UFIR State Estimator for Network Systems with Two-Step Delayed and Lost Data

KAREN URIBE-MURCIA, YURIY S. SHMALIY
Department of Electronics Engineering University of Guanajuato Salamanca, 36885, MEXICO

Abstract—Wireless communication over networks often produces issues associated with delayed and missing data. In this paper, we consider one-step and two-step delays. The state space model is transformed to have no delay with new system and observation matrices. To mitigate the effect, we develop the unbiased finite impulse response (UFIR) filter, Kalman filter (KF), and game theory H∞ filter for Bernoulli-distributed delays with possible packet dropouts. A comparative study of the filters developed is provided under the uncertain noise and transmission probability. Numerical simulation is conducted employing a GPSbased tracking network system. A better performance of the UFIR filter is demonstrated experimentally.

Keywords: Delayed data, missing data, UFIR filter, Kalman filter, game theory H∞ filter

Received: December 27, 2020. Revised: July 28, 2021. Accepted: August 21, 2021. Published: August 31, 2021.

1. Introduction

Errors in state estimation over wireless sensor networks (WSN) affected by uncertain, delayed, and missing data have been investigated in the last years by many authors [1]–[4]. It has been revealed that factors such as the environmental changes, failures in measurement equipment, congestion in transmission channels, and limited communication bandwidth cause data to arrive at the received with latency and packet dropout. Furthermore, ignoring such phenomena may cause crucial consequences for WSN operation. In [5], [6], the problem was solved under the supposition that the delays are known and deterministic. That applied to the transmission with time-stamped data and cannot be used otherwise.

In many cases, it is required to consider randomly delayed and missing data as, for example, has been made in [7] for unreliable WSN channels. In many cases, the binary stochastic Bernoulli distribution is employed to describe the intermittent random faults in the received signal [8]. For randomly delayed and missing data, the Kalman filter (KF) was developed in [9], [10], the H_∞ filter in [11], and the optimal estimation problem solved in [12]. The problem with lost data was also investigated. An innovated compensation of the lost data is developed in [13] by solving an optimal linear filtering problem. A modified model based on the Bernoulli distribution is presented in [14] to substitute lost data by using an estimator that processes one or two packets at once. The problem with multi-step delays has been considered in [15] using the Bernoulli distribution, an H_∞ filter was designed in [16], and some other relevant solutions can be found in [17]–[19].

To improve the estimation accuracy under delayed and missing data [20]–[22]. A drawback is that the KF is not robust and thus does not guarantee an optimal performance under uncertain conditions. The unbiased finite impulse response (UFIR) filter was designed in [23] as a robust alternative to the KF [24], [25] and other methods such as the game theory H_∞ filter [26] developed under the parameter uncertainties. The UFIR filter required no information about zero-mean noise and initial values and is thus more robust than other linear filters. In [27]–[29] the UFIR filter was used to process data with multi-step known deterministic delays and lost data.

In this paper, the UFIR filter is developed for WSN with one- or two-step random delays and lost data. Similarly to [30], the observation equation is modeled using the Bernoulli distribution with known probabilistic parameters. An innovation system transformation is presented to apply the conventional estimators such as the KF and H_∞ filter which derivation does not depend on latency. Experimental testing is provided based on the Global Positioning System (GPS) tracking problem.

2. State Space Model and Problem Formulation

Consider a dynamic quantity measured and observed in discrete-time state space with equations

$$x_n = Fx_{n-1} + w_n \,, \quad (1)$$

$$y_n = Hx_n + v_n \,, \quad (2)$$

where n is a discrete time index, $x_n \in \mathbb{R}^K$ is the state vector, $y_n \in \mathbb{R}^M$ is the observation vector, $F \in \mathbb{R}^{K\times K}$, $H \in \mathbb{R}^{K\times M}$, and $w_n \sim \mathcal{N}(0,Q) \in \mathbb{R}^K$ and $v_n \sim \mathcal{N}(0,R) \in \mathbb{R}^M$ are zero mean white Gaussian noise vectors with the covariances $Q = E\{w_n w_n^T\} \in \mathbb{R}^{K\times K}$ and $R = E\{v_n v_n^T\} \in \mathbb{R}^{M\times M}$ and the property $E\{w_n v_k^T\} = 0$ for all n and k.

