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Ultrasonic compressional and shear velocities in dry clastic rocks as a function of porosity, clay content, and confining pressure

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SUMMARY

For clastic silicate rocks sampled from a Rotliegendes well core the velocities v_P and v_S were obtained at 10 pressures up to 300 MPa using a pulse-transmission technique. The porosities of all rocks (57 sandstones, 26 siltstones, five claystones) ranged from 0.01 and 0.15 by volume fraction, and the clay content varied from less than 0.01 to 0.88 by volume fraction. Both velocities increase with pressure. In the low-pressure range the rate of increase is large, non-linear and is greater for v_P than for v_S. Above 120 MPa both velocities increase linearly. Velocities, porosity, and clay content were fitted by least-squares regression for pressures of 8, 24, 60, 120, 200, and 300 MPa. The fractional effect of porosity and clay on the velocities for dry clastic silicate rocks can be described (above 120 MPa) by $v = A - B\phi - CClay$, where v is the velocity of the P-wave or S-wave, ϕ is the volume fraction of pores, and *Clay* is the volume fraction of clay. From this it is possible to obtain pressure-dependent velocity functions $v = a + pb - c\exp(-dp)$, where a is the crack-free velocity, linear in porosity and clay, b is the velocity slope under high pressure, $a - c$ is the zero-pressure velocity, and d is related to closure of cracks.

Key words: clastic sedimentary rocks, clay content, hydrostatic pressure, porosity, ultrasonic velocities v_P and v_S.

1 INTRODUCTION

Data of ultrasonic velocities of compressional waves, v_P, and shear waves, v_S, from a representative collection of clay-bearing clastic rocks have been given by Han, Nur & Morgan (1986). Their measurements were carried out under differential pressures up to 50 MPa on water-saturated samples. The data were compared with different velocity relations, based on theoretical models, laboratory and well-logging data. Consequently, both v_P and v_S have been found to vary linearly with porosity and clay content,

$$v = A - B\phi - CClay, \qquad (1)$$

where v is the velocity of the P-wave or S-wave, ϕ is the volume fraction of pores, and *Clay* is the volume fraction of clay.

Theoretical considerations and laboratory data indicate that P-waves in porous rocks are sensitive to the type of pore fluid, while S-waves are almost unaffected by different pore fluids. The difference between the P-wave velocities of a rock containing liquid and a rock containing gas is mainly caused by the difference in the bulk modulus. Because gases are more compressible than liquids, the v_P value of rock containing gas is smaller than v_P of the same rock saturated with water. The small difference between the S-wave velocities in the same rock, filled with gas or with water, is produced by the different bulk densities. In addition, the velocities of water-saturated sedimentary rocks are influenced by water/clay interaction.

Obviously, there is a lack of investigations of gas-saturated sedimentary rocks. Velocities, obtained from experiments on dry rocks, can serve as the most simple simulation of the velocities in the pure gas-filled state.

Under hydrostatic pressure up to 100 MPa the increase of the velocities of elastic waves is more pronounced for dry sedimentary rocks than for water-saturated samples. This behaviour is similar to that of crystalline rocks and is caused by the closure of cracks and the change of the pore shape distribution without drastic diminution of the bulk porosity. Therefore, strong correlations between the ultrasonic compressional and shear velocities and the mineral content and porosity are not expected in this low-pressure range. What is known for certain about the influence of the clay content on the elastic behaviour of clastic silicate rocks is that an increase of the clay content reduces the bulk rigidity and stiffness and, consequently, the velocities, particularly the shear velocity (Toksöz, Cheng & Timur 1976; Tosaya 1982; Castagna, Batzle & Eastwood 1985; Han *et al.* 1986).

2 SAMPLE DESCRIPTION

The 88 samples investigated in this study are from the well SALZWEDEL 2/64 (in the northern part of Sachsen-Anhalt, Germany) at depths of −3340 to −3670 m and belong to the formation of Rotliegendes. The sample collection consists of strongly consolidated clastic sedimentary rocks: five claystones, 26 siltstones, and 57 sandstones.

Their porosities range from 0.5 to 15 per cent by volume and were determined by standard techniques. The clay content (illite, chlorite) was obtained by thin section point-counting and X-ray analysis and varied from less than 1 to 88 per cent by volume.

The parameter median *Md* characterizes the distribution of the grain diameter. It was obtained by granulometric analysis using standard sedimentological methods. Table 1

Table 1. Porosity, mineral content, grain size parameter median and density for all samples.

No	porosity vol. %	clasts vol. %	clay vol. %	carbonate vol. %	anhydrite vol. %	median (mm)	density (g/cm^3)
8	3.7	55.1	25.7	12.5	3.0	0.070	2.62
10	5.1	48.8	41.4	4.7	0.0	0.060	2.59
17	1.5	38.4	58.0	0.3	1.8		2.68
21	3.6	48.6	39.2	1.9	6.7	0.070	2.66
22	3.3	54.3	37.3	0.5	4.6	0.072	2.63
24	3.3	65.4	11.1	13.5	6.7	0.100	2.65
25	1.8	60.2	21.6	13.4	3.0	0.060	2.66
27	1.8	38.2	58.0	0.2	1.8		2.68
34	4.3	33.1	61.1	1.5	0.0		2.65
38	2.6	72.9	16.3	5.0	3.2	0.073	2.62
41	3.6	40.3	53.2	2.9	0.0		2.67
43	1.8	45.4	51.1	0.0	1.7		2.68
44	1.7	50.3	41.2	6.8	0.0	0.025	2.72
48	2.2	76.9	6.1	4.8	10.0	0.048	2.65
51	3.7	46.5	45.0	4.8	0.0	0.040	2.63
52	5.0	56.1	26.6	10.4	1.9	0.040	2.60
53	4.3	49.2	40.8	5.7	0.0	0.020	2.63
55	4.4	49.9	39.0	4.8	1.9	0.050	2.63
56	5.0	73.2	7.5	7.8	6.5	0.102	2.64
58	2.2	49.3	41.7	6.8	0.0	0.060	2.67
60	5.0	27.3	64.9	2.8	0.0		2.64
62	4.8	47.9	41.6	3.8	1.9	0.038	2.60
65	4.4	39.2	44.0	3.8	8.6	0.065	2.62
73	3.6	26.9	63.7	5.8	0.0		2.66
74	3.6	42.7	48.9	4.8	0.0	0.035	2.64
78	2.6	37.4	51.2	4.9	3.9		2.67
81	3.3	46.7	42.3	2.9	4.8	0.040	2.66
90	6.6	46.4	41.4	1.9	3.7	0.040	2.56
91	6.9	56.0	16.4	0.8	19.9	0.111	2.56
95	1.1	66.5	17.8	5.7	8.9	0.090	2.68
102	3.7	71.8	11.0	5.8	7.7	0.114	2.61
107	2.6	64.3	18.0	2.3	12.8	0.097	2.66
108	2.6	69.2	6.4	1.8	20.0	0.153	2.64
116	3.7	56.7	20.5	0.8	18.3	0.102	2.64
120	3.6	76.4	0.1	1.2	18.7	0.145	2.66
123	2.2	58.7	24.4	13.7	1.0	0.085	2.68
126	1.1	11.0	87.1	0.0	0.8		2.78
131	4.1	80.4	1.0	0.1	14.4	0.130	2.57
134	9.0	85.6	0.9	3.9	0.6	0.173	2.44
135	11.6	81.0	3.3	1.7	2.4	0.165	2.37
139	4.7	76.9	4.5	0.8	13.1	0.175	2.61
140	14.3	75.5	1.9	0.1	8.2	0.180	2.34
146	8.6	87.1	1.9	1.2	1.2	0.200	2.46
159	4.4	52.1	37.9	4.8	0.8	0.040	2.61
165	4.3	46.4	39.5	7.7	2.1	0.028	2.62
172	1.0	14.0	81.1	3.9	0.0		2.77
180	2.9	56.3	37.8	1.8	1.2	0.090	2.64
191	4.7	73.4	0.2	0.2	21.5	0.167	2.56
196	3.2	18.6	74.5	2.9	0.8		2.72
202	2.6	68.1	26.6	0.5	2.2	0.120	2.63
205	2.6	51.2	39.4	6.8	0.0	0.034	2.67
206	4.3	15.0	77.1	1.5	2.1		2.68
213	7.8	72.7	17.2	0.0	2.3	0.132	2.48
216	6.7	91.6	0.6	1.1	0.0	0.221	2.51
218	6.7	91.3	1.6	0.0	0.4	0.145	2.51
219	7.8	81.7	9.0	0.4	1.1	0.127	2.48
221	7.3	88.1	2.5	1.2	0.9	0.144	2.53
222	6.3	89.2	1.7	1.4	1.4	0.145	2.47
223	11.2	86.5	1.2	0.6	0.5	0.150	2.42
230	3.6	68.0	5.9	3.0	19.5	0.154	2.69
235	2.2	61.5	3.9	25.3	7.1	0.126	2.68
240	6.9	69.7	4.8	5.5	13.1	0.097	2.50
246	3.6	48.0	47.2	0.0	1.2	0.034	2.65
253	2.6	95.7	1.4	0.1	0.2	0.184	2.59
254	3.0	90.5	4.4	0.2	1.9	0.142	2.61
256	8.9	81.9	6.6	0.5	2.1	0.154	2.47
258	10.7	82.2	3.8	0.3	3.0	0.210	2.44
261	3.8	93.0	2.6	0.2	0.4	0.190	2.56
265	7.3	87.8	4.2	0.1	0.6	0.195	2.46
266	6.4	90.3	1.3	1.0	1.0	0.180	2.50
267	10.6	81.3	5.0	0.4	2.7	0.160	2.35
272	3.5	88.2	2.8	0.7	4.8	0.145	2.54
281	4.8	84.2	5.2	1.6	4.2	0.130	2.56
287	12.3	80.6	4.9	0.5	1.7	0.192	2.35
288	11.2	84.8	2.0	0.8	1.7	0.205	2.37
294	14.9	80.4	1.7	1.0	2.0	0.200	2.28
296	11.0	84.2	1.8	1.1	1.9	0.200	2.35
299	7.5	84.2	4.3	0.5	3.5	0.175	2.48
302	9.7	83.6	4.0	0.7	2.0	0.130	2.41
303	7.9	87.1	3.8	0.3	0.9	0.210	2.45
304	9.1	82.1	5.0	1.0	2.8	0.130	2.41
305	13.0	81.5	3.3	0.1	2.1	0.165	2.35
306	9.2	81.8	5.4	0.5	3.1	0.210	2.43
307	12.7	83.0	3.3	0.4	0.6	0.171	2.33
308	11.4	82.0	3.7	0.1	3.5	0.176	2.36
309	8.6	82.5	7.4	0.3	1.2	0.173	2.45
314	1.4	74.2	3.1	2.2	19.1	0.175	2.69
320	4.1	90.0	4.2	0.0	1.7	0.148	2.57

