Uncertainty analysis of rainfall depth duration frequency curves using the bootstrap resampling technique

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Rainfall depth duration frequency (DDF) curves are used extensively in many engineering designs. However, due to the sampling error and the uncertainty associated with the parameter estimation process, the DDF curves are subjected to parameter uncertainty. In this study, an evaluation of the uncertainty of the DDF curves in the Kelantan river basin was performed using the bootstrap resampling method. Annual maximum rainfall series for durations of 24, 48, 72, 96 and 120 h were derived from the stochastic rainfall model outputs and fitted to the generalised extreme value (GEV) distribution. The bootstrap samples were generated by resampling with replacement from the annual maximum rainfall series. The relationships that describe the GEV parameters as a function of duration were used to establish the DDF curves. The 95% confidence intervals were used as an indicator to quantify the uncertainty in the DDF curves. The bootstrap distribution of the rainfall depth quantiles was represented by a normal probability density function. The results showed that uncertainty increased with the return period and there was significant uncertainty in the DDF curves. The suggested procedure is expected to contribute to endeavours in obtaining reliable DDF curves, where the uncertainty features are assessed.

Keywords. Depth duration frequency curves; uncertainty; bootstrap resampling; generalised extreme value distribution.

1. Introduction

Rainfall depth duration frequency (DDF) curves are graphical representations of the rainfall depths at a given duration having desired probabilities of exceedance or return period. They are particularly needed to model the extreme rainfall characteristics for hydrological and engineering applications, such as the design of sediment control structures,

management of storm water facilities, risk analysis for flood hazards and the operations for urban drainage works (Brath *et al.* 2003; Jiang and Tung 2015; Sarhadi and Soulis 2017). This is especially true in Malaysia, which experiences extreme rainfall events (flash flood and monsoon flood) nearly every year, resulting in high economic losses, significant damages to property, degradation of water quality and loss of lives. However, the

quantification of uncertainty associated with the DDF curves is often neglected or simply disregarded. This may affect the reliability of the hydrological modelling and leads to significant errors that may affect the decisions drawn from the research analysis. Therefore, it is important to assess and incorporate the uncertainty features when deriving the rainfall DDF relationships to obtain reliable estimations of extreme rainfall.

In general, the rainfall DDF curves are constructed based on extreme rainfall series at the location of interest for various durations. The typical steps involved are as follows: (i) fitting of the probability distribution function to the annual maximum rainfall series for a specific duration; (ii) estimating the quantile function of extreme rainfall for each duration with the corresponding return period and (iii) deriving rainfall DDF relationships between the rainfall depth and duration. Previously, the Gumbel distribution has been widely used to fit and describe the characteristics of annual maximum rainfall (Wotling *et al.* 2000; Borga *et al.* 2005; Nadarajah and Choi 2007). However, several studies have shown that the Gumbel distribution tends to underestimate the extreme rainfall quantiles and extreme events like floods appear to have heavier tails than the Gumbel distribution (Coles *et al.* 2003; Malamud and Turcotte 2006; Sisson *et al.* 2006; Papalexiou and Koutsoyiannis 2013). This may consequently lead to the inaccurate estimation of hydrological risk, particularly for the design of hydraulic structures or water resource projects that involve high return periods. Alternatively, the generalised extreme value (GEV) distribution is used as a consistent probabilistic model, where the additional parameter can describe better the upper tail of the extreme rainfall. For this reason, GEV distribution was applied in this study. GEV distribution has been applied in several regions to model the rainfall extremes and shown to be the appropriate distribution to model the behaviour of the extreme events, such as in Malaysia (Zalina *et al.* 2002), Korea (Nadarajah and Choi 2007), India (Guhathakurta *et al.* 2011), Canada (Burn 2014), Iran (Mirzaei *et al.* 2015) and China (Yin *et al.* 2016).

The evaluation of uncertainty is often overlooked for the typical estimation of the DDF curves. The main limitation of the typical DDF procedure is that the parameter uncertainty is not determined. In hydrological modelling, uncertainty is important to provide information on the hydrological models for future risk assessments. The bootstrap resampling approach has been adopted for uncertainty assessment in hydrology due to its simplicity and versatility. Overeem *et al.* (2008) assessed the uncertainty associated with the DDF curves by using the bootstrap method and obtained the uncertainty bands and the bootstrap distribution of uncertainty. Hailegeorgis *et al.* (2013) utilised the bootstrap method to analyse the sampling uncertainty associated with the quantile estimation of extreme rainfall events in Norway. The rainfall samples were drawn with replacement from original rainfall series and the confidence intervals were derived from the resamples. The confidence bounds of the rainfall quantiles showed that the uncertainty was high in the estimates. Similar findings were reported by Mirzaei *et al.* (2015) who assessed the uncertainty associated with the DDF curves in Iran by using the bootstrap method and calculated the confidence bands of the DDF curves. It was observed that the uncertainty increases with the return periods. Hu *et al.* (2015) estimated the uncertainty of hydrological design values by making use of the bootstrap method to estimate the standard deviation. The bootstrap method can be used to evaluate uncertainty in terms of bias, standard deviation and confidence bands. It is frequently applied in various fields, such as flood forecasting, water resources planning and climate change modelling (Prudhomme *et al.* 2003; Tiwari and Chatterjee 2010; Noguchi *et al.* 2011).

Another notable issue in the uncertainty assessment is the lack of adequate sample information, such as small sample sizes and short maximum rainfall series. The insufficient information inevitably leads to inaccurate quantification of the uncertainty related to the whole hydrological process, especially in the modelling of extreme rainfall values where the available observation data sets are scarce. For this reason, a long and complete series of rainfall output generated from the stochastic rainfall model is particularly useful. The stochastic rainfall model is a statistical tool used to generate arbitrarily long and complete synthetic rainfall series, enabling a more thorough representation of rainfall variability. The stochastic generation of rainfall is based on a random process where the sequences of random numbers simulated from computer algorithms are transformed into sequences of synthetic data which are statistically similar to the historical data. There are wide varieties of the stochastic weather generator developed in different regions, consisting of different simulation procedures according to the weather condition of

the location of interest. The common examples are WGEN (Richardson 1981), LARS-WG (Semenov and Brooks 1999), MV-WG (Fodor *et al.* 2010) and WeaGETS (Chen *et al.* 2012).

