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# Ultra-relativistic bulk motion and radio flux variability of Active Galactic Nuclei

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**Summary.** We have analysed 10 published prominent events of metre-wavelength flux variability associated with seven extragalactic radio sources in terms of the hypothesis that the variable components in these sources have a bulk relativistic motion and radiate by the incoherent synchrotron process. The Doppler factor for each variable component is derived by considering that synchrotron self-absorption effects limit the surface brightness of the component, which is assumed to be homogeneous and spherical in shape and to have a magnetic energy density not much lower than the energy density due to relativistic electrons (as expected on theoretical grounds). Lower limits to the Doppler factors thus estimated for our sample of 10 variability events have a median value of $\delta \sim 70$. A similarly high median value of $\delta \sim 45$ is estimated for a large sample comprising all 35 events of metre-wavelength flux variability reported for the 23 sources of known redshift covered in the Bologna monitoring programme. On the other hand, the same approach yields a median Doppler factor of only $\sim 3.5$ for a sample of 11 prominent centimetre-wavelength variable sources, which is consistent with the Doppler factors inferred in the past from observations of superluminal motion and also from the observed lack of X-ray emission from variable radio sources. It is noted, however, that even at centimetre wavelengths a small number of events of ultra-rapid but relatively large ($\gtrsim 10$ per cent) flux variation have been reported in literature. For eight such rare events, with a variability time-scale of $\sim 1$ day, the minimum values of Doppler factor are again inferred to be very large ($\delta \sim 50$).

## 1 Introduction

The intensities of many compact extragalactic radio sources at metre wavelengths are known to vary substantially on time-scales of months to years (Hunstead 1972; McAdam 1978; Condon *et al.* 1979; Fanti *et al.* 1983). Employing the causality argument, such time-scales frequently imply brightness temperatures for the variable sources, which are a few orders of magnitude higher than the limit of $\sim 10^{12}$ K set for an incoherently radiating synchrotron source by the inverse Compton losses (see Kellermann & Pauliny-Toth 1969). Explanations suggested for circumventing this dilemma include coherent radiation processes (e.g. Cocke & Pacholczyk 1975; Colgate & Petschek 1978), slow interstellar scintillations (Shapirovskaya 1982; Rickett, Coles & Bourgois 1984) and variable absorbing screens (Marscher 1979; Norman & Miley 1984). But a widely discussed explanation invokes a bulk relativistic motion of the variable component, directed at a small angle from the line-of-sight towards the observer (Rees 1966; Jones & Burbidge 1973; Burbidge, Jones & O'Dell 1974; Blandford & Königl 1979). The Doppler factors of the motion needed to make the excessive apparent brightness temperatures consistent with the limit of $\sim 10^{12}$ K are typically found to be $\delta \lesssim 20$ (e.g. Fanti *et al.* 1983). Similar values of $\delta$ have been inferred in the past from the observed superluminal motions inside several active galactic nuclei at centimetre wavelengths (e.g. Kellermann & Pauliny-Toth 1981; Cohen & Unwin 1982), as well as from the observed lack of X-ray emission from variable radio sources (Marscher *et al.* 1979). It should be noted, however, that the values of $\delta$ inferred from flux variations could have been underestimates, as the true brightness temperatures at the frequencies at which the observed radiation was emitted might in fact be significantly below the theoretical upper limit of $\sim 10^{12}$ K. In this paper we shall be pursuing an alternative approach for deriving lower limits to $\delta$ for several reported events of rapid flux variations at metre wavelengths, assuming the radiation to be from the incoherent synchrotron process. These values of $\delta$ follow from the attempt to reconcile the excessive apparent brightness inferred from the flux variation with the maximum surface brightness attainable for the varying component, given some minimum plausible magnetic field strength. For example, a rough lower limit to the magnetic field could come from the supposition that the magnetic energy density is not much lower than the energy density of the microwave background, or the energy density of relativistic electrons within the source, as discussed in Section 4.

In this paper we employ this approach to estimate Doppler factors for $\sim 40$ events of flux variabilty reported at metre wavelengths and compare these values with those estimated for some prominent events of flux variability reported at centimetre wavelengths.

