On the Influence of Gravitation on the Propagation of Light

By A. Einstein.

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There are two translations of this paper that I know of:

1. In *The principle of relativity, a collection of original memoirs on the special and general theory of relativity,* by H. A. Lorentz, A. Einstein, H. Minkowski and H. Weyl, with notes by A. Sommerfeld, tr. by W. Perrett and G. B. Jeffery, Dover Publications, Inc., 1923. This paper appears on pp. 97-108.
2. In *The Collected Papers of Albert Einstein, Volume 3: The Swiss Years: Writings, 1909-1911,* Albert Einstein, vol. 3 edited by Martin J. Klein, A. J. Kox, Jürgen Renn, and Robert Schulmann, Series ed. John Stachel, Princeton University Press, 1987-.

Neither of these translations is readily available outside major research libraries. The Dover publication is available through AbeBooks.com (typical price $4.) and Princeton vol. 3 is, I think, still in print. (The current AbeBooks price is $168.) It appears that Princeton Press has no interest in making such material available on the web.

The original Annalen der Physik publication is available on the web at Gallica (http://gallica.bnf.fr/ark:/12148/bpt6k15338w.image) or on the Wiley website, which, however, requires subscription.

The two English translations appear to have been done independently, although much of the Princeton text is quite similar to the Dover text. Both just used "cut and paste" to produce the Figures. Neither translation seemed to me to be sufficiently accurate to fully convey what Einstein wrote. For these reasons I felt that a new, and freely available, translation would be helpful. In doing the translation I made substantial use of the Dover text, which seems somewhat better than the Princeton one. Where problems occurred I also checked the Princeton text. For anyone with reasonable familiarity with German it is, I think, still a good idea to download a copy of the original. Einstein was, as is well-known, a quite original writer. His exact choice of phrases and individual wording is often important. The purely technical content is, in any case, in the equations. Both translations almost always correctly transcribed the equations, although they both just transcribed typos of the math symbols in the text.

I have checked this text carefully, but it is likely that errors have been missed. Any corrections or suggestions for clarification would be welcome.

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On the Influence of Gravitation on the Propagation of Light

By A. Einstein.

In a contribution published four years ago* I tried to answer the question whether the propagation of light is influenced by gravitation. I return to this theme because my previous presentation of the subject does not satisfy me, but even more because I now see that one of the most important consequences of my former treatment is capable of being tested experimentally. For it follows from the theory to be presented here, that light-rays passing close to the sun are deflected by its gravitational field so that the apparent angular distance between the sun and a visible fixed star near to it is increased by nearly a second of arc.

In the course of these investigations further results which relate to gravitation are shown. But, as the exposition of the entire group of considerations would be rather difficult to follow, only a few quite elementary investigations will be given in the following pages, from which the reader will readily be able to orient himself as to the direction and train of thought of the theory. The relations here deduced, even though the theoretical foundation is sound, are valid only to a first approximation.

§ 1. A Hypothesis as to the Physical Nature of the Gravitational Field

In a homogeneous gravitational field (acceleration of gravity γ) let there be a stationary system of co-ordinates K, orientated so that the lines of force of the gravitational field run in the negative direction of the z-axis. In a space free of gravitational fields let there be a second system of co-ordinates K', moving with uniform acceleration (γ) in the positive direction of its z-axis. To avoid unnecessary complications, let us for the present disregard the theory of relativity, and regard both systems from the customary point of view of kinematics, and the movements occurring in them from that of ordinary mechanics.

Relative to K, as well as relative to K', material points which are not subjected to the action of other material points, move according to the equations:

$$\frac{d^2x_\nu}{dt^2} = 0, \quad \frac{d^2y_\nu}{dt^2} = 0, \quad \frac{d^2z_\nu}{dt^2} = -\gamma\,.$$

For the accelerated system K' this follows directly from Galileo's principle, but for the system K, at rest in a homogeneous gravitational field, it follows from the experience that all bodies in such a field are equally and uniformly accelerated. This experience, of the equal falling of all bodies in the gravitational field, is one of the most universal which the observation of nature has yielded to us; but in spite of this, this law has found no place in the foundations of our world view (Weltbildes) of the physical universe.

