

Type IIB orientifolds with NS-NS antisymmetric tensor backgrounds

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We consider six dimensional $\mathcal{N}=1$ space-time supersymmetric type IIB orientifolds with a nonzero untwisted NS-NS sector B field. The B field is quantized due to the requirement that the type IIB spectrum be left-right symmetric. The presence of the B field results in rank reduction of both 99 and 55 open string sector gauge groups. We point out that in some of the models with a nonzero B field there are extra tensor multiplets in the \mathbf{Z}_2 twisted closed string sector, and we explain their origin in a simple example. Also, the 59 open string sector states come with a multiplicity that depends on the B field. These two facts are in accord with anomaly cancellation requirements. We point out relations between various orientifolds with and without the B field, and also discuss the F-theory duals of these models. [S0556-2821(98)05116-9]

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I. INTRODUCTION

In recent years a lot of progress has been made in understanding the web of dualities between different string theories in various dimensions. Among the five consistent string theories in ten dimensions, type I string theory (which is a theory of unoriented open plus closed strings) remains largely unexplored, especially upon compactification with a nontrivial background. This is due in part to the lack of modular invariance which is necessary for perturbative consistency of oriented closed string theories.

Various unoriented open plus closed string vacua have been constructed using orientifold techniques [1–9]. Some of these vacua exhibit novel features that are inaccessible perturbatively in other string theories. For instance, upon compactification to six dimensions it is possible to construct $\mathcal{N}=1$ orientifolds with multiple tensor multiplets [1,3,4]. Similarly, enhanced gauge symmetries due to small instantons, which cannot be described within conformal field theory in heterotic compactifications, have a perturbative description in terms of coincident D5-branes in the type I language. Using the conjectured type-I–heterotic duality [11], one can understand these nonperturbative heterotic phenomena as perturbative effects on the type I side [7]. In fact, this approach has been taken in constructing nonperturbative chiral heterotic string vacua in four dimensions [8].

In this paper we will focus on six¹ dimensional $\mathcal{N}=1$ type IIB orientifolds with constant nonzero Neveu-Schwarz–Neveu-Schwarz (NS–NS) antisymmetric tensor backgrounds. Toroidal compactifications of type I strings with nonzero B field have been studied in [12] (and also recently in [13,14]). There it was shown that the rank of the Chan-

Paton gauge group is reduced by a factor of $2^{b/2}$ where $b \in 2\mathbf{Z}$ is the rank of the matrix B_{ij} corresponding to the compactified directions. In Sec. II we extend these arguments to orbifold compactifications and show that when there are D9- and/or D5-branes, both the 99 and 55 gauge group ranks are reduced by a factor of $2^{b/2}$. We point out that in some of the models with nonzero B field there are extra tensor multiplets that arise in the \mathbf{Z}_2 twisted closed string sector, and we explain their origin in Sec. III in a simple example. Also, the 59 open string sector states come with a multiplicity that depends on the B field. Appearance of extra tensor multiplets (as well as a nontrivial multiplicity of states in the 59 open string sector) is in accord with anomaly cancellation. Thus, cancellation of the $\text{Tr}(R^4)$ gravitational anomalies requires [15]

$$n_H - n_V = 244 - 29n_T, \quad (1)$$

where n_H , n_V and $n_T + 1$ are the numbers of hypermultiplets, vector multiplets, and tensor multiplets, respectively. In the presence of the B field, the rank of the gauge group is reduced and hence the numbers of vector multiplets and hypermultiplets are reduced accordingly. Unless the difference $n_H - n_V$ is unchanged, this implies that the number of tensor multiplets depends upon the quantized value of the B field. We will show that some of the fixed points in the \mathbf{Z}_2 twisted closed string sector become odd under the orientifold projection once we turn on the B field. As a result, tensor multiplets (instead of hypermultiplets) are kept at those fixed points. This gives precisely the correct number of tensor multiplets to cancel the anomalies.

In Sec. IV we point out relations between various orientifolds with and without the B field, and also discuss the F-theory duals of these models. In particular, some of the models with the B field are on the same moduli as some of the orientifolds without the B field.

II. QUANTIZED B FIELD AND RANK REDUCTION

Type IIB orientifolds are generalized orbifolds that involve the world-sheet parity reversal Ω along with other geo-

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¹Four dimensional $\mathcal{N}=1$ supersymmetric orientifolds will be discussed in [10].

metric symmetries of the theory. Orientifolding results in a theory with unoriented closed strings. In order to cancel space-time anomalies, the massless tadpoles must be canceled. Thus, one generically has to introduce open strings that start and end on D-branes. The global Chan-Paton charges associated with the D-branes give rise to gauge symmetry in space-time.