We consider transmission over a WSN with random delays with multi-step delays. To deal with lost data, a packet is transmitted several times but an estimator process only the first arrived packet at each time instant. The following model is adopted to describe the measured information at the estimator,

$$\begin{aligned} z_n = {} & \xi_{0,n} y_n + (1-\xi_{0,n})\left\{\xi_{1,n} y_{n-1} + (1-\xi_{1,n})\right. \\ & \left\{\xi_{2,n} y_{n-2} + \cdots + (1-\xi_{k_n-2,n})\left\{\xi_{k_n-1,n} y_{n-k_n-1}\right.\right. \\ & \left.\left.\left. + (1-\xi_{k_n-1,n}) y_{n-k_n}\right\}\cdots\right\}\,, \end{aligned} \quad (3)$$

where $z_n \in \mathbb{R}^M$ is the transmitted measurement vector and $\xi_{i,n}$, $i \in [0, k_n - 1]$, is a binary random variable with known probabilities $\mathcal{P}\{\xi_{i,n} = 1\} = \bar{\xi}_{i,n}$ and $\mathcal{P}\{\xi_{i,n} = 0\} = 1 - \bar{\xi}_{i,n}$, where $0 \le \xi_i \le 1$. Although model (3) is valid for an arbitrary delay step, in this paper we consider a special case of $k_n = 2$ that yields

$$z_n = \xi_{0,n} y_n + (1 - \xi_{0,n}) \left\{ \xi_{1,n} y_{n-1} + (1 - \xi_{1,n}) \, y_{n-2} \right\} . \quad (4)$$

Model (4) suggests that a packet received on time $z_n = y_n$ with the probability $\bar{\xi}_{0,n}$ when $\xi_{0,n} = 1$. Otherwise, if $\xi_{0,n} = 0$, one-step delayed data are received $z_n = y_{n-1}$ with the probability $(1 - \bar{\xi}_{0,n})\bar{\xi}_{1,n}$ when $\xi_{1,n} = 1$ or two-step delayed data are received $z_n = y_{n-2}$ with the probability $(1-\bar{\xi}_{0,n})(1-\bar{\xi}_{1,n})$ when $\xi_{1,n} = 0$. The latest data transmitted is used when the current data is lost and the Bernoulli distribution guarantees $z_n = \xi_{0,n} + (1 - \xi_{0,n})\, \xi_{1,n} + (1 - \xi_{0,n})\,(1 - \xi_{1,n})$.

Typical scenarios with delayed data are listed in Table I, where y_1, y_5, and y_6 are received on time, y_7 and y_8 are one-step delayed, y_2 is two-step delayed, and y_4, y_9 and y_{10} are lost. Given the model (1)–(3), our aim is modify the UFIR, KF, and H_∞ state estimators under two-step delayed and missing data in z_n. We also wish to investigate the trade-off in accuracy and robustness of these estimators.

TABLE I
TYPICAL SCENARIOS WITH TWO-STEP DELAYED DATA AND PACKET DROPOUTS

n	1	2	3	4	5	6	7	8	9	10
θ_0	1	0	1	0	1	1	0	0	0	0
θ_1	–	1	–	0	–	–	0	1	1	0
Z_n	y_1	y_1	y_3	y_2	y_5	y_6	y_5	y_7	y_8	y_8