## 2 Derivation of the Doppler factors

Consider a radio component ejected from a galactic nucleus with a cosmological redshift $z$, in a direction making a small angle $\psi$ from the line-of-sight towards the observer. The Doppler factor $\delta$ of the component is given by $[\gamma(1-\beta\cos\psi)]^{-1}$, in terms of the bulk Lorentz factor $\gamma$ of the component, where $\gamma=(1-\beta^2)^{-1/2}$. Any variation in the flux density of this moving component would appear $[\delta/(1+z)]^3$ times boosted and shifted to a frequency $(\delta/1+z)$ times higher (Behr *et al.* 1976). Moreover, the time-scale of the variation, $\tau$, would appear shortened by a factor $(\delta/1+z)$, and accordingly the maximum angular size $\theta_0$ of the component, inferred by the observer employing the causality argument, would be related to the maximum intrinsic angular size $\theta$ in the comoving frame as

$$\theta=\theta_0(\delta/1+z)=(2c\tau/D)(\delta/1+z). \qquad (1)$$

Here $D$ is the cosmological distance which can be expressed as $D(\mathrm{Gpc})=6z(1+z/2)/(1+z)^2$ for $q_0=0$ and $H_0=50\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (Sandage & Tammann 1982). This expression would be applicable even for the case when the component responsible for the observed outburst behaves as a homogeneous sphere expanding relativistically about the nucleus with a Lorentz factor $\gamma$. The parameters $\theta_0$ and $\delta$ would then pertain to the most rapidly approaching portion of the sphere, whose radiation would be strongly boosted. This relevant part of the sphere would be contained within an angle $\sim\gamma^{-1}$ from the line-of-sight and it would thus be a small fraction of the entire source whose extent is inferred from the causality argument.

Now, from the theory of synchrotron radiation, the surface brightness of a homogeneous, incoherently radiating source with a power-law distribution of electron energies is given by (Terrell 1966; Scheuer & Williams 1968) $S_\nu/\theta^2 \lesssim 1.6\times10^{-4}\nu^{2.5}B^{-0.5}$, where $S_\nu$(Jy) is the flux density at a frequency $\nu$(MHz), $B$(G) is the magnetic field and $\theta$(arcsec) is the angular size of the source (all parameters measured in the comoving frame of the source). In the event of a motion of the variable component towards the observer with a Doppler factor $\delta$, this relation for the observed parameters becomes

$$\delta \gtrsim 33\left(\frac{B}{\mathrm{G}}\right)^{1/5}\left(\frac{\nu_0}{\mathrm{MHz}}\right)^{-1}\left(\frac{S_0}{\mathrm{Jy}}\right)^{2/5}\left(\frac{\theta_0}{\mathrm{arcsec}}\right)^{-4/5}(1+z). \tag{2}$$

All the parameters on the right are directly measurable, except $B$. If a minimum plausible value could be assigned to $B$, one could obtain a lower limit to $\delta$ for the variable component. It is clear from equation (2) that an order of magnitude uncertainty in $B$ would only make a marginal difference to the estimate of $\delta$. As a broad indicator, it is already known that the strength of magnetic field inside the nuclear cores of active galaxies is typically $10^{-3\pm1}$ G (Kellermann & Pauliny Toth 1981). Such fields are stronger than the equivalent magnetic field due to the microwave background $B_{\mathrm{bg}}\sim3(1+z)^2\mu$G, ensuring that the relativistic electrons lose energy primarily by synchrotron radiation and not by inverse Compton scattering the background photons to optical/X-ray energies. Similar conditions may also prevail inside the compact radio components responsible for the flux variations. Another constraint to the magnetic field strength comes from the theoretical view that in a synchrotron source the magnetic energy density $u_{\mathrm{B}}$ should not be much lower than the relativistic electron energy density $u_{\mathrm{e}}$ (Section 4). This condition, namely, $B^2/8\pi \gtrsim B_{\mathrm{eq}}^2/8\pi = u_{\mathrm{e}}$ will be employed in the present work.