But we arrive at a very satisfactory interpretation of this empirical law, if we assume that the systems K and K' are physically exactly equivalent, that is, if we

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* A. Einstein, Jahrbuch für Radioact, und Electronik, 4, 1907.

assume that we may just as well regard the system K' as being in a space free from gravitational fields; then we must regard K as uniformly accelerated. This assumption of exact physical equivalence makes it impossible for us to speak of the *absolute acceleration* of the system of reference, just as the usual theory of relativity forbids us to talk of the *absolute velocity* of a system;[1] This assumption also makes the equal falling of all bodies in a gravitational field seem obvious.

As long as we restrict ourselves to purely mechanical processes in the realm where Newton's mechanics is valid, we are certain of the equivalence of the systems K and K'. But our view of this will not have any deeper significance unless the systems K and K' are equivalent with respect to all physical processes, that is, unless the laws of nature with respect to K are in entire agreement with those with respect to K'. By assuming this to be so, we arrive at a principle which, if it is really true, has great heuristic importance. For by theoretical consideration of processes which take place relative to a system of reference with uniform acceleration, we obtain information as to the behavior of processes in a homogeneous gravitational field.[2] We shall now show, first of all from the standpoint of the ordinary theory of relativity, that our hypothesis has considerable probability.

§ 2. On the Gravitation of Energy

The theory of relativity shows that the inertial mass of a body increases with the energy it contains; if the increase of energy amounts to E, the increase in inertial mass is equal to E/c^2, where c denotes the velocity of light. Now, is there an increase of gravitational mass corresponding to this increase of inertial mass? If not, then a body would fall in the same gravitational field with varying acceleration according to the energy it contained. And then the highly satisfactory result of the theory of relativity, by which the law of the conservation of mass leads to the law of conservation of energy, could not be maintained, because it would compel us to abandon the law of the conservation of mass in its old form for *inertial* mass, but maintain it for gravitational mass.

This must be regarded as very improbable. On the other hand, the usual theory of relativity does not provide us with any argument from which to infer that the weight of a body depends on the energy contained in it. But we shall show that our hypothesis of the equivalence of the systems K and K' gives us gravitation of energy as a necessary consequence.

Let two material systems S_1 and S_2 (Fig. 1), each provided with measuring instruments, be situated on the z-axis of K at the distance h from each other,[3] so that the gravitational potential at S_2 is greater than that at S_1 by γh. Let a definite quantity of

[1] Of course, we cannot replace an *arbitrary* gravitational field by a state of motion of a system without a gravitational field, just as we cannot transform to rest all the points of an arbitrarily moving medium by means of a relativistic transformation.

[2] It will be shown in a subsequent paper that the gravitational field considered here is homogeneous only to a first approximation.

[3] S_1 and S_2 are regarded as infinitely small in comparison with h.

energy E be emitted from S_2 towards S_1. Let the quantities of energy in S_1 and S_2 be measured by devices which – brought to *one* location in the system z and there compared – are perfectly alike. As to the process of this energy transmission by radiation we can make no a priori assertion because we do not know the influence of the gravitational field on the radiation and the measuring instruments at S_1 and S_2.

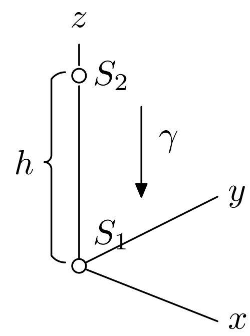

Fig. 1.

But by our postulate of the equivalence of K and K' we are able, in place of the system K in a homogeneous gravitational field, to set the gravitation-free system K', which moves with uniform acceleration in the direction of positive z, and by the z-axis of which the material systems S_1 and S_2 are rigidly connected.

