working paper
department
of economics

UNCERTAINTY AND OPTIMAL SOCIAL SECURITY SYSTEMS

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Number 225 October 1978

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1. Introduction

The effects of social security have been discussed within the framework of an overlapping generations model by Barro (1974).[1] His analysis can be summarized quite simply. If social security is fully funded, i.e. contributions to the system are invested at the market rate of interest, it is a perfect substitute for private savings. Consequently, a forced increase in social security will reduce private savings by an equal amount. Consumption, bequests and aggregate savings will be unaffected. If social security is financed on a pay-as-you-go basis, i.e. taxes on the currently working population are used to finance benefits to the retired population, it is a perfect substitute for private bequests. Hence, a forced increase in social security will reduce bequests by an equal amount. Consumption, private savings and aggregate savings will be unaffected. In such models the optimal level of social security is clearly undetermined.

In this paper we analyze social security in a somewhat more realistic context. The duration of life is assumed uncertain and the annuity

* The Hebrew University of Jerusalem and Tel-Aviv University. This research was partially supported by National Science Foundation Grant SOC-00587.

aspect of social security benefits is incorporated explicitly. We use a simplified two-period version of Yaari's model (1965) to determine the demand for annuities. Optimal social security is defined as the amount of annuities demanded by a representative individual. Our working hypothesis is that all such insurance is provided publicly. Although in principle the private market could also satisfy this demand, in practice the public supply appears to be dominant.[2]

The demand for social security depends upon the mode of financing. Under a fully funded system, demand is determined by each generation so as to maximize its lifetime utility, taking into account the welfare of future generations. A pay-as-you-go system involves transfers from the younger generation to the old. Demand is therefore defined from the point of view of the old, taking into account the social security taxes paid by the young.

Under an actuarially fair, fully funded social security system, perfect insurance against life uncertainty is feasible. The representative individual will prefer this option which enables him to equate the marginal utility of bequests across states of nature. Consequently, at the optimum private savings are reserved for bequests while social security benefits are used solely for consumption. The same pattern appears in an optimal pay-as-you-go system.

A steady-state is defined as a path along which all the choice variables, in per-capita terms, remain fixed. There is a unique steady-state path associated with each mode of financing the social security system. It is shown that for any values of the exogeneous variables, these steady-state paths are equivalent in the following sense:

(a) Consumption, aggregate savings and individual utility are the same in both systems;

(b) the sum of private savings and social security contributions under a fully-funded system is equal to private savings under a pay-as-you-go system and

(c) the product of the population growth rate and the level of social security taxes under a pay-as-you-go system is equal to the product of the rate of return and the level of social security taxes under a fully-funded system. As an immediate implication, when the rate of return exceeds the population growth rate then the optimal level of social security under a pay-as-you-go system exceeds the one under a fully-funded system. These conclusions are made consistent through adjustments in the optimal level of bequests.

We analyze the effects of two demographic changes which were the subject of recent discussion: an increase in life expectancy and a decrease in the birth rate. Under certain conditions, an increase in life expectancy is shown to increase the optimal level of social security, to decrease private savings and to increase aggregate savings. These results are expected since, with a given income, an increase in life expectancy calls for a decrease in the flow of lifetime consumption but to an increase in total consumption during the retirement period, which is financed by social security benefits. The implications of a decrease in the birth rate are unambiguous: bequests, private and aggregate savings decrease, while the optimal level of social security increases. The reason for these results is that a reduction in the birth rate and, correspondingly, in the size of optimal bequests, increases consumption during retirement and hence social security benefits. The above results apply to the long-run, comparing steady-states, and to the short-run, with given initial endowments.

Finally, we consider the short-run effects of an imposed change in the level of social security. Starting at the optimum, an increase in the level of a fully-funded social security system is shown to be only partially compensated by a decrease in private savings, thereby increasing aggregate savings. Full compensation occurs only in the absence of uncertainty, which is the case discussed by Barro (1974). Under additional assumptions concerning risk-aversion, the same conclusions apply when the initial level of social security exceeds the optimal level. An imposed increase in the level of a social security system based on pay-as-you-go will increase aggregate savings and bequests. However, under uncertainty, the increase in bequests does not fully compensate for the increased taxes on the younger generation.