Extreme rainfall event that leads to flooding is the most significant natural hazard in Malaysia which occurs due to very high intensity monsoon rainfall, heavy convective tropical rainstorms, rapid construction of buildings and other local factors. The Kelantan river basin is one of the areas in Malaysia where the monsoon flood has been occurring annually due to the northeast monsoon that brings heavy rainfalls and strong winds. The DDF curves or intensity duration frequency (IDF) curves have been established to describe the characteristics of the extreme rainfall and the information is used to aid in the design of flood mitigation infrastructures and assessment of flooding risk. The procedures of DDF and IDF curves construction are similar, except the former uses the maximum rainfall depth whereas the latter uses the maximum rainfall intensity. It is always important to evaluate the uncertainty of the DDF/IDF curves as the uncertainty is equally vital as the estimates themselves in any hydrological modelling. While many studies have been carried out to develop the DDF/IDF curves in this region (Ariff *et al.* 2012; Chang *et al.* 2013), researches on the evaluation of the uncertainty associated with the DDF/IDF curves are still rare. Lacking of the uncertainty information may lead to misleading predictions in the hydrological models and consequently pose much higher risk of failures in infrastructure design and implementation. It is crucial to carry out a comprehensive assessment of uncertainty estimates after the DDF model is constructed. Limited work on the uncertainty quantification of stochastic rainfall output had been done. In this study area which experiences tropical climate, no work on such matters has ever been carried out. The existing uncertainty analysis approaches usually quantify the uncertainty based on the historical rainfall series. For this study, the stochastic realisations of synthetic rainfall series were generated from the historical rainfall series using the stochastic rainfall generator. Therefore, the results of the uncertainty quantification or the estimated uncertainty were also from the stochastic rainfall generator. This paper is about the bootstrap-based methodological framework to investigate and evaluate the uncertainty of DDF curves based on the synthetic rainfall series. The bootstrap resampling technique was chosen due to its ease of implementation, conceptual simplicity and capability to provide accurate inferences without any distributional assumptions. The synthetic rainfall series were generated from a stochastic rainfall model, which was developed to generate the complete sequences of synthetic rainfall in the Kelantan river basin, Malaysia. The uncertainty analysis was carried out for 17 selected rainfall stations of the Kelantan river basin. The outcomes from this study can be used as a reference for the hydrological forecasting, infrastructure design, risk assessment and agricultural planning.

2. Study area

The Kelantan river basin is located at the northeastern part of Peninsular Malaysia between the latitudes $4°40'–6°12'$N and longitudes $101°20'–102°20'$E (figure 1). The region has a typical tropical climate due to the maritime exposure and proximity to the equator. The weather is hot and humid all the year round with uniform high temperature between $21°$C and $32°$C and high relative humidity ranging from 82% to 86%. It has an annual rainfall of about 2700 mm, which is mainly caused by the northeast monsoon that occurs from November to February. The northeast monsoon usually brings heavy rain to the study area and consequently leads to the monsoon floods. The southwest monsoon is considered as a drier period because the area is virtually sheltered by the Titiwangsa Range which runs down the peninsula. There are two inter-monsoon periods from March to April and September to October and convectional rain is expected during these periods.

3. Methodology

3.1 *Stochastic generation of rainfall series*

In this study, the historical daily rainfall series of 17 rainfall stations of the Kelantan river basin, Malaysia, were acquired from the Malaysian Meteorological Department (MMD). The spatial location and the summary statistics of all stations are given in figure 2 and table 1, respectively. A stochastic rainfall model was developed to stochastically generate a long and complete series of synthetic rainfall series based on the statistical characteristics of the historical rainfall series. Firstly, a simple but parsimonious two-state, first-order Markov model was used to simulate the rainfall
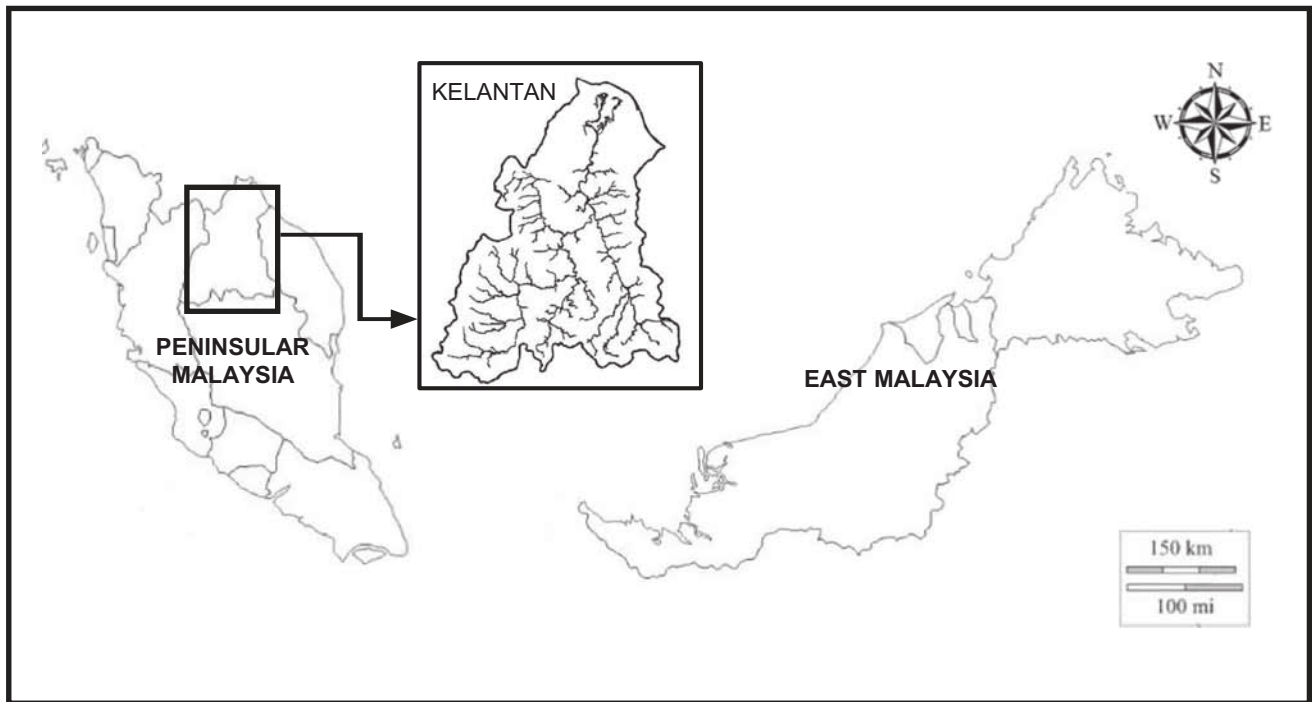
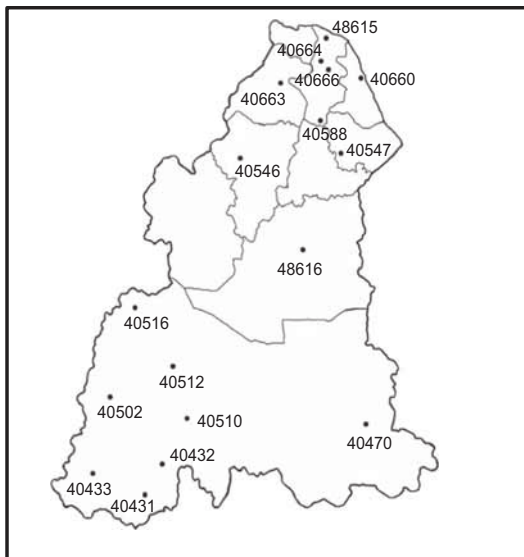Figure 1. Location of the Kelantan river basin in Malaysia.