3. Filtering Under Randomly Delayed Data

To design a FIR filter based on model (1)–(4), the latter can be transformed to have no delay. To this end, we first represent model (1) as

$$x_{n-1} = F^{-1}(x_n - w_{n-1}) . \quad (5)$$

Then, substituting the delayed states $x_{n-1} = F^{-1}\left(x_n - w_n\right)$ and $x_{n-2} = F^{-2}\left(x_n - w_n - F^{-1} w_{n-1}\right)$ the observation equation can be written for $k_n = 2$ as

$$y_n = \bar{H}_n x_n + \bar{v}_n \, , \quad (6)$$

where the modified observation matrix $\bar{H}$ and noise vector $\bar{v}_n$ are defined as

$$\begin{aligned} \bar{H} &= \xi_{0,n} H + (1 - \xi_{0,n}) \{ \xi_{1,n} H F^{-1} \\ &\quad + (1 - \xi_{1,n}) H F^{-2} \} \, , \quad (7) \\ \bar{v}_n &= \xi_{0,n} v_n + (1 - \xi_{0,n}) \left\{ \xi_{1,n} v_{n-1} + (1 - \xi_{1,n}) v_{n-2} \right\} \\ &\quad - (1 - \xi_{0,n}) \left[\xi_{1,n} H F^{-1} + (1 - \xi_{1,n}) H F^{-2} \right] w_n \\ &\quad - (1 - \xi_{0,n})(1 - \xi_{1,n}) H F^{-1} w_{n-1} \quad (8) \end{aligned}$$

and the covariance $R = E\left\{ \bar{v}_n \bar{v}_n^T \right\}$ of noise $\bar{v}_n$ is given by

$$\begin{aligned} \bar{R}_n &= \bar{\xi}_{0,n} R_n + (1 - \bar{\xi}_{0,n}) \left\{ \bar{\xi}_{1,n} R_{n-1} + (1 - \bar{\xi}_{1,n}) R_{n-2} \right\} \\ &\quad + (1 - \bar{\xi}_{0,n}) \bar{\xi}_{1,n} H F^{-1} Q_n {F^{-1}}^T H^T \\ &\quad + (1 - \bar{\xi}_{0,n})(1 - \bar{\xi}_{1,n}) H F^{-2} Q_n {F^{-2}}^T H^T \\ &\quad + (1 - \bar{\xi}_{0,n})(1 - \bar{\xi}_{1,n}) H F^{-1} Q_{n-1} {F^{-1}}^T H^T . \quad (9) \end{aligned}$$

From (8) we see that noise $\bar{v}_n$ is time-correlated with noise $w(n)$ and the cross covariance is

$$\begin{aligned} E\{\bar{v}_n w_n^T\} &= -[(1 - \bar{\xi}_{0,n})\bar{\xi}_{1,n} H F^{-1} + (1 - \bar{\xi}_{0,n}) \\ &\quad (1 - \bar{\xi}_{1,n}) H F^{-2}] Q_n \, . \quad (10) \end{aligned}$$

The time-correlation may reduce the estimator efficiency. To avoid this issue, a de-correlation can be provided using the Lagrange multiplier method [31], [32] as will be shown next.

3.1 De-correlation of w_n and $\bar{v}_n$

Rewrite model (1) as follows

$$\begin{aligned} x_n &= F x_{n-1} + w_n + \Lambda_n \left(z_n - \bar{H}_n x_n - \bar{v}_n \right) \quad (11) \\ &= A_n x_{n-1} + u_n + \zeta_n \, , \end{aligned}$$

where $A_n = F - \Lambda_n \bar{H}_n F$, $u_n = \Lambda_n z_n$,

$$\zeta_n = (I - \Lambda_n \bar{H}_n) w_n - \Lambda_n \bar{v}_n \, , \quad (12)$$

and Λ_n is the Lagrange multiplier.

To make the noise vector white $\zeta_n \sim \mathcal{N}(0, Q_\zeta) \in \mathbb{R}^K$, the cross-covariance between the new measurement noise Q_ζ and the state noise $\bar{v}_n$ should be zero, $E\{\zeta_n \bar{v}_n^T\} = 0$. Then, transform the covariance $Q_\zeta = E\{\zeta_n \zeta_n^T\}$ as follows,

$$\begin{aligned} Q_\zeta &= (I - \bar{\xi}_{0,n} \Lambda_n H) Q_n (I - \bar{\xi}_{0,n} \Lambda_n H)^T + (1 - \bar{\xi}_{0,n}) \\ &\quad (1 - \bar{\xi}_{1,n}) \Lambda_n H F^{-1} Q_{n-1} {F^{-1}}^T H^T \Lambda_n^T \\ &\quad + (1 - \bar{\xi}_{0,n}) \Lambda_n R_n \Lambda_n^T + (1 - \bar{\xi}_{0,n}) \bar{\xi}_{1,n} \Lambda_n R_{n-1} \Lambda_n^T \\ &\quad + (1 - \bar{\xi}_{0,n})(1 - \bar{\xi}_{1,n}) \Lambda_n R_{n-2} \Lambda_n^T . \quad (13) \end{aligned}$$

