Ultra-Wideband Compressed Sensing: Channel Estimation

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Abstract—In this paper, ultra-wideband (UWB) channel estimation based on the theory of compressive sensing (CS) is developed. The proposed approach relies on the fact that transmitting an ultra-short pulse through a multipath UWB channel leads to a received UWB signal that can be approximated by a linear combination of a few atoms from a pre-defined dictionary, yielding thus a sparse representation of the received UWB signal. The key in the proposed approach is in the design of a dictionary of parameterized waveforms (atoms) that closely matches the information-carrying pulseshape leading thus to higher energy compaction and sparse representation, and, therefore higher probability for CS reconstruction. Two approaches for UWB channel estimation are developed under a data-aided framework. In the first approach, the CS reconstruction capabilities are exploited to recover the composite pulse-multipath channel from a reduced set of random projections. This reconstructed signal is subsequently used as a referent template in a correlator-based detector. In the second approach, from a set of random projections of the received pilot signal, the Matching Pursuit algorithm is used to identify the strongest atoms in the projected signal that, in turn, are related to the strongest propagation paths that composite the multipath UWB channel. A Rake like receiver uses those atoms as templates for the bank of correlators in the detection stage. The bit error rate performances of the proposed approaches are analyzed and compared to that of traditional correlator-based detector. Extensive simulations show that for different propagation scenarios and UWB communication channels, detectors based on CS channel estimation outperform traditional correlator using just 1/3 of the sampling rate leading thus to a reduced use of analog-to-digital resources in the channel estimation stage.

Index Terms—Channel estimation, compressive sensing, detection, ultra-wideband.

I. INTRODUCTION

ULTRA-wideband (UWB) communications has emerged as a promising technology for wireless communications systems that require high bandwidth, low-power consumption and shared spectrum resources with applicability ranging from short-distance high-data-rate applications to long-distance low-data-rate applications like sensor networks and high precision location and navigation systems [1], [2].

In UWB impulse radio communications, an ultra-short duration pulse, typically on the order of nanoseconds, is used as the elementary pulse-shaping to carry information [3]. Transmitting ultrashort pulses leads to several desirable characteristics. Firstly, simplicity is attained in the transmitter since a carry-less baseband signal is used for conveying information [4]. Secondly, the transmitted signal power is spread broadly in frequency having little or not impact on other narrowband radio systems operating on the same frequency [5]. Finally, the received UWB signal is rich in multipath diversity introduced by the large number of propagation paths existing in a UWB channel.

UWB receivers, however, face several challenges including interference cancellation, antenna design, timing synchronization, and channel estimation, among others [2]. Digital UWB receiver architectures have been proposed in [6]–[8] as an alternative to implement UWB receivers since digital detectors offer considerable flexibility and technology scaling benefits. However, the extremely high bandwidth of the received UWB signal (up to 7.5 GHz) requires high-speed analog-to-digital converters (ADCs). These speeds demand the use of interleaved flash ADC [7] or a bank of polyphase ADCs with accurate timing control [6]. The former approach, however, consumes a lot of power, has relatively low resolution, and can be quite expensive, whereas the latter requires precise timing to control the ADCs while incurring high circuit complexity. Furthermore, oversampling of the received UWB signal may be required to improve timing synchronization and channel estimation. For instance, in [4] the required sampling rate is in excess of 25 GHz for accurate UWB channel estimation. Such formidable sampling rates are not feasible with state-of-the-art ADC technology. New approaches for UWB receivers are needed to attain the required sampling rates and bit resolution.

This paper focuses on this goal by casting the problem of UWB channel estimation and detection into the emerging framework of compressive sensing [9], [10]. Compressed sensing (CS) is a new concept based on the theoretical results of signal reconstruction with random basis coefficients. The remarkable result of CS reveals that with high probability, a signal, f, with a large number of data points that is M-sparse[1] in some dictionary Ψ of basis functions or tight-frames, can be exactly reconstructed using only a few number of random projections of the signal onto a random basis Φ that is incoherent with Ψ. The number of projections, in general, is much smaller than the number of samples in the original signal

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[1]By M-sparse, a signal f can be written as a sum of M known basis functions, i.e., $f = \sum_{i=1}^{M} \alpha_i \psi_i$
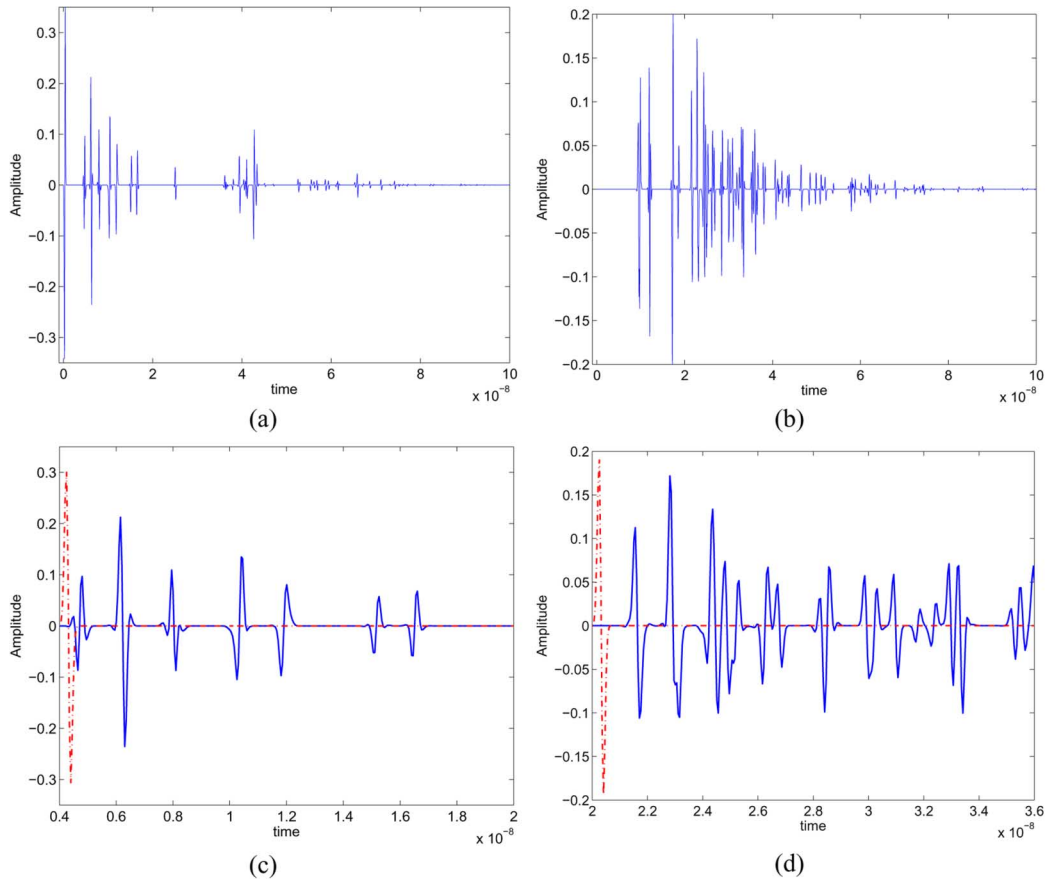
Fig. 1. Effect of UWB channel (indoor propagation in residential environments) on the transmitted pulse for two different propagation scenarios: (a) line-of-sight (LOS); (b) non-line-of-sight (NLOS); (c) zoom-in of (a); and (d) zoom-in of (b). Transmitted pulse $(-\cdot-)$ is also shown in (c) and (d).

leading to a reduced sampling rate and, hence, to a reduced use of ADCs resources [10]. Signal reconstruction from the set of projections is obtained from the solution of a simplex convex optimization problem that can be solved using fast iterative algorithms [11]–[14].

