Ultraslow bright and dark optical solitons in a cold three-state medium

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We show the formation of ultraslow bright and dark optical solitons in a lifetime-broadened three-state atomic system under Raman excitation. We also discuss why such ultraslow optical solitons may not exist under the conditions of the usual electromagnetically induced transparency configuration where zero one- and two-photon detunings are required. © 2004 Optical Society of America

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The term optical soliton[1–4] describes a class of fascinating shape-preserving propagation phenomena of optical fields in nonlinear media. The key to such shape-preserving propagation is the interplay between the nonlinear effects and the dispersion properties of the medium under optical excitation. Most optical soliton generation involves highly intense electromagnetic fields and solid media. The former are necessary to bring out the generally very weak nonlinear effects, whereas the latter provide rich dispersion properties usually seen in typical telecommunication-related devices such as optical fibers. In general, it is desirable to carry out the operation far from any strong transitions to avoid unmanageable optical field attenuation and distortion. As a consequence, optical solitons produced in this way generally travel with a speed very close to the speed of light in vacuum.

In the past few years the technique of electromagnetically induced transparency[5] (EIT) has received a great deal of attention mainly because it can significantly reduce the absorption of an optical field tuned to a strong one-photon resonance. One consequence of such an index-manipulation technique is the significant enhancement of certain nonlinear effects.[6–9] One may naturally ask if such an index-enhancement technique can also be used to facilitate the formation of an optical soliton in such a highly resonant medium. In this Letter we present a systematic study to address this question. We propose a Raman scheme[10,11] and show that this new scheme can provide all the necessary ingredients for generating both bright and dark solitons that traverse the medium with an ultraslow group velocity.[12,13] In addition, we discuss the difficulties of soliton formation by use of the conventional EIT technique. To the best of our knowledge, such an ultraslow optical soliton has never been studied.

We start by writing atomic equations of motion and the wave equation for the time-dependent probe field for the three-state system shown in Fig. 1 as follows:

$$\frac{\partial A_1}{\partial t} = i(\Delta_t + i\gamma_1)A_1 + i\Omega_C{}^* A_2\,, \quad \text{(1a)}$$

$$\frac{\partial A_2}{\partial t} = i(\Delta_s + i\gamma_2)A_2 + i\Omega_C A_1 + i\Omega_p A_0\,, \quad \text{(1b)}$$

$$\frac{\partial \Omega_p}{\partial z} + \frac{1}{c}\frac{\partial \Omega_p}{\partial t} = i\kappa_{02}A_2 A_0{}^*. \quad \text{(1c)}$$

Here, $2\Omega_{p,C}$ and $\omega_{p,C}$ are the Rabi and the optical frequencies of the probe and control fields, respectively, and γ_j is the decay rate of state $|j\rangle$ ($j = 1, 2$). $\kappa_{02} = 2N\omega_p|D_{02}|^2/(\hbar c)$, with N and D_{02} being the concentration and the dipole moment between states $|0\rangle$ and $|2\rangle$, respectively. A_0 is determined by the relation $|A_0|^2 + |A_1|^2 + |A_2|^2 = 1$.

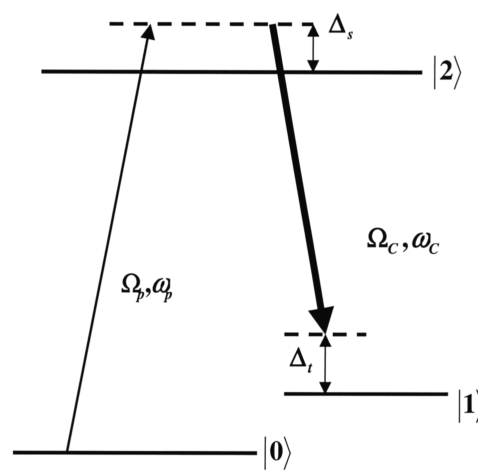

Fig. 1. Lifetime-broadened three-state atomic system interacts with a cw strong control laser (frequency ω_C and Rabi frequency $2\Omega_C$) and a pulsed probe field (frequency ω_p and Rabi frequency $2\Omega_p$).

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In deriving Eqs. (1), we assume a cw control field, apply a slowly varying envelope approximation, and define one- and two-photon detunings as $\Delta_s = \omega_p - \epsilon_2/\hbar$ and $\Delta_t = \omega_p - \omega_C - \epsilon_1/\hbar$, respectively, with ϵ_j being the energy of state $|j\rangle$ ($\epsilon_0 = 0$).

