad \text{and} \qquad p^2+q^2-r^2 = n^2-4, \tag{71}$$

  for some integer $n \geqslant 3$. This has solutions $(p,q,r) = (\pm n, 0, \pm 2)$ with monodromy $\Lambda = (ST^{-n})^{\pm 1}$ but not all other solutions are related by $SL(2,\mathbb{Z})$ [27, 28]. Note that the explicit choice we have made for class II with $(p,q,r) = (p,0,0)$ does not solve the diophantic equation. Thus the R7-brane is not consistent at the quantum level but particular $SL(2,\mathbb{R})$ partners are.

- For class III with $\alpha^2 < 0$ the monodromy matrix reads (using $\alpha = \mathrm{i}m$)

$$\Lambda = \begin{pmatrix} \cos(m2\pi R) + \frac{m_1}{2m}\sin(m2\pi R) & \frac{1}{2m}(m_2 + m_3)\sin(m2\pi R) \\ \frac{1}{2m}(m_2 - m_3)\sin(m2\pi R) & \cos(m2\pi R) - \frac{m_1}{2m}\sin(m2\pi R) \end{pmatrix}. \tag{72}$$

Here we find that there are three distinct possibilities for Λ to be an element of $SL(2,\mathbb{Z})$. For the first possibility we must have

$$\text{class III:} \quad \tilde{m} = \frac{1}{4R} \qquad \text{and} \qquad p^2 + q^2 - r^2 = -4. \tag{73}$$

This is the explicit choice we have made for the T7- and G7-brane solution with $(p, q, r) = (0, 0, \pm 2)$ and $\Lambda = S^{\pm 1}$. In fact all other solutions to the diophantic equation are related by $SL(2,\mathbb{Z})$ [27–29]. For the second possibility one must require

$$\text{class III:} \quad \tilde{m} = \frac{1}{3\sqrt{3}R} \qquad \text{and} \qquad p^2 + q^2 - r^2 = -3. \tag{74}$$

We have not explicitly considered this case but one solution is $(p, q, r) = (\pm 1, 0, \pm 2)$ with monodromy $\Lambda = (T^{-1}S)^{\pm 1}$. Again all other solutions are related by $SL(2,\mathbb{Z})$ [27–29]. If neither of these two possibilities applies, one can always choose

$$\text{class III:} \quad \tilde{m} = \frac{1}{R} \qquad \text{and} \qquad p^2 + q^2 - r^2 = -4, \qquad (p, q, r) \in \mathbb{R}, \tag{75}$$

where (p, q, r) are not required to be integer-valued. This gives rise to trivial monodromy $\Lambda = 1$.

6. Conclusions

One of the aims of this paper is to study domain wall solutions in a nine-dimensional setting. The advantage of picking out nine dimensions is that it is simple enough to investigate in full detail but also shares some of the complications of the lower-dimensional supergravities. In section 4 we constructed several half-supersymmetric domain wall solutions and we gave their uplifting to ten dimensions. This uplifting introduces a non-trivial dependence of the $D = 10$ solution on the compact coordinate.

In the IIB case we thus found three classes of 7-brane solutions in ten dimensions, which are all characterized by two holomorpic functions (25). One class contains the D7-brane and its $SL(2,\mathbb{R})$-related partners but the R7-brane and T7-brane solutions we found for the other two classes are not related to the D7-brane by $SL(2,\mathbb{R})$ duality. It would be interesting to see their interpretation in terms of the type IIB superstring theory. Together the solutions provide a set of half-supersymmetric 7-branes with arbitrary charges that are consistent in the sense that the monodromies of the scalars and Killing spinors coincide. Our method of uplifting domain walls also leads to half-supersymmetric conical G7-brane solutions with deficit angle $3\pi/2$ or $5\pi/3$, not carried by any scalars. The non-trivial monodromies sit in the fermionic sector. The G7-brane solution can also be uplifted to the Minkowski spacetime, in which case we have supersymmetry enhancement upon uplifting. It would be interesting to further study the properties of the $D = 9$ domain wall solutions and their $D = 10$ 7-brane origins and to see whether some of the features we find also occur for $D < 9$ domain walls.