We begin with the basic assumption that when the short duration (high frequency) pulses used in UWB communications propagate through multipath channels, the received signals remain sparse in some domain and thus compressed sensing is indeed applicable. To illustrate this concept, consider a Gaussian monocycle as the information carrier UWB pulse having a duration of 0.65 ns [3]. Furthermore, consider the pulse propagating through two different noiseless propagation scenarios. Fig. 1(a) shows the received signal per frame for a UWB channel that models an indoor residential environment with line-of-sight — IEEE 802.15.4a channel model 1 (CM1) [15], while Fig. 1(b) shows the received signal per frame for the same communication environment but with non-line-of-sight — IEEE 802.15.4a channel model 2 (CM2) [15]. The time observation window is 100 ns, that is a typical frame time for UWB systems [2], [16]. Both propagation scenarios use the models in [15]. Fig. 1(c) and Fig. 1(d) show a zoom-in of Fig. 1(a) and (b), respectively. For comparative purposes, the transmitted pulse, has been rescaled and also shown in these figures in dashed-dotted lines.

As depicted in Fig. 1(a)–(d), the received UWB signal is composed of sets of spaced clusters of the transmitted pulse which, in turn, captures the statistical characteristics of multipath ar-

rivals in a UWB channel [17], [18]. Upon closer examination of these figures, it can be seen relatively long time intervals between clusters and rays where the signal takes on zero or negligible values. It is precisely this signal sparsity of the received UWB signals that is exploited in this work aimed at UWB channel estimation and detection.

In this paper, we show that the CS framework can indeed be used for the processing of UWB signal. We not only show that the received UWB signals can be reconstructed from a set of random projections, leading to a reduced sampling rate but also that the CS framework can be extended to other statistical inference tasks suitable in UWB communications. More precisely, two approaches for UWB channel estimation are proposed that exploit the sparsity of the received UWB signal. The first approach, named CS correlator, explores the reconstruction capability of CS using the Matching Pursuit (MP) algorithm. This approach first estimates the composite pulse-multipath channel $(p(t) * h(t))$ by randomly projecting a set of received pilot waveforms for training. This reconstructed signal is subsequently used as a template for correlator-based detector that demodulates the transmitted information symbols. In the second approach, a CS-based UWB channel parameter estimation followed by a Rake like receiver is developed. In this approach, the MP algorithm is used to approximate the received pilot signal by a reduced set of elements of a pre-defined dictionary. These elements are closely related to the signal contribution of the strongest UWB propagation paths leading

thus to a UWB channel parameter estimation approach that determines the path gains and path delays of the most relevant UWB propagation paths.

CS requires that the underlying signals are sparse in some dictionary of basis or tight-frames. The key in the use of CS in UWB communications is in the design of a dictionary of parameterized waveforms that closely match the information-carrying pulseshape [19], [20]. Thus, the received UWB signal can be compactly represented in this dictionary leading to a sparsity signal model that is suitable for the CS framework.

To alleviate the effect of additive white Gaussian noise, a data-aided framework is adopted in this paper where a set of training symbols, also known as pilot signals, are used to estimate the channel parameters as in [4], [21] or to estimate a referent template for subsequent correlation detection [2], [16], [22]. In this light of work, we use the CS approach for template reconstruction and UWB channel parameter estimation leading naturally to new methods for signal detection.

II. ULTRA-WIDEBAND COMPRESSIVE SENSING

Compressive sensing is a novel theory recently introduced in [9] and [10] that unifies signal sensing and compression into a single task. In essence, CS theory has shown that a sparse signal can be recovered, with high probability, from a set of random linear projections using nonlinear reconstruction algorithms. The sparsity of the signal can be in any domain (time domain, frequency domain, wavelet domain, etc.) and the number of random measurements, in general, is much smaller than the number of samples in the original signal leading to a reduced sampling rate and, hence, reduced use of ADCs resources. Next, we briefly describe the CS framework proposed in [9] and [10].

A. Compressive Sensing Overview

Suppose f is the N-point discrete-time representation of an analog signal of interest. Also, suppose a set of K measurements, y, are acquired that are linear combinations of the points in f. More precisely, $y = \Phi f$, where Φ is a $K \times N$ matrix, hereafter called measurement matrix, whose rows are basis vectors of the space R^N. It can be shown that if f is sparse, in the sense that f can be written as a superposition of a small number of vector taken from a dictionary $\Psi = [\psi_1, \psi_2, \dots, \psi_Z]$ of basis or tight-frames, such that

$$f = \sum_{i=1}^{M} \theta_{\ell_i} \psi_{\ell_i} = \Psi \Theta \qquad (1)$$

for $K \ll N$, then f can be recovered from y, with high probability as long as the measurement matrix Φ is incoherent with the dictionary Ψ.

In (1), $\Theta = [\theta_1, \theta_2, \dots, \theta_Z]^{\dagger}$ is a vector that contains M nonzeros coefficients where Z is the number of elements (atoms) in the dictionary Ψ. The index of the nonzero coefficient defines which element in the dictionary composes the signal and the coefficient value the contribution of that element in defining the signal f.

The signal f can be recovered from the solution of a convex, nonquadratic optimization problem known as *basis pursuit* [19]

that yields the sparse vector Θ. Formally, with very high probability, Θ is the unique solution to

$$\min \|\Theta\|_1 \text{ subject to } y = \mathbf{V} f \qquad (2)$$

where $\|\cdot\|_1$ denotes the ℓ_1 norm and $\mathbf{V} = \Phi \Psi$ is the holographic dictionary. It was shown in [23] that if the random measurement matrix has i.i.d. entries taken from a normal distribution and the number of random projection K is greater than or equal to $c_1 M \log(N/M)$, the probability of exact reconstruction exceeds $\left(1 - e^{c_2 K}\right)$, where c_1 and c_2 are some constants.

Note that the only *a priori* knowledge required is that f is sparse in some dictionary. Also, note that the measurements are completely independent of the signal itself. That is, a fixed set of random basis vectors are used to acquire the measurements of any signal. Reconstruction then only requires the space in which the signal is sparse. This space, referred before as dictionary, is a collection of parameterized waveforms, called atoms, and may contain Fourier basis, Wavelet basis, cosine packets, chirplets basis, Gabor functions, or even a combination of basis and tight-frame [19]. In general, the structure of the signal of interest leads to the definition of the dictionary [20], for instance if the signal is smooth, a Fourier basis dictionary will yield a sparse representation of this kind of signal, whereas if the signal is piecewise smooth a wavelet-based dictionary is more suitable.

Solving the optimization problem in (2) is computationally expensive and is not suitable for real-time applications. Faster and more efficient reconstruction algorithms exist that use iterative greedy-based algorithms, at the expense of slightly more measurements, among them, matching pursuit [11], orthogonal matching pursuit [12] and tree-based matching pursuit [13], [14].

In particular, MP is a computationally simple iterative greedy algorithm that tries to recover the signal by finding, in the measurement signal, the strongest component (atom of a holographic dictionary), removing it from the signal, and searching again the dictionary for the strongest atom that is presented in the residual signal. This procedure is iteratively repeated until the residual signal contains just insignificant information. Signal reconstruction is then achieved by linearly combining the set of atoms found in the measurements. Table I shows in details the MP algorithm [11] where $\mathbf{V} = \Phi \Psi = [v_1, v_2, \dots, v_Z]$ is the holographic dictionary, T_0 is the maximum number of algorithm iterations and ϵ sets the minimum energy that is left in the residual error signal.