To provide a clear picture of the interplay between the dispersion and nonlinear effects of the atomic system interacting with two optical fields, we first investigate the dispersion properties of the system. This requires a perturbation treatment of the system response to the first order of weak probe field Ω_p while keeping all orders due to control field Ω_C. Below, we demonstrate effects that are due to higher-order Ω_p that are required for balancing the dispersion effect so that the formation of ultraslow solitons can occur.

Let us assume that $A_j = \sum_k A_j^{(k)}$, where $A_j^{(k)}$ is the kth-order part of A_j in terms of Ω_p. Within an adiabatic framework it can be shown that $A_j^{(0)} = \delta_{j0}$ and $A_0^{(1)} = 0$. Taking the time Fourier transform of Eqs. (1) and keeping up to the first order of Ω_p, we have

$$(\omega + \Delta_t + i\gamma_1)\beta_1^{(1)} + \Omega_C{}^*\beta_2^{(1)} = 0\,, \tag{2a}$$

$$(\omega + \Delta_s + i\gamma_2)\beta_2^{(1)} + \Omega_C\beta_1^{(1)} = -\Lambda_p\,, \tag{2b}$$

$$\frac{\partial \Lambda_p}{\partial z} - i\,\frac{\omega}{c}\,\Lambda_p = i\kappa_{02}\beta_2^{(1)}, \tag{2c}$$

where $\beta_j^{(1)}$ and Λ_p are the Fourier transforms of $A_j^{(1)}$ and Ω_p, respectively, and ω is the Fourier-transform variable.

Equation (2) can be solved analytically, yielding

$$\Lambda_p(z,\omega) = \Lambda_p(0,\omega)\exp(iKz)\,, \tag{3}$$

where

$$\begin{aligned} K &= \frac{\omega}{c} + \frac{\kappa_{02}(\omega + \Delta_t + i\gamma_1)}{|\Omega_C|^2 - (\omega + \Delta_t + i\gamma_1)(\omega + \Delta_s + i\gamma_2)} \\ &= K_0 + \frac{\omega}{V_g} + K_2\omega^2 + \cdots\,, \end{aligned} \tag{4}$$

The physical interpretation of Eq. (4) is rather clear. $K_0 = \phi + i\alpha/2$ describes the phase shift ϕ per unit length and absorption coefficient α of the probe field, $K_1 = 1/V_g$ gives the propagation velocity, and K_2 represents the group-velocity dispersion that contributes to the probe pulse's shape change and additional loss of probe field intensity.

With the dispersion coefficients obtained, we now investigate the nonlinear evolution of the probe field. We first show that with the Raman scheme it is indeed possible to obtain a set of rather clean and experimentally achievable parameters that will lead to the formation of ultraslow solitons. Below we discuss fundamental difficulties associated with the EIT method when it is applied for soliton generation.

Following Ref. 1, we take a trial function $\Omega_p(z,t) = \Omega_P(z,t)\exp(iK_0 z)$ and substitute it into Eq. (1) to obtain the nonlinear wave equation of the slowly varying envelope $\Omega_P(z,t)$:

$$-i\left(\frac{\partial}{\partial z} + \frac{1}{V_g}\frac{\partial}{\partial t}\right)\Omega_P + K_2\frac{\partial^2}{\partial t^2}\Omega_P = \mathrm{NLT}\,, \tag{5}$$

where nonlinear term NLT is given by $\mathrm{NLT}\exp(iK_0 z) = \kappa_{02}\{A_2^{(3)}[A_0^{(0)}]^* + A_2^{(1)}[A_0^{(2)}]^*\} = -\kappa_{02}A_2^{(1)}[|A_1^{(1)}|^2 + |A_2^{(1)}|^2]$. Using

$$A_j^{(1)} = \frac{(\Delta_t + i\gamma_1)\delta_{j2} - \Omega_C{}^*\delta_{j1}}{|\Omega_C|^2 - (\Delta_t + i\gamma_1)(\Delta_s + i\gamma_2)}\,\Omega_p\,, \quad j = 1, 2\,, \tag{6}$$

we then obtain the nonlinear evolution equation for Ω_P (we define $\xi = z$ and $\eta = t - z/V_g$):

$$i\,\frac{\partial}{\partial \xi}\,\Omega_P - K_2\,\frac{\partial^2}{\partial \eta^2}\,\Omega_P = W\exp(-\alpha\xi)\,|\Omega_P|^2\Omega_P\,, \tag{7}$$

where absorption coefficient $\alpha = 2\,\mathrm{Im}(K_0)$ and

$$W = \frac{\kappa_{02}(\Delta_t + i\gamma_1)(|\Omega_C|^2 + \Delta_t{}^2 + \gamma_1{}^2)}{D|D|^2}\,, \tag{8}$$

with $D = |\Omega_C|^2 - (\Delta_t + i\gamma_1)(\Delta_s + i\gamma_2)$.