Satisfied $E\{\zeta_n \bar{v}_n^T\} = 0$, the Lagrange multiplier Λ_n becomes

$$\begin{aligned} \Lambda_n &= -Q_n [(1 - \bar{\xi}_{0,n}) \bar{\xi}_{1,n} H F^{-1} + (1 - \bar{\xi}_{0,n})(1 - \bar{\xi}_{1,n}) \\ &\quad \times H F^{-2}]^T [(1 - \bar{\xi}_{0,n})(1 - \bar{\xi}_{1,n}) H F^{-1} Q_{n-1} {F^{-1}}^T \\ &\quad \times H^T + \bar{\xi}_{0,n} R_n + (1 - \bar{\xi}_{0,n}) \bar{\xi}_{1,n} R_{n-1} + (1 - \bar{\xi}_{0,n}) \\ &\quad (1 - \bar{\xi}_{1,n}) R_{n-2}]^{-1} \quad (14) \end{aligned}$$

to guarantee the de-correlation. The covariance matrix Q_ζ can now be used in the algorithms.

3.2 UFIR Filter Algorithm

Unlike the KF, the UFIR filter operates with N most recent data points on a horizon $[m, n]$, where $m = n - N + 1$. To design the UFIR filter, model (1)–(6) needs an extention on $[m, n]$ as shown in [24]. An extension of (1) yields

$$\begin{gathered} x_n = F_n x_{n-1} + B_n W_n \, , \\ x_{n-1} = F_{n-1} x_{n-2} + B_{n-1} W_{n-1} \, , \\ \cdots \\ x_m = F_m x_{m-1} + B_m W_m \, , \end{gathered} \quad (15)$$

and leads to the extended state model

$$X_{m,n} = A_{m,n}X_m + B_{m,n}W_{m,n}\,, \tag{16}$$

where $X_{m,n} = \left[x_m^T\, x_{m+1}^T\, \ldots\, x_n^T\right]^T$ and extended matrices are

$$A_N = \left[I\ F^T\ \ldots\ F^{N-1^T}\right]^T, \tag{17}$$

$$B_N = \begin{bmatrix} I & 0 & \cdots & 0 & 0 \\ F & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ F^{N-2} & F^{N-3} & \cdots & I & 0 \\ F^{N-1} & F^{N-2} & \cdots & F & I \end{bmatrix}. \tag{18}$$

Similarly, the observation equation is extended on $[m, n]$ as

$$y_{m,n} = C_{m,n}x_m + D_{m,n}w_{m,n} + v_{m,n}\,, \tag{19}$$

where the extended observation vector and matrices are $y_{m,n} = \left[y_m^T\, y_{m+1}^T\, \cdots\, y_n^T\right]^T$,

$$C_{m,n} = \begin{bmatrix} \bar{H}_m \\ \bar{H}_{m+1}F \\ \bar{H}_{m+1}F^2 \\ \vdots \\ \bar{H}_nF^{n-1} \end{bmatrix}, \tag{20}$$

$$D_{m,n} = \begin{bmatrix} \bar{H}_m & 0 & 0 & \ldots & 0 \\ \bar{H}_{m+1}F & \bar{H}_{m+1} & 0 & \ldots & 0 \\ \bar{H}_{m+2}F^2 & \bar{H}_{m+2}F & \bar{H}_{m+2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \bar{H}_nF^{N-1} & \bar{H}_nF^{N-2} & \bar{H}_nF^{N-3} & \ldots & \bar{H}_n \end{bmatrix}. \tag{21}$$

The UFIR filter can now be designed in the batch and fast iterative form using recursions.