shows the porosities, densities, grain sizes (*Md*) and mineral constituents. Clasts are differently rounded grains of quartz, feldspars and lithic fragments with diameter >0.02 mm. The average fractions of secondary cemented anhydrite and carbonate amount of 4.5 and 3.0 per cent by volume, respectively. For detailed lithologic and mineralogic description see Freund (1989).

3 EXPERIMENTAL PROCEDURES

After two weeks of careful drying (60 °C) the cylindrical (27 × 30 mm) samples with parallel (±0.1 mm) end faces were encapsulated. The velocities of compressional and shear waves were measured perpendicular to bedding by a pulse-transmission technique. Ceramic transducers with frequencies of 1 MHz were used. The errors in the measurements of the velocities were less than 2 per cent for v_P and 3 per cent for v_S. The measurements were performed in a hydrostatic pressure vessel at the pressure points marked in Fig. 1 and given in Table 2.

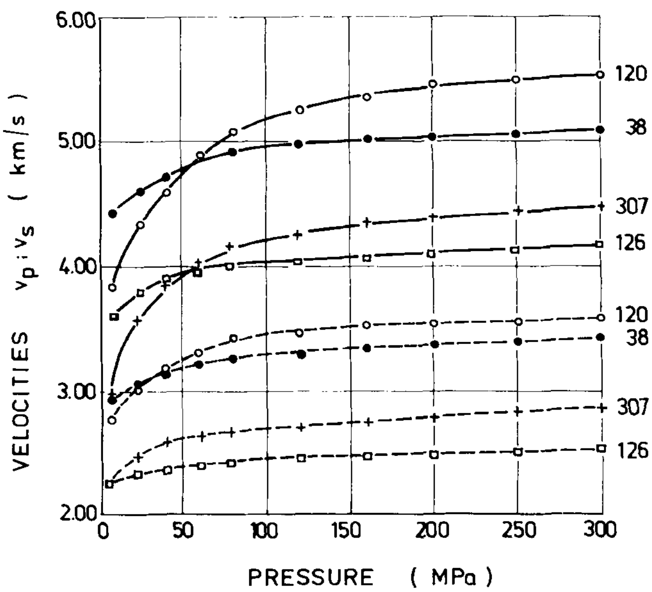

Figure 1. Experimental data of the velocities v_P (full lines) and v_S (dashed lines) versus pressure (samples 38, 120, 126, and 307).

4 RESULTS

Figure 1 compares some examples of representative velocity–pressure functions for three sandstones (Nos 38, 120, 307) and a claystone (No. 126) up to 300 MPa (the velocity data for all samples at increasing pressure are given in Table 2). The velocities v_P (full lines) as well as v_S (dashed lines) show rapid increases at low pressures. The velocity slopes are greater for v_P than for v_S; this is a behaviour typical for dry or gas-saturated sedimentary rocks (Toksöz *et al.* 1976; Johnston & Toksöz 1980; Jones & Wang 1981) and also for crystalline rocks (Gebrande 1982). In the higher pressure range both velocities increase linearly with small gradients. Here the velocities show large scatter, depending on porosity and significant differences in the mineralogic composition.

4.1 The effect of porosity on velocities

The classical time-average equation of Wyllie, Gregory & Gardner (1956) have been used to evaluate porosities for water-saturated rocks from *P*-wave velocities v_P provided from acoustic-log data. This relation is

$$1/v_P = 1/v_m + (1/v_f - 1/v_m)\phi, \tag{2}$$

where v_m is the matrix velocity and v_f the velocity of the pore fluid. This simple equation appears to be valid for porosities from 10 up to 25 per cent. The velocity variations caused by mineral composition and pore geometry are not taken into account. Relation (2) is not acceptable for gas-saturated samples. Pickett (1963) gives another velocity–porosity relation.

$$1/v = A + B\phi, \tag{3}$$

which can be used for different lithologies as well as for the velocities of *S*-waves. When $A = 1/v_m$ and $B = 1/v_f - 1/v_m$ this relation is identical to (2). From acoustic well logging data Raymer, Hunt & Gardner (1980) obtained

$$v_P = (1 - \phi)^2 v_m + \phi v_f, \tag{4}$$

which can also be used for gas-saturated sedimentary rocks.

For obtaining relations between both velocities and porosity and clay content, respectively, the data of all

Table 2. Velocity data v_P and v_S at various pressures.

	8 MPa		24 MPa		40 MPa		60 MPa		80 MPa		120 MPa		160 MPa		200 MPa		250 MPa		300 MPa	
No	Vp	Vs	Vp	Vs	Vp	Vs	Vp	Vs	Vp	Vs	Vp	Vs	Vp	Vs	Vp	Vs	Vp	Vs	Vp	Vs
	(km/s)		(km/s)		(km/s)		(km/s)		(km/s)		(km/s)		(km/s)		(km/s)		(km/s)		(km/s)	
8	4.48	3.06	4.70	3.14	4.81	3.19	4.88	3.22	4.92	3.26	4.98	3.28	5.00	3.29	5.01	3.30	5.04	3.31	5.06	3.32
10	4.11	2.87	4.36	2.94	4.40	2.99	4.50	3.03	4.57	3.07	4.60	3.08	4.61	3.10	4.62	3.12	4.65	3.13	4.67	3.13
17	4.20	2.48	4.38	2.55	4.53	2.59	4.65	2.63	4.72	2.66	4.75	2.70	4.79	2.72	4.81	2.73	4.84	2.75	4.86	2.77
21	4.03	2.75	4.18	2.85	4.30	2.88	4.37	2.91	4.41	2.94	4.54	2.96	4.57	2.98	4.60	2.99	4.64	3.01	4.69	3.02
22	4.27	2.94	4.80	3.07	4.96	3.13	5.04	3.18	5.09	3.20	5.16	3.23	5.17	3.24	5.19	3.25	5.21	3.27	5.22	3.28
24	4.38	3.03	4.56	3.11	4.76	3.19	4.95	3.25	5.06	3.30	5.14	3.35	5.20	3.39	5.22	3.42	5.26	3.44	5.30	3.46
25	4.40	3.06	4.64	3.16	4.80	3.26	4.88	3.30	4.93	3.33	5.00	3.36	5.02	3.37	5.04	3.38	5.05	3.39	5.06	3.40
27	4.29	2.93	4.48	2.96	4.53	2.99	4.58	3.01	4.63	3.04	4.73	3.07	4.75	3.08	4.76	3.09	4.79	3.11	4.82	3.12
34	3.58	2.38	3.80	2.46	3.98	2.51	4.07	2.53	4.13	2.56	4.20	2.60	4.24	2.62	4.28	2.65	4.32	2.66	4.36	2.70
38	4.44	2.97	4.60	3.06	4.72	3.15	4.85	3.23	4.93	3.26	4.99	3.30	5.02	3.34	5.06	3.37	5.08	3.37	5.10	3.38
41	4.20	2.80	4.27	2.83	4.29	2.85	4.33	2.87	4.36	2.89	4.45	2.92	4.48	2.94	4.50	2.96	4.52	2.99	4.54	3.01
43	4.50	2.82	4.66	2.89	4.75	2.94	4.84	2.97	4.89	3.00	4.94	3.04	4.95	3.05	4.96	3.06	4.97	3.07	4.98	3.09
44	4.36	2.96	4.57	3.01	4.63	3.08	4.66	3.10	4.68	3.12	4.71	3.14	4.73	3.16	4.74	3.18	4.76	3.19	4.78	3.20
48	4.56	3.16	4.65	3.22	4.78	3.30	4.87	3.40	5.00	3.44	5.08	3.48	5.12	3.50	5.16	3.52	5.18	3.53	5.20	3.55
51	4.26	2.89	4.36	2.96	4.44	3.01	4.53	3.05	4.60	3.08	4.63	3.12	4.66	3.13	4.68	3.14	4.71	3.15	4.73	3.16