For a source having a straight spectral index $\alpha$ between lower and upper frequency cut-offs of $\nu_1$ and $\nu_2$, the total energy content in the form of relativistic electrons is given by (Pacholczyk 1970)

$$E_{\mathrm{e}} = \frac{2.65\times10^{13}D^2 S_\nu \nu^\alpha}{(2\alpha-1)B^{3/2}}\left[\nu_1^{-\alpha+0.5}-\nu_2^{-\alpha+0.5}\right]\ \mathrm{cgs},$$

where the flux density $S_\nu \propto \nu^{-\alpha}$ and $D$ is the appropriate cosmological distance of the source. Taking both the frequency of observation and the low cut-off frequency $\nu_1$ as the frequency $\nu_{\mathrm{m}}$ where the maximum in flux density, $S_{\mathrm{m}}$, occurs and synchrotron self-absorption begins, this expression becomes

$$E_{\mathrm{e}} = \frac{2.65\times10^{13}D^2 S_{\mathrm{m}}\nu_{\mathrm{m}}^{1/2}}{(2\alpha-1)B^{3/2}}\left[1-\left(\frac{\nu_{\mathrm{m}}}{\nu_2}\right)^{\alpha-0.5}\right]\ \mathrm{cgs}.$$

When account is taken of the motion of the source relative to the observer, we have

$$E_{\mathrm{e}}(\mathrm{erg}) = \frac{2.53\times10^{48}D_{\mathrm{Gpc}}^2}{(2\alpha-1)B_{\mathrm{G}}^{3/2}}\left(\frac{\nu_{\mathrm{m}}}{\mathrm{MHz}}\right)^{1/2}\left(\frac{S_{\mathrm{m}}}{\mathrm{Jy}}\right)\left[1-\left(\frac{\nu_{\mathrm{m}}}{\nu_2}\right)^{\alpha-0.5}\right]\left(\frac{1+z}{\delta}\right)^{7/2}. \tag{3}$$

From the causality argument, the volume, $V$, of the variable component is

$$\frac{V}{\mathrm{cm}^3} = 1.77\times10^{66}\left(\frac{D}{\mathrm{Gpc}}\right)^3\left(\frac{\theta_0}{\mathrm{arcsec}}\right)^3\left(\frac{\delta}{1+z}\right)^3,$$

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so the electron energy density $u_e = E_e/V$ is given by

$$u_e = \frac{1.43\times10^{-18}}{(2\alpha-1)\,B^{3/2}}\left(\frac{\nu_m}{\text{MHz}}\right)^{1/2}\left(\frac{S_m}{\text{Jy}}\right)\left(\frac{D}{\text{Gpc}}\right)^{-1}\left(\frac{\theta_0}{\text{arcsec}}\right)^{-3}\left[1-\left(\frac{\nu_m}{\nu_2}\right)^{\alpha-0.5}\right]\left(\frac{1+z}{\delta}\right)^{6.5}. \tag{4}$$

For the case of equipartition, $u_e$ should equal $B_{eq}^2/8\pi$, where $B_{eq}$ is expressed using equation (2) as

$$B_{eq} \sim 2.43\times10^{-8}\left(\frac{\nu_m}{\text{MHz}}\right)^{5}\left(\frac{\theta_0}{\text{arcsec}}\right)^{4}\left(\frac{S_m}{\text{Jy}}\right)^{-2}\left(\frac{\delta_{eq}}{1+z}\right)^{5}. \tag{5}$$

Thus, eliminating $B_{eq}$ from equations (4) and (5), the Doppler factor $\delta_{eq}$ for the case of equipartition can be expressed as

$$\delta_{eq} \sim 2.66\, C_\alpha^{1/24}\left(\frac{S_m}{\text{Jy}}\right)^{1/3}\left(\frac{\nu_m}{\text{MHz}}\times\frac{\theta_0}{\text{arcsec}}\right)^{-17/24} D_{\text{Gpc}}^{-1/24}(1+z) \tag{6}$$

where $C_\alpha = [1-(\nu_m/\nu_2)^{\alpha-0.5}]/(2\alpha-1)$.

