UFIR Filtering Under Uncertain One-Step Delayed and Missing Data

KAREN URIBE-MURCIA
Dept. of Electronics Engineering
Universidad de Guanajuato
Salamanca, MEXICO

YURIY S. SHMALIY
Dept. of Electronics Engineering
Universidad de Guanajuato
Salamanca, MEXICO

Abstract—This paper develops the unbiased finite impulse response (UFIR) filter for wireless sensor network (WSN) systems whose measurements are affected by random delays and packet dropout due to inescapable failures in the transmission and sensors. The Bernoulli distribution is used to model delays in arrived measurement data with known transmission probability. The effectiveness of the UFIR filter is compared experimentally to the KF and game theory recursive H1 filter in terms of accuracy and robustness employing the GPS-measured vehicle coordinates transmitted with latency over WSN.

Key-Words: Delayed data, packet dropouts, Bernoulli distribution, unbiased FIR filter
Received: July 23, 2020. Revised: December 2, 2020. Accepted: December 23, 2020. Published: December 30 , 2020.

1. Introduction

State estimation of measured quantities provided over wireless sensor networks (WSN) has attracted attention in recent decades due to the ability to create measurement environments over big areas for navigation, target tracking, manufacturing control, and communication [1]–[8]. A common weakness of such systems is associated with the influence of sensor uncertainties, random transmission delays, and packet dropouts causing estimation failures and deteriorating the performance significantly [9], [10].

To avoid large estimation errors caused by latency and packet dropout, various filtering algorithms have been developed during the last decades to address the phenomena separately [11]–[13], even though they practically occur jointly [14], [15]. The game theory H_∞ filter is applied in [16]–[18] to network system considering different phenomena such as noise covariances, one-step delayed data, and multiplicative packed dropouts. Other techniques such as the recursive state estimation [19] and the Kalman filter (KF) [20], [21] have been applied to non-linear WSN-based systems. The particle filter [22] and robust L_1 filter, which minimizes the peak-to-peak errors, have been developed for delayed data in [23].

The Bernoulli distribution is most widely used to model latency and develop estimation algorithms [24], [25]. Even so, a big challenge still exists to design efficient algorithms under lost data. The Bernoulli distribution thus requires a modification to deal with the probability of lost data [26], [27]. Here, the zero-input procedure with a predictive algorithm is considered to include the packet dropouts to the observation model using the Bernoulli distribution.

A drawback of the traditional KF is the dependence on the incomplete knowledge about the system and environmental noise. Accordingly, the KF-based estimators often produce large errors when the conditions of optimality are not obeyed properly. This fact was recently pointed out in [28], where the KF and game theory H_∞ filter [29] were compared for robustness to the unbiased finite impulse response (UFIR) filter [30], which ignores any information about zero-mean noise and initial values and is thus robust by design. For time-stamped delays and missing data, the UFIR filter was originally designed in [31]. For systems with randomly delayed and missing data the UFIR approach still has not been applied that motivates our present work. In this paper, we develop the UFIR filter for systems with Bernoulli-distributed randomly-delayed and missing data. A better performance of the UFIR filter compared with the KF and H_∞ is demonstrated under different scenarios of the tracking problem.

This investigation was partly supported by the Mexican CONACyT-SEP Project A1-S-10287, Funding CB2017-2018.

2. State-Space Model

As has been mentioned above, measurement data transmitted over a WSN typically arrive at a receiver with latency and packet dropouts originated from different sources. Understanding an importance of the mathematical model to design an efficient filtering algorithm, the following model using a Bernoulli distribution is presented to depict the real behavior of the transmitted data. We assume that each data are transmitted and received only once, hence one-step packet dropouts may happen. In Fig.1, we show typical scenarios with the transmitted/received data. It is assumed that data arrive at the correct time t_1 if received as $x_1^{(1)}$ and $x_2^{(1)}$. If at time t_2 data are received as $x_1^{(2)}$, then such data are one-step delayed with respect to time t_1 and this information will be lost at t_1. The observation can be modeled similarly to [35] as

$$\begin{aligned} z_n &= \gamma_{0,n} y_n + (1-\gamma_{0,n}) \left[(1-\gamma_{0,n-1}) \gamma_{1,n} y_{n-1} \right. \\ &\quad \left. + (1-(1-\gamma_{0,n-1})\gamma_{1,n}) \tilde{z}_n \right], \end{aligned} \tag{1}$$

where a zero input compensation with prediction are using to substitute wrong and missing data, $z_n \in \mathbb{R}^M$ is the transmitted measurement vector at the processor, $\tilde{z}_n \in \mathbb{R}^M$ is the predicted measurement, and $\hat{x}_{n-1}$ is the available estimate.