A nonoptimal level of social security leads to a random distribution of bequests, generated by lifetime uncertainty. Consequently, steady-states and the long-run effects of a change in social security should be discussed in terms of stable distributions. We have not included here such an analysis.

The only type of uncertainty considered in this paper relates to the duration of life. The inclusion of other uncertain elements, such as health and future wage income, may change some of the conclusions that we have reached. It will be important in these extensions to specify which decisions by the individual are made *ex ante* and which *ex post*.

2. Optimal Social Security Systems

We shall analyze a model in which all annuities are supplied by the public sector through a social security system and focus on the determination of its optimal level.

Two methods of financing will be considered: a fully-funded system and a system based on a pay-as-you-go principle.

Our point of departure from the previous literature is the emphasis on life uncertainty, which implies that social security is not a perfect substitute for private savings. We extend the standard model of overlapping generations (Samuelson [1958] and Diamond [1965]) to include uncertain lifetime. Life of a representative individual is divided into two periods: a working period of fixed duration and a retirement period whose length is random. Wages in each period are assumed fixed. Utility depends upon own consumption in each period, the length of life and the expected utility of the next generation. Each generation may affect the welfare of the next one through the transfer of bequests.

Two types of assets are available: an annuity which yields a given return for the duration of retirement and regular savings. Due to the uncertain lifetime, the rate of return on annuities is random. We assume that the rate of return on savings is certain.

Let c_0^i and c_1^i be the consumption flows in the first and second periods, respectively, of generation i. Let B^i be the level of per-capita bequests left by generation i to generation $i + 1$. The fraction of the potential retirement period which is actually realized is denoted by θ, $0 \leq \theta \leq 1$. The distribution of θ is assumed to be the same for all generations.

The budget constraint for a representative member of generation i is

$$(2.1) \qquad \begin{aligned} c_0^i &= w + B^{i-1} - a^i - s^i \\ GB^i &= Rs^i + (R'a^i - c_1^i)\theta \end{aligned}$$

where w is first-period earnings, a^i the investment in annuities and s^i the amount of savings, R the return on savings, R' the benefit-investment ratio on annuities and G the number of children. It is assumed that w, R, R' and G are fixed. Note also that there is no reinvestment of funds during the second period.

The determination of R' depends on the method of financing the social security system. In an actuarially fair, fully-funded system, expected benefits are equal to the return on the investment in the system. That is,

$$(2.2) \qquad R' = \frac{R}{\bar{\theta}}$$

where $\bar{\theta}$ is the expected length of the retirement period in the population.

With a pay-as-you-go system, expected benefits to retirees are equal to social security taxes collected from the working population, i.e.,

$$(2.2') \qquad R' = \frac{G}{\bar{\theta}} .$$

Equations (2.2) and (2.2′) assume that risk pooling is feasible. The social security system can therefore offer a nonrandom benefit rate to each individual.

We shall first analyze a funded system.

A. Funded Social Security

Since a funded system does not involve intergenerational transfers, its optimality can be analyzed from the point of view of a single generation.

For a given optimal policy of the next generation, each generation's indirect utility depends on its initial endowment. We can therefore write the expected utility of generation i as a function of its consumption and the level of bequests. We assume the following additive form:

$$V^i = u(c_0^i) + \underset{\theta}{E}[v(c_1^i, \theta) + Gh(B^i)] \tag{2.3}$$

where $u(c_0^i)$ is first-period utility, $v(c_1^i, \theta)$ second-period utility and $h(B^i)$ is the evaluation of the next generation's representative individual indirect utility. Each of these functions is assumed to be invariant over time and to satisfy, for any θ, the usual monotonicity and strict concavity assumptions. Note that both c_1, the fixed flow of consumption, and θ, its duration, affect second period's utility.

All the variables, except B^i, are chosen by the individual *ex ante*, i.e., prior to the realization of θ. To ensure an interior solution, we assume that

(2.4) $$u'(0) = v_1(0,\theta) = h'(0) = \infty, \text{ for any } \theta,$$

where $v_1 = \frac{\partial v}{\partial c_1}$. The maximization of (2.3) subject to (2.1) yields the following first-order conditions (F.O.C.):

(2.5) $$\frac{\partial V^i}{\partial s} = -u'(c_0^i) + RE[h'(B^i)] = 0$$

(2.6) $$\frac{\partial V^i}{\partial a} = -u'(c_0^i) + \frac{R}{\bar{\theta}} E[\theta h'(B^i)] = 0$$

(2.7) $$\frac{\partial V^i}{\partial c_1} = E[v_1(c_1^i, \theta) - \theta h'(B^i)] = 0 .$$

The objective function can be shown to be globally strictly concave in the variables s, a and c_1, and thus the solution to (2.5) - (2.7) is unique.[3] For following reference we shall denote the solution for each i by $(s^{*i}, a^{*i}, c_1^{*i})$, which are functions of B^{i-1}. By (2.1) we obtain the corresponding c_0^{*i} and B^{*i}.