Table 2. (*continued*)

No	8 MPa Vp	8 MPa Vs	24 MPa Vp	24 MPa Vs	40 MPa Vp	40 MPa Vs	60 MPa Vp	60 MPa Vs	80 MPa Vp	80 MPa Vs	120 MPa Vp	120 MPa Vs	160 MPa Vp	160 MPa Vs	200 MPa Vp	200 MPa Vs	250 MPa Vp	250 MPa Vs	300 MPa Vp	300 MPa Vs
	(km/s)		(km/s)		(km/s)		(km/s)		(km/s)		(km/s)		(km/s)		(km/s)		(km/s)		(km/s)	
52	4.13	2.79	4.29	2.90	4.38	2.97	4.50	3.02	4.58	3.06	4.65	3.09	4.70	3.11	4.74	3.13	4.75	3.13	4.76	3.14
53	4.44	2.90	4.51	2.93	4.53	2.95	4.56	2.96	4.59	2.98	4.64	3.00	4.66	3.01	4.68	3.02	4.70	3.03	4.72	3.04
55	4.19	2.86	4.30	2.92	4.46	2.99	4.49	3.02	4.52	3.04	4.56	3.06	4.58	3.08	4.61	3.12	4.65	3.13	4.68	3.15
56	3.94	2.81	4.52	3.04	4.70	3.18	5.00	3.27	5.10	3.32	5.20	3.38	5.36	3.43	5.40	3.48	5.45	3.51	5.50	3.55
58	4.16	2.83	4.46	2.93	4.56	2.97	4.65	3.00	4.70	3.02	4.74	3.04	4.77	3.06	4.80	3.08	4.84	3.10	4.88	3.12
60	3.77	2.42	4.09	2.56	4.16	2.63	4.22	2.66	4.32	2.68	4.40	2.70	4.42	2.71	4.44	2.72	4.46	2.73	4.48	2.75
62	4.40	2.84	4.50	2.90	4.54	2.93	4.57	2.96	4.64	3.00	4.75	3.07	4.77	3.09	4.81	3.10	4.86	3.12	4.90	3.13
65	4.09	2.82	4.44	3.00	4.51	3.02	4.58	3.05	4.61	3.07	4.67	3.09	4.70	3.12	4.74	3.14	4.77	3.15	4.80	3.16
73	3.38	2.36	3.72	2.45	3.82	2.48	3.88	2.51	3.92	2.56	3.97	2.59	4.00	2.60	4.04	2.61	4.08	2.63	4.12	2.66
74	3.89	2.66	4.02	2.72	4.08	2.74	4.14	2.76	4.17	2.77	4.22	2.78	4.23	2.79	4.24	2.80	4.26	2.81	4.28	2.83
78	4.04	2.70	4.24	2.80	4.39	2.92	4.45	2.99	4.48	3.01	4.51	3.03	4.54	3.04	4.58	3.05	4.59	3.05	4.61	3.06
81	3.90	2.68	4.22	2.80	4.32	2.84	4.39	2.87	4.43	2.88	4.48	2.90	4.51	2.92	4.54	2.93	4.56	2.95	4.58	2.96
90	3.49	2.39	3.70	2.44	3.84	2.47	3.91	2.50	3.94	2.52	3.96	2.54	3.97	2.56	3.98	2.57	3.99	2.59	4.00	2.60
91	3.59	2.57	4.20	2.86	4.50	2.97	4.78	3.07	4.88	3.18	4.98	3.27	5.06	3.30	5.14	3.32	5.20	3.34	5.26	3.37
95	4.56	2.79	4.80	2.94	5.00	3.00	5.14	3.03	5.19	3.05	5.29	3.07	5.37	3.10	5.43	3.13	5.46	3.16	5.48	3.19
102	4.02	2.74	4.30	2.86	4.62	3.02	4.97	3.10	5.09	3.16	5.30	3.22	5.32	3.23	5.34	3.24	5.35	3.26	5.37	3.28
107	3.70	2.64	4.30	2.98	4.66	3.16	4.94	3.26	5.08	3.28	5.21	3.30	5.28	3.33	5.33	3.37	5.36	3.38	5.39	3.39
108	3.28	2.45	3.87	2.83	4.28	3.06	4.56	3.20	4.82	3.34	5.17	3.47	5.25	3.49	5.28	3.51	5.29	3.52	5.30	3.53
116	3.68	2.49	4.20	2.70	4.57	2.89	4.85	3.00	4.94	3.08	5.04	3.14	5.12	3.18	5.18	3.20	5.22	3.22	5.26	3.24
120	3.82	2.77	4.33	3.00	4.60	3.18	4.89	3.31	5.09	3.42	5.26	3.48	5.36	3.52	5.47	3.54	5.50	3.55	5.54	3.57
123	4.73	3.11	4.81	3.16	4.90	3.21	5.06	3.28	5.15	3.32	5.24	3.35	5.30	3.38	5.36	3.40	5.40	3.41	5.46	3.43
126	3.62	2.28	3.83	2.32	3.92	2.36	3.97	2.40	4.00	2.43	4.04	2.47	4.08	2.49	4.12	2.50	4.15	2.52	4.18	2.54
131	3.18	2.31	3.95	2.82	4.43	3.05	4.69	3.20	4.83	3.24	4.98	3.29	5.03	3.31	5.07	3.34	5.09	3.35	5.11	3.36
134	3.55	2.53	4.14	2.76	4.45	2.91	4.60	2.98	4.68	3.02	4.76	3.08	4.83	3.10	4.90	3.12	4.93	3.15	4.97	3.20
135	3.04	2.23	3.50	2.42	3.70	2.64	4.00	2.77	4.14	2.89	4.28	2.96	4.37	2.98	4.42	3.00	4.45	3.02	4.49	3.03
139	2.88	2.12	3.67	2.50	4.18	2.78	4.70	3.04	5.01	3.11	5.11	3.18	5.14	3.20	5.17	3.22	5.20	3.23	5.22	3.24
140	3.61	2.54	3.87	2.65	4.00	2.73	4.16	2.80	4.21	2.83	4.38	2.86	4.40	2.88	4.42	2.90	4.44	2.90	4.46	2.91
146	3.76	2.41	4.05	2.53	4.22	2.60	4.30	2.67	4.41	2.72	4.54	2.77	4.65	2.80	4.70	2.82	4.70	2.83	4.71	2.83
159	4.28	2.88	4.47	2.95	4.58	3.01	4.60	3.04	4.70	3.07	4.82	3.10	4.83	3.12	4.84	3.14	4.86	3.14	4.88	3.15
165	4.44	2.89	4.67	3.00	4.73	3.02	4.80	3.05	4.84	3.08	4.90	3.12	4.96	3.14	5.04	3.15	5.10	3.17	5.14	3.18
172	3.37	2.21	3.70	2.30	3.00	3.35	3.86	2.40	3.92	2.42	4.00	2.44	4.02	2.46	4.04	2.48	4.08	2.49	4.12	2.50
180	4.40	2.65	4.53	2.72	4.63	2.77	4.74	2.81	4.78	2.84	4.86	2.86	4.92	2.87	4.96	2.89	4.98	2.90	5.00	2.91
191	3.98	2.70	4.50	3.10	4.72	3.16	4.90	3.20	5.05	3.22	5.20	3.24	5.22	3.26	5.24	3.27	5.28	3.28	5.32	3.29
196	3.35	2.28	3.48	2.31	3.57	2.34	3.65	2.36	3.68	2.38	3.72	2.39	3.76	2.41	3.82	2.44	3.86	2.46	3.90	2.48
202	3.58	2.73	4.18	3.00	4.54	3.08	4.89	3.16	5.03	3.18	5.20	3.21	5.21	3.23	5.22	3.25	5.23	3.27	5.24	3.28
205	4.21	2.87	4.40	2.94	4.50	2.97	4.52	3.00	4.54	3.02	4.56	3.04	4.58	3.06	4.59	3.07	4.60	3.07	4.61	3.08
206	3.37	2.34	3.50	2.43	3.57	2.46	3.64	2.50	3.70	2.52	3.80	2.55	3.82	2.56	3.84	2.58	3.86	2.59	3.87	2.60
213	3.11	2.33	4.08	2.82	4.37	2.97	4.60	3.07	4.65	3.09	4.68	3.11	4.72	3.12	4.76	3.14	4.77	3.16	4.78	3.17
216	3.15	2.32	3.90	2.68	4.30	2.92	4.47	3.08	4.58	3.16	4.70	3.20	4.74	3.28	4.78	3.34	4.84	3.34	4.90	3.35
218	4.22	2.98	4.67	3.19	4.78	3.26	4.86	3.29	4.90	3.32	4.97	3.34	5.01	3.36	5.04	3.37	5.06	3.38	5.09	3.39
219	3.10	2.31	3.90	2.71	4.31	3.00	4.46	3.11	4.60	3.13	4.70	3.18	4.72	3.19	4.74	3.20	4.75	3.21	4.76	3.22
221	3.66	2.62	4.27	3.00	4.57	3.11	4.77	3.23	4.82	3.26	4.88	3.30	4.93	3.32	4.98	3.35	5.00	3.36	5.02	3.38
222	3.05	2.31	4.50	2.72	4.86	2.80	4.96	2.86	5.04	2.88	5.10	2.90	5.13	2.92	5.16	2.94	5.17	2.95	5.18	2.96
223	3.62	2.39	4.03	2.50	4.28	2.55	4.24	2.60	4.30	2.63	4.39	2.66	4.46	2.69	4.52	2.71	4.56	2.74	4.60	2.77
230	3.64	2.66	4.18	3.01	4.55	3.21	4.81	3.34	4.92	3.40	5.09	3.45	5.18	3.47	5.23	3.50	5.25	3.50	5.27	3.51
235	3.89	2.75	4.80	2.92	5.10	3.10	5.35	3.19	5.43	3.22	5.54	3.27	5.59	3.30	5.64	3.33	5.67	3.37	5.70	3.40
240	3.55	2.55	4.27	2.67	4.48	2.74	4.60	2.78	4.67	2.82	4.73	2.87	4.75	2.88	4.78	2.88	4.80	2.89	4.83	2.90
246	4.37	2.80	4.66	2.82	4.72	2.85	4.78	2.88	4.82	2.90	4.86	2.93	4.87	2.95	4.88	2.96	4.89	2.97	4.90	2.98
253	3.55	2.61	4.53	2.81	4.89	3.02	5.14	3.12	5.26	3.15	5.37	3.18	5.41	3.20	5.45	3.22	5.46	3.24	5.48	3.25
254	3.79	2.81	4.73	3.05	4.86	3.18	5.25	3.26	5.31	3.30	5.38	3.35	5.41	3.38	5.45	3.41	5.47	3.41	5.49	3.42
256	3.09	2.31	3.83	2.73	4.26	2.92	4.47	3.00	4.56	3.05	4.61	3.09	4.65	3.11	4.69	3.12	4.71	3.14	4.73	3.16
258	3.07	2.22	3.82	2.64	4.25	2.81	4.36	2.89	4.40	2.91	4.45	2.92	4.47	2.93	4.48	2.94	4.50	2.96	4.51	2.97
261	3.40	2.49	4.23	2.82	4.70	2.97	4.94	3.13	5.02	3.19	5.12	3.26	5.20	3.31	5.26	3.36	5.30	3.40	5.38	3.44
265	3.96	2.75	4.70	2.83	4.80	2.94	4.89	2.98	4.93	2.99	4.98	3.00	5.05	3.01	5.11	3.02	5.14	3.03	5.20	3.04
266	3.45	2.44	4.20	2.71	4.60	2.89	4.78	2.95	4.93	3.00	5.11	3.06	5.13	3.07	5.15	3.09	5.17	3.10	5.18	3.11
267	3.62	2.75	4.21	2.90	4.36	2.96	4.47	3.00	4.58	3.04	4.63	3.06	4.68	3.08	4.73	3.11	4.78	3.13	4.83	3.15
272	3.27	2.48	4.66	2.93	5.10	3.16	5.15	3.25	5.21	3.28	5.34	3.33	5.37	3.37	5.40	3.40	5.44	3.41	5.48	3.42
281	3.29	2.44	4.46	2.74	4.84	3.00	5.05	3.20	5.14	3.39	5.20	3.47	5.25	3.51	5.29	3.56	5.30	3.57	5.32	3.58
287	3.48	2.42	3.97	2.60	4.24	2.65	4.33	2.69	4.37	2.71	4.41	2.73	4.45	2.76	4.48	2.79	4.51	2.80	4.54	2.82
288	2.91	2.16	3.46	2.47	3.86	2.70	4.09	2.90	4.25	2.94	4.39	2.98	4.45	3.02	4.51	3.06	4.55	3.07	4.59	3.08
294	3.61	2.54	3.96	2.73	4.14	2.80	4.26	2.84	4.34	2.87	4.44	2.90	4.53	2.91	4.57	2.92	4.62	2.94	4.65	2.96
296	3.44	2.39	3.93	2.45	4.11	2.50	4.28	2.53	4.38	2.57	4.53	2.64	4.61	2.70	4.66	2.75	4.71	2.82	4.76	2.90
299	3.16	2.30	4.50	2.63	4.74	2.80	4.88	2.95	4.95	2.97	4.98	3.00	5.01	3.03	5.03	3.06	5.06	3.06	5.11	3.07
302	3.32	2.47	3.90	2.87	4.14	2.96	4.37	3.05	4.58	3.13	4.71	3.15	4.76	3.17	4.80	3.20	4.82	3.21	4.84	3.22
303	3.48	2.48	4.35	2.87	4.72	3.04	4.90	3.10	5.00	3.15	5.05	3.19	5.10	3.20	5.16	3.22	5.20	3.23	5.24	3.25
304	3.37	2.66	4.09	2.94	4.57	3.18	4.78	3.27	4.94	3.30	5.00	3.33	5.01	3.34	5.02	3.35	5.02	3.36	5.03	3.37
305	3.20	2.16	3.85	2.47	4.04	2.74	4.20	2.84	4.30	2.90	4.42	2.96	4.52	3.00	4.58	3.03	4.60	3.04	4.61	3.06
306	3.35	2.46	3.80	2.67	4.16	2.80	4.40	2.90	4.47	2.98	4.54	3.03	4.57	3.06	4.60	3.10	4.62	3.12	4.64	3.15
307	2.97	2.26	3.59	2.48	3.83	2.60	4.03	2.65	4.17	2.68	4.26	2.70	4.33	2.75	4.40	2.80	4.45	2.84	4.49	2.88
308	3.84	2.57	4.12	2.76	4.32	2.92	4.58	3.05	4.64	3.14	4.82	3.18	4.88	3.20	4.94	3.22	4.96	3.23	4.99	3.24
309	3.03	2.31	4.00	2.76	4.26	3.06	4.50	3.20	4.71	3.23	4.81	3.25	4.83	3.27	4.86	3.29	4.89	3.31	4.92	3.33
314	3.64	2.55	4.30	2.84	4.86	3.03	5.18	3.16	5.35	3.25	5.50	3.37	5.53	3.39	5.56	3.40	5.60	3.41	5.62	3.42
320	3.70	2.71	4.60	3.03	4.97	3.24	5.17	3.34	5.28	3.43	5.40	3.47	5.41	3.48	5.42	3.50	5.42	3.51	5.43	3.53