TABLE I
DATA RECEIVED WITH PACKET DROPOUTS USING A ZERO-INPUT COMPENSATOR

n	1	2	3	4	5	6	7	8	9	10
θ_0	1	0	0	1	0	1	0	0	1	0
θ_1	–	1	1	–	0	0	0	1	–	1
θ_2	–	–	–	–	0	–	1	–	–	–
$Z(n)$	$y(1)$	$\tilde{z}(2)$	$y(2)$	$y(4)$	$\tilde{z}(5)$	$y(6)$	$y(5)$	$y(7)$	$y(9)$	$\tilde{z}(10)$

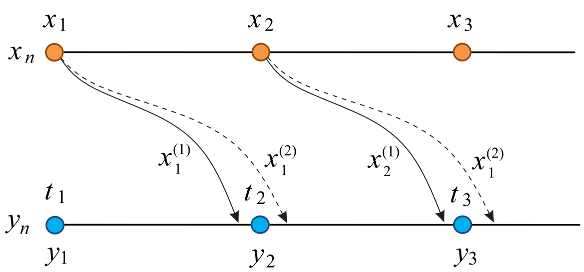

Fig. 1. Typical scenarios in WSN channels with latency. A regular mode is when data are received as $x_1^{(1)}$ and $x_2^{(1)}$. When data are received as $x_1^{(2)}$ and $x_2^{(1)}$, then x_1 will be one-step delayed and x_2 lost. When data are received as $x_1^{(1)}$ and $x_2^{(2)}$, then x_2 will be one-step delayed with no missing information.

Now, suppose that dynamics state estimation and the measurement observation are represented with the linear discrete-time state-space model

$$x_n = Fx_{n-1} + w_n \,, \quad (2)$$
$$y_n = Hx_n + v_n \,, \quad (3)$$
$$\tilde{z}_n = HF\hat{x}_{n-1} \,, \quad (4)$$

where n is the discrete time index, $x_n \in \mathbb{R}^K$ is the state vector, $y_n \in \mathbb{R}^M$ is the measured output vector, $\tilde{z}_n \in \mathbb{R}^M$ is the predictive vector and $F \in \mathbb{R}^{K\times K}$ and $H \in \mathbb{R}^{K\times M}$ are known matrices. White Gaussian noise vectors $w_n \sim \mathcal{N}(0,Q) \in \mathbb{R}^K$ and $v_n \sim \mathcal{N}(0,R) \in \mathbb{R}^M$ have zero mean, the covariances $Q = E\{w_n w_n^T\} \in \mathbb{R}^{K\times K}$ and $R = E\{v_n v_n^T\} \in \mathbb{R}^{M\times M}$. The process w_n and v_n are mutually independent, then $E\{w_n v_n^T\} = 0$ for all n. Here, $\gamma_{0,n}$ and $\gamma_{1,n}$ denote sequences of Bernoulli random variables with known probabilities $\mathcal{P}\{\gamma_0 = 1\} = \overline{\gamma}_0$ and $\mathcal{P}\{\gamma_1 = 1\} = \overline{\gamma}_1$.

It follows from (1) that if $\gamma_0 = 1$ holds at time n then the output is assigned as $z_n = y_n$ with the probability $\overline{\gamma}_{0,n}$. If $\gamma_0 = 0$ holds at n, then the delays and packet drop-outs may happen. In this cases, if $\gamma_0 = 0$ holds at $n-1$ and $\gamma_1 = 1$ at n, then the one-step delayed data y_{n-1} will be processed at n with the probability $(1-\overline{\gamma}_{0,n})(1-\overline{\gamma}_{0,n-1})\overline{\gamma}_{1,n}$. Otherwise, the information will be lost and will be recovered by the predictive algorithm $\tilde{z}_n$ with the probability $1 - \overline{\gamma}_{0,n} - (1-\overline{\gamma}_{0,n})(1-\overline{\gamma}_{0,n-1})\overline{\gamma}_{1,n}$. Examples of such scenarios are given in Table I for data received on time as y_1, y_4, y_6 and y_9; one-step delayed as y_2 and y_7; two-step delayed as y_5, and completely lost as y_3,y_8 and y_{10}.

Given the state-space model (1)-(2), the UFIR filter, KF, and game theory H_∞ filter can be developed for randomly delayed and missing data as will be shown next.

3. State-Space Model Transformation

To simplify the mathematical derivations, the observation equation can be rewritten as

$$z_n = \alpha_{0,n} y_n + \alpha_{1,n} y_{n-1} + \alpha_{2,n} \tilde{z}_n \,, \quad (5)$$

where, taking into account that $\alpha_{0,n} + \alpha_{1,n} + \alpha_{2,n} = 1$, the auxiliary coefficients can be defined by

$$\alpha_{0,n} = \gamma_{0,n} \,, \quad (6)$$
$$\alpha_{1,n} = (1-\gamma_{0,n})(1-\gamma_{0,n-1})\gamma_{1,n} \,, \quad (7)$$
$$\alpha_{2,n} = (1-\gamma_{0,n})[1-(1-\gamma_{0,n-1})\gamma_{1,n}] \,, \quad (8)$$

for which the properties of the Bernoulli distribution with $i = 0, 1, 2$ are denoted as

$$E\{(\alpha_i)(\alpha_i)\} = \bar{\alpha}_i \,,$$
$$E\{(1-\alpha_i)(1.\alpha_i)\} = 1 - \bar{\alpha}_i \,, \quad (9)$$

To represent the k_n-step delayed state x_{n-k_n} via the current state x_n, we reorganize (1) to have the backward-in-time solution [36]:

$$x_{n-k_n} = F^{-k_n}\left(x_n - \sum_{i=0}^{k_n-1} F^i w_{n-i}\right) . \quad (10)$$