Due to assumption (2.4), only interior solutions need to be considered. It follows directly from these conditions that $c_0^* > 0$, $c_1^* > 0$ and $B^* > 0$ for all θ. Furthermore, a^* and s^* must be strictly positive. First, $B^* > 0$ implies, by (2.1), that either a^* or s^* is positive. Second, suppose that $s^* = 0$ and $\frac{\partial V}{\partial s} \leq 0$. It then follows from (2.5) - (2.6) that $\text{Cov}[h'(B), \theta] \geq 0$. However, by (2.1), $Ra^* - c_1^* > 0$ and hence B is increasing in θ. Due to risk aversion, $h''(B) < 0$, the above covariance must therefore be negative, which is a contradiction. A similar argument shows that $a^* > 0$.

The condition

$$\text{(2.8)} \qquad \mathrm{Cov}[\theta, h'(B)] = E[\theta h'(B)] - \bar{\theta}E[h'(B)] = 0$$

which follows from (2.5) and (2.6) implies, since B is monotone in θ and $h'' < 0$, that B^* is <u>constant</u> for all θ. This, of course, is a well-known result. Annuities enable the individual to transfer consumption across states of nature and it is optimal to equate the marginal utility of bequests, $h'(B)$, in all states. Further, it follows from (2.1) that

$$\text{(2.9)} \qquad c_1^{*i} = \frac{R}{\bar{\theta}} a^{*i} \quad \text{and} \quad GB^{*i} = Rs^{*i}.$$

That is, annuities are used exclusively for consumption during retirement while all savings are reserved for bequests. Clearly, in the absence of a bequest motive, $h'(B) \equiv 0$, the individual's portfolio would contain only annuities.

Substituting (2.9) into (2.3), the objective function becomes

$$\text{(2.10)} \qquad V^i = u(c_0^i) + E\left[v\left(\frac{R}{\bar{\theta}} a^i, \theta\right)\right] + Gh\left(\frac{Rs^i}{G}\right),$$

which is to be maximized subject to the constraint $c_0^i + a^i + s^i = w + B^{i-1}$. The problem is thus reduced to a problem of choice under certainty. This does not imply, of course, that uncertainty is redundant in our model. On the contrary, in the absence of life uncertainty the optimal level of social security is indeterminate, being a perfect substitute with private savings. However, the achievement of complete insurance at the optimum facilitates the comparative statics analysis.

Two types of comparative statics analyses are of interest. One, a short-run analysis for generation i which holds initial endowments, i.e., $w+B^{i-1}$, constant. Second, a long-run steady-state analysis along a path satisfying $B^{i-1} = B^i$, which implies that all the choice variables remain fixed. In both cases we restrict ourselves to partial equilibrium, holding w and R constant.

We shall discuss here the steady-state effects of two demographic changes which were the subject of recent discussion: an increase in life expectancy and a decrease in the birth rate.

Recalling that $h(B^i) = F(V^{*i+1}(B^i))$, where V^{*i+1} is the optimal level of utility for generation i+1 and F is a monotone increasing function, we can rewrite condition (2.5) in recursive form

(2.5′) $$-u'(c_0^i) + RF'(V^{*i+1})u'(c_0^{i+1}) = 0\,.$$

Hence, in a steady-state, $c_0^i = c_0^{i+1}$, we have $F' = \frac{h'}{u'} = \frac{1}{R}$. A sufficient condition for a unique steady-state is that when marginal utilities of consumption are equalized across generations, the following "selfishness" condition is satisfied:

(2.11) $$u'' - Rh'' > 0\,.$$

In subsequent discussion we shall assume that condition (2.11) holds.