B. Processing UWB Signals Using CS

Consider the simple communications model of transmitting a pulse $p(t)$ throughout a noiseless UWB communication channel $h(t)$. The received UWB signal can be modeled as

$$g(t) = p(t) * h(t) = \sum_{\ell=0}^{L-1} \alpha_\ell p(t - \tau_\ell) \qquad (3)$$

where $p(t)$ is the ultra-short pulse used to convey information with a time duration in the order of nanoseconds. Typically, a Gaussian pulse or its derivatives are used as the UWB pulses. Thus $p(t) = p_n(t) e^{-t^2/2\sigma^2}$ where $p_n(t)$ is a polynomial of

TABLE I
MATCHING PURSUIT ALGORITHM

Step A	Initialize: the residual error $e_0 = y$ the approximation $\hat{\Theta} = 0, \hat{\Theta} \in R^Z$ Interation counter $t = 1$
Step B	Select the atom in the holographic dictionary that best match the residual error. $$\ell_t = \underset{i=1,2,\dots,Z}{\arg\max} \frac{|\langle e_{t-1}, v_i \rangle|}{\|v_i\|}$$
Step C	Update the residual error and the estimate of the coefficient for the selected vector $$e_t = e_{t-1} - \frac{\langle r_{t-1}, v_{\ell_t} \rangle}{\|v_{\ell_t}\|^2} v_{\ell_t}$$ $$\hat{\theta}_{\ell_t} = \hat{\theta}_{\ell_t} + \frac{\langle r_{t-1}, v_{\ell_t} \rangle}{\|v_{\ell_t}\|^2}$$
Step D	Check for convergence If $t < T_0$ and $\|e_t\|_2 > \epsilon \|y\|_2$ then set $t = t + 1$ and go to step B; otherwise, go to step E.
Step E	Reconstruct the signal estimate as: $\hat{f} = \Psi \hat{\Theta}$.

degree n that depends on the order of the derivative used. σ controls the width of the pulse and is chosen to meet the FCC spectral mask requirements [24]. The first, second and fifth derivatives of the Gaussian pulse have been proposed for UWB communications [3], [21]. In [21], for instance, the fifth derivative is used with $\sigma = 5.28 \cdot 10^{-11}$ yielding a pulse duration of 0.5 ns. The principles developed here are general admitting arbitrary types of UWB pulses.

In (3), $h(t)$ is the impulse response of the UWB channel and has been modeled as

$$h(t) = \sum_{\ell=0}^{L-1} \alpha_\ell \delta(t - \tau_\ell) \qquad (4)$$

where $\delta(\cdot)$ is the dirac delta function, τ_ℓ and α_ℓ are, respectively, the delay and gain associated with the ℓth path of the UWB channel and L is the number of propagation paths. Extensive efforts have been devoted to the characterization of UWB channel. The Saleh-Valenzuela model, in particular, provides a useful analysis tool for indoor multipath propagation, where multipath components arrive in Poisson-distributed clusters [17]. Furthermore, within each cluster, the path arrivals are also described by a Poisson process. Both cluster interarrival times and path interarrival times, are thus exponentially distributed with model parameters that are experimentally determined [15], [18].

In our analysis, the set of delays and gains are generated according to the models proposed by the IEEE 802.15.4a working group in [15] for different communication environments: indoor residential, indoor office, industrial, outdoor and farm environments; different propagation scenarios: line-of-sight (LOS) or non-line-of-sight (NLOS); in an operating range greater than 10 m and up to few hundred meters for outdoor environments; and for low data rates (less than a few megabits per second). In the UWB channel model developed in [15], however, a frequency-dependent path loss model is used to describe the per-path pulse distortion yielding a UWB channel model with complex coefficients. We restrict our analysis to real-valued UWB channel models where there is not pulse distortion. Thus far, it is not clear how our approach can be extended to deal with UWB pulse distortion and it is an ongoing research topic that will be reported elsewhere.

Note in (3) that the received UWB signal is composed of scaled and time-delayed versions of the transmitted pulse. Note also that the statistics of the arrival paths define the time-space between pulses. Thus, if the averaged path interarrival time

is greater than the pulse duration, the received UWB signal presents less pulse-overlapping and therefore more sparsity is expected. On the other hand, for dense multipath UWB channel, like NLOS propagation where the multipath components arrive closely spaced, more pulse-overlapping is found. This remark can be further observed on Fig. 1 where the pulse $p(t)$ is transmitted through two different propagation scenarios, Fig. 1(a) depicts the received waveform for an indoor residential environment with LOS propagation (CM1), while Fig. 1(b) shows the same communication environment but with NLOS propagation (CM2). Note that more pulse-overlapping occurs on the NLOS channel.

Next the CS framework is used to reconstruct the waveform modeled by (3) which, as will be seen shortly, plays an important role on signal detection. Hereafter, this waveform is called the composite pulse-multipath channel.

1) UWB Signal Reconstruction Using Time Sparsity Model: A first approach to reconstruct the composite pulse-multipath channel from a set of random projections assumes that the signal is sparse in the time domain. That is, transmitting an ultra-short pulse through a UWB communications channel leads to a received signal that has a few nonzero values. This signal model is adequate for the UWB channel provided that there are only a few propagation paths as is the case of UWB channels in industrial environments with LOS propagation [15].

Let $\mathbf{g}$ be a discrete-time representation of the composite pulse-multipath channel. That is, $\mathbf{g} = [g(0), g(T), \dots, g((N - 1)T)]^\dagger$, where T is the sampling period, N the number of samples and $\dagger$ denotes the transpose operator. Define the measurement matrix, Φ, as $K \times N$ random matrix with entries i.i.d. taken from a normal distribution with zero-mean and unit variance. Since we are assuming sparsity in the time domain, the dictionary Ψ reduces to the identity matrix. Running the matching-pursuit algorithm with the holographic dictionary $\mathbf{V} = \Phi$ and the random projections $\mathbf{y} = \Phi \mathbf{g}$ yields the results show in Fig. 2.

Fig. 2(a) shows the 2048-point composite pulse-multipath channel for a realization of an indoor residential channel with LOS propagation obtained from [15]. This is the signal targeted for reconstruction from a reduced set of random projections. Fig. 2(b) depicts the reconstructed signal obtained using 500 random measurements. Note that although CS is able to recover the most significant values of the underlying signal, it fails to recover many of the signal details yielding, in general, a poor performance. Note also that several spurious components are introduced in the reconstructed signal leading to a cumulative square error of 0.9276.

To improve the CS reconstruction performance, one may be tempted to increase the number of random projections since a large number of random projections increases the probability of exact reconstruction [9], [10], [12], [13]; this, however, leads to a higher sampling rate, and therefore more demanding ADC resources.

A more appealing approach is to design a dictionary of parameterized waveforms where the received UWB signal can be compactly represented, increasing thus the sparsity of the underlying signal. This approach is motivated by the fact that the
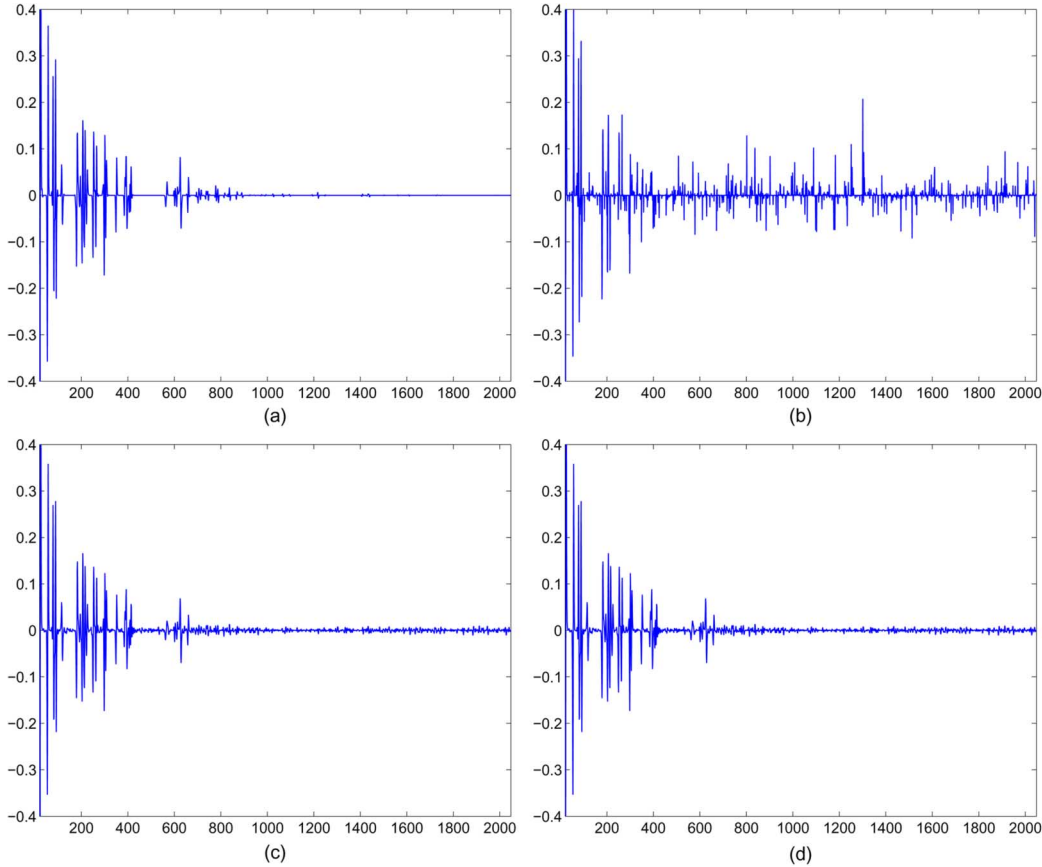
Fig. 2. (a) Received UWB signal for a realization of an indoor residential channel with LOS propagation (CM1). (b) CS reconstruction using time-sparsity model, with 500 random projections. (c) CS reconstruction using multipath diversity, with 500 random projections. (d) CS reconstruction using multipath diversity, with 250 random projections.

received UWB signal given by (3) can be thought of as a linear combination of the signal contributions of the various propagation paths that compose the UWB multipath channel.