Inspection of Eq. (8) shows that, if a reasonable and realistic set of parameters can be found so that $\exp(-\alpha L) \simeq 1$, $K_2 = K_{2r} + iK_{2i} \simeq K_{2r}$, and $W = W_r + iW_i \simeq W_r$, then Eq. (7) can be reduced to the standard nonlinear Schrödinger equation:

$$i\,\frac{\partial}{\partial \xi}\,\Omega_P - K_{2r}\,\frac{\partial^2}{\partial \eta^2}\,\Omega_P = W_r|\Omega_P|^2\Omega_P\,. \tag{9}$$

which admits of solutions describing bright and dark solitons,[1–4] including N solitons ($N = 1, 2, 3, \ldots$) for bright solitons,[4] depending on the sign of product $K_{2r}W_r$. The fundamental bright soliton is given by

$$\Omega_P = \Omega_{P0}\,\mathrm{sech}(\eta/\tau)\exp(-i\xi W_r|\Omega_{P0}|^2/2)\,, \tag{10}$$

where $\mathrm{sech}(\eta/\tau)$ is the hyperbolic secant function. Amplitude Ω_{P0} and width τ are arbitrary constants, subject only to the constraint $|\Omega_{P0}\tau|^2 = 2K_{2r}/W_r$. We note that the assumption of $|\Omega_{P0}\tau|^2 \ll |\Omega_C\tau|^2$ has been used to derive Eqs. (7) and (9). Therefore width τ should be chosen to satisfy $2K_{2r}/W_r \ll |\Omega_C\tau|^2$.

We now explain why solitons with ultraslow group velocities cannot be formed under the conditions of normal three-state EIT operation.[14] Notice that ultraslow propagation requires weak driving conditions. This leads to very narrow transparency windows. It is for this reason that the normal EIT configuration with weak driving conditions requires one- and two-photon resonance excitations (i.e., $\Delta_s = \Delta_t = 0$). Deviations from these conditions will result in significant probe field attenuation and distortion. From Eq. (8), however, it is clear that nonlinear coefficient W is purely imaginary under these EIT conditions. This is contradictory to the requirement that W be predominately real to preserve the complete integrability[1] of nonlinear Schrödinger equation (9).

With $|\Delta_t| \gg \gamma_1$, however, it can be seen clearly from Eq. (8) that both bright and dark solitons with

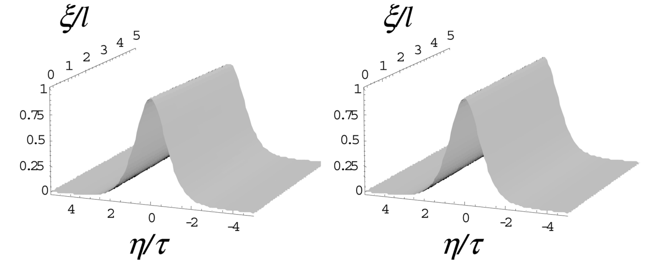

Fig. 2. Surface plots of the relative probe intensity versus dimensionless time η/τ and distance ξ/l. Left, $|\Omega_P/\Omega_{P0}|^2\exp(-\alpha\xi)$, where $|\Omega_P|^2$ is the numerical solution to Eq. (7); right, single bright soliton of Eq. (9) as given by $|\Omega_P/\Omega_{P0}|^2 = \mathrm{sech}^2(\eta/\tau)$ in Eq. (10). $l = 1$ cm, $\tau = 1.0 \times 10^{-6}$ s, and the other parameters are explained in the text.

ultraslow group velocities can be formed. In addition, a larger one-photon detuning Δ_s results in an asymmetric shift of the resonance and can further reduce the loss under weak driving conditions.[10,11] As we demonstrate below, these two features lead to a Raman scheme that is drastically different from the conventional EIT scheme.