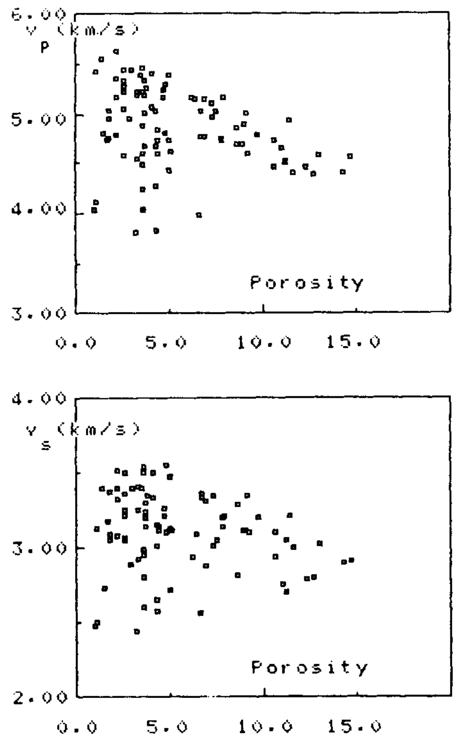

Figure 2. Velocities of compressional wave (top) and shear wave (bottom) versus porosity (left) and clay content (right) for all samples at 200 MPa.

samples have been fitted by least-squares regression. But only small correlation coefficients were obtained, independent of pressure. For example, at a pressure of 200 MPa the linear relations are found to be

$$v_P = 5.04 - 3.32\phi, \qquad v_P = 5.07 - 1.01\,Clay, \tag{5}$$

for the compressional velocity and

$$v_S = 3.18 - 1.42\phi, \qquad v_S = 3.25 - 0.63\,Clay \tag{6}$$

for the shear velocity (Fig. 2). The corresponding correlation coefficients are $r = 0.28$, $r = 0.58$ for (5) and $r = 0.19$, $r = 0.55$ for (6). This is not acceptable.

4.2 Effect of porosity and clay content on velocities

Based on the above results, it must be concluded that for successful correlation of velocities and porosity of lithologically different sedimentary rocks, the mineralogical composition also has to be taken into account. As a first-order effect, different mineral velocities change the matrix velocity. These can be calculated from single crystal or aggregate velocities, resulting, for example, from the VRH approximation with regard to the volume fraction of the mineral constituents. But these calculations are inaccurate and their error increases with the number of mineral constituents. Between the end members of the suite sandstone–limestone quantitative relations exist for the velocities v_P and v_S. This is possible because of the great differences in the grain velocities of quartz and calcite. Good correlations can be achieved when the porosity varies in a small range (Wilkens, Simmons & Caruso 1984). It is not possible to extrapolate the aggregate velocities of quartz and calcite from the zero-porosity velocities or the zero-porosity traveltimes estimated from siliceous limestone data (Domenico 1983; Wilkens *et al.* 1984). Data from Kopf (1977) show that the *P*-wave velocity of a clean sandstone ($\phi = 0.01$) is changed in the event of a substitution by 0.5 fraction per volume of the following components:

dolomite	$+0.6\ \mathrm{km\,s^{-1}}$,
calcite	$+0.3\ \mathrm{km\,s^{-1}}$,
anhydrite	$+0.2\ \mathrm{km\,s^{-1}}$,
clay	$-0.4\ \mathrm{km\,s^{-1}}$.