Assuming that $k_n = 1$ and substituting the delayed state $x_{n-1} = F^{-1}(x_n - w_n)$ and y_n (1) into (5), we obtain the new observation equation, which has no latency,

$$\begin{aligned} z_n &= \alpha_{0,n}(Hx_n + v_n) + \alpha_{1,n}(Hx_{n-1} + v_{n-1}) \\ &\quad + \alpha_{2,n}(HFx_{n-1}) \,, \\ &= \alpha_{0,n}(Hx_n + v_n) + \alpha_{1,n}(H(F^{-1}(x_n - w_n)) \\ &\quad + v_{n-1}) + \alpha_{2,n}(HF(F^{-1}(x_n - w_n)) \,, \\ &= \left(\alpha_{0,n}H + \alpha_{1,n}HF^{-1} + \alpha_{2,n}H\right)x_n + \\ &\quad \alpha_{0,n}v_n + \alpha_{1,n}v_{n-1} - \left(\alpha_{1,n}HF^{-1} + \alpha_{2,n}H\right)w_n \,, \\ &= \bar{H}_n x_n + \bar{v}_n \,, \end{aligned} \quad (11)$$

where the modified observation matrix $\bar{H}_n$ and white Gaussian noise vector $\bar{v}_n$ are given by

$$\bar{H}_n = (\alpha_{0,n} + \alpha_{2,n})H + \alpha_{1,n}HF^{-1} \,, \quad (12)$$
$$\bar{v}_n = \alpha_{0,n}v_n + \alpha_{1,n}v_{n-1} - (\alpha_{1,n}HF^{-1} + \alpha_{2,n}H)w_n \quad (13)$$

and the noise covariances $\bar{R}_n = E\{\bar{v}_n\bar{v}_n^T\}$ and $Q_n = E\{w_n w_n^T\}$ are given with

$$\bar{R}_n = \bar{\alpha}_{0,n}R_n + \bar{\alpha}_{1,n}R_{n-1} + \bar{\alpha}_{1,n}HF^{-1}QH^T F^{-1^T} + \bar{\alpha}_{2,n}HQH^T \,. \quad (14)$$

Note that the noise vector $\bar{v}_n$ and w_n are time-correlated in (13). Since an optimal estimate is guaranteed under the independence of noise vectors, the de-correlation is required as will be shown next.

3.1. De-Correlation of w_n and $\bar{u}_n$

To de-correlate the noise vectors $\bar{v}_n$ and w_n, the Lagrange multiplier method can be used as proposed in [32], [33]. The approach suggests that the state equation must be subjected to a de-correlating condition as

$$\begin{aligned} x_n &= Fx_{n-1} + w_n + \Lambda_n \left(z_n - \bar{H}_n x_n - \bar{v}_n\right) \\ &= A_n x_{n-1} + u_n + \zeta_n \,, \end{aligned} \tag{15}$$

where z_n is a vector of real data, $A_n = F - \Lambda_n \bar{H}_n F$, $u_n = \Lambda_n z_n$, and $\zeta_n = (I - \Lambda_n \bar{H}_n) w_n - \Lambda_n \bar{v}_n$. Our desire is to have a noise vector ζ_n such that $\zeta_n \sim \mathcal{N}(0, Q_\zeta) \in \mathbb{R}^K$ has the covariance $Q_\zeta = E\{\zeta_n \zeta_n^T\}$ defined by

$$\begin{aligned} Q_\zeta &= (I - \bar{\alpha}_{0,n}\Lambda_n H) Q (I - \bar{\alpha}_{0,n}\Lambda_n H)^T \\ &\quad + \bar{\alpha}_{0,n}\Lambda_n R_n \Lambda_n^T + \bar{\alpha}_{1,n}\Lambda_n R_{n-1}\Lambda_n^T \,. \end{aligned} \tag{16}$$

The Lagrange multiplier Λ_n can now be derived to satisfy the desired property of

$$\begin{aligned} 0 &= E\{\zeta_n \bar{v}_n^T\} \\ &= E\{[(I - \Lambda_n \bar{H}_n) w_n - \Lambda_n \bar{v}_n] \bar{v}_n^T\} \\ &= E\{[(I - \alpha_0 \Lambda_n H_n) w_n - \alpha_0 \Lambda_n v_n - \alpha_1 \Lambda_n v_{n-1}] \\ &\quad [\alpha_0 v_n + \alpha_1 v_{n-1} - (\alpha_1 H_n F_n^{-1} + \alpha_2 H_n) w_n]^T\} \end{aligned}$$

and further transformations yield

$$\begin{aligned} \Lambda_n &= -Q_n (\bar{\alpha}_{1,n} H_n F_n^{-1} + \bar{\alpha}_{2,n} H_n)^T (\bar{\alpha}_{0,n} R_n \\ &\quad + \bar{\alpha}_{1,n} R_{n-1})^{-1} \,. \end{aligned} \tag{17}$$

Provided the de-correlation of noise vectors ζ_n and $\bar{v}_n$, the estimation algorithms can be developed accordingly.