The three distinct massive supergravities corresponding to the IIB case are $SL(2,\mathbb{R})$-covariant and characterized by the $SL(2,\mathbb{R})$-invariant α^2. One of them, with $\alpha^2 = 0$, has a singular mass matrix and therefore, following a similar statement made in [30], does not seem to correspond to a gauged supergravity theory. The class with $\alpha^2 < 0$ has been shown to be an $SO(2)$-gauged supergravity [18]. We conjecture that the remaining class with $\alpha^2 > 0$ is an

$SO(1, 1)$-gauged supergravity. Interestingly, in a recent paper it is stated that both the $\alpha^2 = 0$ and the $\alpha^2 > 0$ cases correspond to $SO(1, 1)$-gauged supergravities [29]. The distinction between different theories does not occur in the compact case, i.e. when the symmetry group would be $SU(2)$ rather than $SL(2, \mathbb{R})$. Such a situation occurs, for instance, when gauging the $U(1) \subset SU(2)$ R-symmetry group in $N = 2, D = 5$ supergravity coupled to vector multiplets. Here all choices for the mass parameters are physically equivalent leading to a single-gauged supergravity theory (see, e.g., [31]).

In the IIA case we performed two reductions, one leading to the m_4 deformation and one leading to class I. In the case of the m_4 deformation we find that there is no domain wall solution preserving any supersymmetry. The reason that we could not perform both IIA reductions at the same time was that the $SO(1, 1)$ symmetry is only valid for $m_{\mathrm{R}} = 0$. One might change this situation by replacing m_{R} by a scalar field $M(x)$ and a 9-form Lagrange multiplier $A^{(9)}$ [19] via

$$\mathcal{L}(m_{\mathrm{R}}) \to \mathcal{L}(M(x)) + M(x)\partial A^{(9)}. \tag{76}$$

Unfortunately, the reduction of the second term leads to an additional term in nine dimensions containing a 9-form Lagrange multiplier. The equation of motion of this Lagrange multiplier leads to the constraint $M(x)m_4 = 0$ which brings us back to the previous situation. A similar thing happens in 11 dimensions if one tries to use the same trick to convert the scale symmetry of the equations of motion to a symmetry of the action by replacing the gravitational constant by a scalar field. The elimination of the Lagrange multiplier brings us back to the analysis of [32].

Let us finally comment on the relation between different massive deformations of $N = 2$ $D = 9$ supergravity and T-duality. The massless theory can be obtained from the reduction of both IIA and IIB massless supergravity [33]. This follows from the T-duality between the underlying IIA and IIB string theories. However, the Scherk–Schwarz reductions of IIA and IIB supergravity to nine dimensions give rise to four different massive deformations of the unique massless theory. Only one of these deformations (class I) can be reproduced by both IIA and IIB supergravity. It is not clear what the IIA or M-theory origin is of the other two deformations (classes II and III)[16]. Similarly, it is not clear what the IIB origin is of the class IV deformation.

To understand massive T-duality it might be necessary to explicitly include massive winding multiplets[17] (while in supergravity reduction one only keeps the states without winding). Massive T-duality suggests the existence of a maximally supersymmetric massive supergravity theory containing *all four* mass parameters (m_1, m_2, m_3, m_4). The existence of such a theory is not implied by the massive supergravities with seperate deformations (m_1, m_2, m_3) and m_4. This massive supergravity has already been suggested for different reasons in [13] and it would be interesting to see whether it can be constructed [37].

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[16] We do not consider here the use of Killing vectors in the Lagrangian. Assuming that the IIA theory has such explicit Killing vectors a massive T-duality map can be constructed [13]. We neither consider a further reduction to $D = 8$ dimensions [15].