The broadest and most continuous distribution of the different mineral constituents in our sample collection is available for the clay fraction (0–88 per cent by volume). For the less abundant and sporadically distributed content of carbonate and anhydrite quantitative relations with the velocities were not expected to exist.

The decrease of the velocities v_P and v_S due to clay is undisputed, but quantitative estimates are different (Jones & Wang 1981; Tosaya 1982; Kowallis, Jones & Wang 1984; Han *et al.* 1986).

To separate the effect of clay content from the influence of porosity, both parameters must be limited by statistical classification (Freund 1988). Fig. 3 shows the distribution of porosity and clay content for all samples investigated and demonstrates that high clay content and high porosities are not coexistent.

The combined and fractional influence of porosity and clay content on the velocities of compressional and shear waves can be more exactly proved by statistical means with regard to both of these variables. As a basic theoretical formulation the time-average relation has been extended by Millican (1960) with a clay content term, *Clay*:

$$1/v = (1 - \phi - Clay)/v_m + (Clay/v_{Clay}) + (\phi/v_f). \tag{7}$$

Analogously, Pickett's relation can be modified according to

$$1/v = A + B\phi + C Clay. \tag{8}$$

For our experimental data equation (1), tested by Han *et al.* (1986), was preferred because of its good correlation. This equation was fitted by multiple regression analysis at pressures of 8, 24, 60, 120, 200, 300 MPa, resulting in a Linear Velocity–Porosity–Clay Empirical Relationship, called LIVEPOCER in the following. The pressure-dependent coefficients are listed in Table 3. For pressures of

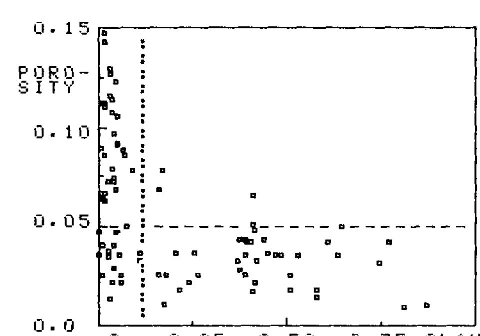

Figure 3. Distribution of porosity versus clay content for all samples. High values of these parameters exclude each other.

Table 3. Coefficients of LIVEPOCER at different pressures. Subscripts 1 and 2 correspond to v_P and v_S, respectively (SD means standard deviation).

P	A_1	A_2	B_1	B_2	C_1	C_2	r_1	r_2	SD_1	SD_2
(MPa)	(km/s)		(km/s)		(km/s)				(km/s)	
8	4.06	2.87	6.62	4.12	0.24	0.17	0.545	0.487	0.36	0.23
24	4.80	3.17	7.62	4.43	0.69	0.56	0.632	0.626	0.22	0.17
60	5.41	3.45	8.99	4.86	1.37	0.92	0.834	0.780	0.22	0.15
120	5.68	3.58	9.68	5.28	1.67	1.03	0.895	0.805	0.20	0.15
200	5.76	3.63	9.53	5.27	1.74	1.07	0.897	0.820	0.17	0.14
300	5.82	3.66	9.51	5.15	1.74	1.07	0.896	0.820	0.17	0.14

24, 60, 120, 300 MPa LIVEPOCER is shown in Fig. 4. Based on this figure and the coefficients given in Table 3 the following can be concluded.

(i) The correlation coefficients demonstrate that above 120 MPa the relation between the velocities v_P, v_S, the porosity and clay content may be described by the model LIVEPOCER.

(ii) LIVEPOCER suggests that increasing porosity and clay content lowers the velocities of elastic waves.

(iii) Above 120 MPa v_P is better described by LIVEPOCER than v_S, according to the correlation coefficients $r_1 = 0.90$ and $r_2 = 0.82$.

(iv) From LIVEPOCER above 120 MPa the effect on reducing velocities is stronger for porosity than for clay content by a factor of 5.6 for v_P and 4.9 for v_S.

(v) All coefficients show strong variations in the pressure range up to 120 MPa. Above 120 MPa their change is small.

(vi) The pressure-induced changes in velocities are larger for the lithologic components with small clay contents.

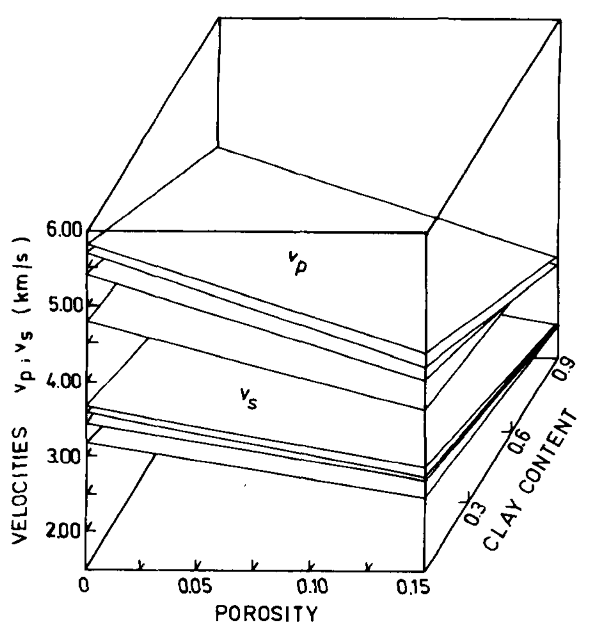

Figure 4. The Linear Velocity–Porosity–Clay Empirical Relationship = LIVEPOCER (see Table 3) for the compressional (above) and shear wave (below) at pressures of 24, 60, 120, and 300 MPa (from the bottom upwards).

(vii) LIVEPOCER is insensitive to the type or the location of clay. Apparently, the influence of clay-surrounded clasts does not differ from the effect of clay as matrix or clay which is distributed in pores.

In Fig. 5 relative velocity deviations for LIVEPOCER for 200 MPa are shown as a function of porosity (top) and clay content (bottom), respectively. The velocity deviations in LIVEPOCER above 120 MPa are larger than for the relations given by Han *et al.* (1986). This may be due to the more variable mineralogical composition caused by carbonatic and anhydritic components.

A comparison between LIVEPOCER for 120 MPa and the results, obtained by Tosaya (1982) (80 MPa) and Han *et al.* (1986) (40 MPa) on rocks of similar lithology, is given in Fig. 6. The different saturants are responsible for the differences in the dependence of v_P on porosity. v_P clearly shows lower values for the higher porosities than v_P for water-saturated samples investigated by the authors mentioned above. When the v_P porosity line according to LIVEPOCER for 120 MPa is converted into a function for the water-saturated equivalents by using a method by Schön (1983) we obtained a function which is congruent with the straight line 1 in Fig. 6. The velocity–porosity line for v_S is nearly the same.

Both velocities also decrease with increasing clay content. But the clay-rich components have higher velocities than water-saturated samples described in the literature, particularly for the velocity of the shear wave. Except for the fact that differences in the composition of the clay component (and additionally other mineralogical constituents) are evident, the higher velocities are due to the dry state of the samples investigated. For a given sandstone–clay matrix the reduction of bulk rigidity and consequently of the shear wave velocity is small without the interaction between water and clay minerals. The higher velocities of shear waves for our clay-rich clastic rocks are in accordance with results obtained by Han *et al.* (1986). They found that the shear modulus (G_{sat}) of well-consolidated borehole cores (samples P) saturated with water is lower than the shear modulus (G_{dry}) of the dry samples. Furthermore the ratio G_{sat}/G_{dry} tends to decrease with increasing clay content. It also means that the ratio $v_S(\text{dry})/v_S(\text{sat})$ changes with increasing clay content. In this way the great deviation of our v_S clay content line (3) from Han *et al.*'s line (2) is explainable.