4. Filters Design for Delayed and Missing Data

In this section, the UFIR filter will be developed for systems represented with an uncertain observations model (12). Due to the ability of the UFIR filter to ignore zero mean noise [32] and because time-correlation does not produce bias errors, this filter can be applied straightforwardly. On the contrary, the KF and H_∞ filter require the noise de-correlation.

4.1. Batch UFIR Filter

The UFIR filter is the convolution-based structure that satisfies the unbiasedness condition $E\{x_n\} = E\{\hat{x}_n\}$ to ensure the unbiasedness. The batch UFIR filter produces an estimate over an averaging horizon $[m, n]$ of N points, where $m = n - N + 1$ and the horizon length is required to be optimal as N_{opt} to minimize the mean square error (MSE). This filter does not require any information about zero mean noise and initial values [34].

To design the batcg UFIR filter, we consider the following model expanded on $[m, n]$ [34],

$$X_{m,n} = A_N x_m + B_N W_{m,n} \,, \tag{18}$$

$$Y_{m,n} = H_{m,n} x_m + G_{m,n} W_{m,n} + V_{m,n} \,, \tag{19}$$

where $X_{m,n} = [x_m^T \, x_{m+1}^T \dots x_n^T]^T$ and $Y_{m,n} = [y_m^T \, y_{m+1}^T \dots y_n^T]^T$ are extended vectors and the extended matrices are defined as

$$A_N = [I \; F^T \dots F^{N-1^T}]^T, \tag{20}$$

$$B_N = \begin{bmatrix} I & 0 & \dots & 0 & 0 \\ F & I & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ F^{N-2} & F^{N-3} & \dots & I & 0 \\ F^{N-1} & F^{N-2} & \dots & F & I \end{bmatrix}, \tag{21}$$

$$H_{m,n} = \begin{bmatrix} \bar{H}_m \\ \bar{H}_{m+1} F \\ \vdots \\ \bar{H}_n F^{n-1} \end{bmatrix}, \tag{22}$$

$G_{m,n} = \bar{H}_{m,n} D_{m,n}$, $\bar{H}_{m,n} = \text{diag}(\bar{H}_m \, \bar{H}_{m+1} \dots \bar{H}_n)$, and matrix $\bar{H}_n$ is specified with (14).

The batch UFIR filtering estimate is given by [34]

$$\hat{x}_n = F^{N-1} (H_{m,n}^T H_{m,n})^{-1} H_{m,n}^T Y_{m,n} \tag{23a}$$

$$= \mathcal{G}_n \mathcal{C}_{m,n}^T \mathcal{C}_{m,n}^T Y_{m,n} \tag{23b}$$

$$= \mathcal{H}_{m,n} Y_{m,n} \,, \tag{23c}$$

where $\mathcal{H}_{m,n}$ is the filter gain, the generalized noise power gain (GNPG) is given by

$$\mathcal{G}_n = (\mathcal{C}_{m,n}^T \mathcal{C}_{m,n})^{-1} \,, \tag{24}$$

$Y_{m,n}$ is a vector of real data, matrix $\mathcal{C}_{m,n}$ is defined as

$$\mathcal{C}_{m,n} = \begin{bmatrix} \bar{H}_m F^{-N+1} \\ \vdots \\ \bar{H}_{n-1} F^{-1} \\ \bar{H}_n \end{bmatrix}, \tag{25}$$

and the latency-dependent matrix $\bar{H}_n$ is given by (14).

4.2. Iterative UFIR Algorithm Using Recursions

Given the coefficients $\alpha_{0,n}$, $\alpha_{1,n}$, and $\alpha_{2,n}$ and the initial estimate at $s = n - N + K$ computed in a short batch form (23c) over $[m, s]$, the iterative UFIR filter computes estimates recursively on a horizon $[m, n]$ for the initial estimate at $s = n - N + K$. A pseudo code of the iterative UFIR filtering algorithm for delayed and missing data is listed as Algorithm 1.

5. Applications

In this section, a numerical example of the maneuvering vehicle tracking is considered to test the performances of the UFIR, Kalman, and H_∞ filtering algorithms under the randomly delayed and missing measurement data. The purpose is to investigate advantages and disadvantages of the algorithms

Algorithm 1: Iterative UFIR Filtering Algorithm for Delayed and Missing Data

Data: y_n, $\alpha_{0,n}$, $\alpha_{1,n}$, $\alpha_{2,n}$, N, κ_n
Result: $\hat{x}_n$
begin
  for $n = N-1 : \infty$ **do**
    $m = n - N + 1, \quad s = n - N + K$;
    if $\kappa_n = 0$ **then**
      $y_n = HF\hat{x}_{n-1}$
    end if
    $\bar{H}_n = (\alpha_{0,n} + \alpha_{2,n})H + \alpha_{1,n}HF^{-1}$;
    Compute $\mathcal{C}_{m,s}$ by (20);
    $\mathcal{G}_s = (\mathcal{C}_{m,s}^T \mathcal{C}_{m,s})^{-1}$;
    $\tilde{x}_s = \mathcal{G}_s \mathcal{C}_{m,s}^T Y_{m,s}$;
    for $l = s+1 : n$ **do**
      $\mathcal{G}_l = [\bar{H}_l^T \bar{H}_l + (F\mathcal{G}_{l-1}F^T)^{-1}]^{-1}$;
      $K_l^{\mathrm{U}} = \mathcal{G}_l \bar{H}_l^T$;
      $\tilde{x}_l = F\tilde{x}_{l-1} + K_l^{\mathrm{U}}(y_l - \bar{H}_l F\tilde{x}_{l-1})$;
    end for
    $\hat{x}_n = \tilde{x}_n$;
  end for
end
† Data y_0, y_1,... and $Y_{m,s}$

under the same operation conditions. We consider a vehicle trajectory measured using a GPS reader in the Cook County of Illinois and available from the University of Illinois at Chicago [37]. The GPS-based vehicle trajectory in the local north and east coordinates is shown in Fig. 2.