From equation (6) it is clear that, if the values of $S$ and $\nu$ to be substituted in this expression do not correspond to the turnover frequency, the derived values of $\delta$ would be underestimates. This would be even more so if $\nu$ is higher than $\nu_m$, which is true in general, because the spectra of metre-wavelength variable components are not usually found to be highly opaque and are often steep (Fisher & Erickson 1980; Spangler & Cotton 1982; Gopal-Krishna, Singal & Krishnamohan 1984). In the next section, we shall apply equation (6) to some observed events of rapid flux variability at metre and centimetre wavelengths. For all these events, the Doppler factors $\delta_{12}$ are also estimated in the conventional way by equating the inferred comoving brightness temperatures to the upper limit of $10^{12}$ K which is set by inverse Compton losses (Section 1). In terms of the maximum apparent brightness temperature $T_{app}$, $\delta_{12}$ can then be expressed as (see Blandford & Königl 1979; Gopal-Krishna *et al.* 1984)

$$\delta_{12} \gtrsim (T_{app}/10^{12}\,\text{K})^{1/3}(1+z) \simeq 1.21[S_0(\text{Jy})\,\nu^{-2}(\text{MHz})\,\theta_0^{-2}(\text{arcsec})]^{1/3}(1+z). \tag{7}$$

## 3 Results

For the purpose of illustration, we have selected from the published records 10 events of rapid flux variations at metre wavelengths (Table 1). These events are associated with seven sources whose redshifts are known and all of which happen to be quasars. For three of the events, simultaneous recordings at two or more frequencies are available. From the references quoted in Table 1, we have plotted in Fig. 1 the flux density measurements for all the events. Values of variability time-scales for individual events are derived using the rising and/or falling portions of the curves fitted to the data, following the definition given by Fanti *et al.* (1983):

$$\tau \sim S_{max}(\Delta S/\Delta t)^{-1}.$$

Here $S_{max}$ is the peak flux density attained during the variability event, and $\Delta S$ is the amplitude of variation over a time interval $\Delta t$ measured between the epochs corresponding to the peak and the adjacent minimum of the flux density, as marked with arrows in Fig. 1. Note that this definition is conservative as it leads to an overestimate of $\tau$ by ignoring the fact that part of the contribution to $S_{max}$ may be from a non-varying underlying component (see Fanti *et al.* 1983). For each event, Table 1 gives the cosmological distance $D_{\text{Gpc}}$, the frequency of observation, the epoch of peak flux density, reference, variability time-scales (of rise and/or of fall), peak flux density $S_{max}$ and inferred angular size $\theta_0$ computed from equation (1). The subsequent columns in Table 1 give the values of $\delta_{eq}$, $\delta_{12}$ and $B_{eq}$, computed from equations (6), (7) and (5), respectively. In accordance

**Table 1.** The observed and derived parameters for the 10 events of flux variability at metre wavelengths.

| Source | z | D (Gpc) | $\nu$ (MHz) | Epoch | References | $\tau$* (yr) | $S_{max}$ (Jy) | $\theta_o$ (mas) | $\delta_{eq}$ | $\delta_{12}$ | $B_{eq}$ ($\mu$G) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0333+321<br>(NRAO 140) | 1.26 | 2.41 | 408 | 1978.7 | Fanti et al. (1981) | (3.6) | 3.2 | 0.19 | 50 | 22 | 171 |
| 1055+018<br>(4C 01.28) | 0.89 | 2.16 | 80 | 1983.3 | Slee (1984) | 6.2 | 14.5 | 0.36 | 137 | 59 | 13 |
| | | | 160 | 1983.3 | Slee (1984) | (1.2) | 8.5 | 0.07 | 224 | 93 | 20 |
| | | | 365 | 1977.3 | Spangler & Cotton (1981) | 1.2 | 4.25 | 0.07 | 99 | 43 | 84 |
| | | | 365 | 1977.3 | Spangler & Cotton (1981) | (1.5) | 4.25 | 0.09 | 85 | 37 | 93 |
| 1253-055<br>(3C 279) | 0.54 | 1.74 | 408 | 1978.4 | Fanti et al. (1981) | (1.9) | 15.0 | 0.14 | 70 | 31 | 89 |
| 1504-166<br>(OR-102) | 0.88 | 2.15 | 408 | 1972.5 | McAdam (1982) | 1.0 | 3.5 | 0.06 | 97 | 42 | 97 |
| | | | 408 | 1972.5 | McAdam (1982) | (0.4) | 3.5 | 0.02 | 186 | 77 | 64 |
| 1611+343<br>(DA 406) | 1.40 | 2.48 | 365 | 1977.4 | Cotton & Spangler (1979) | 2.8 | 3.7 | 0.14 | 73 | 32 | 122 |
| | | | 408 | 1982.0 | Aller & Aller (1982) | 2.4 | 3.4 | 0.12 | 73 | 32 | 137 |
| | | | 318 | 1982.0 | Altschuler et al. (1984) | 3.4 | 4.1 | 0.17 | 73 | 32 | 105 |
| | | | 430 | 1982.0 | Altschuler et al. (1984) | 3.3 | 3.8 | 0.17 | 59 | 26 | 165 |
| 1641+399<br>(3C 345) | 0.59 | 1.81 | 408 | 1978.2 | Fanti et al. (1981) | (3.8) | 8.8 | 0.26 | 38 | 18 | 143 |
| 2251+158<br>(3C 454.3) | 0.86 | 2.13 | 408 | 1978.4 | Fanti et al. (1981) | 1.9 | 15.2 | 0.11 | 99 | 44 | 80 |
| | | | 408 | 1978.4 | Aller & Aller (1982) | 2.5 | 15.2 | 0.15 | 81 | 36 | 91 |
| | | | 408 | 1981.1 | Aller & Aller (1982) | 3.9 | 14.5 | 0.23 | 59 | 27 | 113 |
| | | | 318 | 1981.1 | Altschuler et al. (1984) | (5.4) | 14.1 | 0.32 | 55 | 25 | 92 |
| | | | 430 | 1981.1 | Altschuler et al. (1984) | 3.6 | 14.1 | 0.21 | 59 | 27 | 119 |
| | | | 430 | 1981.1 | Altschuler et al. (1984) | (5.0) | 14.1 | 0.30 | 47 | 22 | 139 |