4.3 The effect of crack closure on velocities

According to LIVEPOCER for 120 MPa the velocities v_P and v_S are diminished for a porosity of 15 per cent by volume from 5.68 to 4.23 km s^{-1} ($Dv_P = 1.45$ km s^{-1}) and from 3.58 to 2.79 km s^{-1} ($Dv_S = 0.79$ km s^{-1}), respectively. The differences between A_1, A_2 at 120 MPa and A_1, A_2 at 8 MPa ($D_{A_1} = 1.60$ km s^{-1}, $D_{A_2} = 0.79$ km s^{-1}) are nearly the same (Table 3). But this does not mean that the pressure-induced increase of coefficients A_1, A_2 is caused by the elimination of the bulk porosity. It is known from literature data that the differences between initial bulk porosities and residual porosities at 40 MPa do not exceed 1 per cent (Han *et al.* 1986; Zimmermann, Somerton & King 1986). For investigations up to 150 MPa (Avchan,

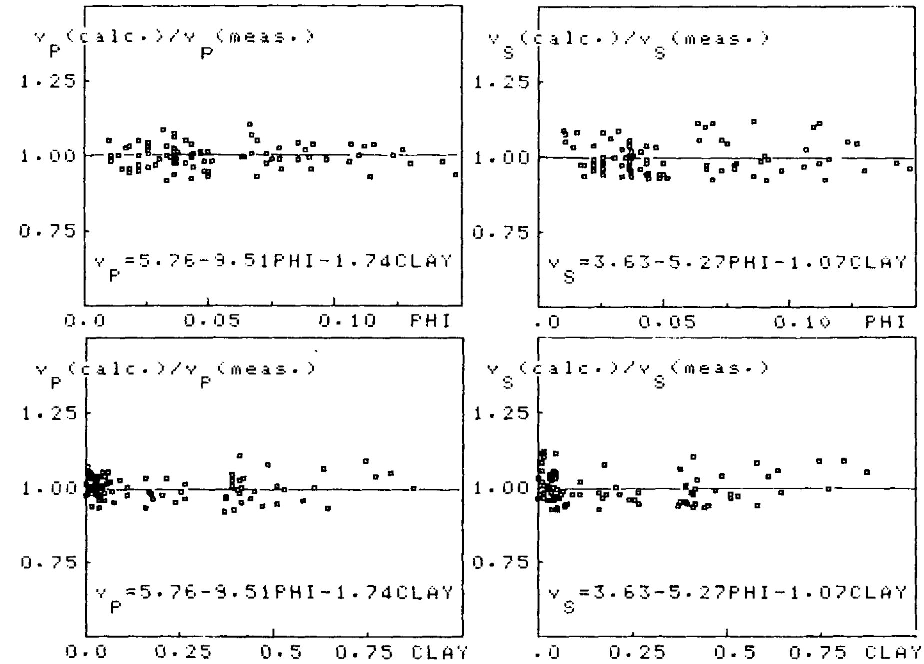

Figure 5. The deviations of the velocities of the compressional (left) and shear wave (right) from LIVEPOCER 200 MPa versus porosity (top) and clay content (bottom).

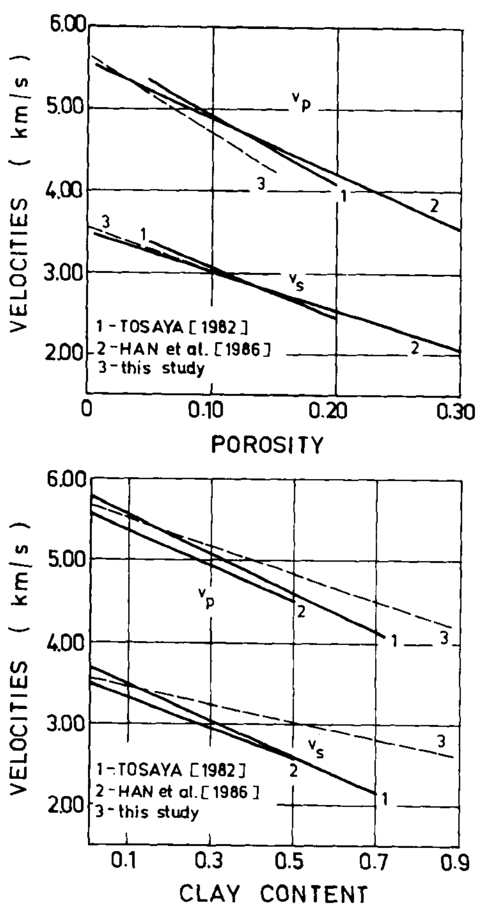

Figure 6. LIVEPOCER 120 MPa (3), compared with results on water-saturated clastic sedimentary rocks obtained by Tosaya (1982)(1) and Han *et al.* (1986)(2), shows a considerable deviation for the v_S-clay content line. (Above—without clay content; below—without porosity.)

Matveenko & Stefankevich 1978) these differences are not more than 2.5 per cent and are independent of initial porosities (Freund 1989). Therefore the strong increase of velocities up to pressures of 120 MPa cannot only be explained by the reduction of the bulk porosities.

From the examination of theoretical considerations by experimentally determined velocity data (Eshelby 1957; Wu 1966; Walsh 1969; Garbin & Knopoff 1973; Anderson, Minster & Cole 1974; O'Connel & Budiansky 1974; Toksöz *et al.* 1976; Nishizawa 1982) it is known that for a given concentration of pores, the flatter pores (cracks) are more responsible for the reduction of the velocities than rounded or spherical ones. In the calculations of these authors, mainly tested on crystalline rocks, pores are described as ellipsoidal voids with an aspect ratio ranging from 1 (sphere) to 10^{-5} (very thin cracks). The boundary conditions of these calculations are different with respect to porosity, saturation, the interaction between pores and the concentration and spatial distribution of inhomogeneities (isotropic or with preferred orientation), respectively. Nevertheless, the following common features exist for the effect of cracks on the velocities.

(1) The velocity of compressional waves is considerably diminished, especially for rocks with low bulk modulus saturants (gases).

(2) The velocity of shear waves is equally reduced, but without great differences for various saturants.

(3) Hydrostatic pressure causes the closure of cracks. First of all the very thin cracks are eliminated and the closure proceeds with pressure increase to cracks with higher aspect ratios.

For the data obtained from this study it must be concluded that the strong non-linear increase of both velocities under

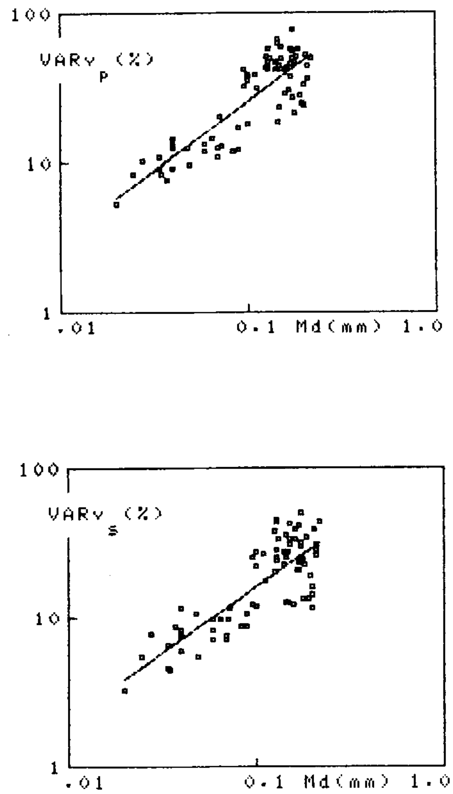

Figure 7. *$VARv_P$* and *$VARv_S$* (see text) versus grain size parameter median *Md* (mm) for 76 from 88 samples.

low pressure (Fig. 1) is a consequence of crack closure which ends at the beginning of the linear slope.

For those samples (76 from 88) for which the grain size parameter *Md* is available, the velocity changes caused by crack closure are illustrated quantitatively in Fig. 7.

$VARv_P$ means $(v_P/\bar{v}_P)-1$ and $VARv_S$ is $(v_S/\bar{v}_S)-1$, where $\bar{v}_P$, $\bar{v}_S$ are the initial velocities (at 8 MPa) and v_P, v_S are the first velocity value on the linear portion of the velocity-pressure curve. $VARv_P$ ranges from 6 to 78 per cent, and $VARv_S$ varies form 4 to 50 per cent. It is evident that pressure-induced changes of both velocities increase with grain size. It can be concluded that the initial velocities of coarse grained clastic sedimentary rocks are more influenced by cracks than the velocities of finer grained rocks. The cracks are formed at grain boundaries as a result of unloading after the recovery of drill cores.

4.4 The v_P/v_S ratio

It is possible to differentiate various lithologies of sedimentary rocks with the help of the v_P/v_S ratio. This is shown for the binary mixture of sandstone and limestone by Pickett (1963), Tatham (1982), Domenico (1983) and for clay-bearing sandstones by Tosaya (1982) and Han *et al.* (1986). Furthermore, the v_P/v_S ratio of liquid-saturated clastic sedimentary rocks increases with the bulk porosity (Gregory 1976; Tosaya 1982; Ostrander 1984; Han *et al.* 1986). For the dry-rock v_P/v_S the opposite can be true (Kowallis *et al.* 1984). But the dependence of v_P/v_S on the

Table 4. Pressure-dependent coefficients of relation (9).

p (MPa)	a (km/s)	b (km/s)	r
8	1.68	−0.63	0.90
24	1.27	0.67	0.80
60	1.24	0.92	0.81
120	1.29	0.83	0.85
200	1.29	0.85	0.85
300	1.31	0.79	0.85

bulk porosity must be specified by the pore shape commonly described in terms of the aspect ratio, as shown above.

In contrast to v_P/v_S in gas-saturated rocks, the v_P/v_S ratio of the same rock, saturated with a liquid, is higher (Gregory 1976; Toksöz *et al.* 1976; Jones & Wang 1981; Castagna *et al.* 1985). The v_P/v_S ratio of liquid-saturated rocks decreases with increasing pressure (Toksöz *et al.* 1976; Christensen & Wang 1985; Castagna *et al.* 1985), whereas in dry or gas-filled rocks v_P/v_S increases with pressure (Gregory 1976; Johnston & Toksöz 1980) or is independent of pressure (Aktan & Farouq Ali 1975; Castagna *et al.* 1985).