2) UWB Signal Reconstruction Using Multipath Diversity: UWB channels in general are rich in multipath diversity motivating the construction of basis functions offering higher energy compaction, sparseness, and higher probability for CS reconstruction. Since CS theory relies on the fact that the underlying signal is sparse in some dictionary of basis or tight-frames, it is important to define a suitable dictionary to represent the underlying UWB signal. We can explore a great variety of dictionaries that have been defined in the context of atomic decomposition to find the best basis that match our problem [19], [25]. Alternatively, we can generate a new dictionary just inspecting the characteristic of the received UWB waveform. In particular, the transmitted pulse shapes as well as their spread caused by closely spaced channel taps, suggest the use of atom basis representations (tight-frames) that can provide a better sparse representation of the received UWB signals.

Since the received UWB signal is formed by scaled and delayed versions of the transmitted pulse and since the dictionary should contain elements (atoms) that can fully represent the signal of interest, it is natural to think that the elementary function to generate the atoms of the dictionary should be closely related to the pulse waveform used to covey information, i.e., the Gaussian pulse or its derivatives. Therefore, the dictionary is

generated by shifting with minimum step Δ the generating function, $p(t)$, leading to a set of parameterized waveforms given by

$$d_j(t) = p(t - j\Delta) = p_n(t - j\Delta)e^{-\frac{(t-j\Delta)^2}{2\sigma^2}}$$
$$j = 0, 1, 2, \ldots \tag{5}$$

that define the dictionary $\mathcal{D} = \{d_0(t), d_1(t), d_2(t), \ldots, \}$. The atoms in the dictionary are thus delayed versions of the UWB transmitted pulse. Note that by setting Δ greater than the time support of the basic pulse, atoms in (5) become orthogonal to each other. Although the orthogonal property is a desirable characteristic of a basis to guarantee unique representation of a signal, the rich multipath diversity introduced by the UWB channel produces pulse spread that is better captured by a redundant dictionary. Thus, Δ is set such that overlapping between atoms occurs.

Although (5) is expressed in terms of continuous time t and Δ, in practice both parameters are discretized, setting Δ to a multiple of the sampling period. In particular, if Δ is set to the sampling period, the dictionary becomes a complete and redundant dictionary of tight-frames [19], [20].

It should be pointed out that the use of Gaussian pulses and their derivatives as generating functions to generate atoms in a dictionary is not new. In [26], for instance, a two dimensional overcomplete dictionary that has as generating functions the first

and second derivatives of the Gaussian pulse is proposed to characterize the edge component of images. Furthermore, defining a dictionary that matches the signal of interest has been reported in [20] in the context of processing acoustic waves scattered from submerged elastic targets.

Having defined a suitable dictionary that matches the UWB signal, we next return to the reconstruction problem.

Consider the composite pulse-multipath channel given by (3) that has been sampled to define the discrete-time vector $\mathbf{g} = [g(0), g(T), \ldots, g((N-1)T)]^\dagger$. Furthermore, let $\mathbf{y} = \Phi\mathbf{g}$ be the random projected signal where Φ is the $K \times N$ measurement matrix with $\phi_{i,j} \sim N(0,1)$. The MP algorithm is then applied on the random projected signal, $\mathbf{y}$, and the dictionary Ψ, where Ψ is the discrete-time dictionary defined by uniformly sampling the atoms of the dictionary $\mathcal{D}$.

Fig. 2(c) and (d) show the reconstructed signal using 500 and 250 random measurements, respectively. As it can be seen from Fig. 2(c) and (d), CS successfully recovers the desired signal from random projections yielding cumulative square errors of 0.0262 and 0.1110 using just 500 and 250 measurements, respectively. Furthermore, comparing Fig. 2(b) and (c), it can be seen that UWB signal reconstruction using multipath diversity outperforms UWB signal reconstruction using time sparsity model yielding a reconstruction error that is 35-fold smaller. Therefore, by building a dictionary that is closely matched to the underlying waveform, a notable performance gain is achieved in the reconstruction.

Note that by having just 1/8 of the original samples, CS can reconstruct each resolvable delay bin carrying significant amount of energy in the composite pulse-multipath channel waveform. This can be further seen as follows. By sampling the random projected signal at a notably reduced sampling rate, it is possible to reconstruct the unprojected signal with a very small cumulative reconstruction error. Therefore, from a reduced set of random projections, the composite pulse-multipath channel at much higher sampling rate can be reconstructed. Obviously, this leads to a reduced use of ADCs resources and improved subsequent signal detection as it will be seen shortly.

Note also that the signal prior to the projection stage does not have to be a discrete-time signal, since the random projections can be performed in the analogous domain by a bank of synchronized high speed analog mixers with PAM waveform random generators followed by low-rate sampling.

Next, the probability of successful reconstruction as a function of the number of measurements is presented. In order to compute this probability, a signal is considered to be successfully reconstructed if the reconstruction error is less than 1% of the signal's energy, i.e., $\|g - \hat{g}\|_2^2 < 0.01\|g\|_2^2$. Fig. 3 depicts the probability of successful reconstruction as a function of the number of measurements (averaged over 1000 trails) for two different propagation scenarios (LOS and NLOS) in an indoor residential environment. The same UWB channel realizations used to obtain Fig. 1 are used in this simulation. As expected, the NLOS propagation scenario requires more random measurements than that required by the LOS propagation scenario. This observation is in concordance with the fact that NLOS channels are more dispersive and thus, have more multipath components
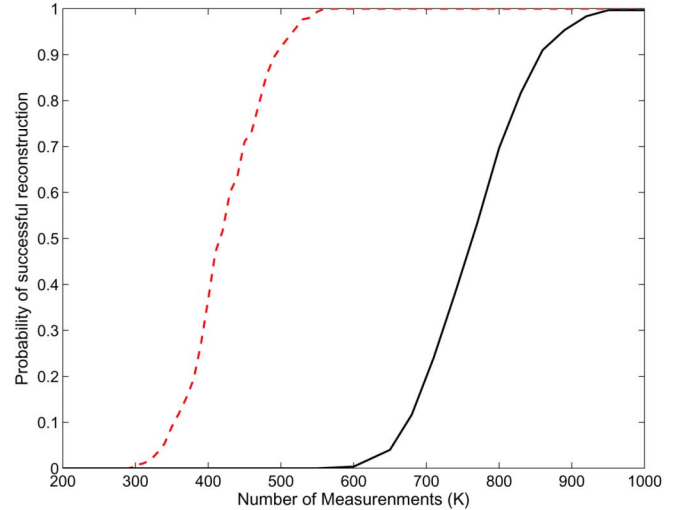

Fig. 3. Probability of success reconstruction for UWB signal for two different propagation scenarios: LOS (- - -) and NLOS (—).

than LOS channels, consequently the NLOS received signal is less sparse than LOS received signal demanding more measurements. Note that to achieve 95% of success in reconstruction, the MP algorithm needs about 500 random measurements for LOS channels and about 900 random measurements for NLOS channels.

C. UWB Channel Estimation Using CS

While CS research has focused primarily on signal reconstruction and approximation, the CS framework can be extended to a much broader range of statistical inference tasks, well suited for applications in wireless UWB communications. UWB channel estimation is one of those applications which will be used extensively in Section III and Section IV and it is addressed next.