1) Batch UFIR Filter : In a batch form, the UFIR filter operates on $[m, n]$ to satisfy the unbiased condition $E\{x_n\} = E\{\hat{x}_n\}$ and can be written similarly to the least squares as [24],

$$\hat{x}_n = (H_{m,n}^T H_{m,n})^{-1} H_{m,n}^T Y_{m,n}\,, \tag{22}$$

where the observation vector $Y_{m,n}$ and $H_{m,n}$ are given by

$$Y_{m,n} = \left[y_m^T \quad y_{m+1}^T \quad \cdots \quad y_n^T\right], \tag{23}$$

$$H_{m,n} = \begin{bmatrix} \bar{H}F^{-N+1-k_m} \\ \vdots \\ \bar{H}F^{-1-k_{n-1}} \\ \bar{H}F^{-k_n} \end{bmatrix}. \tag{24}$$

The UFIR Filter can also be written as

$$\hat{x}_n = G_n H_{m,n}^T Y_{m,n}\,, \tag{25}$$

where $G_n = (H_{m,n}^T H_{m,n})^{-1}$ is the *generalized noise power gain* (GNPG) responsible for denoising.

2) Iterative UFIR Filtering Algorithm: The iterative UFIR filtering algorithm operates similarly to the KF in two phases, predict and update. The initial state is self-computed in a short batch form (25) on $[m, s]$, where $s = m + K - 1$. A pseudo code of the iterative UFIR filtering algorithm is listed as Algorithm III-B2. It is implied that data arrive with delays having the Bernoulli distribution. When data contain only noise, data prediction is organized in lines 4–6 with $\kappa = 0$. Note that in this algorithm matrix $\bar{H}$ given by (7) is a function of the delay probability ξ. Provided the modified state-space model, the KF and H_∞ filter can be applied straightforwardly.

Algorithm 1 Iterative UFIR Filtering Algorithm for Delayed and Missing Data

Data: y_n, k_n, N, ξ, κ_n
Result: $\hat{x}_n$
begin
  for $n = N-1 : \infty$ **do**
    $m = n - N + 1, \quad s = m + K - 1;$
    if $\kappa = 0$ **then**
      $y_n = HF\hat{x}_{n-1}$
    end
    $\bar{H} = \xi_{0,n}H + (1-\xi_{0,n})\left\{\xi_{1,n}HF^{-1} + (1-\xi_{1,n})HF^{-2}\right\};$
    $G_s = (\mathcal{C}_{m,s}^T\mathcal{C}_{m,s})^{-1};$
    $\tilde{x}_s = G_s\mathcal{C}_{m,s}^T y_{m,s};$
    for $l = s+1 : n$ **do**
      $G_l = [\bar{H}^T\bar{H} + (FG_{l-1}F^T)^{-1}]^{-1};$
      $K_l^{\mathrm{UF}} = G_l\bar{H}^T;$
      $\tilde{x}_l = F\tilde{x}_{l-1} + K_l^{\mathrm{UF}}(y_l - \bar{H}F\tilde{x}_{l-1});$
    end
    $\hat{x}_n = \tilde{x}_n;$
  end
end

† Data y_0, y_1,..., y_{N-1} must be available.

4. Experimental Example

In this section, we consider an experimental example of tracking over a network, where the measurement information is transmitted with latency and lost data. Measurements are obtained from the Beijing's county and available from [33]. The GPS coordinates of a vehicle are transmitted via a wireless communication channel to a central station. The main results obtained in this example using the UFIR filter are compared to the performances of the KF and H_∞ filter. The vehicle trajectory in the north-east direction in coordinates x and y is shown in Fig.1.