For all samples investigated in this study an increase of v_P/v_S with pressure is recorded. Data of v_P versus v_S were fitted for various pressures according to

$$v_P = av_S + b. \quad (9)$$

The coefficients in (9) are summarized in Table 4. Above 120 MPa the functions are nearly the same. The relation

$$v_P\,(\mathrm{km\,s^{-1}}) = 1.16\,v_S + 1.36 \quad (10)$$

represents the 'mudrock-line' (Castagna *et al.* 1985) compiled from laboratory measurements, sonic logs, field seismics, and velocity models for liquid-saturated clastic silicate rocks. Fig. 8 demonstrates for a better distinction of the pressure-dependent functions the relation (9) in the form (see Castagna *et al.* 1985)

$$v_P/v_S = av_P/(v_P - b) \quad (11)$$

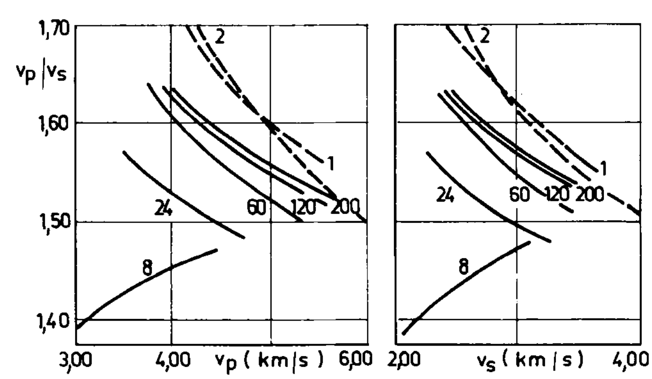

Figure 8. v_P/v_S versus v_P and v_S, respectively according to equations (11) and (12) at pressures of 8, 24, 60, 120, and 200 MPa, compared with results of Han *et al.* (1986)(1) and the 'mudrock'-line (2) obtained by Castagna *et al.* (1985).

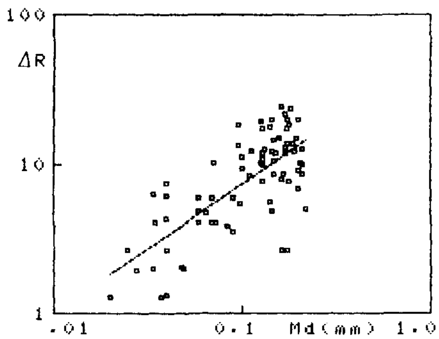

Figure 9. The pressure-induced change ΔR of the v_P/v_S ratio tends to increase ($r = 0.72$) with grain size parameter *Md*.

and

$$v_P/v_S = a + b/v_S, \tag{12}$$

respectively. The functions at pressures steps of 8, 24, 60, 120 and 200 MPa are compared with the mudrock-line and data taken from Han *et al.* (1986). Even for the highest pressures the curves for water-saturated samples of comparable composition are never reached by our data. At 8 MPa initial values of $v_P/v_S < 1.41$ exist, pointing to the fact that an elastic anisotropy must be present (Gregory 1976), generated by preferred crack orientation (Thomsen 1986).

The stronger change of v_P, compared with the change of v_S, in the low-pressure range also results in a strong increase of the v_P/v_S ratio. Our velocity measurements are performed only in the direction perpendicular to the bedding plane. It can be concluded that ellipsoidal cracks, which are mainly aligned parallel to bedding, and their closing under pressure cause the increase of v_P/v_S. The crack model, developed for the effect of oriented cracks on velocities of granite by Anderson *et al.* (1974), indicates that in the direction perpendicular to the longest axis of ellipsoidal cracks v_P/v_S decreases most strongly, in particular if the bulk modulus of the crack-filling fluid approaches that of a gas. This means that in our dry (air-saturated) clastic sedimentary rocks investigated the v_P/v_S ratio can have the minimum at the lowest pressure (8 MPa). In this connection note that in Fig. 9 the changes in v_P/v_S ratio (ΔR) tend to increase with the grain size parameter median. ΔR is defined as $(R/\bar{R}) - 1$, where $\bar{R} = v_P/v_S$ at 8 MPa and $R = v_P/v_S$ is the ratio at the end of crack closure.

Han *et al.* (1986) found by least-squares regression the dependence

$$v_P/v_S = 1.55 + 0.56\,\phi + 0.43\,Clay \tag{13}$$

for water-saturated shaly sandstones at 40 MPa without sufficient correlation ($r = 0.70$). For our data we similarly obtained no acceptable correlation between v_P/v_S and porosity and clay content, respectively.

4.5 Velocity–pressure functions for dry clastic siliceous rocks

By using the pressure-dependent coefficients of LIVEPOCER (Table 3) velocity–pressure functions were fitted according to

$$v_P = a_1 + pb_1 - c_1 \exp(-d_1 p) \tag{14}$$

and

$$v_S = a_2 + pb_2 - c_2 \exp(-d_2 p), \tag{15}$$

where a is the crack-free velocity resulting from extrapolation of the high-pressure straight line to the velocity axis, d is related to the crack closure, $a - c$ is the zero pressure velocity, and b is the velocity slope under high pressure. For clay-free sandstones with residual porosities of 0, 0.05, 0.10, and 0.15 the velocities v_P and v_S versus pressure are plotted in Fig. 10. Analogous functions for clastic rocks with clay contents of 0, 0.2, 0.4, 0.6, and 0.8 (without porosity) are given in Fig. 11. As a result of the low correlation coefficients of LIVEPOCER in the low-pressure range the curves are marked by dashed lines. For clay-free rocks and for clastic sediments without porosity the velocity–pressure functions are identical. Further, it is evident that the total change in velocities under pressure and their slopes in the low-pressure range are more pronounced for clay-free rocks than for rocks with higher clay contents. The coefficients a, b, c, and d of the functions shown in Figs 10 and 11 are given in Table 5.

Finally, all velocity–pressure data (Table 1) were fitted according to relations (14) and (15). By least-squares regression we find that the coefficients a_1 and a_2 are linearly related to porosity and clay content according to

$$a_1 = 5.46 - 10.36\,\phi - 1.75\,Clay \tag{16}$$

and

$$a_2 = 3.56 - 5.73\,\phi - 1.03\,Clay \tag{17}$$

with correlation coefficients $r_1 = 0.90$ and $r_2 = 0.81$, respectively.

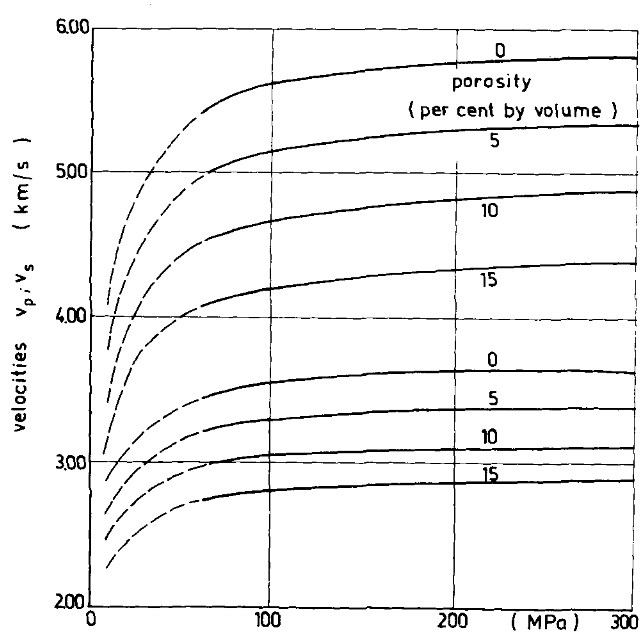

Figure 10. Velocity–pressure curves for clay-free sandstones with porosities of 0, 0.05, 0.10, and 0.15 according to relations (14) and (15).

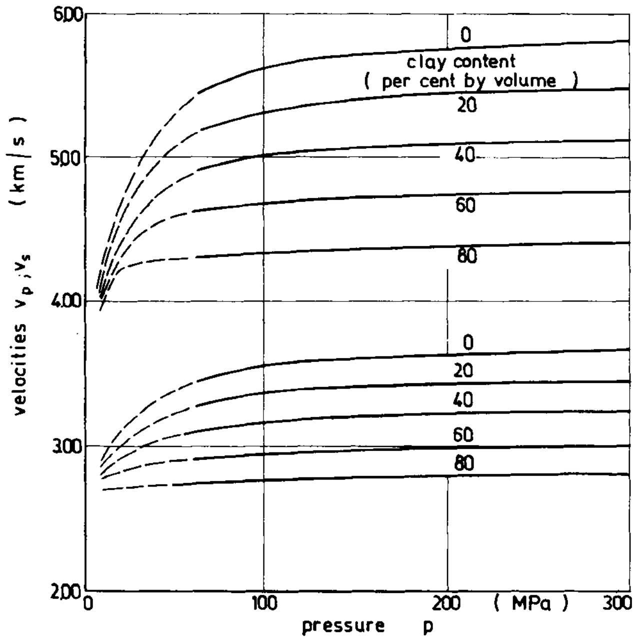

Figure 11. Velocity–pressure curves for zero-porosity clastic sedimentary rocks with clay contents of 0, 0.2, 0.4, 0.6, and 0.8 according to (14) and (15).