We consider the process of transmission of energy by radiation from S_2 to S_1 from a system K_0, which is free of acceleration. At the moment when the radiation energy E_2 is emitted from S_2 toward S_1, let the velocity of K' relative to K_0 be zero. The radiation will arrive at S_1 when the time h/c has elapsed (to a first approximation). But at this moment the velocity of S_1 relative to K_0 is $\gamma h/c = v$. Therefore by the ordinary theory of relativity the radiation arriving at S_1 does not possess the energy E_2, but a greater energy E_1, which is related to E_2, to a first approximation, by the equation[1]:

$$E_1 = E_2\left(1 + \frac{v}{c}\right) = E_2\left(1 + \frac{\gamma h}{c^2}\right). \tag{1}$$

By our assumption exactly the same relation holds if the same process takes place in the system K, which is not accelerated, but is provided with a gravitational field. In this case we may replace γh by the potential Φ of the gravitation vector in S_2, if the arbitrary constant of Φ in S_1 is set to zero. We then have the equation:

$$E_1 = E_2 + \frac{E_2}{c^2}\Phi. \tag{1a}$$

This equation expresses the energy law for the process under observation. The energy E_1 arriving at S_1 is greater than the energy E_2, measured by the same means, which was emitted from S_2, the excess being the potential energy of the mass E_2/c^2 in the gravitational field. This shows that in order to satisfy the energy principle we have to ascribe to the energy E, before its emission from S_2, a potential energy, due to gravity, which corresponds to the (gravitational) mass E/c^2. Our assumption of the equivalence of K and K' thus removes the difficulty mentioned at the beginning of this Section, which is left unsolved by the ordinary theory of relativity.

The meaning of this result is shown particularly clearly if we consider the following cycle of operations: –

[1] A. Einstein, Ann. d. Phys. 17, p. 913 – 914. 1905.

1. The energy E, as measured in S_2, is emitted in the form of radiation from S_2 towards S_1, where, by the result just obtained, the energy $E(1 + \gamma h/c^2)$ (as measured in S_1) is absorbed.
2. A body W of mass M is lowered from S_2 to S_1, work $M\gamma h$ thereby being done.
3. The energy E is transmitted from S_1 to the body W while W is in S_1. The gravitational mass M is thereby changed so that it acquires the value M'.
4. Let W be again raised to S_2, work $M'\gamma h$ being done as a result.
5. Let E be transmitted from W back to S_2.

The effect of this cycle is simply that S_1 has undergone an energy increase of $E(\gamma h/c^2)$, and that the quantity of energy

$$M'\gamma h - M\gamma h$$

has been supplied to the system in the form of mechanical work. By the energy principle, we must therefore have

$$E\frac{\gamma h}{c^2} = M'\gamma h - M\gamma h$$

or

$$M' - M = \frac{E}{c^2}. \tag{1b}$$

The increase in *gravitational* mass is thus equal to E/c^2, and therefore equal to the increase in *inertial* mass as given by the theory of relativity.

This result emerges still more directly from the equivalence of the systems K and K', according to which the *gravitational* mass of K is exactly equal to the *inertial* mass of K'; energy must therefore possess a *gravitational* mass which is equal to its *inertial* mass. If a mass M_0 be suspended on a spring balance in the system K' the balance will indicate the apparent weight $M_0\gamma$ on account of the inertia of M_0. If the quantity of energy E be transmitted to M_0, the spring balance, by the law of the inertia of energy, will indicate $\left(M_0 + \frac{E}{c^2}\right)\gamma$. By reason of our fundamental assumption exactly the same thing must occur when the experiment is repeated in the system K, that is, in the gravitational field.

§ 3. Time and the Velocity of Light in the Gravitational Field

If the radiation emitted in the uniformly accelerated system K' in S_2 toward S_1 had the frequency ν_2 relative to the clock at S_2, then, relative to S_1, at its arrival at S_1 it no longer has the frequency ν_2 relative to an identical clock at S_1, but a greater frequency ν_1, such that, to a first approximation

$$\nu_1 = \nu_2\left(1 + \gamma\frac{h}{c^2}\right). \tag{2}$$

If we again introduce the unaccelerated reference system K_0, relative to which at the time of the emission of light, K' has no velocity, then S_1, at the time of arrival of the radiation at S_1 has, relative to K_0, the velocity $\gamma(h/c)$ from which, by Doppler's principle, the relation as given results immediately.