There are two massless antisymmetric tensor fields in the perturbative spectrum of type IIB theory: one coming from the NS-NS sector, and another coming from the Ramond-Ramond (R-R) sector. Under the world-sheet parity reversal, the NS-NS two-form $B_{\mu\nu}$ is projected out, while the R-R two-form $B'_{\mu\nu}$ is kept. Although the fluctuations of B_{ij} (the components of $B_{\mu\nu}$ in the compactified dimensions) are projected out of the perturbative unoriented closed string spectrum, a quantized vacuum expectation value of B_{ij} is allowed. To see this, consider the left- and right-moving momenta in the d dimensions compactified on a torus T^d :

$$P_{L,R} = \tilde{e}^j (m_i - B_{ij} n^j) \pm e_i \frac{n^i}{2}, \quad (2)$$

where m_i and n^i are integers, e_i are constant vielbeins such that $e_i \cdot e_j = G_{ij}$ is the constant background metric on T^d , and $e_i \cdot \tilde{e}^j = \delta_i^j$. Note that the components of B_{ij} are defined up to a shift $B_{ij} \rightarrow B_{ij} + 1$ (which can be absorbed by redefining m_i). With this normalization, only the values $B_{ij} = 0$ and $1/2$ are invariant under Ω , hence quantization of B_{ij} .

This quantized B field has nontrivial consequences on the open string spectrum. The effect of nonzero B field in toroidal compactifications of type I string theory has been studied in [12]. Recall that open strings and D-branes are introduced to cancel the massless tadpoles coming from the one-loop Klein bottle amplitude (or, equivalently, the tree channel amplitude for a cylinder with two cross-caps). In the presence of unoriented open strings there are two other one-loop diagrams which contribute to the tadpoles: the Möbius strip and annulus amplitudes. The massless tadpoles are divergences in the tree channel due to the exchange of R-R closed string states between Dp-branes and/or orientifold planes both of which carry R-R charges and are sources for the corresponding $(p+1)$ -form potential. In the case where there are both D9- and D5-branes, there are three types of massless tadpoles due to the untwisted R-R ten-form, untwisted R-R six-form and twisted R-R six-form potentials, respectively.

To be specific, let us consider orientifolds of type IIB on the orbifold limits of K3: T^4/\mathbf{Z}_N ($N=2,3,4,6$).² Let g be the generator of \mathbf{Z}_N . The action of the orbifold on the Chan-Paton factors is described by matrices $\gamma_{k,9}$ and $\gamma_{k,5}$ corresponding to the action of the g^k ($k=0, \dots, N-1$) element of \mathbf{Z}_N on the D9- and D5-branes, respectively. Note that

²Orientifolds of type IIB on smooth K3 with nonzero B field have been studied in [16]. The orbifold cases we are discussing here have extra tensor multiplets and qualitatively differ from the smooth K3 cases corresponding to type I compactifications with only one tensor multiplet.

$\text{Tr}(\gamma_{0,9}) = n_9$ and $\text{Tr}(\gamma_{0,5}) = n_5$ are the numbers of D9- and D5-branes, respectively.

First, recall the known results for the case with no B field [2,3]. The tadpoles for the untwisted R-R ten-form potential are proportional to

$$(\text{Tr}(\gamma_{0,9}))^2 - 64\text{Tr}(\gamma_{0,9}) + 32^2, \quad (3)$$

and the tadpole cancellation requires presence of $n_9 = 32$ D9-branes.

For even N the tadpoles for the untwisted R-R six-form potential are proportional to

$$(\text{Tr}(\gamma_{0,5}))^2 - 64\text{Tr}(\gamma_{0,5}) + 32^2, \quad (4)$$

and the tadpole cancellation implies that there are $n_5 = 32$ D5-branes. This could also be seen from T-duality between D9- and D5-branes. For odd N there are no tadpoles for the untwisted R-R six-form potential, and there are no D5-branes.

Finally, the tadpole cancellation for the twisted R-R six-form potential constrains the action of the twists on the Chan-Paton factors, that is, the Chan-Paton matrices $\gamma_{k,9}$ and $\gamma_{k,5}$ ($k=1, \dots, N-1$). Since the twisted closed string states propagating in the tree channel do not have momenta or windings, the twisted tadpoles will not be affected by the B field (the effect of which is to shift the left- plus right-moving momentum lattice). For this reason, we will not need the explicit expressions for the twisted tadpoles here.