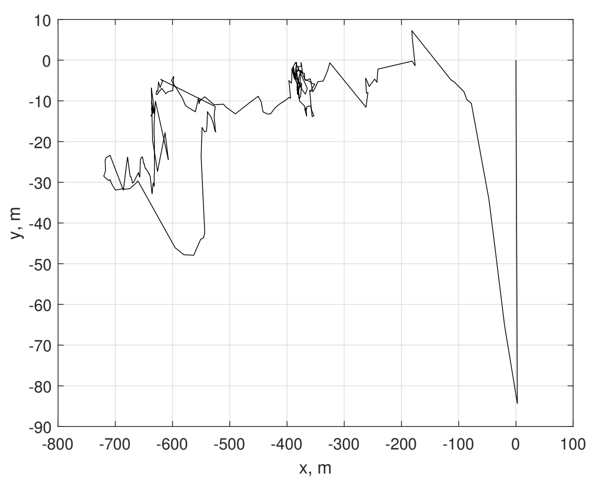

Fig. 2. GPS-measured vehicle trajectory in the local north (y) and east (x) coordinates.

5.1. Tracking State-Space Model

Because information about the vehicle and its measurement is limited and no noise statistics are provided, we specify the vehicle trajectory and the process model as in the following.

- The state model (2) is described to have two states in each of the directions, $K = 4$, the state vector $x_n = [x_{1n}\ x_{2n}\ x_{3n}\ x_{4n}]^T$ has the components $x_{1n} = \mathrm{x}_n$, $x_{2n} = \mathrm{x}'_n$, $x_{3n} = \mathrm{y}_n$, and $x_{4n} = \mathrm{y}'_n$, and matrices are

$$F = \begin{bmatrix} 1 & \tau & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{\tau}{2} & 0 \\ 1 & 0 \\ 0 & \frac{\tau}{2} \\ 0 & 1 \end{bmatrix}.$$

- The observation is represented with (1) with the probabilities $\bar{\gamma}_0 = 0.7$ and $\bar{\gamma}_1 = 0.8$ and observation matrix

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

- The system noise and observation noise are supposed to be zero mean and white Gaussian. Supposing that the vehicle has an average speed of about $15\,\mathrm{m/s}$, we assign the velocity noise standard deviation as $\sigma_{2w} = 1.5\,\mathrm{m/s}$ and neglect noise in the distance, $\sigma_{1w} = 0\,\mathrm{m}$. The GPS service guarantees an error of smaller than 15 meters with the probability of 95% in the 2σ sense. We thus set $\sigma_v = 3.75\,\mathrm{m}$ and define the noise covariances as

$$Q = \sigma_{w2}^2 \begin{bmatrix} \frac{\tau^2}{4} & \frac{\tau}{2} & 0 & 0 \\ \frac{\tau^2}{2} & 1 & 0 & 0 \\ 0 & 0 & \frac{\tau^2}{4} & \frac{\tau}{2} \\ 0 & 0 & \frac{\tau^2}{2} & 1 \end{bmatrix}, \quad R = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}.$$

- The tuning factor of the UFIR filter (N_{opt}) is determined to minimize the MSE in the UFIR filter by solving the minimization problem

$$N_{\mathrm{opt}} = \arg\min_N [\mathrm{tr}P_n(N)],$$

where P_n is the error covariance. For the trajectory shown in Fig. 1, the optimal horizon was found to be $N_{\mathrm{opt}} = 5$.

- The optimum tuning factor θ_{opt} for the H_∞ filter is determined by minimizing the MSE. The value θ_{opt} needs to be kept accurately in view of a high sensitivity of the H_∞ filter to θ. Otherwise, this filter may diverge [28], [36]. In this paper, we apply $\theta_{\mathrm{opt}} \cong 0.02$ found experimentally to the entire trajectory.

The estimates produced by the properly tuned UFIR filter, KF, and H_∞ filter are sketched in Fig 3. It follows that all estimates are consistent with poorly distinguishable differences despite data failures. The prediction option is used when some data are lost as shown in Fig 3. Effect of the missing measurement phenomenon is illustrated in Fig 4 and it is seen that the estimation errors grow due to fast changes in the trajectory. Even so, the UFIR filter demonstrates an ability to converge faster to the regular mode than other filters due to the inherently bounded input bounded output (BIBO) stability.