*Values within and without the parentheses refer to the falling and rising portion of the intensity profile, respectively.

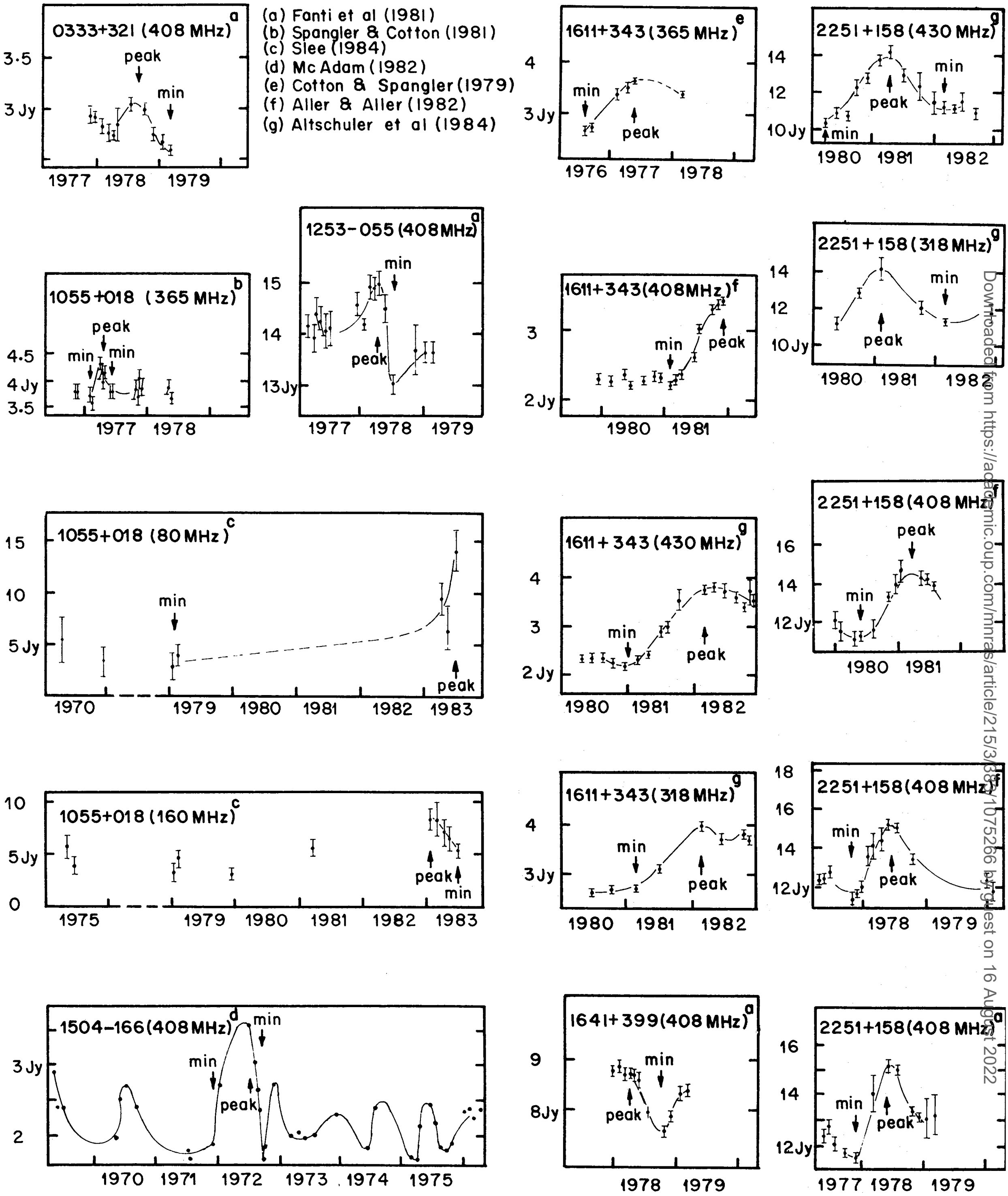

**Figure 1.** Flux densities measured at different epochs are plotted for the sources forming the present sample (Section 3; Table 1) and the frequencies of observations as well as the references to the data are given. For each variability event, the adopted epochs of peak and minimum of flux density are indicated by arrows. The interval, $\Delta t$, between the two epochs has been used in Section 3 for computing the variability time-scale for each event.

**Table 2.** The observed and derived parameters for the eight events of ultra-rapid flux variability at centimetre wavelengths.

| Source | z | D (Gpc) | $\nu$ (GHz) | Epoch | References | $\tau$* (day) | $S_{max}$ (Jy) | $\theta_o$ (mas) | $\delta_{eq}$ | $\delta_{12}$ | $B_{eq}$ (mG) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0851+203 | 0.306 | 1.24 | 85.3 | May 23, 1971 | Kinman & Conklin (1971) | 2.1 | 6.0 | $6.0\times10^{-4}$ | 50 | 21 | 27.8 |
| (OJ287) | | | 85.7 | Feb 18, 1972 | Epstein et al. (1972) | 0.25 | 16.0 | $7.0\times10^{-5}$ | 309 | 121 | 7.6 |
| | | | 4.2 | Feb 8, 1973 | Kikuchi et al. (1973) | (0.39) | 5.6 | $1.1\times10^{-4}$ | 1350 | 475 | 0.2 |