Let us now see what happens when we turn on the B field. Consider the vacuum amplitude for the open string stretched between two D9-branes. The corresponding graph is an annulus whose boundaries lie on D9-branes. The open strings stretched between D9-branes satisfy Neumann boundary conditions. This implies that in the tree-channel (the corresponding amplitude being a cylinder with closed strings propagating between the D9-branes) the closed strings must satisfy the condition of ‘‘no momentum flow’’ through the boundaries (that is, D9-branes):

$$P_L + P_R = 0. \quad (5)$$

This implies the following constraints on the momenta and windings of the closed strings propagating along the cylinder:

$$m_i + B_{ij} n^j = 0. \quad (6)$$

For $B_{ij} = 0$ this simply states that there are no momentum states propagating between the D9-branes, but the winding number n^i is arbitrary. Let us now consider the cases where some of the elements $B_{ij} = 1/2$, so that the rank b of the B_{ij} matrix is nonzero. (Note that since B_{ij} is an antisymmetric matrix, its rank b is always even. For compactifications on orbifold limits of K3 the allowed values of b are $b=0,2,4$.) For simplicity we will assume that $T^4 = T^2 \otimes T^2$ (so that G_{ij} has a 2×2 block-diagonal form), and the B_{ij} matrix has a 2×2 block-diagonal form as well. Thus, if $B_{ij} = 0$, then closed string states with both odd and even windings n^j contribute into the cylinder amplitude. On the other hand, if

$B_{ij}=1/2$, then only closed string states with even windings n^j contribute. To see how the cylinder amplitude is modified, let us extract the piece that depends on P_L and P_R (here we are suppressing the contributions from the corresponding oscillators as well as all the other world-sheet degrees of freedom, for they are not affected by the B field):

$$\mathcal{C} \sim k \sum_{\mathbf{n}} e^{-(\pi t/4)G_{ij}n^i n^j}, \quad (7)$$

where $n^j \in \mathbf{Z}$ if $B_{ij}=0$, and $n^j \in 2\mathbf{Z}$ if $B_{ij}=1/2$. Here t is a real modulus (related to the length of the cylinder), and k is a normalization constant to be determined. Under the modular transformation $t \rightarrow 1/t$ (which maps the closed string tree channel to the open string loop channel) we obtain the annulus amplitude ($G^{ij} = \tilde{e}^i \cdot \tilde{e}^j$):

$$\mathcal{A} \sim \frac{1}{2^b k} \sum_{\mathbf{m}} e^{-(4\pi t)G^{ij}m_i m_j}, \quad (8)$$

where $m_j \in \mathbf{Z}$ if $B_{ij}=0$, and $m_j \in \frac{1}{2}\mathbf{Z}$ if $B_{ij}=1/2$. The annulus amplitude must be properly normalized so that the overall normalization factor has the interpretation of the number of D9-branes. That is, we must have $k/2^b = (\text{Tr}(\gamma_{0,9}))^2$ [recall that $\text{Tr}(\gamma_{0,9}) = n_9$ is the total number of D9-branes]. This implies that the cylinder amplitude (7), as well as the corresponding tadpole, is proportional to $2^b (\text{Tr}(\gamma_{0,9}))^2$ [instead of $(\text{Tr}(\gamma_{0,9}))^2$ as in the case with $b=0$]. Similar considerations for the Möbius amplitude show that the corresponding tree channel amplitude (which is a cylinder with closed strings propagating between a D9-brane and a cross-cap) is proportional to $2^{b/2} \text{Tr}(\gamma_{0,9})$. The Klein bottle contribution into the tadpoles for the untwisted R-R ten-form potential is not modified as the orientifold projection keeps the left-right symmetric states ($P_L = P_R$) which are insensitive to the presence of the B field as can be seen from Eq. (2). Putting all of the above together we conclude that the tadpole cancellation condition for the untwisted R-R ten-form potential becomes

$$2^b (\text{Tr}(\gamma_{0,9}))^2 - 2^{b/2} 64 \text{Tr}(\gamma_{0,9}) + 32^2 = 0. \quad (9)$$

Therefore, the number of D9-branes is given by $n_9 = 32/2^{b/2}$.