Using (2.9), the F.O.C. (2.5)-(2.7) and rewriting $\theta = \overline{\theta} + \varepsilon$, where ε is a random variable independent of $\overline{\theta}$ with zero mean,[4] we obtain

$$\frac{da^*}{d\bar{\theta}} = \frac{1}{\Delta} h'\left(u'' - \frac{R}{G}(u'' - Rh'')\right)\left(- \frac{E[v_{11}]c_1}{E[v_1]} + \frac{E[v_{12}]\bar{\theta}}{E[v_1]} - 1\right) \tag{2.12}$$

$$\frac{ds^*}{d\bar{\theta}} = \frac{1}{\Delta} h'u''\left(\frac{E[v_{11}]c_1}{E[v_1]} - \frac{E[v_{12}]\bar{\theta}}{E[v_1]} + 1\right) \tag{2.13}$$

where $\Delta = - \frac{R\bar{\theta}}{G} h''u'' - \left(u''\left(1 - \frac{R}{G}\right) + \frac{R^2 h''}{G}\right) \frac{R}{\bar{\theta}} E[v_{11}] < 0$.

Similar to standard models of savings under uncertainty, the effect of a spread preserving shift depends upon whether the relative change in expected marginal utility of future consumption, $- \frac{E[v_{11}]c_1}{E[v_1]} + \frac{E[v_{12}]\bar{\theta}}{E[v_1]}$ is larger or smaller than one. An increase in $\bar{\theta}$ affects expected marginal utility directly and indirectly through the decrease in the flow of social security benefits during retirement. It is reasonable to assume that $v_{12} > 0$ and thus the sum of the above two expressions will be positive. It is only with stronger conditions, however, that one obtains the "intuitive" outcome $\frac{da^*}{d\bar{\theta}} > 0$ and $\frac{ds^*}{d\bar{\theta}} < 0$.[5] The effect on aggregate savings, i.e., $a^* + s^*$, also depends on the same condition. The source of the ambiguity concerning the optimal level of social security is quite clear. For given labor inputs and income, an increase in life expectancy calls for a reduction in the flow of second-period consumption, c_1. However, expected total consumption, $\bar{\theta}c_1$, which is provided by social security taxes, may increase or decrease.

The implications of a decrease in the birth rate are unambiguous. A reduction in G leads to a decrease in the size of bequests and hence in private savings. Per-capita bequests will, however, increase and so will second-period consumption and thus the optimal level of social security. While a^* and s^* change in opposite directions, their sum, i.e., aggregate savings, will decrease. Specifically,

$$\text{(2.14)} \qquad \frac{ds^*}{dG} = -\frac{Rs^*}{\Delta G^2}\left(\bar{\theta}u''h'' - \frac{R}{\bar{\theta}}E[v_{11}](u''-Rh'')\right) > 0$$

$$\text{(2.15)} \qquad \frac{dc_1^*}{dG} = \frac{R}{\bar{\theta}}\frac{da^*}{dG} = \frac{R^2 s}{\Delta G^2}h''u'' < 0$$

$$\text{(2.16)} \qquad \frac{dB^*}{dG} = \frac{R^2 s^*}{\Delta G^2 \bar{\theta}}u''E[v_{11}] < 0$$

$$\text{(2.17)} \qquad \frac{d(a^* + s^*)}{dG} = \frac{R^2 s^*}{\Delta G\bar{\theta}}E[v_{11}](u''-Rh'') > 0 .$$

The short-run effects of changes in $\bar{\theta}$ and in G are qualitatively identical to the long-run effects presented above. The usual relation between the short-run and long-run effects holds in this case: the long-run elasticities, across steady-states, exceed (in absolute values) the short-run elasticities.

B. Pay-As-You-Go System

A pay-as-you-go system involves transfers across generations. An optimal social security must weigh the welfare of the retired recipients against that of working population. We assume that the social planner's point of view is identical in this respect with that of the older generation at each point of time. At a point in time when the population consists of generations i and i+1, the problem solved by a member of the older generation i is

$$\max_{a^i, c_1^i} E[v(c_1^i, \theta) + Gh(B^i - a^i)] \tag{2.18}$$

subject to

$$GB^i = Rs^i + \left(\frac{Ga^i}{\bar{\theta}} - c_1^i\right)\theta \tag{2.19}$$