5.2. Effect of Probabilistic Errors in $\bar{\gamma}_0$

Assuming that measurements are transmitted with the probabilities $\bar{\gamma}_0 = 0.7$ and $\bar{\gamma}_1 = 0.8$ at each n, we next investigate the estimation errors caused by errors in $\bar{\gamma}_0$. The aim is to

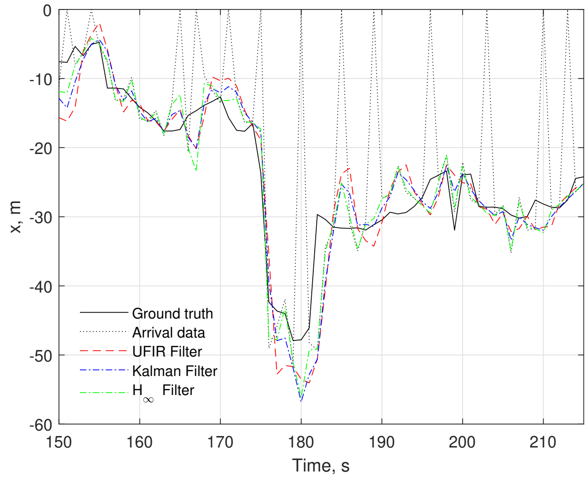

Fig. 3. GPS-based vehicle tracking in the x, m direction by the UFIR filter, KF, and H_∞ filter using model (1)–(10).

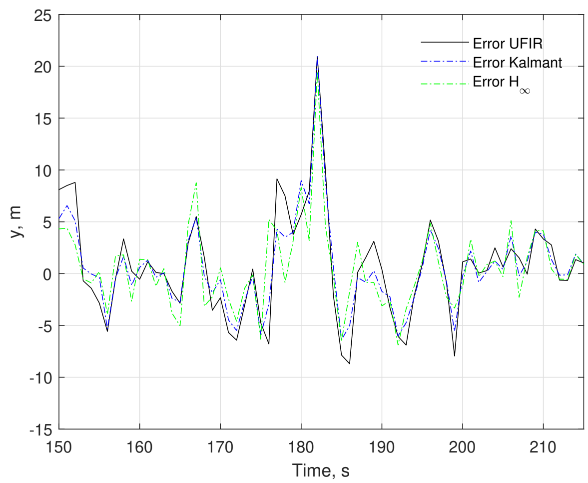

Fig. 4. Tracking error produced by the UFIR filter, KF, and H_∞ filter in the x, m direction (1)–(4).

investigate the impact of the transmission probability $\overline{\gamma}_0$ on the RMSE. We thus assume that $\overline{\gamma}_0$ varies from 0.1 to 0.9 and sketch the RMSEs as functions of $\overline{\gamma}_0$ in Fig 5. What follows now is that the decrease in $\overline{\gamma}_0$ results in growing errors, which means that the minimum tracking errors can be obtained only if the data transmission probability is equal to that in the algorithms. The KF errors range lower than in the UFIR filter for all γ. The H_∞ improves the performance when $\gamma > 0.4$, but produces rapidly growing large errors when $\gamma < 0.3$ and diverges when $\gamma < 0.1$.

5.3. Effect of Errors in Noise Covariances

The noise statistics and error matrices are typically not well defined in object tracking. To investigate effects of the relevant errors on the estimator performance, we next introduce a scalar scaling error factor β [36] and substitute in the algorithms $Q = \check{Q}$ with Q/β^2 and $R = \check{R}$ with $\beta^2 R$.

The RMSEs produced for the UFIR filter, KF, and H_∞ filter under $\beta \neq 1$ are displayed in Fig 6 as functions of β.

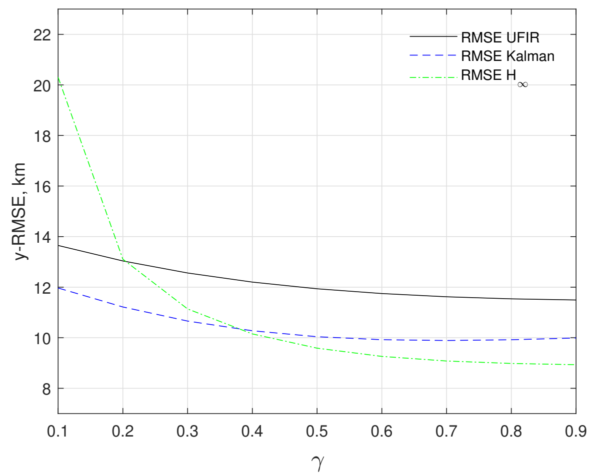

Fig. 5. Effect of a scalar scaling error factor β on the RMSEs produced by the UFIR filter, KF, and H_∞ filter in the y direction.

Inherently, the UFIR filter performance is β-invariant, while the KF and H_∞ filter have different sensitivities to β that affect their performances. It is especially true for the H_∞ filter, which diverges beyond the interval of $0.5 < \beta < 1.0$.