Consider the composite pulse-multipath channel, given by (3), where the channel parameters $\{\alpha_i, \tau_i\}_{i=1}^L$ related to the various propagation paths have to be estimated. The number of multipath components in (4) that form the UWB channel can be quite large, leading to a large time dispersion of the transmitted pulse [3]. For instance, in an LOS indoor residential environment [15], the average number of multipaths for 1000 realizations of the channel is about 1160 paths. This seems to lead to an untractable channel model with 2320 channel parameters, however many of those paths are negligible thus reducing the channel complexity. In fact, for the above example, the average number of paths capturing 85% of the energy is just 70 paths. Therefore, we limit ourselves to estimate the L_c most significant paths that composes the UWB channel impulse response, as in [4] and [21].

Furthermore, consider the UWB signal reconstruction problem discussed in the previous section where the random projections of the noiseless composite pulse-multipath waveform and the dictionary $\mathcal{D}$ are input to the MP algorithm.

Upon closer examination of the MP algorithm, it can be noticed that, the holographic dictionary is searched for the strongest delayed version of the transmitted signal that is contained in the residual signal (random projected signal for the

first iteration). That delayed version of the transmitted pulse is an atom of the tailored dictionary and is related to a propagation path in the UWB multipath channel. Next, the signal contribution of that path is canceled out from the residual signal. This searching-and-canceling stage is repeated T_0 iterations or after a target residual error is reached. Finally, the MP algorithm outputs a sparse vector $\Theta = [\theta_1, \theta_2, \ldots, \theta_Z]^{\dagger}$ that contains the signal contribution of the various propagation paths.

Furthermore, the reconstruction step in the MP algorithm can be thought of as a weighted sum of the elements in the dictionary, that is $\sum_{i=1}^{Z} \theta_i d_i(t)$. Since each element in the dictionary is a shifted version of the transmitted pulse, it turns out that θ_i is an estimate of the path gain related to the ith propagation path. Furthermore, the path delay is directly determined by observing the time-location of the ith atom found in the received UWB signal. This leads naturally to a method to estimate the channel parameters (path delays and path gains) of the most significant paths using the MP algorithm, as follows.

Let $\Theta = [\theta_1, \theta_2, \ldots, \theta_Z]^{\dagger}$ be the sparse vector yielded by the MP algorithm after T_0 iterations[2]. Let $\theta_{(k)}$ for $k = 1, 2, \ldots, Z$ be the sorted elements of the set $\{|\theta_1|, |\theta_2|, \ldots, |\theta_Z|\}$ defined such that $\theta_{(1)} = \max\{|\theta_1|, \ldots, |\theta_{(Z)}|\}$, $\theta_{(Z)} = \min\{|\theta_1|, \ldots, |\theta_{(Z)}|\}$, and $\theta_{(i)} \geq \theta_{(j)}$ for $i \leq j$. Furthermore, let $\ell_{(k)}$ be the index in the sparse vector of the kth sorted element. The estimate path gain and path delay for the ith propagation path are, respectively

$$\hat{\alpha}_i = \theta_{\ell_{(i)}}$$
$$\hat{\tau}_i = \ell_{(i)} \Delta \qquad (6)$$

for $i = 1, 2, \ldots, L_c$, where Δ denotes the minimum shifting step of the transmitted pulse as defined in (5). The number iterations used in the algorithm is a parameter that has to be suitably selected. At first glance it seems that it should be set equal to the number of most significant paths, L_c. However, L_c is, in general, unknown. Furthermore, it is quite possible that, as the MP algorithm progresses, those previously selected atoms are chosen again and the corresponding path gains are updated. As it will be seen in Section III, a subsequent stage of a UWB receiver (symbol detection) provides a reliable criterion that can be used to select the number of MP iterations.

Nevertheless, T_0 is set to a value much smaller than necessary for reconstruction, since our motivation is not to fully reconstruct the composite pulse-multipath waveform. Rather, by using a dictionary that is closely matched to the underlying transmitted pulse, we anticipate to find the most correlated atoms in the dictionary that, in turn, are related to the strongest propagation paths. This proposed approach is similar, in spirit, to the successive cancelation algorithm introduced in [27] for DS-CDMA channel estimation, where at each iteration the parameters of a given path are estimated and a delayed version of the transmitted signal corresponding to the estimated tap is subtracted from the received sequence and the algorithm is repeated.

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[2]Since we are not interested in signal reconstruction, the target residual energy is no longer used to end the MP algorithm.

III. ULTRAWIDEBAND DETECTION BASED ON COMPRESSIVE SENSING

Thus far, we have focused on CS reconstruction of noiseless UWB signals. Furthermore, a UWB channel estimation based on CS has been also proposed that relies on the assumption that the noiseless composite pulse-multipath waveform is sparse in a pre-designed dictionary. In a more realistic UWB communication scenario, however, the received signal is contaminated with noise and interferences, and the challenges fall in the design of a UWB receiver with the ultimate goal of signal detection.

The CS theory can be further extended to address the detection problem under the framework of data-aided channel estimation followed by symbol demodulation [2], [4], [16], [21], [22]. In this framework, a set of training symbols, also known as pilot signals, are used to estimate the channel parameters [4], [21] or to construct a referent template for subsequent correlation detection [2], [16], [22]. In this light of work, we use the CS framework for template reconstruction and channel parameter estimation leading naturally to new methods for signal detection.

A. UWB Signal Models

Consider a peer-to-peer UWB communication system where the kth binary information symbol is transmitted by sending N_f ultra-short pulses in the symbol interval T_s, that is [22]

$$s(t) = \sum_k b(k) \sum_{j=0}^{N_f - 1} p(t - jT_f - kT_s) \qquad (7)$$

where $T_f = T_s / N_f$ is the frame time, i.e., the time interval between two consecutive pulses, and $b(k) \in \{-1, 1\}$ is the binary information symbol that modulates the amplitude of the pulse stream. $p(t)$ is the pulse used to convey information with a pulse duration, T_p, much smaller than the frame time ($T_p \ll T_f$), hence N_f nonoverlapped pulses are transmitted for each information symbol.

Following [3], [4], [16], and [22], consider that the channel is static during a burst of N_s consecutive symbols, that is the channel parameters, τ_ℓ's and α_ℓ's in (4), remain invariant over several data symbols. Thus, $h(t)$ is assumed fixed during the burst of N_s symbols but it may vary during the subsequent symbol burst. Furthermore, let $T_f \geq \tau_{L-1} + T_p$ such that there is not interpulse interference between consecutive transmitted pulses, where τ_{L-1} is the maximum delay spread of the multipath channel. The received waveform during the first frame of the kth transmitted information symbol can be expressed as

$$r_f(t) = b(k) \cdot p(t - kT_s) * h(t) + \eta(t)$$
$$= b(k) \cdot \sum_{l=0}^{L-1} \alpha_l p(t - kT_s - \tau_l) + \eta(t) \qquad (8)$$

where $*$ denotes convolution and $\eta(t)$ is a zero-mean additive white Gaussian (AWG) noise that models thermal noise and other interferences like multi-user interference for a large number of users [4]. The received signal is thus made up of a sum of attenuated and delayed replicates of the transmitted pulse, $p(t)$. Furthermore, the ultra-short pulse of duration T_p is spread over the frame interval T_f as a result of the multipath. Note that the received UWB waveform in (8) is an amplitude

modulated version of the composite pulse-multipath channel corrupted with AWG noise.

Since $T_f \geq \tau_{L-1} + T_p$ and the UWB channel is invariant over several information symbol, the received signal during the kth information symbol can be represented by periodically repeating the noiseless part of $r_f(t)$ every T_f seconds, leading to the received signal

$$r(t) = \sum_{j=0}^{N_f - 1} r_f(t - jT_f) + \eta(t). \qquad (9)$$

The rich multipath diversity embedded in (9) by the UWB channel has to be exploited at the receiver in some optimal fashion. Two approaches have been commonly used to address the detection problem. They are the correlator-based detector [2], [16], [22] and the Rake receiver [3], [4], [21]. In the first approach, the received UWB signal per frame, $r_f(t)$, is correlated with a reference template to decode the transmitted information symbol in the corresponding frame. The optimal template for demodulation is the compose pulse-multipath channel, $p(t) * h(t) = \sum_{\ell=1}^{L} \alpha_\ell p(t - \tau_\ell)$. Thus, the receiver performs frame-rate sampling on the correlator output to generate sufficient statistics for the detection of the transmitted information symbol [2], [3], [16], [22]. The second approach commonly used for UWB detection is the RAKE receiver. In this case, a bank of L_r correlators exploit the multipath diversity capturing the energy in the most significant propagation paths [3]. The correlators' output are, then, combined via maximum ratio combining (MRC) [28] to obtain sufficient statistics for symbol detection.