[17] The inclusion of the full tower of (massive) multiplets of higher Fourier and winding number has been discussed in [34–36].

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Appendix. Conventions

We mostly use plus signature $(- + \cdots +)$. Hatted fields and indices are ten dimensional while unhatted ones are nine dimensional. Greek indices $\hat{\mu}, \hat{\nu}, \hat{\rho} \ldots$ denote world coordinates and Latin indices $\hat{a}, \hat{b}, \hat{c} \ldots$ represent tangent spacetime. They are related by the vielbeins $\hat{e}_{\hat{\mu}}{}^{\hat{a}}$ and inverse vielbeins $\hat{e}_{\hat{a}}{}^{\hat{\mu}}$. Explicit indices x, y are underlined when flat and not underlined when curved. We antisymmetrize with weight one, for instance, $(\partial \hat{A})_{\hat{\mu}\hat{\nu}} = \frac{1}{2}(\partial_{\hat{\mu}} \hat{A}_{\hat{\nu}} - \partial_{\hat{\nu}} \hat{A}_{\hat{\mu}})$. Omitted indices are contracted without numerical factors, e.g., $(\partial \hat{A})^2 = (\partial \hat{A})_{\hat{\mu}\hat{\nu}} (\partial \hat{A})^{\hat{\mu}\hat{\nu}}$. The covariant derivative on fermions is given by $D_{\hat{\mu}} = \partial_{\hat{\mu}} + \hat{\omega}_{\hat{\mu}}$ with the spin connection $\hat{\omega}_{\hat{\mu}} = \frac{1}{4} \hat{\omega}_{\hat{\mu}}{}^{\hat{a}\hat{b}} \Gamma_{\hat{a}\hat{b}}$.

We have chosen all Γ-matrices real. Curved indices of hatted Γ-matrices $\hat{\Gamma}_\mu$ refer to the ten-dimensional metric while curved indices of unhatted Γ-matrices Γ_μ refer to the nine-dimensional metric. Furthermore

$$\Gamma_{11} = \Gamma^{\underline{0} \cdots \underline{9}}, \qquad {\Gamma_{11}}^2 = 1. \tag{77}$$

In the IIA theory we have real Majorana spinors of indefinite chirality. In the IIB theory we have complex spinors of definite chirality. To switch between Majorana and Weyl fermions in nine dimensions one must use

$$\begin{aligned}
\tfrac{1}{2}(1 + \Gamma_{11}) \psi_\mu^M &= \mathrm{Re}\left(\psi_\mu^W\right), & \tfrac{1}{2}(1 - \Gamma_{11}) \psi_\mu^M &= \mathrm{Im}\left(\Gamma_{\underline{x}} \psi_\mu^W\right), \\
\tfrac{1}{2}(1 + \Gamma_{11}) \lambda^M &= \mathrm{Im}(\Gamma_{\underline{x}} \lambda^W), & \tfrac{1}{2}(1 - \Gamma_{11}) \lambda^M &= \mathrm{Re}(\lambda^W), \\
\tfrac{1}{2}(1 + \Gamma_{11}) \tilde{\lambda}^M &= \mathrm{Im}(\Gamma_{\underline{x}} \tilde{\lambda}^W), & \tfrac{1}{2}(1 - \Gamma_{11}) \tilde{\lambda}^M &= \mathrm{Re}(\tilde{\lambda}^W), \\
\tfrac{1}{2}(1 + \Gamma_{11}) \epsilon^M &= \mathrm{Re}(\epsilon^W), & \tfrac{1}{2}(1 - \Gamma_{11}) \epsilon^M &= \mathrm{Im}(\Gamma_{\underline{x}} \epsilon^W),
\end{aligned} \tag{78}$$

for positive $\left(\psi_\mu^W, \epsilon^W\right)$ and negative $(\lambda^W, \tilde{\lambda}^W)$ chirality Weyl fermions.

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