| 1226+023 | 0.158 | 0.76 | 90.0 | Oct 31, 1969 | Epstein et al. (1982) | (4.1) | 41.0 | 0.002 | 35 | 16 | 26.9 |
| (3C273) | | | 22.2 | Mar 22, 1976 | Efanov et al. (1977) | (1.0) | 41.5 | $4.5\times10^{-4}$ | 260 | 104 | 1.8 |
| | | | 22.2 | Mar 29, 1976 | Efanov et al. (1977) | (0.92) | 38.0 | $4.1\times10^{-4}$ | 263 | 107 | 1.8 |
| 1921-293 | 0.352 | 1.36 | 90.0 | Dec 9, 1978 | Epstein et al. (1980) | 2.3 | 6.5 | $6.0\times10^{-4}$ | 51 | 22 | 29.3 |
| (OV-236) | | | | | | | | | | | |
| 2200+420 | 0.07 | 0.38 | 10.7 | Oct 29, 1971 | Harvey et al. (1972) | (9) | 13.8 | 0.008 | 36 | 16 | 3.4 |
| (BL Lac) | | | | | | | | | | | |

* Values within and without the parentheses refer to the falling and rising portion of the intensity profile,respectively.

with the remarks following equation (6), we have adopted a spectral index $\alpha=0.75$. The derived values of $\delta_{\mathrm{eq}}$ are insensitive to uncertainties in the spectral index (see equation 6) and in the parameter $\nu_2/\nu_{\mathrm{m}}$ for which we have adopted a value of 4, considering that the metre-wavelength flux variations usually do not extend to frequencies beyond ~1 GHz (e.g. Spangler & Cotton 1982). The values of $\delta$ estimated for these 10 variability events are compared in the next section with estimates similarly obtained for other prominent variability events reported at metre and centimetre wavelengths.