The vehicle dynamics is represented with the four-state vector $x_n = \left[x_{1n} \quad x_{2n} \quad x_{3n} \quad x_{4n}\right]^T$, where $x_{1n} = x_n$, $x_{2n} = \dot{x}_n$, $x_{3n} = y_n$ and $x_{4n} = \dot{y}_n$. Accordingly, the system matrix and the observation matrix are specified as

$$F = \begin{bmatrix} 1 & \tau & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

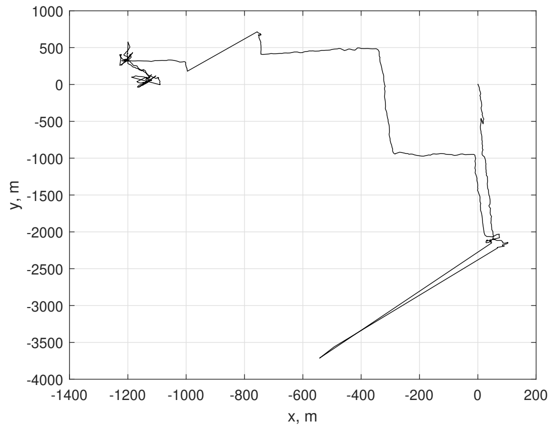

Fig. 1. GPS-measured vehicle trajectory in the north y and east x coordinates.

4.1 Tuning Factors and Noise Covariances

The UFIR filter require N_{opt} to minimize the noise variance. We determine the optimal horizon N_{opt} by solving the minimization problem

$$N_{\text{opt}} = \arg\min_{N}[\text{tr}P_n(N)]\,, \tag{26}$$

where the error covariance matrix $P = E\left\{(\varepsilon_{1\ldots n})(\varepsilon_{1\ldots n})^T\right\}$ is represented with

$$P = \begin{pmatrix} \varepsilon_1^2 & 0 & \cdots & 0 \\ 0 & \varepsilon_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \varepsilon_n^2 \end{pmatrix}, \tag{27}$$

where $\varepsilon_i = x_i - \hat{x}_i$ is the estimation error of the ith state. In this example the optimal horizon was found to be $N_{\text{opt}} = 5$.

The parameter θ_{opt} is the principal tuning factor of the game theory H_∞ filter that is introduced to minimize errors and we notice that it is highly sensible to delays. If not properly tuned, the H_∞ filter produces large errors and can diverge. In our experiment, we found $\theta_{\text{opt}} \cong 0.0192$.

Big efforts are commonly required to specify the noise covariances. Because no information about noise is available in [33], we do it based on a general knowledge. A vehicle in the residential district moves with an average speed of $11\,\text{m/s}$. Based upon, we suppose that the optimal filter performance will be obtained with the standard deviation in the acceleration noise of $\sigma_{3w} = 0.2\,\text{m/s}$ by neglecting noise in the first and second states, $\sigma_{1w} = 0\,\text{m}$ and $\sigma_{2w} = 0\,\text{m/s}$. The GPS navigation service produces an error of less than 15 meters with a probability of 95%. Accordingly, we assign $\sigma_v = 3.75\,\text{m}$ and form the noise covariance matrices as

$$Q = \sigma_{w2}^2 \begin{bmatrix} \frac{\tau^2}{4} & \frac{\tau}{2} & 0 & 0 \\ \frac{\tau^2}{2} & 1 & 0 & 0 \\ 0 & 0 & \frac{\tau^2}{4} & \frac{\tau}{2} \\ 0 & 0 & \frac{\tau^2}{2} & 1 \end{bmatrix}, \quad R = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}.$$

4.2 State Estimation

Setting the probabilistic parameters as $\xi_0 = 0.7$ and $\xi_1 = 0.5$, the vehicle trajectory estimated by the UFIR filter, KF, and H_∞ filter is sketched in Fig. 2, where a consistent estimation is observed with identical development. The three filters have