Next, let us see what happens to the tadpoles for the untwisted R-R six-form potential (which determine the number of D5-branes) in the cases where the orbifold group \mathbf{Z}_N has even order $N \in 2\mathbf{N}$ (for odd N there are no D5-branes present). The open strings stretched between D5-branes satisfy Dirichlet boundary conditions. This implies that in the tree channel the closed strings must satisfy the condition of ‘no winding flow’ through the boundaries (that is, D5-branes):

$$P_L - P_R = 0. \quad (10)$$

This does not impose any new constraints on the 55 cylinder amplitude in the presence of the B field. However, the Klein bottle contribution into the tadpoles for the untwisted R-R six-form potential is modified as the corresponding projec-

tion (that is, ΩR where R is the generator of the \mathbf{Z}_2 subgroup of \mathbf{Z}_N) keeps the states with $P_L = -P_R$. The net result of this is that the Klein bottle tadpole is reduced by 2^b . The Möbius strip amplitude can be analyzed similarly, and the tadpole cancellation condition for the untwisted R-R six-form potential becomes

$$(\text{Tr}(\gamma_{0,5}))^2 - \frac{64}{2^{b/2}} \text{Tr}(\gamma_{0,5}) + \frac{1}{2^b} 32^2 = 0. \quad (11)$$

Therefore, the number of D5-branes is given by $n_5 = 32/2^{b/2}$. This was also expected from T-duality between D9- and D5-branes.

So far we have focused on the 99 and 55 open string sectors. Let us understand the effect of the B field on the 59 open string sector. From the 99 and 55 cylinder amplitudes one can construct the boundary states corresponding to D9- and D5-branes ($p=9,5$):

$$C_{pp} = \langle B, p | \exp(-\pi t(L_0 + \bar{L}_0)) | B, p \rangle. \quad (12)$$

Let $|B, p\rangle_0$ be the corresponding boundary states without the B field. Then from the previous discussion it should be clear that in the presence of the B field the boundary states are given by $|B, 9\rangle = 2^{b/2} |B, 9\rangle_0$ and $|B, 5\rangle = |B, 5\rangle_0$. The 59 cylinder amplitude therefore reads

$$C_{59} = \langle B, 5 | \exp(-\pi t(L_0 + \bar{L}_0)) | B, 9 \rangle = 2^{b/2} C_{59}^{(0)}, \quad (13)$$

where $C_{59}^{(0)}$ is the 59 cylinder amplitude without the B field. The interpretation of the extra factor of $2^{b/2}$ in the 59 loop channel amplitude is that the 59 open string sector states come with a multiplicity $\xi = 2^{b/2}$ in the presence of the B field. (Recall that without the B field the multiplicity of states in the 59 open string sector was one per given configuration of Chan-Paton charges [2,3].)

III. ORIENTIFOLDS WITH NONZERO B FIELD

In this section we discuss orientifolds of type IIB on T^4/\mathbf{Z}_N ($N=2,3,4,6$) in the presence of the B field. For illustrative purposes we will focus our discussion on the \mathbf{Z}_2 case, and state the results for the other cases.

First consider the \mathbf{Z}_2 orientifold model of [1,2] without the B field. The closed string sector gives 20 neutral hypermultiplets (4 from the untwisted sector and 1 from each of the 16 fixed points in the twisted sector) and 1 tensor multiplet (from the untwisted sector). The open string sector gives gauge bosons and charged hypermultiplets. The 99 gauge group is $U(16)$ in the absence of Wilson lines, and the 55 gauge group is $U(16)$ when all the D5-branes are located at the same fixed point. The spectrum of this model is given in Table I. The states charged under both 99 and 55 gauge groups correspond to the 59 open string sector.

Let us turn on the B field with $b=2$. From our discussions in Sec. II, numbers of both the D9- and D5-branes are reduced to 16. The twisted tadpole cancellation conditions remain the same in the presence of the B field, and are given by [2]:

TABLE I. The massless spectrum of the six dimensional type IIB orientifolds on T^4/\mathbf{Z}_N for $N=2,4$, and various values of b (the rank of B_{ij}). The semi-colon in the column ‘‘Charged hypermultiplets’’ separates 99 and 55 representations.