In agreement with our assumption of the equivalence of the systems K' and K, this equation also holds for a stationary system of co-ordinates K_0 in a uniform gravitational field, if in it the transmission by radiation takes place as described. It follows, then, that a light-ray emitted from S_2 with a definite gravitational potential, and possessing at its emission the frequency ν_2 – compared with a clock at S_2 – will, at its arrival at S_1, possess a different frequency ν_1 measured by an identical clock at S_1. For γh we substitute the gravitational potential Φ of S_2 – that of S_1 being taken as zero – and assume that the relation which we have deduced for the *homogeneous* gravitational field also holds for other forms of field. Then

$$\nu_1 = \nu_2\left(1 + \frac{\Phi}{c^2}\right) \tag{2a}$$

This result (which by our derivation is valid to a first approximation) permits, first, the following application. Let ν_0 be the oscillation-number of an elementary light-generator, measured by a clock U at the same location. This oscillation-number is then independent of the locations of the light-generator and the clock. Let us imagine them both at a position on the surface of the Sun (where our S_2 is located). Of the light emitted from there a portion reaches the Earth (S_1), where we measure the frequency ν of the arriving light with a clock U of exactly the same properties as the one just mentioned. Then by (2a),

$$\nu = \nu_0\left(1 + \frac{\Phi}{c^2}\right)$$

where Φ is the (negative) difference of gravitational potential between the surface of the Sun and the Earth. Thus according to our view the spectral lines of sunlight, as compared with the corresponding spectral lines of terrestrial light sources, must be somewhat displaced toward the red, in fact by the relative amount

$$\frac{\nu_0 - \nu}{\nu_0} = \frac{-\Phi}{c^2} = 2 \times 10 - 6\,.$$

If the conditions under which the solar lines arise were exactly known, this shifting would be susceptible of measurement. But as other influences (pressure, temperature) affect the position of the centers of the spectral lines, it is difficult to discover whether the inferred influence of the gravitational potential really exists.[1]

On superficial consideration equation (2) or (2a), respectively, seems to assert an absurdity. If there is constant transmission of light from S_2 to S_1, how can any other number of periods per second arrive at S_1 than is emitted from S_2? But the answer is

[1] L. F. Jewell (Journ. de Phys., **6**, p. 84, 1897) and particularly Ch. Fabry and H. Boisson (Compt. rend. **148**. p. 688-690, 1909) have actually found such displacements of fine spectral lines toward the red end of the spectrum, of the order of magnitude here calculated, but have ascribed them to an effect of pressure in the absorbing layer.

simple. We cannot regard ν_2 or respectively ν_1 simply as frequencies (as the number of periods per second) since we have not yet determined a time in system K. What ν_2 denotes is the number of periods per second with reference to the time-unit of the clock U at S_2, while ν_1 denotes the number of periods per second with reference to the identical clock at S_1. Nothing compels us to assume that the clocks U in different gravitation potentials must be regarded as going at the same rate. On the contrary, we must certainly define the time in K in such a way that the number of wave crests and troughs between S_2 and S_1 is independent of the absolute value of time: for the process under observation is by nature a stationary one. If we did not satisfy this condition, we should arrive at a definition of time such that by its application time would enter explicitly into the laws of nature, and this would certainly be unnatural and inappropriate. Therefore the two clocks at S_1 and S_2 do not both give the "time" correctly. If we measure time at S_1 with the clock U, *then we must measure time at S_2 with a clock which goes $1 + \Phi/c^2$ times more slowly than the clock U when compared with U at one at the same location.* For when measured by such a clock, the frequency of the light-ray which is considered above is at its emission from S_2

$$\nu_2\left(1 + \frac{\Phi}{c^2}\right),$$

and is therefore, by (2a), equal to the frequency ν_1 of the same light-ray on its arrival at S_1.

This has a consequence which is of fundamental importance for our theory. For if we measure the velocity of light at different locations in the accelerated, gravitation-free system K', employing clocks U of identical properties we obtain the same magnitude at all these locations. The same holds good, by our fundamental assumption, for the system K as well. But from what has just been said we must use clocks of unlike properties for measuring time at locations with differing gravitation potential. For measuring time at a location which, relative to the origin of the co-ordinates, has the gravitation potential Φ, we must employ a clock which – when transferred to the co-ordinate origin – goes $(1 + \Phi/c^2)$ times more slowly than the clock used for measuring time at the origin of co-ordinates. If we call the velocity of light at the origin of co-ordinates c_0, then the velocity of light c at a location with the gravitation potential Φ will be given by the relation

$$c = c_0\left(1 + \frac{\Phi}{c^2}\right). \tag{3}$$

The principle of the constancy of the velocity of light holds good according to this theory in a different form from that which usually underlies the ordinary theory of relativity.