Table 5. Coefficients for the pressure-dependent velocity functions in (a) Fig. 10 and (b) Fig. 11.

(a)

porosity	a_1	a_2	b_1	b_2	c_1	c_2	d_1	d_2
(vol.-%)	(km/s)		(10^{-4}km/s MPa)		(km/s)		(MPa^{-1})	
0	5.60	3.55	7.52	3.78	2.09	0.89	0.038	0.034
5	5.10	3.28	8.54	3.99	1.90	0.84	0.040	0.037
10	4.60	3.01	9.32	4.53	1.71	0.76	0.044	0.041
15	4.11	2.73	9.62	5.30	1.53	0.70	0.047	0.046

(b)

clay	a_1	a_2	b_1	b_2	c_1	c_2	d_1	d_2
(vol.-%)	(km/s)		(10^{-4}km/s MPa)		(km/s)		(MPa^{-1})	
0	5.60	3.55	7.52	3.78	2.09	0.89	0.038	0.034
20	5.25	3.35	7.46	3.32	1.74	0.66	0.042	0.032
40	4.91	3.14	7.34	3.14	1.43	0.43	0.051	0.031
60	4.58	2.93	6.28	3.11	1.18	0.19	0.070	0.027
80	4.29	2.72	4.33	2.57	1.23	--	0.130	--

5 CONCLUSIONS

From laboratory velocity measurements up to pressures of 300 MPa a Linear Velocity–Porosity–Clay Empirical Relationship (LIVEPOCER) is presented, which describes the fractional effect of porosity and clay content on the velocities of compressional and shear waves. LIVEPOCER is valid above pressures of 120 MPa. The velocity decrease according to LIVEPOCER above 120 MPa, caused by the clay content, is insensitive to the type of clay minerals and their textural arrangement. The velocities v_P, v_S extrapolated for pure clay are higher than data of analogous water-saturated rocks; the difference is greater for v_S than for v_P. LIVEPOCER failed at low-pressure conditions because of the presence of cracks in the siliceous clastic rocks. The effect of cracks in reducing both velocities is greater for coarse-grained rocks than for clay-rich rocks and does not depend on the bulk porosities.

A strong increase in the low-pressure range is not only observed for both velocities, but also for the v_P/v_S ratio. In all cases the v_P/v_S ratio is lower than for analogous water-saturated samples from literature data. This shows that it is possible to differentiate between pure gas-filled (dry) and liquid-saturated clastic rocks. According to the effective pressure law the effect of high pore pressure on petrophysical properties is comparable to the effect of low confining pressure. From this and LIVEPOCER below 120 MPa it can be concluded that by increase of pore pressure in gas-saturated clastic siliceous rocks the effect of cracks on the velocities may dominate and that a correlation between velocities and bulk porosity and clay content does not exist.

REFERENCES

Aktan, T. & Farouq Ali, C. M., 1975. Effect of cyclic and in situ heating on the absolute permeabilities, elastic constants, and electrical resistivities of rocks, *Soc. Petrol. Eng. J.*, 5633.

Anderson, D. L., Minster, B. & Cole, D., 1974. The effect of oriented cracks on seismic velocities, *J. geophys. Res.*, **79**, 4011–4015.

Avchan, G. M., Matveenko, A. A. & Stefankevich, Z. B., 1978. Fizicheskie svojstva osadochnykh porod pri vysokikh davleniyakh i temperaturakh, in *Spravochnik po Fizicheskim Svojstvam Mineralov i Gornykh Porod pri Vysokikh Termodinamicheskikh Parametrakh*, pp. 181–219, Nedra, Moscow (in Russian).

Castagna, J. P., Batzle, M. L. & Eastwood, R. L., 1985. Relationship between compressional-wave and shear-wave velocities in clastic silicate rocks, *Geophysics*, **50**, 571–581.

Christensen, N. I. & Wang, H. F., 1985. The influence of pore pressure and confining pressure on dynamic elastic properties of Berea sandstone, *Geophysics*, **50**, 207–213.

Domenico, S. N., 1983. Sandstone and limestone porosity determination from shear and compressional velocity, *Bull. Austr. Soc. Expl. Geophys.*, **4**, 81–90.

Eshelby, J. D., 1957. The determination of the elastic field of an ellipsoidal inclusion, and related problems, *Proc. R. Soc. Lond.*, A, **241**, 376–396.

Freund, D., 1988. Velocity–porosity and velocity–clay content relationships for dry clastic sedimentary rocks from ultrasonic measurements under pressure, in *High Pressure Geosciences and Material Synthesis*, Proc. XXV. Ann. Mtg. EHPRG, pp. 55–59, ed. Vollstädt, H., Akademie Verlag, Berlin.

Freund, D., 1989. Ausbreitungsgeschwindigkeiten der Kompressions—und der Scherwelle an trockenen Sedimentiten der lithologischen Reihe Sandstein-Tonstein unter hydrostatischem Druck, *PhD thesis*, Forschungsstelle für Hochdruckforschung, Potsdam, Germany.

Garbin, H. D. & Knopoff, L., 1973. The compressional modulus of a material permeated by a random distribution of circular cracks, *Q. Appl. Math.*, **30**, 453–464.

Gebrande, H., 1982. Elastic wave velocities and constants of elasticity of rocks at room temperature and pressures up to 1 GPa, in *Landolt-Börnstein, New Series, Group V*, vol. 1b, pp. 35–99, ed. Angenheister, G., Springer, Berlin.

Gregory, A. R., 1976. Fluid saturation effects on dynamic elastic properties of sedimentary rocks, *Geophysics*, **41**, 895–996.

Han, De-hua, Nur, A. & Morgan, D., 1986. Effects of porosity and clay content on wave velocities in sandstones, *Geophysics,* **51,** 2093–2107.

Johnston, D. H. & Toksöz, M. N., 1980. Ultrasonic *P* and *S* wave attenuation in dry and saturated rocks under pressure, *J. geophys. Res.*, **85,** 925–936.

Jones, L. E. A. & Wang, H. F., 1981. Ultrasonic velocities in cretaceous shales from the Willistone basin, *Geophysics,* **46,** 288–297.

Kopf, M., 1977. Die Teufenabhängigkeit der petrophysikalischen Parameter Dichte, Geschwindigkeit, Schallhärte und Reflexionskoeffizient, *ScD thesis*, University of Leipzig.

Kowallis, B. J., Jones, L. E. A. & Wang, H. F., 1984. Velocity–porosity–clay content systematics of poorly consolidated sandstones, *J. geophys. Res.*, **89,** 10 355–10 364.

Millican, M. L., 1960. The sonic log and the Delaware sand, *J. Petrol. Technol.*, **12,** 71–79.

Nishizawa, O., 1982. Seismic velocity anisotropy in a medium containing oriented cracks-transversely isotropic case, *J. Phys. Earth*, **30,** 331–347.

O'Connell, R. & Budiansky, B., 1974. Seismic velocities in dry and saturated cracked solids, *J. geophys. Res.*, **79,** 5412–5426.

Ostrander, W. J., 1984. Plane-wave reflection coefficients for gas sands at nonnormal angles of incidence, *Geophysics,* **49,** 1637–1648.

Pickett, G. R., 1963. Acoustic character logs and their application in formation evaluation, *J. Petrol. Technol.*, **15,** 659–667.

Raymer, D. S., Hunt, E. R. & Gardner, J. S., 1980. An improved sonic transit time-to-porosity transform, *Soc. Prof. Well Log Analyst. 21st Annual Meeting,* Paper P.

Schön, J., 1983. *Petrophysik*, Akademie Verlag, Berlin.

Tatham, R. H., 1982. v_P/v_S and lithology, *Geophysics,* **47,** 336–344.

Thomsen, L., 1986. Weak elastic anisotropy, *Geophysics,* **51,** 1954–1966.

Toksöz, M. N., Cheng, C. H. & Timur, A., 1976. Velocities of seismic waves in porous rocks, *Geophysics,* **41,** 621–645.

Tosaya, C. A., 1982. Acoustical properties of clay-bearing rocks, *PhD thesis,* Stanford University.

Walsh, J. B., 1969. New analysis of attenuation in partially melted rocks, *J. geophys. Res.*, **74,** 4333–4337.

Wilkens, R., Simmons, G. & Caruso, L., 1984. The ratio v_P/v_S as a discriminant of composition for siliceous limestone, *Geophysics,* **49,** 1850–1860.

Wu, T. T., 1966. The effect of inclusion shape on the elastic moduli of two-phase material, *Int. J. Solids Struct.*, **2,** 1–8.

Wyllie, M. R. J., Gregory, A. R. & Gardner, L. W., 1956. Elastic wave velocities in heterogeneous and porous media, *Geophysics,* **21,** 41–70.

Zimmermann, R. W., Somerton, W. A. & King, M. S., 1986. Compressibility of porous rocks, *J. geophys. Res.*, **91,** 12 765–12 777.