Model	b	Gauge group	Charged hypermultiplets	Neutral hypermultiplets	Extra tensor multiplets
\mathbf{Z}_2	0	$U(16)_{99} \otimes U(16)_{55}$	$2(\mathbf{120}; \mathbf{1})$ $2(\mathbf{1}; \mathbf{120})$ $(\mathbf{16}; \mathbf{16})$	20	0
\mathbf{Z}_2	2	$U(8)_{99} \otimes U(8)_{55}$	$2(\mathbf{28}; \mathbf{1})$ $2(\mathbf{1}; \mathbf{28})$ $2(\mathbf{8}; \mathbf{8})$	16	4
\mathbf{Z}_2	4	$U(4)_{99} \otimes U(4)_{55}$	$2(\mathbf{6}; \mathbf{1})$ $2(\mathbf{1}; \mathbf{6})$ $4(\mathbf{4}; \mathbf{4})$	14	6
\mathbf{Z}_4	0	$[U(8) \otimes U(8)]_{99} \otimes [U(8) \otimes U(8)]_{55}$	$(\mathbf{28}, \mathbf{1}; \mathbf{1}, \mathbf{1})$ $(\mathbf{1}, \mathbf{28}; \mathbf{1}, \mathbf{1})$ $(\mathbf{8}, \mathbf{8}; \mathbf{1}, \mathbf{1})$ $(\mathbf{1}, \mathbf{1}; \mathbf{28}, \mathbf{1})$ $(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{28})$ $(\mathbf{1}, \mathbf{1}; \mathbf{8}, \mathbf{8})$ $(\mathbf{8}, \mathbf{1}; \mathbf{8}, \mathbf{1})$ $(\mathbf{1}, \mathbf{8}; \mathbf{1}, \mathbf{8})$	16	4
\mathbf{Z}_4	2	$[U(4) \otimes U(4)]_{99} \otimes [U(4) \otimes U(4)]_{55}$	$(\mathbf{6}, \mathbf{1}; \mathbf{1}, \mathbf{1})$ $(\mathbf{1}, \mathbf{6}; \mathbf{1}, \mathbf{1})$ $(\mathbf{4}, \mathbf{4}; \mathbf{1}, \mathbf{1})$ $(\mathbf{1}, \mathbf{1}; \mathbf{6}, \mathbf{1})$ $(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{6})$ $(\mathbf{1}, \mathbf{1}; \mathbf{4}, \mathbf{4})$ $2(\mathbf{4}, \mathbf{1}; \mathbf{4}, \mathbf{1})$ $2(\mathbf{1}, \mathbf{4}; \mathbf{1}, \mathbf{4})$	14	6
\mathbf{Z}_4	4	$[U(2) \otimes U(2)]_{99} \otimes [U(2) \otimes U(2)]_{55}$	$4(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})$ $(\mathbf{2}, \mathbf{2}; \mathbf{1}, \mathbf{1})$ $(\mathbf{1}, \mathbf{1}; \mathbf{2}, \mathbf{2})$ $4(\mathbf{2}, \mathbf{1}; \mathbf{2}, \mathbf{1})$ $4(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{2})$	13	7

$$\text{Tr}(\gamma_{1,9}) = \text{Tr}(\gamma_{1,5}) = 0. \tag{14}$$

Using the solutions for the Chan-Paton gamma matrices, we can find the open string spectrum. The gauge group is reduced to $U(8)_{99} \otimes U(8)_{55}$. The massless spectrum of this model is summarized in Table I. The multiplicity $\xi=2$ in the 59 sector [that is, the presence of *two* (instead of one as in the previous case) bi-fundamentals $(\mathbf{8}; \mathbf{8})$ of $U(8)_{99} \otimes U(8)_{55}$] is related to the corresponding multiplicity in the 59 cylinder amplitude as we discussed in Sec. II.

Before we discuss the closed string spectrum, let us see what we would expect from anomaly cancellation. Since we are compactifying type IIB on an orbifold limit of K3 (which has 80 moduli), the number of closed string hypermultiplets plus extra tensor multiplets must be 20:

$$n_H^c + n_T = 20. \tag{15}$$

The anomaly cancellation condition becomes

$$n_H^o - n_V = 224 - 28n_T, \tag{16}$$

where n_H^o is the number of open string hypermultiplets. In the \mathbf{Z}_2 model with $b=2$, the above condition implies that $n_T=4$. Therefore, in addition to rank reduction, some of the closed string hypermultiplets must be converted to tensor multiplets when we turn on the B field.

To see that there are precisely 4 extra tensor multiplets when we turn on the B field with $b=2$, let us analyze the closed string spectrum more carefully. Recall that in the case without the B field, the untwisted sector gives 4 hypermultiplets and one tensor multiplet. At each fixed point of the twisted sector, the NS and R sector massless states transform in the following representations of the six dimensional little group $SO(4) \approx SU(2) \otimes SU(2)$:

Sector	$SU(2) \otimes SU(2)$	rep.	
NS	$2(\mathbf{1}, \mathbf{1})$,		
R	$(\mathbf{1}, \mathbf{2})$.		(17)

The twisted sector spectrum is obtained by taking products of states from the left- and right-moving sectors. The orientifold projection Ω keeps symmetric combinations in the NS-NS sector and antisymmetric combinations in the R-R sector. This gives $4(\mathbf{1}, \mathbf{1})$ which is the bosonic content of a hypermultiplet. Since there are 16 fixed points in the \mathbf{Z}_2 orbifold, the twisted sector gives total of 16 hypermultiplets.