As mentioned earlier, there are reports in the literature of ultra-rapid flux variations of a few sources at centimetre wavelengths, on time-scales of days or even hours. As the amplitude of variations in these exceedingly rare events exceeds ~10 per cent, these events perhaps have an origin different from the 'flickering' activity reported by Heeschen (1984) to be a common phenomenon among compact radio sources at centimetre wavelengths. From the literature to the end of 1983, we have selected eight events of ultra-rapid flux variations. These events recorded at wavelengths in the range 0.33–7.1 cm are associated with four extragalactic sources, namely OJ 287, 3C 273, OV-236 and BL Lac. We have employed the method described earlier in this paper to estimate the values of $\delta_{\mathrm{eq}}$. The relevant parameters are given in Table 2 which is arranged like Table 1. Note that we have adopted $\alpha=0.75$ and $\nu_2/\nu_{\mathrm{m}}=50$ for all these events, considering that the centimetre flux variations are normally known to extend from about a few GHz to ~100 GHz. The derived values of $\delta_{\mathrm{eq}}$ are insensitive to the uncertainties in these adopted values (see equation 6).

## 4 Discussion

As seen from Table 1, the equipartition magnetic fields $B_{\mathrm{eq}}$ estimated for the metre-wavelength variable components are of the order $10^{-4}$ G. These are similar to the fields estimated for the compact radio components found near the nuclei of quasars and active galaxies by interpreting their observed spectral turnovers in terms of synchrotron self-absorption, but without invoking any bulk motion relative to the nucleus (see Kellermann & Pauliny-Toth 1981). Note that the values of $B_{\mathrm{eq}}$ given in Table 1 imply that magnetic energy densities in these sources are higher than the energy density of the microwave background photons expected at the respective redshift $[u_{\mathrm{bg}}\sim 3\times 10^{-13}(1+z)^4\,\mathrm{erg\,cm^{-3}}]$. In these circumstances the relativistic electrons are expected to lose energy mainly via synchrotron radiation.

It is also interesting to note from Table 1 that the estimated value of $\delta_{\mathrm{eq}}$ are systematically higher than those of $\delta_{12}$ by a factor of ~2, indicating that the comoving brightness temperatures of these sources at frequencies corresponding to the observations are probably substantially lower than the inverse Compton limit of $10^{12}$ K (Section 1; equation 7). Also, the values of $\delta_{\mathrm{eq}}$ are found to be $\geq 40$ in all 10 cases and $>100$ in a few cases. These limiting values of $\delta$ are considerably higher than the values of $\lesssim 20$ inferred from the centimetre-wavelength observations of flux variations of active galactic nuclei, from their apparent superluminal expansions, or from the lack of X-ray emission (Section 1). Note that the values of $\delta_{\mathrm{eq}}$ given in Table 1 have probably been underestimated because (i) the causality argument only yields an upper limit to the true physical size (equation 1), (ii) the values of $S$ and $\nu$ used may not correspond to the frequency of turnover due to synchrotron self-absorption [as discussed following equation (6)], and (iii) the adopted definition for $\tau$ is conservative and leads to an overestimate of this parameter (Section 3). It should further be noted that a Hubble constant of $100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, i.e. twice the presently adopted value, would reduce the derived values of $\delta_{\mathrm{eq}}$ and $\delta_{12}$ by a factor of $2^{2/3}\sim 1.6$ only (equations 6 and 7).

Further, to extend our analysis to a larger sample of metre-wavelength variables, we have computed $\delta_{\mathrm{eq}}$ using equation (6) for all the 35 events of variability at 408 MHz which are reported in the sample of Fanti *et al.* (1983) and are associated with those 23 sources in the sample whose redshifts are given. The input parameters to equation (6) are directly taken from Table 1 of Fanti *et al.* (1983). Of the 35 events, four yield $\delta > 100$ and the median value of $\delta$ for the entire sample is found to be 45.

In order to facilitate a comparison, we have employed equation (6) to compute $\delta_{\mathrm{eq}}$ for events of flux variations at centimetre wavelengths reported for 11 prominent variable sources of known redshifts, as tabulated by Marscher *et al.* (1979). For this purpose, the values of the various input parameters to equation (6) were directly taken from their tabulation. The derived values of $\delta_{\mathrm{eq}}$ for these 11 events are found to be $\leq 16$, with a median value of 3.5.