In the UWB correlator-based detector, it is assumed that the channel impulse response is completely known at the receiver to define the reference template that is subsequently used in the demodulation stage. Likewise, for the RAKE-based receiver the channel taps $\{\alpha_{\ell_i}, \tau_{\ell_i}\}|_{i=1}^{L_r}$ related to the most significant propagation paths are assumed to be known *a priori* to define the set of templates for the bank of correlators and the weights for MRC [28]. In either case, the need for UWB channel estimation arises.

Impulse response estimation for UWB channels has been developed in [16], where a data-aided framework is used to estimate the optimal template in the analog domain. Their approach employs analog delay units that delay and average symbol-long segments of the received waveform, $r(t)$, during the training stage to yield an symbol-long estimate of the compose pulse-multipath channel. This estimate is subsequently used as the correlator template to decode the received information-conveying waveform at a symbol-rate sampling. Similarly, in [2], [22] the correlator is implemented in the analog domain

after a previous estimate of the frame-long template based on the received pilot waveforms. By sampling a frame rate, the correlator output provides the statistic for detection. Although these approaches avoid the path-by-path channel estimation and do not require high sampling rate, their implementations demand the use of analog delay units that consume high power. Moreover, for low bit rate, these analog delay elements are not available at the present time.

On the other hand, the estimation of UWB channel parameters, $\{\alpha_{\ell_i}, \tau_{\ell_i}\}|_{i=1}^{L_r}$, has been addressed in [4] and [21] following also a data-aided framework. In particular, in [4], the maximum likelihood (ML) approach is used to derive the optimal values for the path gains and path delays sampling the received signal at subpulse rate. To achieve a good performance, the ML channel estimator requires 12–25 samples per monocycle pulse, leading to a sampling rate in the order of tens of GHz. Such speeds mandate the use of a bank of polyphase ADCs with accurate timing control that tends to consume high power [5]. Furthermore, the computational complexity of the ML channel estimator increases as the number of significant multipath components increase and becomes prohibitively expensive for a realistic NLOS propagation scenario.

Next, we address the problem of UWB channel estimation using CS under the data-aided framework. The setting is as follows. We use N_p known pilots symbols in each packet to estimate the channel impulse response. All pilots symbols are transmitted at the beginning of the data packet[3]. Based on these pilots, the channel is estimated either by CS template reconstruction (Section II-B2) or CS channel tap estimation (Section II-C). The remaining $(N_s - N_p)$ symbols that convey information are decoded based on the acquired channel characteristics. Under this setting, the received UWB signal (9) can be conveniently rewritten as shown in (10), at the bottom of the next page, where $b_p(\cdot)$ and $b_i(\cdot)$ are the pilot and the information symbols, respectively, N_w is the total number of pilot waveforms given by $N_w = N_p N_f$, and T_w is the time duration of the pilot waveforms. $\lfloor x \rfloor$ denotes the greatest integer value smaller or equal to x. Note that for $0 < t \leq T_w$, (10) models training data, whereas for $t > T_w$, it models information-carrying signals. Furthermore, since N_f pulses are transmitted for each symbol (pilot or information), $N_p N_f$ waveforms received during T_w seconds are available for channel estimation.

Consider that the received UWB signal is observed over nonoverlapped time intervals $kT_f \leq t < (k+1)T_f$ for $k = 0, 1, \ldots, N_w - 1$. Assuming perfect timing synchronization, each time interval encloses a modulated composite

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[3]For clarity in the presentation, we assume that the pilot symbols are at the beginning of each packet, however, they can be located anywhere in the packet as in [22].

$$r(t) = \begin{cases} \sum_{k=0}^{N_w - 1} b_p\left(\left\lfloor \frac{k}{N_f} \right\rfloor\right) \sum_{l=1}^{L} \alpha_l p(t - kT_f - \tau_l) + \eta(t) & \text{for } 0 < t \leq T_w \\ \sum_{k=N_w}^{(N-N_p)N_f - 1} b_i\left(\left\lfloor \frac{k}{N_f} \right\rfloor\right) \sum_{l=1}^{L} \alpha_l p(t - kT_f - T_w - \tau_l) + \eta(t) & \text{for } T_w < t \leq N_s N_f T_f \end{cases}. \qquad (10)$$

pulse-multipath channel contaminated with AWG noise. Thus, the received pilot waveform in a frame time is

$$r_k(t) = b_p \left(\left\lfloor \frac{k}{N_f} \right\rfloor \right) \sum_{l=1}^{L} \alpha_l p(t - kT_f - \tau_l) + \eta(t). \quad (11)$$

With these settings, two channel-estimation approaches are derived next that lead naturally to two different demodulation schemes.

B. CS Correlator-Based Detector

A first approach exploits the CS reconstruction of the optimal template that is subsequently used by a correlator-based detector. As was mentioned above, the optimal template for demodulation is the composite pulse-multipath channel given by (3)[2], [16]. Furthermore, the received UWB signal is composed of shifted versions of that template, modulated by the symbols (pilot or information) and contaminated by AWG noise. Therefore, by observing the received UWB signal in a frame-long interval and random projecting the observed signal, a noisy template can be recovered using MP algorithm. Since N_w pilot waveforms are used for channel estimation, the estimate composite pulse-multipath channel is formed by averaging over N_w noisy templates. This approach is computationally demanding as a noisy template is recovered for each received pilot waveform. Alternatively, the random projected signals corresponding to the received pilot waveforms can be averaged and input to the MP algorithm for template reconstruction. This latter approach requires less computation since the MP algorithm is performed just once. Furthermore, by ensemble averaging the random projected signals, the effect of AWG noise is mitigated.

Thus, CS template reconstruction is achieved by random projecting the frame-long received signals, ensemble averaging the random projected signals, and using MP algorithm to recover an estimate of the composite pulse-multipath channel.

Note that in the reconstruction of the optimal template using the MP algorithm, a denoising operation is implicitly performed. To be more precise, by building a dictionary that is closely matched to the transmitted signal, we expect that the dictionary will initially be most correlated with the underlying transmitted pulse than to the noise. Furthermore, since the reconstructed template is a linear combination of the atoms in the dictionary, it does not contain any AWG noise components. However, other type of errors may appear in the reconstructed template coming from spurious atoms that may have been wrongly identified in the received pilot signal as a consequence of the AWG noise. In fact, the noise components in the projected signal may drive the MP algorithm to find erroneous atoms in the received pilot signal that are not part of the noiseless signal. The signal contribution of those "wrong" atoms add spurious components on the reconstructed template.

To overcome this limitation, the temporal correlation between consecutive received pilot waveforms can be exploited by increasing the observation window to enclose more than one received pilot waveforms leading, thus, to the projection of several received pilot signals at the same time[4]. The MP algorithm is

then suitably adapted such that the observed signal is compared to each elements in the dictionary that is periodically repeated several times to match the signal length. Thus, an atom is found in the observed signal if its signal contribution appears periodically each T_f seconds. Therefore, it is less likely that the noise components erroneously drive the MP algorithm to find spurious atoms in the received pilot signal, thus adding robustness to the signal reconstruction. Note that, indeed, the MP algorithm works on the projected signal using the holographic dictionary, $\mathbf{V} = \Phi\Psi$, however its effect can be better understood on the original signal and dictionary.