However, the above counting is based on the fact that the orbifold fixed points are invariant under Ω . If a fixed point picks up a minus sign under the Ω projection (i.e., if it is odd under the Ω projection), then the states that are kept after the Ω projection are antisymmetric combinations in the NS-NS sector and symmetric combinations in the R-R sector. This gives rise to $(\mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3})$ which is the bosonic content of a tensor multiplet.

Therefore, we expect that four of the \mathbf{Z}_2 fixed points are not Ω invariant for $b=2$. To see that this is indeed the case, let us go to the enhanced symmetry point such that the momentum lattice $\{(P_L, P_R)\} \cong \Gamma^{4,4}$ is the $SO(8)$ lattice, i.e., $P_L, P_R \in \tilde{\Gamma}^4$ [$SO(8)$ weight lattice], and $P_L - P_R \in \Gamma^4$ [$SO(8)$ root lattice]. This is achieved by choosing specific values of B_{ij} and G_{ij} :

$$B_{ij} = \begin{pmatrix} 0 & 1/2 & 0 & -1/2 \\ -1/2 & 0 & 1/2 & 0 \\ 0 & -1/2 & 0 & 1/2 \\ 1/2 & 0 & -1/2 & 0 \end{pmatrix},$$

$$G_{ij} = \begin{pmatrix} 2 & -1 & 0 & 1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 1 & 0 & -1 & 2 \end{pmatrix}. \quad (18)$$

Note that the NS-NS antisymmetric tensor B_{ij} has rank $b=2$ in this case.

At this special point with $SO(8)$ enhanced symmetry, we can express the 16 \mathbf{Z}_2 fixed points as points in the $\Gamma^{4,4}$ lattice. If we write the $SO(8)$ representations in the $SU(2)^4$ basis, the fixed points are given by $P_L = P_R = (\mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{1})$, $(\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1})$, $(\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1})$, and $(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2})$.

Let us focus on the four fixed points with $P_L = P_R = (\mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{1})$ as the analysis for the other fixed points is similar. We can form three linear combinations of these four fixed points such that they are invariant under the Ω projection (i.e., $P_L \leftrightarrow P_R$). The remaining linear combination is not Ω invariant but has $\Omega = -1$. Therefore, there are altogether 12 fixed points with $\Omega = +1$ (which give 12 hypermultiplets) and 4 fixed points with $\Omega = -1$ (which give 4 tensor multiplets).

Although our arguments are made at the enhanced symmetry point, the results hold for generic points as well. This

is because rank reduction of the gauge group depends only on the rank of B_{ij} and not on its precise form nor the values of G_{ij} . We can reach other points in the moduli space with different $\Gamma^{4,4}$ lattices by changing G_{ij} . In particular, the number of tensor multiplets in the \mathbf{Z}_2 twisted sector is always 4 as long as $b=2$.

Since we are compactifying type IIB on T^4/\mathbf{Z}_2 , we can consider turning on B_{ij} with $b=4$. We can immediately deduce that the number of extra tensor multiplets in this case is $n_T=6$. To see this, first consider a compactification on $(T^2 \otimes T^2)/\mathbf{Z}_2$. Let the B field in the first 2-torus be nonzero, while keeping the B field in the second 2-torus zero. The fixed points decompose as $\mathbf{16} = \mathbf{4} \otimes \mathbf{4}$, where the second factor of $\mathbf{4}$ is Ω invariant as there is no B field in the second 2-torus. But from the above analysis we know that the total number of fixed points for $b=2$ (which is the case here) must be 12. This implies that the first factor of $\mathbf{4}$ (coming from the first 2-torus with the B field) decomposes as $\mathbf{4} = \mathbf{3}_+ \oplus \mathbf{1}_-$, where the subscript indicates whether the corresponding fixed points are even or odd under Ω . Now consider compactification on $(T^2 \otimes T^2)/\mathbf{Z}_2$ with nonzero B field in both 2-tori. This corresponds to $b=4$. The fixed points now decompose into $\mathbf{16} = (\mathbf{3}_+ \oplus \mathbf{1}_-) \otimes (\mathbf{3}_+ \oplus \mathbf{1}_-) = \mathbf{10}_+ \oplus \mathbf{6}_-$. The number of D9-branes as well as the number of D5-branes in this case is 8. The maximal unbroken gauge group is $U(4) \otimes U(4)$. The massless spectrum of this model is given in Table I. Note that the anomaly is canceled in this model.