where s^i is predetermined. The F.O.C. are given by

$$E[h'(B^i - a^i)(\theta - \bar{\theta})] = 0 \tag{2.20}$$

and

$$E[v_1(c_1^i, \theta) - \theta h'(B^i - a^i)] = 0\,. \tag{2.21}$$

We shall denote the solution to (2.20) and (2.21) by $(\hat{a}^i, \hat{c}_1^i)$ which depends on s^i. The nature of the optimal solution is similar to the one obtained in the case of the funded system; namely, social security benefits are used solely for consumption. Bequests are adjusted to the level of private savings and are thus nonrandom:

$$G\hat{a}^i = \hat{c}_1^i \bar{\theta}\,, \qquad \hat{B}^i = \frac{Rs^i}{G}\,. \tag{2.22}$$

The long-run, steady-state effects can be analyzed with the additional restriction that $\hat{a}^i$ is time invariant, which implies that all the other choice variables are also constant. Accordingly, in addition to conditions (2.20)-(2.21) we also have condition (2.23) which determines the optimal level of savings

$$(2.23) \qquad -u'(c_0) + RE[h'(B-a)] = 0$$

where $c_0 = w + \frac{Rs}{G} - a - s$. The endogeneous steady-state level of s is denoted by $\hat{s}$.

There is a simple correspondence between the steady-state solutions of the two methods of financing the social security system:

$$(2.24) \qquad \hat{a} = \frac{R}{G} a^* \quad \text{and} \quad \hat{s} = s^* + a^* .$$

Using relation (2.24), it can be verified that the system of equations (2.5)-(2.7) and (2.21)-(2.23) are equivalent.

When $R = G$ the two systems yield identical optimal levels of social security and the same aggregate savings. When $R > (<) G$, the optimal social security tax will be larger (smaller) under a pay-as-you-go system than under a funded system. However, bequests adjust so that the net transfer between generations remains unchanged. Consequently, the steady-state levels of consumption, aggregate savings and utility are identical under the two systems.

This result has a direct bearing on the Feldstein-Barro controversy (Feldstein [1974, 1976] and Barro [1974, 1976]) with regard to the effect of social security on aggregate savings. Our discussion indicates that when social security is optimally chosen, the method of financing has no long-run effects. The concept of an optimal level of social security played no role in the above controversy since lifetime uncertainty has been disregarded. The discussion focused on imposed changes in the level of social security. We shall turn to this question in the next section.

Short-run analysis for the pay-as-you-go system can be applied to the older generation, for which private savings are given. Thus, condition (2.23) need not hold. The results w.r.t. the optimal level of social security are qualitatively the same as in the long-run.

3. Departures from the Optimal Level of Social Security

There are marked differences in the effects of an imposed social security system under conditions of certainty and uncertainty. Under certainty, the maximum level of utility is independent of the level of a. If social security is funded then a is a perfect substitute for s and if financed on a pay-as-you-go basis then a is a perfect substitute for B. Hence any change in a will be exactly compensated by either s or B. None of the other variables will be affected. Under undertainty, there is no perfect substitute for a which, by assumption, is the only form of insurance available. Consequently, the maximum level of utility depends on a as well as initial resources. As a simplifying assumption, however, we shall continue to assume that the utility of bequests depends on B in the case of a funded and on $B - a$ in the case of a pay-as-you-go system. This is equivalent to the assumption that each generation is concerned with the expected wealth, rather than expected utility, of the next generation.[6] In other words, each generation disregards the risk aversion of subsequent generations.

When a is imposed at an arbitrary level, the definition of a steady-state is considerably more complicated than when a is optimally chosen. When complete insurance is not feasible, bequests become random. A steady state can then be defined as a stable distribution of B. We shall not attempt to characterize this distribution. Instead, the analysis will be confined to first-impact effects for a given generation, starting from the optimum a^*.

As in the previous section, we shall discuss in turn a funded system and a pay-as-you-go system.

The maximization problem which we discuss is identical to the one in the previous section except that a is predetermined.