We now present numerical examples to demonstrate the existence of bright and dark solitons. We consider a system in which the decay rates are $\Gamma_1 = 2\gamma_1 \simeq 2.0 \times 10^4\ \mathrm{s}^{-1}$ and $\Gamma_2 = 2\gamma_2 \simeq 1.2 \times 10^8\ \mathrm{s}^{-1}$, typical for transitions in hyperfine-split Na D lines.[7]

We first consider the case of dark solitons. Take $\kappa_{02} = 1.0 \times 10^9\ \mathrm{cm}^{-1}\,\mathrm{s}^{-1}$, $2\Omega_C = 2.0 \times 10^8\ \mathrm{s}^{-1}$, $\Delta_s \simeq -1.0 \times 10^9\ \mathrm{s}^{-1}$, and $\Delta_t \simeq 2.0 \times 10^6\ \mathrm{s}^{-1}$, and we have $V_g/c \simeq 4.8 \times 10^{-4}$, $K_2 \simeq -(5.77 - 0.189i) \times 10^{-15}\ \mathrm{s}^2\,\mathrm{cm}^{-1}$, $W \simeq (1.16 + 0.0164i) \times 10^{-17}\ \mathrm{s}^2\,\mathrm{cm}^{-1}$, and $\alpha \simeq 0.00041\ \mathrm{cm}^{-1}$. With these parameters, standard nonlinear Schrödinger equation (9) with $K_{2r}W_r < 0$ is well characterized, and hence we have demonstrated the existence of dark solitons that travel with ultraslow group velocities in a cold atomic medium.

For bright solitons we take $\Delta_s \simeq 1.0 \times 10^9\ \mathrm{s}^{-1}$ with all other parameters given above unchanged. In this case we obtain $V_g/c \simeq 2.1 \times 10^{-4}$, $K_2 \simeq (1.95 + 0.212i) \times 10^{-14}\ \mathrm{s}^2\,\mathrm{cm}^{-1}$, $W \simeq (3.90 + 0.083i) \times 10^{-17}\ \mathrm{s}^2\,\mathrm{cm}^{-1}$, and $\alpha \simeq 0.0106\ \mathrm{cm}^{-1}$. As can be seen from Fig. 2, these parameters and results again show that standard nonlinear Schrödinger equation (9) with $K_{2r}W_r > 0$ is well characterized and that formation of bright solitons occurs. Indeed, Fig. 2 shows excellent agreement between Eqs. (7) and (9).

It is worth noting that the above-described parameter sets also lead to negligible loss of the probe field for both the bright and the dark solitons described, as can be seen from Fig. 2. This figure shows negligible probe field attenuation for a propagation distance of $\xi = 5$ cm. This is a remarkable propagation effect in such a highly resonant system.

We further note that the fundamental soliton given in Eq. (10) has a width and amplitude that satisfy $|\Omega_{P0}\tau| = \sqrt{2K_{2r}/W_r} \simeq 31.6$. We thus have established the existence of bright N solitons[4] with ultraslow group velocity ($V_g/c \sim 10^{-4}$).

Finally we note that choosing different control field Rabi frequency and detunings may further decrease the group velocities of bright and dark solitons. For instance, taking $2\Omega_C = 6.0 \times 10^7\ \mathrm{s}^{-1}$, $\Delta_s \simeq 5.0 \times 10^8\ \mathrm{s}^{-1}$, $\Delta_t \simeq 2.0 \times 10^5\ \mathrm{s}^{-1}$ and with κ_{02} unchanged, we obtain $K_2 \simeq (8.7 + 1.6i) \times 10^{-13}\ \mathrm{s}^2\,\mathrm{cm}^{-1}$, $W \simeq (3.5 + 0.25i) \times 10^{-16}\ \mathrm{s}^2\,\mathrm{cm}^{-1}$, and $\alpha \simeq 0.0356\ \mathrm{cm}^{-1}$. In this case the fundamental bright soliton with its width and amplitude satisfying $|\Omega_{P0}|\tau \simeq 70$ has group velocity $V_g/c \simeq 2.4 \times 10^{-5}$.

The ultraslow optical solitons discussed in the present work may lead to important applications such as high-fidelity optical delay lines and optical buffers. The Raman scheme described represents a drastic departure from the conventional EIT scheme and may lead to other new phenomena that manifest themselves under well-controlled balance of dispersion and nonlinear effects. These include, but are not limited to, simultaneous formation of multiple solitons and soliton–soliton interactions in the ultraslow propagation regime.

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