Next, we briefly discuss other \mathbf{Z}_N orientifolds. Note that the only sector where the number of tensor multiplets can vary with the B field is the \mathbf{Z}_2 twisted sector. This is because the action of Ω interchanges the g^k twisted sector with the g^{N-k} twisted sector for $2k \neq 0, N$ [3]. Each of the fixed points in such sectors, therefore, gives rise to one hypermultiplet and one tensor multiplet regardless of the B field. Thus, in the \mathbf{Z}_3 case the number of extra tensor multiplets is always $n_T=9$. In the \mathbf{Z}_4 case in the \mathbf{Z}_2 twisted sector 4 (of the original 16) fixed points are also fixed under \mathbf{Z}_4 (i.e., they are invariant under \mathbf{Z}_4). The other 12 fixed points pair up into 6 \mathbf{Z}_4 invariant pairs. For $b=2$, two of these 6 pairs have $\Omega = -1$ (which give 2 tensor multiplets). Therefore, the total number of extra tensor multiplets in this case is $n_T=6$. For $b=2$, three of the 6 pairs have $\Omega = -1$ (which give 3 tensor multiplets), and the total number of extra tensor multiplets in this case is $n_T=7$. In the \mathbf{Z}_6 case 1 (of the original 16) fixed point in the \mathbf{Z}_2 twisted sector is also fixed under \mathbf{Z}_3 (i.e., it is invariant under \mathbf{Z}_3). The other 15 fixed points form 5 linear combinations invariant under \mathbf{Z}_3 , 5 linear combinations that pick up a phase $\omega = \exp(2\pi i/3)$ under \mathbf{Z}_3 , and 5 linear combinations that pick up a phase ω^2 under \mathbf{Z}_3 . The $\Omega = -1$ states are always among the 10 combinations which are not invariant under \mathbf{Z}_3 . That is, all the \mathbf{Z}_2 fixed points which are not invariant under Ω in the presence of the B field do not contribute to the massless spectrum. Thus, the number of extra tensor multiplets in the \mathbf{Z}_6 model is always $n_T=6$ regardless of the value of b .

The massless open and closed string spectra of the \mathbf{Z}_N orientifolds with and without the B field are given in Table I (for $N=2,4$) and Table II (for $N=3,6$).

TABLE II. The massless spectrum of the six dimensional type IIB orientifolds on T^4/\mathbf{Z}_N for $N=3,6$, and various values of b (the rank of B_{ij}). The semi-colon in the column ‘‘Charged hypermultiplets’’ separates 99 and 55 representations.

Model	b	Gauge group	Charged hypermultiplets	Neutral hypermultiplets	Extra tensor multiplets			
\mathbf{Z}_3	0	$[U(8) \otimes SO(16)]_{99}$	(28,1) (8,16)	11	9			
	2	$U(8)_{99}$	36	11	9			
	4	$SO(8)_{99}$	—	11	9			
\mathbf{Z}_6	0	$[U(4) \otimes U(4) \otimes U(8)]_{99} \otimes [U(4) \otimes U(4) \otimes U(8)]_{55}$	(6,1,1;1,1,1)	14	6			
			(1,6,1;1,1,1)					
			(4,1,8;1,1,1)					
			(1,4,8;1,1,1)					
			(1,1,1;6,1,1)					
			(1,1,1;1,6,1)					
			(1,1,1;4,1,8)					
			(1,1,1;1,4,8)					
			(4,1,1;4,1,1)					
			(1,4,1;1,4,1) (1,1,8;1,1,8)					
2	$[U(4) \otimes U(4)]_{99} \otimes [U(4) \otimes U(4)]_{55}$	(6,1,1,1)	14	6				
		(1,6,1,1)						
		(4,4,1,1)						
		(1,1,6,1)						
		(1,1,1,6)						
		(1,1,4,4)						
		2(4,1,4,1)						
		2(1,4,1,4)						
		4			$U(4)_{99} \otimes U(4)_{55}$	2(6;1)	14	6
						2(1;6)		
4(4;4)								

IV. COMMENTS

In this section we point out relations between various orientifolds with and without the B field, and also discuss the F-theory duals of these models. For convenience we will refer to the \mathbf{Z}_N model with the rank- b B -field as the $[N, b]$ model. (Here we note that the $[6,4]$ model was discussed in the third reference in [1], while the $[3,4]$ model was discussed in [17].)