Differentiating totally the F.O.C. (2.5) and (2.7) w.r.t. a, we obtain

$$\text{(3.1)} \qquad \frac{ds^*}{da} = \frac{G}{\Delta'}\Big\{-Gu''(c_0)E[v_{11}(c_1,\theta)+\theta^2 h''(B)] - \frac{R^2}{\bar{\theta}}E[v_{11}(c_1,\theta)]E[\theta h''(B)]\Big\} < 0$$

and

$$\text{(3.2)} \qquad \frac{dc_0^*}{da} = -\left(\frac{ds^*}{da}+1\right) = -\frac{R^2}{\Delta'}\Big\{\frac{GE[v_{11}(c_1,\theta)]}{\bar{\theta}}(\bar{\theta}E[h''(B)]-E[\theta h''(B)]) + (E[h''(B)]E[\theta^2 h''(B)] - E[\theta h''(B)]^2)\Big\}$$

where

$$\text{(3.3)} \qquad \Delta' = Gu''(c_0)(GE[v_{11}(c_1,\theta)+E[\theta^2 h''(B)]) + R^2 GE[v_{11}(c_1,\theta)]E[h''(B)] + R^2(E[h''(B)]E[\theta^2 h''(B)] - E[\theta h''(B)]^2) > 0\,.$$

The last term on the R.H.S. of (3.2) and (3.3) is positive by Schwartz's inequality. It follows that $\frac{ds^*}{da}+1 > 0$ if Cov $[h''(B),\theta] \geqslant 0$. We have already seen that at (a^*, s^*, c_1^*) B is nonrandom and thus

$\mathrm{Cov}[h''(B),\theta] = 0$, satisfying the above condition. Hence, if the level of social security happens to be equal to the optimal annuity holding level for the individual, then $\frac{ds}{da} + 1 > 0$. This result means that a forced increase in the level of social security will reduce private savings but increase total savings.

Under the additional assumption of nonincreasing absolute risk aversion in the utility of bequests, which implies $h'''(B) \geq 0$, one can extend the previous result to levels of social security which exceed the optimum level a^*. This can be shown as follows.

By concavity, $\frac{\partial V}{\partial a} \gtrless 0$ as $a \lessgtr a^*$, when (2.5) and (2.7) are satisfied. Hence, by (2.5) and (2.6), $\mathrm{Cov}(h'(B),\theta) \lessgtr 0$ as $a \gtrless a^*$. Consider the case $a \geq a^*$. Since $h'(B)$ is strictly monotone in θ it must be decreasing in θ in order to satisfy $\mathrm{Cov}(h'(B),\theta) \leq 0$. Therefore, B is increasing in θ and if $h'''(B) \geq 0$ then $\mathrm{Cov}(h''(B),\theta) \geq 0$. Hence, by (3.2), $\frac{ds^*}{da} + 1 > 0$ for $a \geq a^*$.[7]

The above result may be interpreted as follows. Consider an experiment in which a is increased and s decreases in equal amount. Under certainty such a shift in asset composition does not change the consumption and bequest possibilities and thus this would be the optimal adjustment. However, under uncertainty such a change affects the probability distribution of bequests. If $a > a^*$ and thus $\frac{Ra}{\bar{\theta}} - c_1^* > 0$, then for a given level of c_1, the variance of bequests increases while the mean remains unchanged. The assumption that $h''' > 0$ implies in this case that the expected marginal utility of bequests increases. This result is retained when c_1 adjusts so as to satisfy (2.7).[8]

Another question of interest is the effect of social security on expected bequests. Using the F.O.C. (2.5) and (2.7) we find that

$$(3.4)\qquad \frac{dE(B)}{da} = R\left(\frac{ds^*}{da}+1\right) - \frac{dc_1^*}{da}$$

$$= -\frac{1}{\Delta'}\left\{\frac{R^3GE[v_{11}]}{\bar{\theta}}(E[\theta h''(B)] - \bar{\theta}E[h''(B)]) + \frac{Gu''(c_0)}{R}(E[\theta^2 h''(B)] - \bar{\theta}E[\theta h''(B)])\right\}.$$

In general, the sign of (3.4) is indeterminate. However, at a^*, B -- and hence $h''(B)$ -- is independent of θ, which implies that (3.4) is negative, $\frac{dE(B)}{da} < 0$. Notice that under certainty $\frac{dE(B)}{da} = 0$ for all a, which is in accordance with the observation made previously that the sum $s+a$ remains constant for changes in a.