Figure 2. Location of 17 rainfall stations in the Kelantan river basin.

occurrence by generating the sequences of the wet and dry days. The probability of the occurrence of rainfall depends on the condition of the previous day, which can be expressed by two transition probabilities:

$$P_{01} = \Pr \{ \text{precipitation on day } t \,| $$
$$\text{no precipitation on day } t-1 \}, \quad (1)$$

$$P_{11} = \Pr \{ \text{precipitation on day } t \,| $$
$$\text{precipitation on day } t-1 \}. \quad (2)$$

Based on the simulated wet days, the rainfall amounts were simulated by using two-parameter log-normal distribution. Its probability density function is given as

$$f(x) = \frac{1}{xq\sqrt{2\pi}} \exp\left(-\frac{(\text{In } x - s)^2}{2\,q^2} \right), q > 0, s > 0, \quad (3)$$

where q and s are the standard deviation and mean of the logarithmic transformed precipitation amount, respectively.

The parameters of the distributions were estimated using the maximum likelihood approach and subsequently used to simulate the synthetic daily rainfall series of five times the length of the historical rainfall series. In order to preserve the low-frequency variability of the rainfall, a spectral correction method suggested by Chen *et al.* (2010) was used to correct the monthly and interannual variability of the synthetic rainfall series.

The annual maximum rainfall series for event durations (D) of 24, 48, 72, 96 and 120 h (hour) were extracted from the synthetic rainfall series. Since the frequency analysis of the hydrological time series is based on the stationarity assumption,

Table 1. *List of rainfall stations used in this study.*

Station code	Station name	Record period	Latitude	Longitude
40431	Pos Blau	1978–2013	04°39′N	101°41′E
40432	RPS Kuala Betis	1974–2013	04°42′N	101°45′E
40433	Pos Hau	1978–2013	04°42′N	101°32′E
40470	Pos Lebir	1978–2013	04°56′N	102°23′E
40502	Pos Bihai	1978–2013	05°00′N	101°341′E
40510	Pos Tehoi	1978–2013	05°03′N	101°45′E
40512	Pos Wias	1978–2013	05°07′N	101°49′E
40516	Pos Gob	1978–2013	05°17′N	101°38′E
40547	Mardi Jeram Pasu	1984–2013	05°48′46″N	102°20′40″E
40546	Pusat Ternakan Haiwan Tanah Merah	1980–2013	5°48′40″N	102°00′33″E
40588	Pusat Pertanian Melor	1968–2013	05°58′N	102°18′E
40660	Pusat Pertanian Bachok	1974–2013	6°03′N	102°24′E
40663	Pusat Pertanian Pasir Mas	1976–2013	06°02′N	102°07′E
40664	Pusat Pertanian Lundang	1974–2013	06°06′N	02°14′E
40666	Mardi Kubang Keranji	1982–2013	06°05′N	102°17′E
48615	Kota Bharu	1954–2013	06°10′N	102°18′E
48616	Kuala Krai	1986–2013	05°32′N	102°12′E

the augmented Dickey–Fuller test (Dickey and Fuller 1979) was used to examine the stationarity of the annual maximum series. The augmented Dickey–Fuller test examines the temporal trends of the annual maximum series and tests for the presence of unit roots. The null hypothesis of the augmented Dickey–Fuller tests stated that there is a unit root for the data sets, indicating non-stationarity of the annual maximum series. The tests were conducted at the level of 5% significance.

3.2 *Bootstrap resampling technique*

In this study, the bootstrap resampling technique was employed to estimate the sampling distribution of statistics and quantify the uncertainty of the rainfall DDF curves. The main idea of the bootstrap is to generate large samples with replacement by resampling from the original samples based on the assumption that the samples are independent and identically distributed. The sample values are treated as similarly as possible to the underlying true distribution from which it is drawn. This method has been advocated not just because of its computational efficiency, but as an easy-to-implement approach that generates bootstrap replications without relying on the assumption of true distribution. It is applicable by just depending on the information obtained from the sample values.

The steps of the bootstrap method employed in this study are described as follows:

(1) Assume that the annual maximum rainfall series for the durations of 24, 48, 72, 96 and 120 h are the original samples, which are denoted as $X = (X_1, X_2, \ldots, X_n)$ for n = total length of annual maximum rainfall records.

(2) Produce bootstrap samples of annual maximum rainfall series by the bootstrapping process, which involves randomly picking values with replacement from the original sample. Resampling was carried out at all rainfall stations for each duration under consideration.

Repeat the bootstrap resampling process 10^4 times to generate 10^4 bootstrap samples, which are $X^* = (X_1^{*(i)}, X_2^{*(i)}, \ldots, X_n^{*(i)})$, where $i = 1, 2, \ldots, m$ and m is the size of the resampling (10^4). Such a large sample size of the bootstrap replications can ensure a reliable and accurate estimation of the statistical parameter of interest.

3.3 *Frequency analysis of extreme rainfall series*

3.3.1 *Fitting the GEV distribution*

To perform the uncertainty analysis of the DDF curves, a very crucial step is the frequency analysis of extreme rainfall events. Therefore, GEV distribution was adopted in this study to fit the 10^4 bootstrap samples of the annual maximum rainfall series. The cumulative distribution function of GEV distribution (Jenkinson 1955) can be expressed as

$$F(x) = \exp\left\{-\left[1+\frac{\gamma(x-\mu)}{\sigma}\right]^{-\frac{1}{\gamma}}\right\} \text{ for } \gamma \neq 0, \tag{4}$$

$$F(x) = \exp\left\{-\exp\left[-\frac{(x-\mu)}{\sigma}\right]\right\} \text{ for } \gamma = 0, \tag{5}$$

where x is the rainfall amount, γ, μ and σ are the shape, location and scale parameters, respectively. The quantile function of GEV distribution with the cumulative probability z is defined as

$$x_z = \mu + \frac{\sigma}{\gamma}\left[1-(-\ln(z))^{\gamma}\right], \quad \text{for } \gamma \neq 0, \tag{6}$$

$$x_z = \mu - \sigma\ln(-\ln(z)), \quad \text{for } \gamma = 0. \tag{7}$$

3.3.2 *Parameter estimation using L-moments*

The selection of the parameter estimation method usually considers the description of a wider range of distributions with the least computational efforts. The method of L-moments is preferable over the conventional moments because it is more robust to the sampling variability and outliers of data, relatively less biased in the estimation process, and free from sample-size-related bounds (Hosking 1990; Kyselý and Dubrovský 2005; Borujeni and Sulaiman 2009; Ngongondo *et al.* 2011; Hailegeorgis *et al.* 2013). Therefore, the method of L-moments was used in this study to derive the parameters of the GEV distribution as it can describe the location, skewness, kurtosis, shape and other aspects of the probability distribution. The L-moments are based on the probability weighted moment which can be expressed as (Hosking 1990)