Once the template has been estimated, it can be used as correlator template to enable integrate-and-dump demodulation at frame-rate sampling. Since each symbol is present in N_f frames, the decision statistics for the kth symbol is formed by adding up the N_f correlator output samples related to the transmitted symbol. That is

$$z(k) = \sum_{j=0}^{N_f-1} \int_{jT_f+kT_s}^{(j+1)T_f+kT_s} r(t)g_{cs}(t - jT_f - kT_s)dt \quad (12)$$

where $g_{cs}(t)$ is the CS estimate of the composite pulse-multipath channel. In (12), the integral term implements the correlation operation between the received UWB signal and the estimate template. Note that the extension of this frame-rate sampling detector to a symbol-rate sampling detector is straightforward since the optimal symbol-long template is generated by periodically extending $g_{cs}(t)$ every T_f second, N_f times. This symbol-long template is then correlated with the received signal (10), and the correlator output is sampled at symbol-rate to derive the decision statistic for detection [16].

C. CS Rake Receiver

Rake-based detectors relies on the assumption that the UWB channel parameters, path delays and path gains, related to the most significant propagation paths are known at the receiver [4], [21]. This calls for an extension of the CS channel parameter estimation approach described above to address the UWB channel estimation problem in a noisy communication environment.

Consider the received pilot waveform given by (11) for $k = 1, 2, \ldots, N_w$, where α_ℓ and τ_ℓ are the UWB channel taps to be estimated. At first, one may use each received pilot waveform to estimate the channel parameters related to the strongest propagation paths following the approach described in Section II-C, thus yielding N_w sets of noisy channel parameters, $\{\hat{\alpha}_\ell, \hat{\tau}_\ell\}|_{\ell=1}^{L_c}$. Those sets are then averaged to form estimates of path gains and path delays.

However, the noise component of the received pilot waveforms may drive the MP algorithm to misplacing the strongest atoms in the received pilot signal and to a wrong estimate of the atoms' contribution to the signal. Consequently, this leads to errors in the estimation of the UWB channel since each atom found in the random projected signal represents the signal contribution of a propagation path which, in turn, is related to a couple of channel parameters (α_ℓ, τ_ℓ).

To reduce the effect of AWG noise on the estimation of the UWB channel parameters, the CS projected pilot signals are averaged to obtain a reduced-noise projected signal that, in turn,

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[4]In this case, the observation time interval reduces to $kT_f < t < (k+W+1)T_f$, where W is the number of consecutive received pilot waveform observed.

is used in the MP algorithm to estimate the channel parameters as described in Section II-C. Thus, CS channel estimation is performed using the ensemble average of the random projections leading to a reduced computational cost and minimizing the noise effect.

Having estimated the path delays and path gains related to the strongest propagation paths, the CS Rake receiver is as follows. Let $\{\hat{\alpha}_\ell, \hat{\tau}_\ell\}|_{\ell=1}^{L_c}$ be the channel parameters related to the L_c strongest paths obtained using CS channel estimation.

Much like in the traditional Rake receiver, in the CS Rake-based detector, the received signal, $r(t)$, is fed to a bank of L_c correlators with templates given by the atoms $p(t - \hat{\tau}_\ell)$ for $\ell = 1, 2, \ldots, L_c$. The outputs of these correlators contain the energy captured by the strongest paths and are combined via maximum ratio combining (MRC) [29] to obtain sufficient statistic for detecting the kth bit transmitted during the jth frame. That is

$$z_R(k,j) = \sum_{\ell=1}^{L_c} \hat{\alpha}_\ell \int_{kT_s + jT_f + \hat{\tau}_\ell}^{kT_s + jT_f + \hat{\tau}_\ell + T_p} r(t)$$
$$\times p(t - kT_s - jT_f - \hat{\tau}_\ell) dt. \quad (13)$$

Note that each correlator's output, defined by the integral term in (13), is weighted by the CS estimated path gains to form the MRC output. Note also that the energy of the received signal is captured by matching the received signal to L_c delayed versions of the transmitted pulse. Thus, by combining the information of the L_c strongest paths, the sufficient statistic is derived for symbol detection. Furthermore, this two-step approach, correlator followed by weighted combination, requires a frame-rate sampling per correlator.

Recalling that N_f pulses are used to transmit an information symbol, the decision statistic for symbol detection is thus formed by summing up the MRC outputs for N_f consecutive frames, leading to the estimate of the kth transmitted information symbol as

$$\hat{b}(k) = \operatorname{sgn}\left(\sum_{j=0}^{N_f-1} z_R(k,j)\right). \quad (14)$$

In presenting the CS Rake receiver, we have assumed that the number of fingers (branches in the bank of correlators) is equal to the number of strongest paths. In practice, the number of fingers is a design parameter and is often chosen as a tradeoff between complexity and performance. It is expected that a large number of correlators will capture most of the energy distributed on the various UWB propagation paths at expensive of more complexity [4], [29]. In our approach, however, the complexity in the channel estimation stage is driven by the complexity of MP algorithm. It can be shown that the complexity of MP is approximately $\mathcal{O}(CL_cT_0)$, where T_0 is the number of MP iterations and C is a constant that depends on the dictionary size [13]. Therefore, UWB channel estimation using CS has a complexity that increases linearly with the number of Rake's fingers. Furthermore, the number of MP iterations has to be set greater than the number of Rake's fingers, to allow the MP algorithm to find the L_c strongest path and possibly revisit those previously selected paths for further updated.

IV. SIMULATION RESULTS

In this section, extensive numerical results are presented showing the potential of CS for UWB signal detection. The performance of the proposed CS-based detectors are compared to that of a correlator detectors used in [16], [22]. All the UWB communication environments (indoor residential, indoor office, industrial and open outdoor environments) and propagation scenarios (line-of-sight and non-line-of-sight) proposed by the IEEE 802.154.4a in [15] are used as channel models for testing the various UWB receivers. We use the average bit error rate (BER) at the receiver as a function of signal-to-noise ratio (SNR) as a performance criterion. The experimental setup used in our simulation is as follows.

We select the first derivative of the Gaussian pulse as the transmitted pulse waveform, $p(t)$, that has been normalized to have unit energy and a pulse duration of 0.650 ns. Further, following [16] and [22], the transmitted parameters in (7) are set to $T_f = 100$ ns and $N_f = 25$. Furthermore, a 2-PAM modulation scheme is adopted in our simulations where the information bits, $b(k)$, are independent binary symbols with equal probability. The sampling frequency before the projection stage in all the simulation was 20 GHz, which is higher than the Nyquist rate. Thus, the continuous-time signals are simulated with a time resolution of 50 ps.

The UWB multipath channel has been simulated following the parameterized model proposed by the IEEE 802.15.4a working group [15]. The channel parameters for the various communication environments and propagation scenarios are set according to the recommended values obtained from measurement campaigns in [15]. For the sake of simplicity in our simulation, the frequency-dependency path loss is taken out from the model in [15], yielding thus UWB channels with real-valued impulse responses. Furthermore, as in [22] the negligible taps at the tail of the multipath impulse response are cut off to make the maximum delay spread of the multipath channel equal to 99.35 ns. The remaining channel taps are normalized such that the channel energy is set to one.

We evaluate the performance of the proposed approaches over a large number of channel realizations (200 random realizations for each channel model). Thus, for each channel realization, 10 000 symbols are transmitted, N_p of these symbols are used as pilot symbols to estimate the channel parameters (CS-Rake) or for template reconstruction (CS-Correlator). These estimated channels are subsequently used for the corresponding receivers' detector to demodulate the $10\,000 - N_p$ information symbols. The BER is determined by averaging the BER obtained on each channel realization.

Furthermore, the parameters for the matching pursuit algorithm are set as follows. The maximum number of MP iterations is set to 400 and the target residual energy is set to 0.01% of the energy of the projected signal, i.e., $\epsilon = 10^{-4}$. Further, since CS-Rake requires less iterations, the sparse vector Θ is examined after $1.5L_c$ iterations and is used to estimate the channel parameters as described above, where L_c is the number of Rake's fingers. Thus, the MP algorithm is run once for both CS-based detectors.
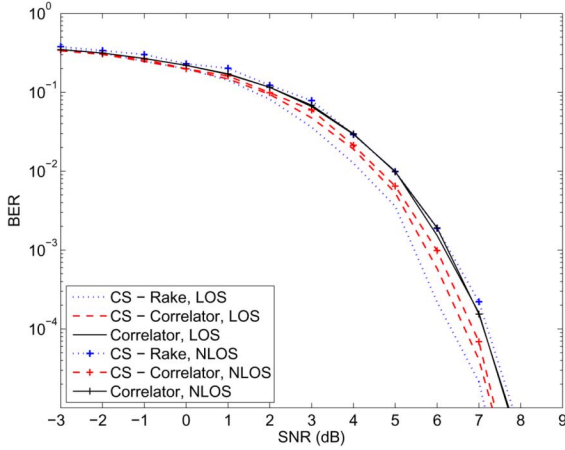
Fig. 4. Indoor residential BER performance for CS-Correlator, CS-Rake, and traditional correlator with $K/N = 0.36$..