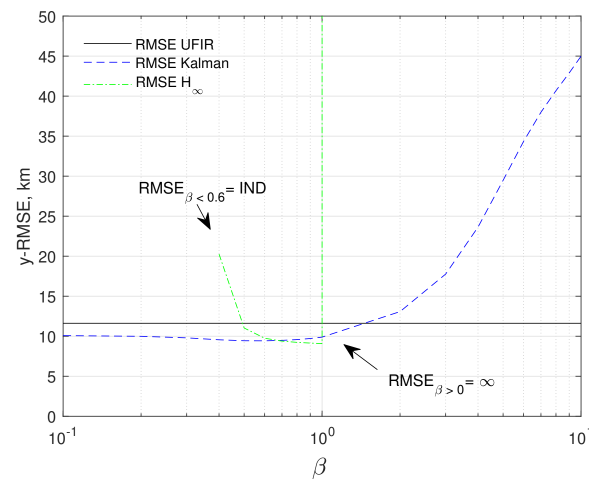

Fig. 6. Effect of the data transmission probability γ on the RMSEs produced by the UFIR filter, KF, and H_∞ filter in the y direction.

6. Conclusions

The UFIR filter, KF, and game theory H_∞ filter developed in this paper for randomly delayed binary Bernoulli-distributed data with packet dropouts have demonstrated a better performance than the standard filters. The effect was achieved by transforming the discrete-time state-space model to have no latency and extending the transformed model on a horizon of N past data points. The problem with missing data was solved using a prediction option. An experimental example of GPS-based vehicle tracking has demonstrated that the UFIR filter generally outperforms both the KF and H_∞ filter and is not prone to divergency.

References

[1] M. Li and Y. Chen, "Robust tracking control of networked control systems with communication constraints and external disturbance," *IEEE Trans. Ind. Electron.*, vol.64, pp. 4037–4047, 2017.

[2] A. Ahmadi, F. R. Salmasi, M. Noori-Manzar and T. A. Najafabadi, "Speed sensorless and sensor-fault tolerant optimal PI regulator for networked DC motor system with unknown time-delay and packet dropout," vol. 61, pp. 708–717, 2014.

[3] M. Daniel, R. Michailas, H. Iván and Z. Ralf, "Robust position and velocity estimation methods in integrated navigation systems for inland water applications," *2016 IEEE/ION Position, Location and Navigation Symp. (PLANS)*, pp. 491–501, 2016.

[4] L. Feng, G. Tianyangyi, H. Jinquan and Q. Xiaojie, "A novel distributed extended Kalman filter for aircraft engine gas-path health estimation with sensor fusion uncertainty," *Aerospace Sci. Techn.*, vol. 84, pp. 90–106, 2019.

[5] M. Batou, G. J. Tsekouras and A. Moronis, "Load estimation of a small autonomous island power system using k-means method," *2019 3rd European Conference on Electrical Engineering and Computer Science (EECS), Athens, Greece*, pp. 60-65, 2019.

[6] Y. Xu, Y. S. Shmaliy, C. K. Ahn, and Y. Zhuang, "Tightly-coupled integration of INS and UWB using fixed-lag UFIR smoothing for quadrotor localization," *IEEE Internet of Things J.*, vol. 8, no. 3, pp. 1716–1727, Feb. 2021.

[7] M. Vazquez-Olguin, Y. S. Shmaliy, and O. Ibarra-Manzano, "Distributed unbiased FIR filtering with average consensus on measurements for WSNs," *IEEE Trans. Ind. Informat.*, vol. 13, no. 3, pp. 1440–1447, Jun. 2017.

[8] M. Vazquez-Olguin, Y. S. Shmaliy, and O. Ibarra-Manzano, "Distributed UFIR filtering over WSNs with consensus on estimates, *IEEE Trans. Ind. Informat.*, vol. 16, no. 3, pp. 1645–1654, Mar. 2020.

[9] C. Dongyan, X. Long, and D. Junhua, "Optimal filtering for systems with finite-step autocorrelated process noises, random one-step sensor delay and missing measurements," *Commun. Nonlin. Sci. Numer. Simulation*, vol. 32, pp. 211–224, 2016.

[10] S. Shuli, "Linear minimum variance estimators for systems with bounded random measurement delays and packet dropouts," *Signal Process.*, vol. 89, no.7, pp. 1457–1466, 2009.

[11] J. Du, "Optimized state estimation of uncertain linear time-varying complex networks with random sensor delay subject to uncertain probabilities," *IEEE Access*, vol. 7, pp. 113005-113016, 2019.

[12] Z. Ben-Mabrouk, A. Abid, M. Ben-Hamed, and L. Sbita, "Estimation for random sensor failure of networked control system subject to random packet dropout," *10th Int. Multi-Conf. Systems, Signals Devices 2013 (SSD13)*, pp. 1-6, 2013.

[13] S. Bruno, S. Luca, F. Massimo, P. Kameshwar, J. Michael and S. Shankar, "Kalman filtering with intermittent observations," *IEEE Trans. Autom. Contr.*, vol. 49, no. 9, pp. 1453–1464, 2004.

[14] K. Ma, L. Xu and H. Fan, "Unscented Kalman filtering for target tracking systems with packet dropout compensation," *IET Contr. Theory Appl.*, vol. 13, no. 12, pp. 1901–1908, 2019.

[15] J. Ma and S. Sun, "Optimal linear filter for systems with random delay and packet dropout compensations," *IEEE Access*, vol. 8, pp. 145268–145277, 2020.