$$P_r = (r+1)^{-1}\left\{\mu+\frac{\sigma}{\gamma}\left[1-(r+1)^{-\gamma}\Gamma(1+\gamma)\right]\right\}, \tag{8}$$

where Γ is the gamma function. The unbiased estimator of P_r is expressed as (Landwehr *et al.* 1979; Hosking and Wallis 1995)

$$\hat{P}_r = \frac{1}{n}\sum_{i=1}^{n}\frac{(i-1)(i-2)\ldots(i-r)}{(n-1)(n-2)\ldots(n-r)}x_i, \tag{9}$$

where x_i is the data value of the annual maximum arranged in ascending order. The sample L-moments are given as

$$\ell_1 = \hat{P}_0, \tag{10}$$

$$\ell_2 = 2\hat{P}_1 - \hat{P}_0, \tag{11}$$

$$\ell_3 = 6\hat{P}_2 - 6\hat{P}_1 + \hat{P}_0. \tag{12}$$

The estimates of the GEV parameters are subsequently estimated as

$$\hat{\gamma} = 7.8590c + 2.9554c^2, \tag{13}$$

$$\hat{\sigma} = \frac{\ell_2\hat{\gamma}}{\Gamma(1+\hat{\gamma})(1-2^{-\hat{\gamma}})}, \tag{14}$$

$$\hat{\mu} = \ell_1 + \frac{\hat{\sigma}}{\hat{\gamma}}\left[1-\Gamma(1+\hat{\gamma})\right], \tag{15}$$

where

$$c = \frac{2}{3+\ell_3/\ell_2} - \frac{\ln 2}{\ln 3}. \tag{16}$$

For the present study, $\hat{\alpha} = \hat{\sigma}/\hat{\mu}$ was considered. The annual maximum rainfall series of durations 24, 48, 72, 96 and 120 h at all stations were fitted to the GEV distribution by means of L-moments.

3.4 *Development of the rainfall DDF curves*

In this section, the rainfall DDF curves were derived from the relationships incorporating the GEV parameters as functions of duration. The rainfall depth, x (mm) was plotted against all considered durations for return periods of 2, 5, 10, 20, 50 and 100 yr. The general linear regression model was used to model the GEV parameters

Table 2. *Results of the validation tests (Mann–Whitney, K–S and squared ranks tests).*

	Monthly			Yearly		
	Mann–Whitney tests	K–S tests	Squared ranks tests	Mann–Whitney tests	K–S tests	Squared ranks tests
Wet spells	204/204	202/204	202/204	16/17	14/17	6/17
Dry spells	204/204	202/204	202/204	16/17	11/17	8/17
Rainfall amounts	203/204	189/204	202/204	17/17	15/17	17/17

$$\lambda = X\beta + e, \qquad (17)$$

where

Table 3. *Augmented Dickey–Fuller test results for the annual maximum rainfall series (D = 24 h).*

Station name	P-value
Pos Blau	<0.0001
RPS Kuala Betis	0.0002
Pos Hau	0.00271
Pos Lebir	<0.0001
Pos Bihai	<0.0001
Pos Tehoi	0.0006
Pos Wias	0.0044
Pos Gob	<0.0001
Mardi Jeram Pasu	0.0005
Pusat Ternakan Haiwan Tanah Merah	<0.0001
Pusat Pertanian Melor	0.0003
Pusat Pertanian Bachok	0.0010
Pusat Pertanian Pasir Mas	0.0002
Pusat Pertanian Lundang	<0.0001
Mardi Kubang Keranji	<0.0001
Kota Bharu	0.0001
Kuala Krai	<0.0001

$$X = \begin{pmatrix} 1 & \ln D_1 \\ . & . \\ 1 & \ln D_6 \end{pmatrix}, \qquad (18)$$

$$\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}. \qquad (19)$$

In general, λ is a $(n \times 1)$ vector of the GEV parameters of length n, X is a $(n \times k)$ known matrix of full rank, β is a $(p \times 1)$ vector of regression coefficients, e is a $(n \times 1)$ vector of random errors and D is the rainfall duration. The regression coefficients (β_1 and β_2) were determined by using the generalised least-squares method (Madsen and Rosbjerg 1997; Madsen *et al.* 2002; Haddad *et al.* 2010). The generalised least-squares method is able to consider the sampling variability explicitly and give an accurate estimation of the parameter sampling variances and parameter estimators. The
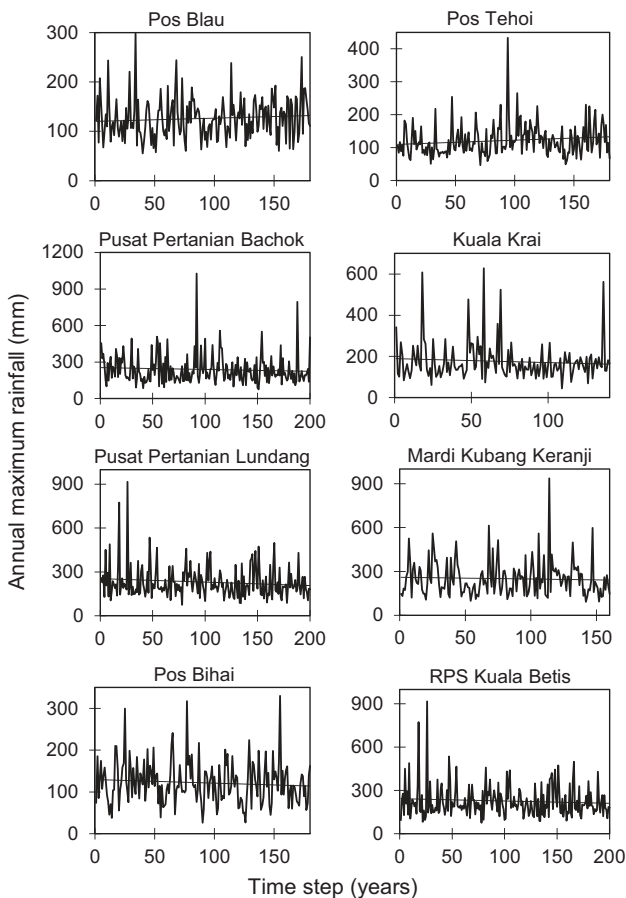