§ 4. Bending of Light-Rays in the Gravitational Field

From the proposition which has just been proved, that the velocity of light in the gravitational field is a function of the location, we may easily infer, by means of Huygens's principle, that light-rays propagated across a gravitational field undergo deflection. For let ε be a wave front of a plane light-wave at the time t, and let P_1 and P_2 be two points in that plane at unit distance from each other. P_1 and P_2 lie in the plane of the paper, which is chosen so that the differential coefficient of Φ, taken in the direction of the normal to the plane, and therefore also that of c, vanishes. We obtain the corresponding wave front at time $t+dt$, or, rather, its intersection with the plane of the paper, by describing circles round the points P_1 and P_2 with radii $c_1 dt$ and $c_2 dt$ respectively, where c_1 and c_2 denote the velocity of light at the points P_1 and P_2 respectively, and by drawing the tangent to these circles. The angle through which the light-ray is deflected on the path cdt is therefore

$$\frac{(c_1 - c_2)dt}{1} = -\frac{\partial c}{\partial n'}dt,$$

if we calculate the angle positively when the ray is bent toward the side of increasing n'.

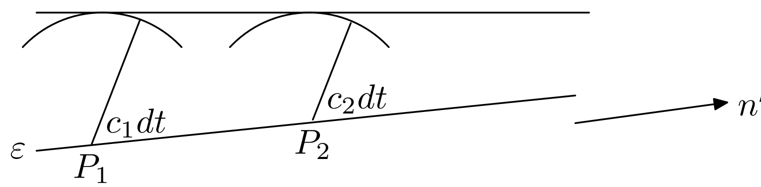

Fig. 2.

The angle of deflection per unit of path of the light-ray is thus

$$-\frac{1}{c}\frac{\partial c}{\partial n'}$$

or by (3) it is

$$-\frac{1}{c^2}\frac{\partial \Phi}{\partial n'}.$$

Finally, we obtain for the deflection α, which a light-ray experiences toward the side n' on any path (s) the expression

$$\alpha = -\frac{1}{c^2}\int \frac{\partial \Phi}{\partial n'}ds. \tag{4}$$

We might have obtained the same result by directly considering the propagation of a light-ray in the uniformly accelerated system K', and transferring the result to the system K, and thence to the case of a gravitational field of any form.

By equation (4) a light-ray passing by a heavenly body suffers a deflection to the side of the diminishing gravitational potential, that is, to the side directed toward the heavenly body, of the magnitude

$$\alpha = \frac{1}{c^2}\int_{\vartheta=-\frac{\pi}{2}}^{\vartheta=+\frac{\pi}{2}} \frac{kM}{r^2}\cos(\vartheta)ds = \frac{2kM}{c^2\Delta},$$

where k denotes the constant of gravitation, M the mass of the heavenly body, Δ the distance of the ray from the center of the body (and r and ϑ are as shown in Fig. 3). *A light-ray going past the Sun would accordingly undergo deflection by the amount of* $4 \times 10^6 = 0.83$ *seconds of arc.* The angular distance of the star from the center of the

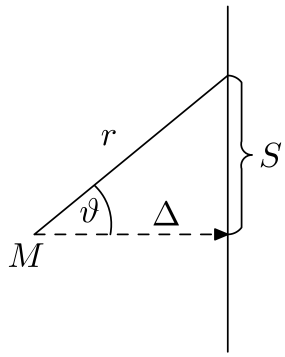

Fig. 3.

Sun appears to be increased by this amount. As the fixed stars in the parts of the sky near the Sun are visible during total eclipses of the Sun, this consequence of the theory may be compared with experimental evidence. With the planet Jupiter the displacement to be expected reaches to about 1/100 of the amount given. It would be urgently wished that astronomers take up the question here raised, even though the considerations presented above may seem insufficiently established or even bizarre. For, apart from any theory, there is the question whether it is possible with the equipment at present available to detect an influence of gravitational fields on the propagation of light.

Prague, June 1911.

(Submitted 21 June 1911.)