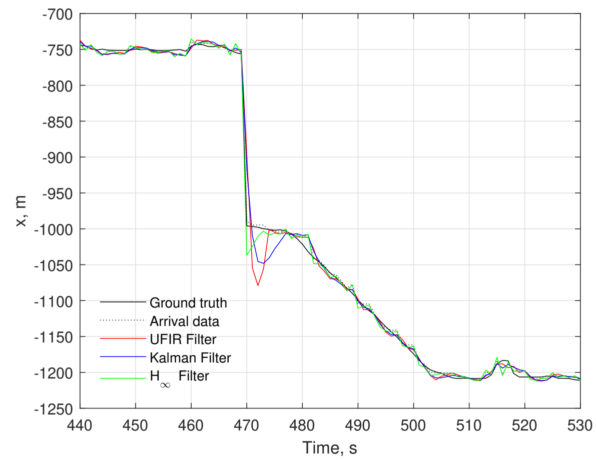

Fig. 2. GPS-based vehicle tracking in the x, m direction by the UFIR filter, KF, and H_∞ filter using model (1)–(4).

the ability to track the ground truth. However, when the vehicle rapidly maneuvers, the filters produce different transients. The UFIR filter has the higher capacity to converge to the trajectory due to the inherently bounded input bounded output (BIBO) stability. The effect can be seen at the 470th second of the movement. The convergence time is shorter in the UFIR filter, but the errors are smaller in the KF. The H_∞ output turned out to be more noisy, but with the shortest convergence time. The errors produced by the filters in the y direction can be seen in Fig. 3.

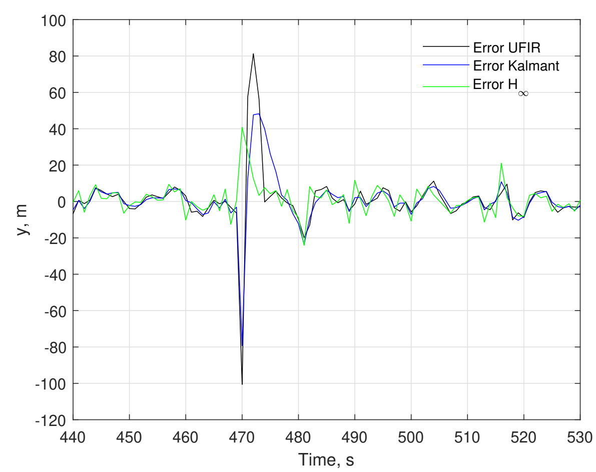

Fig. 3. Tracking error produced by the UFIR filter, KF, and H_∞ filter in the y, m direction (1)–(4).

We next analyze the trade-off in robustness between the filters under the real operation conditions assuming uncertain information in two feasible scenarios.

1) The First Scenario: The noise statistics are typically not well-defined that degrades the estimator performance. We thus suppose that the noise covariances are not known exactly and introduce as error factor in the algorithms. The actual matrices Q and R are substituted in the algorithms with $\alpha^2 Q$ and $\beta^2 R$, where $\alpha = \frac{1}{\beta}$ and β indicates an error in the noise standard deviation. Effect of errors in the noise covariances is sketched in Fig. 4. As can be seen, the UFIR filter is

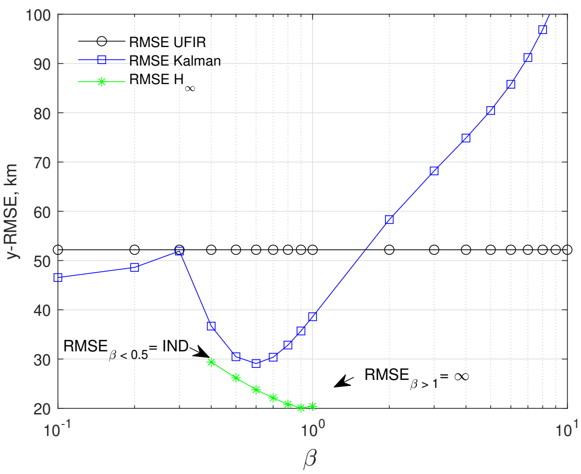

Fig. 4. Effect of a scalar scaling error factor β on the RMSEs produced by the UFIR filter, KF, and H_∞ filter in the y direction.

invariant to β, while the KF demonstrates a big sensibility to β that leads to large errors when β deviates from unity. The KF produces smaller errors when $\beta < 1$ and it performance degrades dramatically when $\beta > 1$. It is also seen that the H_∞ filter produces the smallest errors in the normal mode when $\beta = 1$. But even an insignificant deviation of β from unity makes this filter highly unstable and leads to divergence. We thus conclude that if the operation conditions are uncertain under delayed and missing data, then the robust UFIR filter is the best estimator.