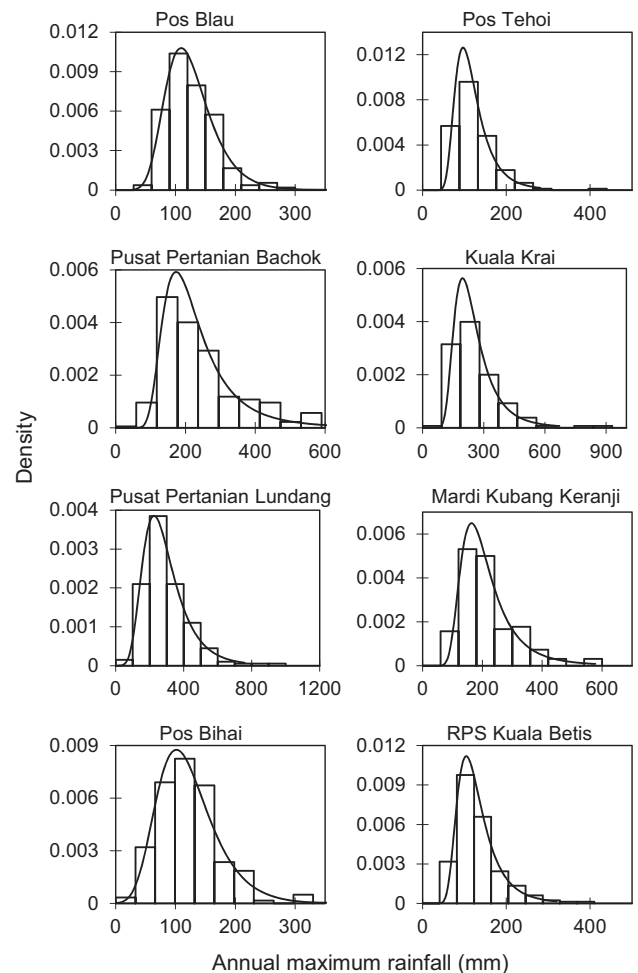Figure 3. Trends of annual maximum rainfall ($D = 24$ h) at eight randomly selected stations.

Figure 4. Histograms of annual maximum rainfall series ($D = 24$ h) fitted to the GEV distribution at eight randomly selected stations.

Table 4. *Estimated GEV parameters for $D = 24, 48, 72, 96$, and 120 h at eight randomly selected stations. Their corresponding standard deviations are given in brackets.*

	Pos Blau			Pos Tehoi		
D (h)	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\propto}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\propto}$
24	0.081 (0.057)	108.369 (3.011)	0.320 (0.018)	0.094 (0.068)	99.321 (2.817)	0.325 (0.020)
48	0.098 (0.060)	123.550 (3.274)	0.312 (0.018)	0.042 (0.046)	116.300 (2.710)	0.290 (0.023)
72	0.172 (0.104)	131.640 (3.701)	0.314 (0.020)	−0.003 (0.049)	127.008 (3.226)	0.310 (0.019)
96	0.186 (1.255)	135.740 (3.261)	0.325 (0.019)	−0.028 (0.059)	130.006 (3.724)	0.340 (0.018)
120	0.143 (0.072)	132.857 (4.060)	0.380 (0.028)	−0.044 (0.060)	125.222 (4.126)	0.385 (0.021)
	Pusat Pertanian Bachok			Kuala Krai		
D (h)	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\propto}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\propto}$
24	0.174 (0.06)	182.750 (5.875)	0.389 (0.023)	0.233 (0.078)	134.390 (4.340)	0.348 (0.028)
48	0.152 (0.059)	215.851 (6.506)	0.372 (0.022)	0.165 (0.071)	166.216 (5.315)	0.349 (0.027)
72	0.109 (0.046)	237.338 (7.003)	0.364 (0.021)	0.165 (0.065)	186.402 (6.116)	0.356 (0.026)
96	0.067 (0.048)	251.735 (7.761)	0.390 (0.024)	0.113 (0.068)	203.179 (6.679)	0.350 (0.023)
120	0.029 (0.047)	263.998 (8.565)	0.412 (0.025)	0.055 (0.071)	213.516 (7.598)	0.386 (0.029)
	Pusat Pertanian Lundang			Mardi Kubang Keranji		
D (h)	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\propto}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\propto}$
24	0.161 (0.059)	178.908 (5.329)	0.371 (0.023)	0.112 (0.064)	194.166 (7.449)	0.412 (0.022)
48	0.076 (0.057)	214.702 (6.906)	0.396 (0.022)	0.094 (0.066)	239.076 (8.172)	0.386 (0.024)
72	0.061 (0.056)	242.296 (7.492)	0.389 (0.021)	0.052 (0.054)	270.115 (8.775)	0.373 (0.025)
96	−0.001 (0.049)	262.682 (8.161)	0.397 (0.023)	−0.012 (0.055)	290.522 (10.878)	0.415 (0.023)
120	−0.022 (0.052)	274.365 (9.277)	0.425 (0.025)	−0.030 (0.058)	314.369 (12.237)	0.438 (0.028)
	Pos Bihai			RPS Kuala Betis		
D (h)	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\propto}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\propto}$
24	−0.075 (0.063)	100.220 (3.567)	0.425 (0.025)	0.119 (0.052)	107.422 (2.863)	0.335 (0.021)
48	−0.066 (0.067)	118.227 (3.972)	0.407 (0.025)	0.115 (0.057)	126.338 (3.140)	0.315 (0.020)
72	−0.089 (0.074)	132.460 (4.576)	0.418 (0.026)	0.099 (0.061)	139.931 (3.462)	0.313 (0.019)
96	−0.136 (0.055)	143.379 (5.136)	0.423 (0.024)	0.047 (0.063)	151.545 (3.929)	0.328 (0.019)
120	−0.114 (0.054)	151.243 (5.196)	0.410 (0.023)	−0.012 (0.054)	156.039 (4.240)	0.345 (0.020)

generalised least-squares estimate of the parameter vector is given as

$$\hat{\beta}_{\text{gls}} = \left(X^{\text{T}}C^{-1}X\right)^{-1} X^{\text{T}}C^{-1}\lambda, \qquad (20)$$

where C is the covariance matrix of λ. The generalised least-squares method was applied to the 10^4 bootstrap samples to obtain the regression coefficients and subsequently establish the relationship between the GEV parameters for various duration periods. Subsequently, the relationship between the GEV parameters as functions of the duration to derive the DDF curves is defined as:

3.5 *Uncertainty quantification of the rainfall DDF curves*

Based on the steps of modelling the rainfall DDF relationships as shown above, the estimation of the GEV parameters is subjected to uncertainty, which is known as the parameter uncertainty. The parameter uncertainty is due to the sampling error or the inability of the GEV distribution to describe the model inputs and parameters completely. For each duration (24, 48, 72, 96 and 120 h) of time step 1 h, there are 10^4 rainfall depths derived from the 10^4 rainfall DDF curves

$$\hat{x}\left(T\right) = \exp\left(\hat{\beta}_{1\mu} + \hat{\beta}_{2\mu} \ln D\right) \times \left(1 + \left(\hat{\beta}_{1\propto} + \hat{\beta}_{2\propto} \ln D\right) \frac{\left\{1 - \left[-\ln\left(1 - T^1\right)\right]^{\left(\hat{\beta}_{1\gamma} + \hat{\beta}_{2\gamma} \ln D\right)}\right\}}{\left(\hat{\beta}_{1\gamma} + \hat{\beta}_{2\gamma} \ln D\right)}\right). \qquad (21)$$