To begin with note the following.

The $[2,2]$ and $[4,0]$ models are on the same moduli. This can be seen by starting from the $[4,0]$ model and giving VEVs to the fields $(\mathbf{8}, \mathbf{8}; \mathbf{1}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{1}; \mathbf{8}, \mathbf{8})$ such that the 99 gauge group is broken to $[U(8)_{\text{diag}}]_{99} \subset [U(8) \otimes U(8)]_{99}$, and similarly for the 55 gauge group.

The $[2,4]$, $[4,2]$, $[6,0]$, $[6,2]$ and $[6,4]$ models are on the same moduli. This is obvious for the $[4,2]$ and $[6,2]$ models as their massless spectra are identical. Similarly, the massless spectra of the $[2,4]$ and $[6,4]$ models are the same. We can obtain the $[2,4]$ model from the $[4,2]$ model by giving VEVs to the fields $(\mathbf{4}, \mathbf{4}; \mathbf{1}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{1}; \mathbf{4}, \mathbf{4})$ such that the 99 gauge group is broken to $[U(4)_{\text{diag}}]_{99} \subset [U(4) \otimes U(4)]_{99}$, and

similarly for the 55 gauge group. In fact, we can break the gauge group by the Higgs mechanism completely. After using the Higgs mechanism there are 56 neutral hypermultiplets in the open string sector. Note that in the $[6,0]$ model the gauge group can also be broken by the Higgs mechanism completely, and there are 56 neutral hypermultiplets in the open string sector in this case as well. This implies that the $[6,0]$ model is on the same moduli as the other four models.

The $[3,2]$ and $[3,4]$ models are on the same moduli. This can be seen by starting from the $[3,2]$ model and giving a VEV to the field in the symmetric $(\mathbf{36})$ representation of $U(8)$ such that the 99 gauge group is broken to $SO(8)$.

Here we can extend some of the analyses of [18] to the orientifolds with nonzero B field. In particular, let us compactify these models further on T^2 . The resulting four dimensional models have $\mathcal{N}=2$ space-time supersymmetry. Let us go to a generic point in the moduli space where the gauge group is maximally broken by the Higgs mechanism (so that the gauge group is either completely broken or consists of Abelian factors only). Let $r(V)$ be the number of open string sector vector multiplets after using the Higgs mechanism.

TABLE III. Hodge numbers for F-theory duals.

Model	b	Gauge group	$(h^{1,1}, h^{2,1})$	χ
\mathbf{Z}_2	0	$U(16) \otimes U(16)$	(3,243)	-480
	2	$U(8) \otimes U(8)$	(7,127)	-240
	4	$U(4) \otimes U(4)$	(9,69)	-120
\mathbf{Z}_3	0	$U(8) \otimes SO(16)$	(20,14)	12
	2	$U(8)$	(16,10)	12
	4	$SO(8)$	(16,10)	12
\mathbf{Z}_4	0	$[U(8) \otimes U(8)]^2$	(7,127)	-240
	2	$[U(4) \otimes U(4)]^2$	(9,69)	-120
	4	$[U(2) \otimes U(2)]^2$	(10,40)	-60
\mathbf{Z}_6	0	$[U(4) \otimes U(4) \otimes U(8)]^2$	(9,69)	-120
	2	$[U(4) \otimes U(4)]^2$	(9,69)	-120
	4	$U(4) \otimes U(4)$	(9,69)	-120

Then the total number of vector multiplets is given by $r(V) + T + 2$, where $T = n_T + 1$ is the total number of tensor multiplets in six dimensions. Let H^0 be the number of neutral hypermultiplets that are uncharged with respect to the leftover Abelian gauge group. Then, if we assume that the four dimensional models have type IIA duals, the Hodge numbers of the corresponding Calabi-Yau three-folds are given by [19]

$$h^{1,1} = r(V) + T + 2, \quad (19)$$

$$h^{2,1} = H^0 - 1. \quad (20)$$

From the various dualities between type IIA, heterotic, type I, and F-theory, it is not difficult to see that these Calabi-Yau three-folds must be elliptically fibered, and F-theory com-

pactifications on these spaces should be dual to the original six dimensional orientifold models. In Table III we give the Hodge (and Euler) numbers of the Calabi-Yau three-folds for each of the $[N, b]$ models.

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