[16] Q. Huifang and Y. Fuwen, "Distributed H_∞-consensus filtering for target state tracking over a wireless filter network with switching topology, channel fading and packet dropouts," *Neurocomputing*, vol. 400, pp. 401–411, 2020.

[17] H. Dong, Z. Wang and H. Gao, "Robust H_∞ filtering for a class of nonlinear networked systems with multiple stochastic communication delays and packet dropouts"*IEEE Trans. Signal Process.*, vol.58, pp. 1957–1966, 2010.

[18] Y. Luo, Z. Wang, G. Wei and F. E. Alsaadi, "Robust H_∞ filtering for a class of two-dimensional uncertain fuzzy systems with randomly occurring mixed delays," *IEEE Trans. Fuzzy Syst.*, vol. 25, no. 1, pp. 70–83. 2017.

[19] Z. Hongxu, H. Jun, L. Hongjian, Y. Xiaoyang and L. Fengqiu, "Recursive state estimation for time-varying complex networks subject to missing measurements and stochastic inner coupling under random access protocol," *Neurocomputing*, vol. 346, pp. 48–57, 2019.

[20] Y. Fuwen, W. Zidong, F. Gang and L. Xiaohui, "Robust filtering with randomly varying sensor delay: the finite-horizon case," *IEEE Trans. Circuits Syst. I: Regular Papers*, vol. 56, no. 6, pp. 664–672, 2009.

[21] S. Shu-Li, "Optimal linear estimators for discrete-time systems with one-step random delays and multiple packet dropouts," *Acta Automatica Sinica*, vol. 38, no. 3, pp. 349–354, 2012.

[22] X. Long, M. Kemao, L. Wenshuo, L. Yurong and A. Fuad E, "Particle filtering for networked nonlinear systems subject to random one-step sensor delay and missing measurements," *Neurocomputing*, vol. 275, pp. 2162–2169, 2018.

[23] X. H. Chang, Q. Liu, Y. M. Wang, and J. Xiong, "Fuzzy peak-to-peak filtering for networked nonlinear systems with multipath data packet dropouts," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 3, pp. 436–446, 2018.

[24] Q. Li, Z. Wang, W. Sheng and F. E. Alsaadi, "Dynamic event-triggered mechanism for H non-fragile state estimation of complex networks under randomly occurring sensor saturations," *Inform. Sci.*, vol. 509, pp. 304-316, 2020.

[25] M. Zhang, P. Shi, C. Shen, Z. G. Wu and F. E. Alsaadi, "Static output feedback control of switched nonlinear systems with actuator faults," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 8, pp. 1600-1609. 2020.

[26] L. Xiu-Ying and Sun. Shu-Li, "H_∞ filtering for networked linear systems with multiple packet dropouts and random delays," *Digital Signal Process.*, vol. 46, pp. 59–67, 2015.

[27] C.Á. Raquel, H. C. Aurora and L. P. Josefa, "Networked distributed fusion estimation under uncertain outputs with random transmission delays, packet losses and multi-packet processing," *Signal Process.*, vol. 156, pp. 71–83, 2019.

[28] Y. S. Shmaliy, F. Lehmann, S. Zhao. and C. K. Ahn, "Comparing robustness of the Kalman, H_∞, and UFIR filters," *IEEE Trans. Signal Process.*, vol. 66, no. 13, pp. 3447–3458, 2018.

[29] D. Simon, "Optimal state estimation: Kalman, H infinity, and nonlinear approaches," *John Wiley & Sons*, 2006.

[30] Y. S. Shmaliy, S. Zhao, and C. K. Ahn, "Unbiased finite impluse response filtering: an iterative alternative to Kalman filtering ignoring noise and initial conditions," *IEEE Contr. Syst. Mag.*, vol. 37, no.5, pp. 70–89, 2017.

[31] K. Uribe-Murcia, Y. S. Shmaliy, and L. A. Andrade-Lucio, "UFIR filtering for GPS-based tracking over WSNs with delayed and missing data," *J. Electr. Comput. Eng.*, 2018.

[32] Y. S. Shmaliy, S. Zhao, and C. K. Ahn, "Kalman and UFIR state estimation with colored measurement noise using backward Euler method," *IET Signal Process.*, vol. 14, no. 2, pp. 64–71, 2020.

[33] T. Kailath, A. H. Sayed, and B. Hassibi, "Linear Estimation", *Prentice Hall*, 2000.

[34] Y. S. Shmaliy, "An unbiased FIR filter for TIE model of a local clock in applications to GPS-based timekeeping," *IEEE Trans. Ultrason. Ferroel. Freq. Contr.*, vol. 53, no. 5, pp. 862–870, 2006.

[35] C. Zhu, Y. Xia,L. Xie and L. Yan, "Optimal linear estimation for systems with transmission delays and packet dropouts," *IET Signal Process.*, vol. 7, no. 9, pp. 814-823, 2013.

[36] U. M. Karen, S. Yuriy S, A. Choon Ki and Z. Shunyi, "Unbiased FIR filtering for time-stamped discretely delayed and missing data," *IEEE Trans. Autom. Contr.*, pp. 2155–2162, 2019.

[37] Databases and Mobile Computing Laboratory in University of Illinois at Chicago, *https://www.cs.uic.edu/ wolfson/html/p2p.html*, 2006.