Table 5. *Estimated regression coefficients of eight randomly selected stations.*

GEV parameter	Pos Blau				Pos Tehoi			
	$\hat{\beta}_1$	$\sigma(\hat{\beta}_1)$	$\hat{\beta}_2$	$\sigma(\hat{\beta}_2)$	$\hat{\beta}_1$	$\sigma(\hat{\beta}_1)$	$\hat{\beta}_2$	$\sigma(\hat{\beta}_2)$
$\ln \mu$	4.426	0.012	0.146	0.002	4.275	0.011	0.156	0.003
$\propto$	0.295	0.016	−0.001	0.004	0.100	0.021	0.055	0.004
γ	0.113	0.062	−0.053	0.013	0.380	0.047	−0.088	0.011

GEV parameter	Pusat Pertanian Bachok				Kuala Krai			
	$\hat{\beta}_1$	$\sigma(\hat{\beta}_1)$	$\hat{\beta}_2$	$\sigma(\hat{\beta}_2)$	$\hat{\beta}_1$	$\sigma(\hat{\beta}_1)$	$\hat{\beta}_2$	$\sigma(\hat{\beta}_2)$
$\ln \mu$	4.548	0.037	0.214	0.010	3.953	0.021	0.298	0.003
$\propto$	0.126	0.054	0.063	0.013	0.328	0.023	0.007	0.004
γ	0.726	0.094	−0.150	0.021	0.386	0.097	−0.059	0.019

GEV parameter	Pusat Pertanian Lundang				Mardi Kubang Keranji			
	$\hat{\beta}_1$	$\sigma(\hat{\beta}_1)$	$\hat{\beta}_2$	$\sigma(\hat{\beta}_2)$	$\hat{\beta}_1$	$\sigma(\hat{\beta}_1)$	$\hat{\beta}_2$	$\sigma(\hat{\beta}_2)$
$\ln \mu$	4.349	0.019	0.268	0.004	4.315	0.022	0.299	0.003
$\propto$	0.254	0.025	0.035	0.005	0.351	0.026	0.012	0.006
γ	0.570	0.087	−0.127	0.017	0.419	0.084	−0.090	0.018

GEV parameter	Pos Bihai				RPS Kuala Betis			
	$\hat{\beta}_1$	$\sigma(\hat{\beta}_1)$	$\hat{\beta}_2$	$\sigma(\hat{\beta}_2)$	$\hat{\beta}_1$	$\sigma(\hat{\beta}_1)$	$\hat{\beta}_2$	$\sigma(\hat{\beta}_2)$
$\ln \mu$	3.731	0.026	0.258	0.004	3.811	0.020	0.245	0.002
$\propto$	0.449	0.020	−0.009	0.003	0.413	0.022	−0.018	0.003
γ	−0.035	0.072	−0.016	0.016	0.160	0.039	−0.029	0.010

and ranked in increasing order. The 95% confidence interval was used as an indicator to quantify the uncertainty of the estimation. The 95% confidence interval was denoted as the 2.5th (lower bound) and 97.5th percentiles (upper bound) of the bootstrap estimates. Therefore, the 250th and 9750th values were obtained from the ranked rainfall depths and plotted to estimate the 95% confidence interval for the rainfall depth quantile estimation.

Next, the distribution of the bootstrapped rainfall quantiles was represented by a normal distribution, which consists of two parameters, namely, mean and standard deviation. The standard deviation was generated from the 10^4 bootstrap samples and modelled as a function of the duration and the return period. The fitting procedure involved five durations ($D = 24, 48, 72, 96$ and 120 h) and six return periods ($T = 2, 5, 10, 20, 50$ and 100 yr). The ordinary least-squares method was adopted to estimate the regression coefficients. Finally, the lower bounds and upper bounds of the confidence intervals were fitted to the normal distribution and plotted on the DDF curves to describe the uncertainty of the DDF curves.

4. Results and discussions

4.1 *Analysis of the synthetic rainfall series*

The ability of the stochastic rainfall model to represent the mean, frequency distribution and standard deviation of rainfall was evaluated by using the Mann–Whitney, Kolmogorov–Smirnov (K–S) and squared ranks tests. The results of the validation tests for the station–month (17 stations × 12 months) and station–year (17 stations × 1 yr) combinations are presented in table 2. It appears that most of the station–month and station–year combinations of the rainfall occurrences and rainfall amounts passed all three validation tests. For example, the monthly wet spells and dry spells of all 204 station–month combinations passed the Mann–Whitney tests. This indicated that the first-order Markov model and the log-normal distribution can reproduce the important properties of the rainfall occurrences and rainfall amounts, respectively. Therefore, the synthetic rainfall series were well validated and reliable to be used for further analysis.

Next, the results of the stationarity of the annual maximum rainfall series are presented in

table 3. It can be observed that the computed *P*-values of all rainfall stations were lower than the significance level of 5%. This implied that there were no unit roots for the annual maximum rainfall series and the alternative hypothesis that stated that the time series are stationary was accepted. Figure 3 further shows that the annual maximum rainfall series fitted with its temporal trend. For simplicity and without the loss of generality, the results are presented for the eight randomly selected rainfall stations out of the total 17 rainfall stations. It is apparent from figure 3 that the series appears to be constant over time. This indicated that the trend of the annual maximum rainfall series with time was not statistically significant, and thus, the stationarity assumption was valid.

4.2 *Goodness-of-fit and parameters of the GEV distribution*

GEV distribution has been applied in the frequency analysis of the annual maximum rainfall series where the main concern is to model the behaviour of the upper tail. The histograms of the GEV distribution fit were constructed to inspect the predictive ability of the distribution. The density values were plotted as the *y*-axis values. The density value *D* is defined as:

$$D = \frac{fr}{n*w}, \qquad (22)$$

where fr is the frequency of each interval, n is the total number of samples and w is the width of each interval. Figure 4 shows the excellent fit of the GEV distribution as the histograms of the annual maximum rainfall series were a good match with the probability density function of the fitted GEV model. This confirmed that the selection of the GEV distribution for the frequency analysis of this study area is a practically good choice. GEV distribution has the additional shape parameter other than the scale and location parameters for a better description of the upper tail of the extreme rainfall series. The results are consistent with those of Zalina *et al.* (2002) and Win and Win (2014) who found that GEV distribution gave good performance in fitting the maximum rainfall series in several areas of Malaysia.