It appears, therefore, that the interpretation of flux variability at metre wavelengths in terms of bulk relativistic motion would often require values of $\delta_{\mathrm{eq}}$ much higher than the value $\sim 10$ which is usually thought to be adequate to explain nearly all the observed flux variations at centimetre wavelengths. Of course, as discussed in Section 3, a few events of ultra-rapid variability at centimetre wavelengths have also been reported which indeed suggest very large values of $\delta_{\mathrm{eq}}(>50)$ (Table 2). But observations of such events with variability time-scales of just a few days are scarce and, in view of their serious implications, there is a clear need for additional and more firmly established observations of such events. This is because rapid flux variability at centimetre wavelengths indicating apparent brightness temperatures $>10^{12}$ K is generally interpreted by invoking relativistic bulk motion, whereas alternative explanations have been proposed for the variations observed at metre wavelengths (Section 1).

As stated in Section 2, the present treatment is based on the assumption of equipartition of energy between relativistic electrons and magnetic field. This consideration was uniformly applied to both metre and centimetre wavelength variables in Section 3 and systematically higher values of Doppler factors were found for the metre-wavelength variables. This difference would disappear if magnetic fields in the metre-wavelength variables were a few orders of magnitude weaker than the equipartition estimate of $\sim 10^{-4}$ G (see equation 2 and Table 1), but such a situation is unlikely in synchrotron sources (Ginzburg & Syrovatskii 1969; Singal 1985). If the magnetic energy density were more than an order of magnitude lower than the energy density of relativistic electrons, the diamagnetic effects of the orbiting electrons would be strong, leading to a total screening of the field and an unstable situation for the synchrotron radiation. Accordingly, the magnetic field could be only mildly weaker than $B_{\mathrm{eq}}$ and such a difference would not significantly reduce the lower limit to $\delta_{\mathrm{eq}}$ given in Table 1 (see equation 2). On the other hand, the field may be considerably stronger than the equipartition value, in which case the derived values of $\delta_{\mathrm{eq}}$ would be conservative lower limits.

## 5 Conclusions

We have explored some consequences of the hypothesis that the short time-scales of flux variability of compact extragalactic sources at metre wavelengths can be interpreted in terms of a relativistically moving variable component radiating by incoherent synchrotron process. For a sample of 10 published events of metre-wavelength flux variability of seven quasars, it is found that the excessive apparent surface brightness inferred from the time-scales and amplitudes of these events can be reconciled with the upper limits imposed by synchrotron self-absorption effects provided that a bulk motion with extremely large Doppler factors ($\delta \gtrsim 50$) is invoked (Table 1). This inference involves the assumption that the energy density due to magnetic field is not much smaller than that due to relativistic electrons in the variable component. Such a requirement follows from the need to avoid a total screening of the magnetic field due to diamagnetic effects of the relativistic electrons.

The lower limits to the Doppler factors inferred here for the large sample of metre wavelength outbursts are considerably higher than those inferred in past from similar events ($\delta \lesssim 20$; Section 1). The earlier estimates might be conservative since these were based on the assumption that the comoving brightness temperature of the source at the frequency where the observed radiation was emitted is equal to the maximum attainable value of $10^{12}$ K, which need not always be the case. The presently estimated lower limits to $\delta$ are also much higher than the values inferred either from prominent, well-observed outbursts at centimetre wavelengths, or from X-ray observations, or from the VLBI measurements of superluminal motion in active galactic nuclei (see Section 1; also Cohen & Unwin 1982). Clearly, the high Doppler factors inferred here for the metre-wavelength variables imply a very low probability of order ($\frac{1}{2}\delta^{-2}$) $\lesssim 10^{-4}$ for detection of individual events caused by a relativistically objected plasmon. However, in this picture a very strong beaming ($\propto \delta^3 \gtrsim 10^5$) is also implied, so that in the comoving frame the ejected component would only be required to emit at a level of $\lesssim 100\,\mu$Jy. The fact that rapid metre-wavelength outbursts have been observed for many sources despite the low probability would then indicate that the occurrence of such sub-millijanksy bursts may be a common phenomenon among the population of extragalactic sources.

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