2) The Second Scenario: To learn effects of the data transmission probability on the estimator performance, we consider the RMSEs as functions of ξ_0 and sketch the results in Fig. 5 and Fig. 6. When the one-step delay probability parameter does not vary, the constant value is $\xi_{1,n} = 0.5$. We know that when the transmitted probability and the model probability are equal the minimum errors occur when $\xi_{0,n} = 0.8$. Otherwise, the estimation errors grow. The RMSE produced by the filters become large when $\xi_{0,n}$ decreases because the probability to obtain the one-step or two-step delay grow. It is seen that the KF and UFIR filter are not heavily affected by the possible increase in a lack of information. On the contrary, the H_∞ produces minimum RMSEs. Variations in $\xi_{1,n}$ cause an increase or reduction at the one-step or two-step delay information. One can also notice that a bit more errors occur when $\xi_{1,n}$ decreases, since the two-step delays distort the ground truth vector.

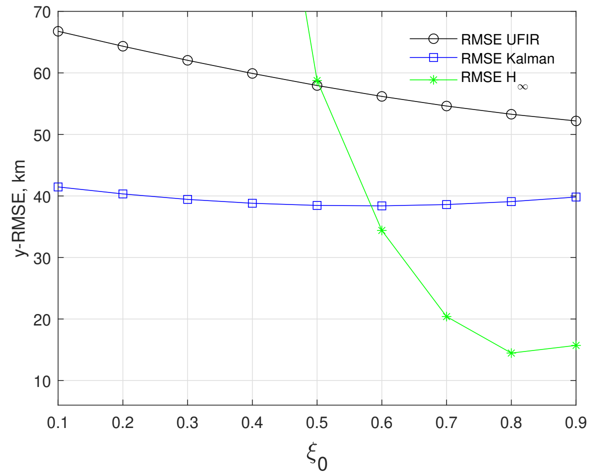

Fig. 5. Effect of the data transmission probability ξ_0 on the RMSEs produced by the UFIR filter, KF, and H_∞ filter in the y direction.

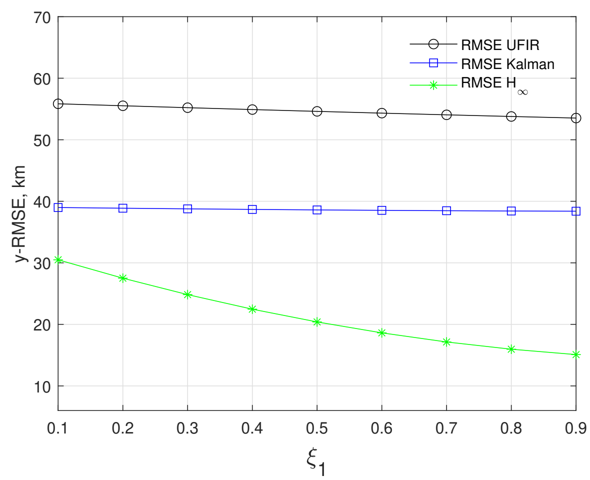

Fig. 6. Effect of the data transmission probability ξ_1 on the RMSEs produced by the UFIR filter, KF, and H_∞ filter in the y direction.

5. Conclusions

In this paper, we have developed the UFIR filter for the information transmission under two-step delayed and lost data. The Bernoulli distribution was used to model the multi-step delayed and missing data. More specifically, we have considered the one-step and two-step delayed data. A transmission protocol where data are sent twice at a central station was considered to avoid lost information. The system state-space model has been reformulated in a way such that the delay factor was removed from the state to the matrices. An experimental example of vehicle tracking was considered to compare the effectiveness of the UFIR filter, KF, and H_∞ filter in terms of accuracy and robustness against errors in the noise statistics and tuning probabilistic parameter. It has been shown that the UFIR filter is not affected by these factors as much as the KF and H_∞ filter.

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