The estimated GEV parameters $(\hat{\gamma}, \hat{\mu}, \hat{\propto})$ and their corresponding standard deviations are presented in table 4. The values of the standard deviation reflect the presence of possible uncertainty arising from sampling variability. It was
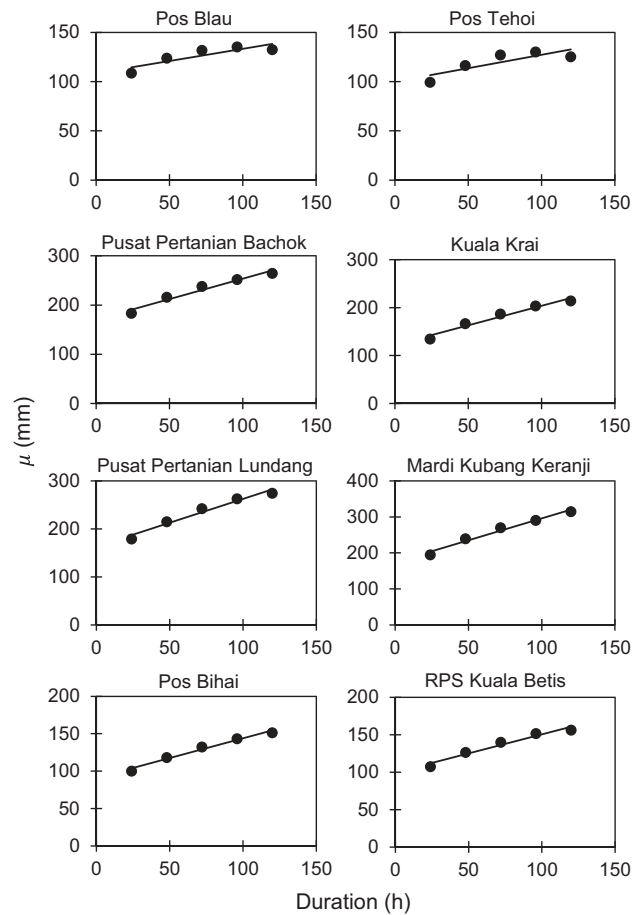
Figure 5. Parameter μ plotted against duration *D* at the eight randomly selected stations.

shown that the values of parameter $\hat{\mu}$ increased with the event durations. This is in good agreement with the findings of Overeem *et al.* (2008) and Mirzaei *et al.* (2013). For the parameters $\hat{\gamma}$ and $\hat{\propto}$, they were unlikely to decrease with decreasing duration. The mean and standard deviation of the estimated regression coefficients were calculated and are presented in table 5. It was shown that there were systematic variations of the GEV parameters with the durations as the estimates of two slopes (β_1 and β_2) were not zero. This is further displayed in figure 5 that the parameter μ increased with the durations.

4.3 *Uncertainty features of DDF curves*

The DDF curves relate the rainfall depth, duration and frequency to generate the rainfall frequency at any intermediate duration and return period. The resulting DDF curves are presented in figure 6 for the eight randomly selected stations. It was shown that the rainfall depths increase with the durations
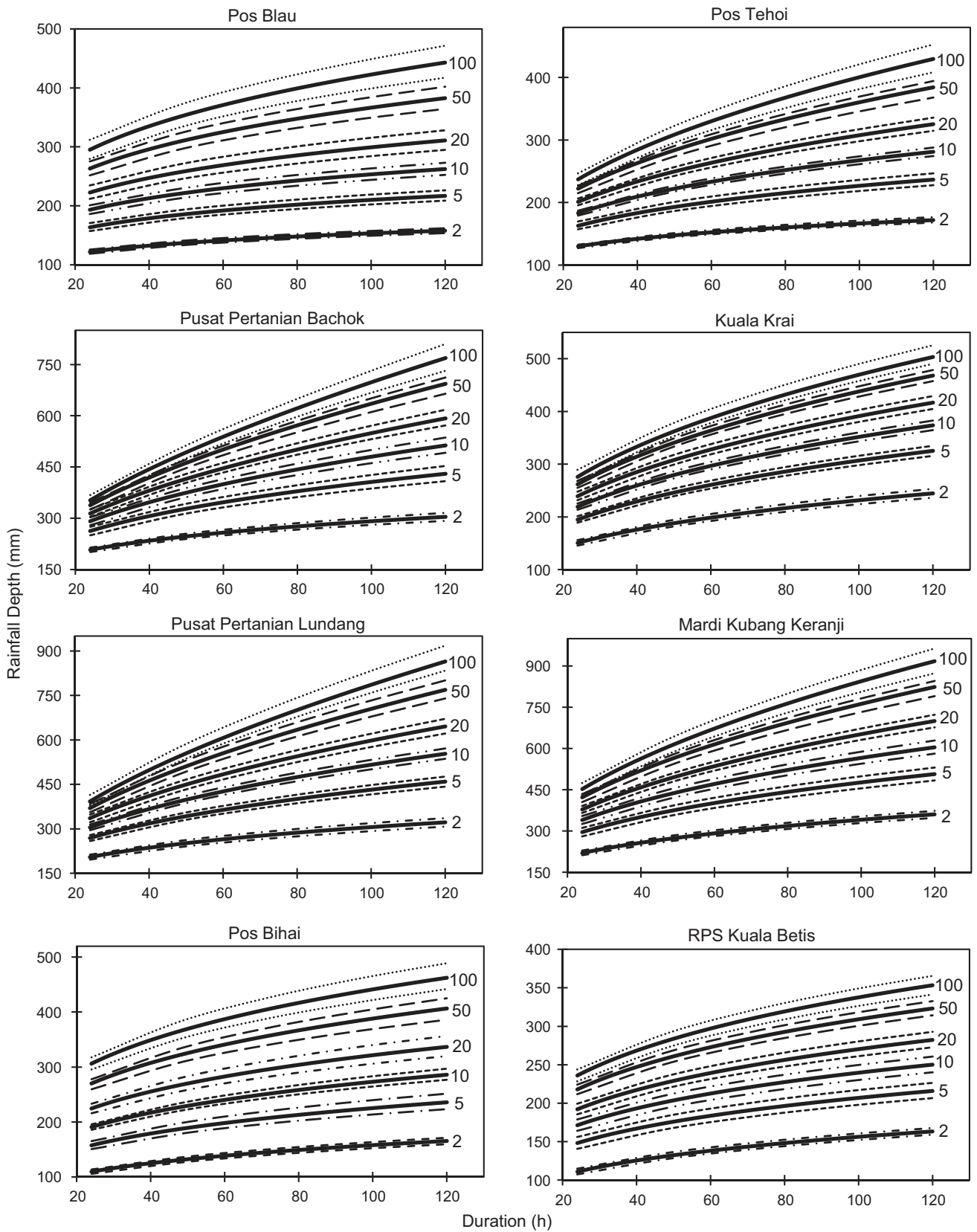
Figure 6. Rainfall DDF curves (solid lines) and 95% confidence intervals (dashed lines) for the return periods of 2, 5, 10, 20, 50, and 100 yr at eight randomly selected stations.
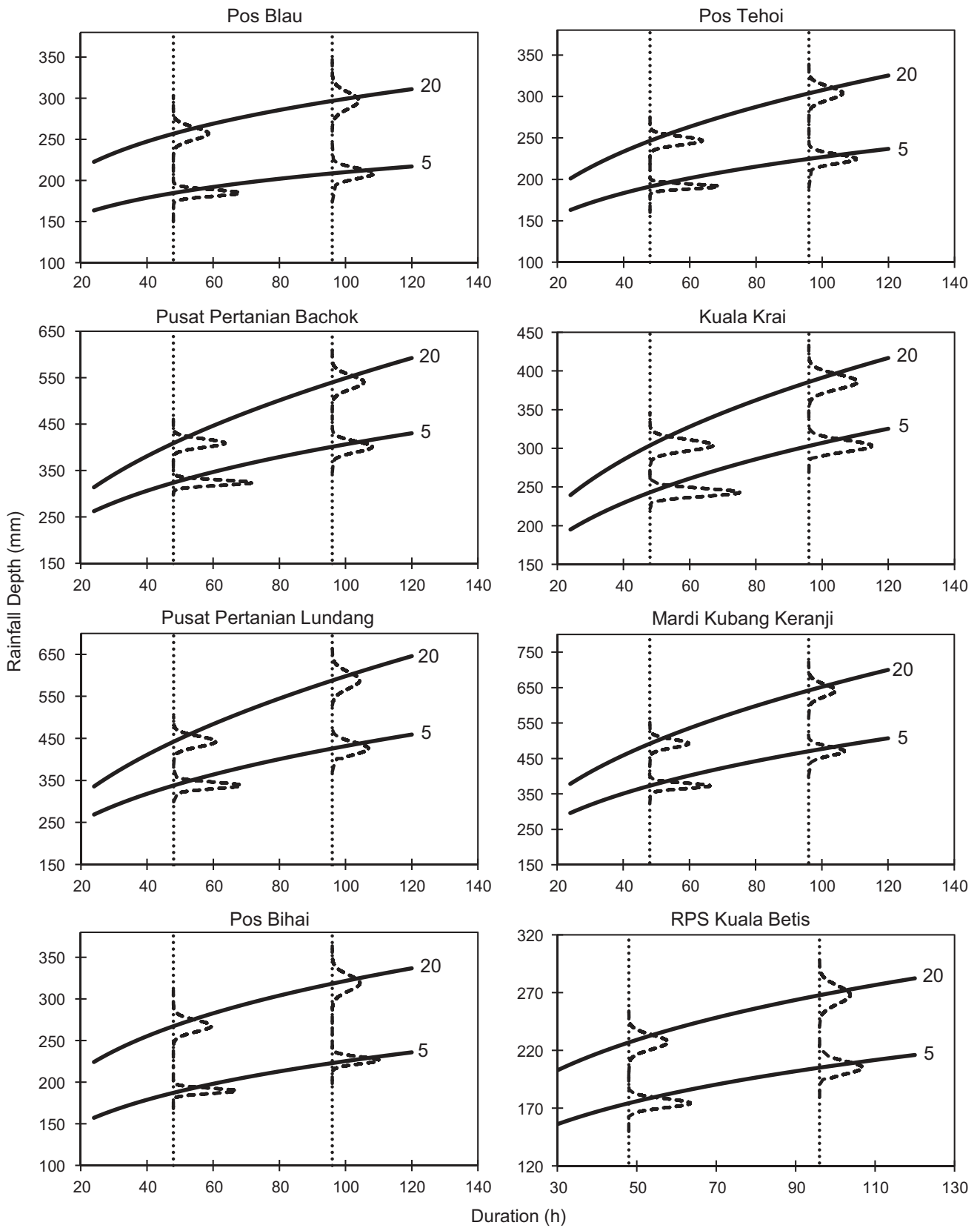
Figure 7. Rainfall DDF curves (solid lines) and the normal probability density functions (dashed lines) which describe the bootstrap distribution at $D = 48$ and 96 h.