Fig. 5. BER performance for different number of pilot symbols, with $K/N = 0.36$.

To compare the performance of the proposed UWB detectors based on compressive sensing, the correlator-based receiver used in [16], [22] is also implemented. In this case, the estimate of the composite pulse-multipath channel is formed at the receiver by averaging over the received pilot waveforms, that is $\tilde{g}(t) = \sum_{k=0}^{N_w-1} r_k(t)/N_w$ where $r_k(t)$ is given by (11). This estimate $\tilde{g}(t)$ is then used as a correlator template to demodulate the transmitted symbol [16], [22]. Note that this approach—template-estimation followed by correlator-based detector—is similar, in spirit, to the CS-Correlator. However, the proposed CS approach performs template reconstruction from the random projected signal sampled a significantly reduced rate, avoiding thus the use of analog delay units needed to implement the analog template-estimate approach.

Furthermore, all BER curves shown next are depicted as a function of signal-to-noise ratio defined as E_p/σ^2, where E_p is the transmitted pulse energy and σ^2 is the variance of the AWG noise. For short notation, CS-Rake denotes the CS Rake receiver, CS-Correlator denotes the CS-Correlator based detector and Correlator is the tradition correlator, i.e., analog-template estimation followed by correlator-based detector.

1) BER Performance for Different Propagation Scenarios: Fig. 4 depicts the BER performance of the proposed approach for different propagation scenarios (LOS and NLOS). In these simulations, the UWB communication channel is modeled as an indoor residential environment, the number of measurements is 36% of the samples in a frame-long interval, the number of fingers for CS-Rake is set to 50, two pilot symbols are used for channel estimation, and 200 channel realizations for each propagation scenario.

As can be seen from Fig. 4, the CS-Correlator outperforms the traditional correlator for all range of SNR tested for both propagation scenarios. This shows that the reconstructed template using CS framework, $g_{cs(t)}$, is more reliable for symbol detection than the one obtained by averaging the received pilot signal, $\tilde{g}(t)$. This performance is expected since a denoising operation is inherently applied on the recovered signal yielding a template that is a linear combination of the transmitted pulses. Furthermore, note that the performance of CS-correlator for LOS
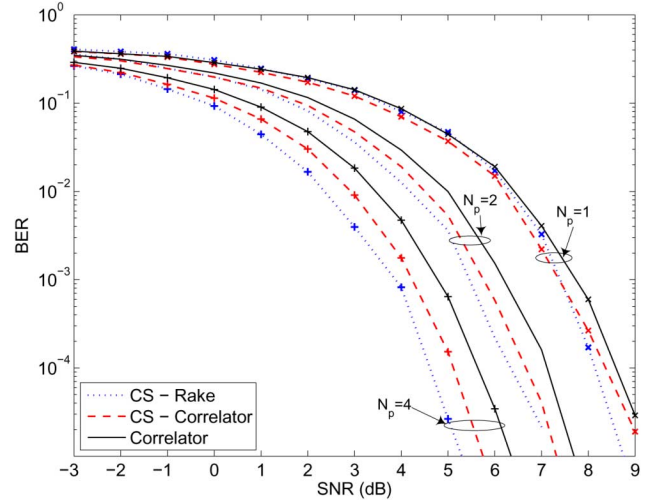
channel is better than that for NLOS channel. This is also expected since NLOS channel introduces more multipath components than LOS channel, yielding thus a received UWB signal with less sparsity. Therefore, for the same number of random projections, the CS-Correlator yields a reconstructed template that is much closer to the optimal one for LOS channel than for NLOS channel.

Note also that CS-Rake outperforms the correlator-based detectors for LOS channel and yields competitive performance to that yielded by the traditional correlator for NLOS channel. As can be seen, CS-Rake degrades its performance for dense multipath channel since the CS channel estimation is unable to resolve the strongest paths among the multiple closely spaced propagation paths. Furthermore, in a dense multipath UWB channel, the energy is distributed over a large number of propagation paths demanding more fingers to capture more energy at the receiver.

2) BER Performance for Different Number of Pilot Symbols: To evaluate the effect of the number of pilot symbols in the performance of the proposed approaches, the same experiment as described above is repeated for different number of pilot symbols. Fig. 5 shows the performance of the various receivers as the number of pilot waveform changes. Note that increasing the number of pilot waveforms, improvement in the channel estimation is achieved, leading to a performance gain on all the methods. For instance, for a BER of 10^{-3}, a performance gain of approximately 3.5, 3.13, and 2.95 dB are achieved, respectively, by the CS-Rake, CS-Correlator and tradition correlator as the number of pilot symbols increases by a factor of 4. Therefore, at the expense of a relatively low energy loss, the channel estimation for all the methods improves notably.

3) BER Performance for Different Number of Projections: Fig. 6 shows the BER performance of CS-Correlator for different number of random projections. For comparative purposes, the BER performance for the traditional correlator is also shown in Fig. 6. As expected, the CS-correlator's performance improves as the number of projections increases. More interestingly, by sampling the random projected signal at 30%
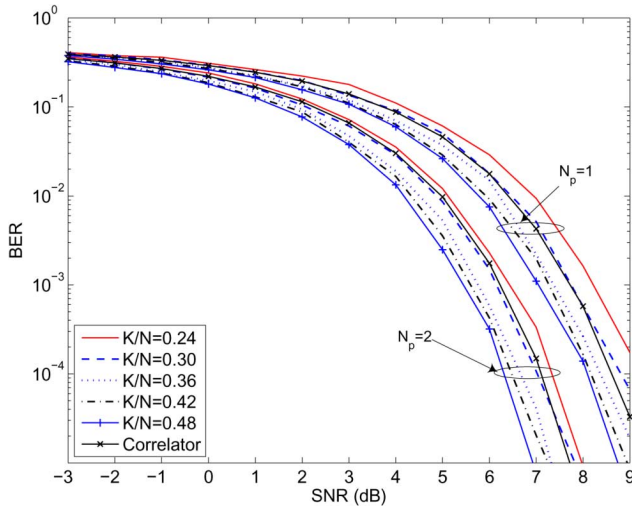
Fig. 6. BER performance for different number of projections.

of the signal's sampling rate, the CS-Correlator achieves the same performance as that yielded by the traditional correlator. Thus, with reduced ADC resources, the CS framework is able to reconstruct a template as good as the one obtained sampling the received UWB signal at a much higher sampling rate.

V. Conclusions

In this paper, we have introduced two novel ultra-wideband channel-estimation approaches based on the theory of compressive sensing. We have shown that a reduced number of random projections of the received UWB signal contains most of the relevant information useful not only for signal reconstruction but also for UWB channel parameter estimation. We also show that by CS reconstructing the composite pulse-multipath channel from the set of random projections, a denoising operation is implicitly applied yielding a performance improvement on a correlator-based detector that uses the reconstructed signal as a referent template. Extensive numerical results show that this approach outperforms the traditional detector using just 30% of the ADC resources.

Furthermore, by constructing a dictionary that closely matches the information-carrying pulse-shape, the signal contributions from the strongest paths of the UWB multipath channel can be recovered from the set of random projections of the received pilot signals, leading thus to an approach for UWB channel-parameter estimation with the advantage of using a reduced number of samples in the channel-estimation stage.

In this paper, the novel theory of Compressive Sensing has been used for UWB channel estimation, and we believe that the CS framework can be extended to a much broader range of statistical inference tasks, well suited for applications in wireless UWB communications. One of those applications is UWB symbol detection, which is an ongoing research topic that will be reported elsewhere.

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