of all return periods. For example, as shown in the Pos Blau station, for a duration of 24–120 h within a 100-yr return period, the rainfall depths showed an increase from 295 to 443 mm.

From the confidence bounds of the DDF curves, it was apparent that with the increasing return periods, the lower bounds and upper bounds of the 95% confidence interval were getting larger. For instance, as shown at the Pos Blau station, the 95% confidence interval of the rainfall depth quantiles for a duration of 24 h varies from 119 to 125 mm for a return period of 2 yr to 280 to 312 mm for a return period of 100 yr. The uncertainty increased with the increase of the return period, implying significant uncertainty present in the extreme rainfall quantiles of the DDF curves. Also, the level of uncertainty at each station is different as there are different topography and characteristics at each particular location.

As shown in figure 7, the normal probability density functions of the DDF curves were plotted at $T = 5$ and 20 yr to describe the uncertainty of the DDF curves. The results indicated that the uncertainty regarding the parameter estimation in the DDF curves owing to the sampling error can be considerable, especially for long return periods. These uncertainty ranges need to be addressed in any hydrological analysis and may have an adverse impact on the design of the hydraulic structures and decision-making for extreme hydrologic events if they are not seriously taken into consideration.

5. Conclusions

The quantification of uncertainty in DDF curves is crucial to investigate the implication of uncertainty on the performance reliability of the hydrologic risk analysis and design. This is especially important for a country like Malaysia where floods are considered the most devastating natural disaster. Thus, the DDF curves with quantified uncertainty bounds would help in the estimation of reliable rainfall DDF relationships and quantify the uncertainty associated with the flood risk assessment tasks.

This study presents the bootstrap procedure to evaluate the uncertainty features of DDF curves in the Kelantan river basin. The annual maximum rainfall series of durations between 24 and 120 h were derived from the stochastic rainfall model outputs. The bootstrap samples were fitted to the GEV distribution and the relevant parameters were determined through the L-moments approach. The

GEV parameters were modelled as a function of duration and these relations were used to establish the rainfall DDF curves. Finally, the uncertainty in DDF curves due to sampling variability is quantified in terms of 95% confidence intervals based on bootstrap resampling and the bootstrap distribution was represented by a normal distribution. The computation results showed that with the increase of the return period, the lower and upper bounds of the 95% confidence intervals were getting larger, which indicated that the uncertainty of the DDF curves was increasing. It was also shown that there was a significant uncertainty in the extreme rainfall quantile estimation due to the sampling of data. This implies that neglecting the uncertainty can result in great risks associated with the hydrological modelling and subsequent design decisions.

In summary, the proposed bootstrap procedure is effective, easy to implement and able to quantify the uncertainty in a statistic. It can be applied for other rainfall frequency analysis to derive the rainfall DDF relationships but the concluding remarks may not be applicable because the results may vary among different model structures. It should be noted that this study only considers the parameter uncertainty, which may not completely describe the uncertainty in a modelling process. In a thorough uncertainty analysis, a comprehensive assessment of various sources of uncertainty should be performed. Therefore, further investigations are needed to study other sources of uncertainty, such as the data uncertainty that includes measurement errors, the operational uncertainty caused by human factors that often are being neglected in a modelling procedure and the natural uncertainty due to the intrinsic randomness of the natural process.

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Corresponding editor: A K Sahai