Consider now the short-run impact of an increase in social security payments to generation i financed by an increased tax on generation $i+1$. The only choice variable for generation i is c_1^i, since savings are predetermined. Differentiating (2.21) w. r. t. a yields

$$(3.5)\qquad \frac{d\hat{c}_1}{da} = \frac{1}{\bar{\theta}}\,\frac{E[\theta(\theta-\bar{\theta})h'']}{E[v_{11}] + \frac{1}{G}E[\theta^2 h'']}.$$

Evaluated at $\hat{a}$, $\frac{d\hat{c}_1}{da} > 0$, since h'' is constant. It follows that

$1 > \frac{dE[B]}{da} > 0$. In contrast to the case of certainty, the increase in bequests does not fully compensate for the increased taxes on the younger generation.

Consider now the effects of the increase in social security taxes on the working population. Starting at the steady-state values $(\hat{a}, \hat{s})$, we differentiate equations (2.21)-(2.23) w.r.t. a. The change in first-period consumption is given by

$$(3.6) \qquad \frac{d\hat{c}_0}{da} = -\left(\frac{d\hat{s}}{da}+1\right) = -\frac{Rh''}{\Delta' G}\left[\left(1+\frac{R}{G}\right)h''V_\theta + RE[v_{11}]\right] < 0$$

where $\Delta' = \left(u'' + \frac{R^2 h''}{G}\right)E[v_{11}] + \frac{u''h''E[\theta^2]}{G} + \frac{R^2 h''^2}{G^2}V_\theta > 0$ and $V_\theta = E[\theta^2] - \bar{\theta}^2$ is the variance of θ. Notice that c_0 would decrease even in the absence of uncertainty. This is because initially endowments are held fixed. In the long-run, endowments will vary with bequests and, in the case of certainty, c_0 will remain invariant.

Aggregate consumption in any period is the sum of the older generation's total consumption, $\bar{\theta}c_1$, and the younger generation's total consumption, Gc_0. Thus, the change in aggregate consumption is, by (3.5) and (3.6),

$$(3.7) \qquad \bar{\theta}\frac{dc_1}{da} + G\frac{dc_0}{da} = \frac{1}{\Delta'\left(E[v_{11}] + \frac{h''E[\theta^2]}{G}\right)}\left[h''V_\theta\left((u''-Rh'')\left(E[v_{11}] + \frac{h''E[\theta^2]}{G}\right) - \frac{R^2h''^2\bar{\theta}^2}{G^2}\right) - R^2E[v_{11}]h''\left(E[v_{11}] + \frac{h''E[\theta^2]}{G}\right)\right] < 0.$$

As in a fully funded system, the initial impact of an increase in social security is to increase aggregate savings.

Footnotes

1. See also Feldstein's paper (1974) and the exchange between Barro, Buchanan and Feldstein (1976). Samuelson's analysis (1975) is different since, following Diamond's discussion of the effects of national debt (1965), he disregards bequest motives and hence voluntary intergenerational transfers.

2. In the U.S., pension funds provide less than fifty percent of the total benefits to retirees.

3. The Hessian Matrix of our problem

$$\begin{bmatrix} u'' + \frac{R^2}{G} E[h''] & u'' + \frac{R^2}{\bar{\theta}G} E[\theta h''] & -\frac{R}{G} E[\theta h''] \\ u'' + \frac{R^2}{\bar{\theta}G} E[\theta h''] & u'' + \frac{R^2}{\bar{\theta}^2 G} E[\theta^2 h''] & -\frac{R}{\bar{\theta}G} E[\theta^2 h''] \\ -\frac{R}{G} E[\theta h''] & -\frac{R}{\bar{\theta}G} E[\theta^2 h''] & E[v_{11}] + \frac{1}{G} E[\theta^2 h''] \end{bmatrix}$$

is negative-definite. The diagonal elements are negative by strict concavity. The principal minor of order 2 is equal to

$$\frac{R^2}{\bar{\theta}^2 G^2} \left[Gu''E[(\theta-\bar{\theta})^2 h''] + R^2 (E[\theta^2 h'']E[h''] - E[\theta h'']^2) \right] > 0 .$$

The second term in the above expression is positive by Schwartz's inequality. The determinant can be reduced to

$$\frac{R^2}{G^2}\left[\left(u'' + \frac{R^2}{\bar{\theta}^2}E[v_{11}]\right)\left(E[\theta^2 h'']E[h''] - E[\theta h'']^2\right) + \frac{Gu''E[v_{11}]}{\bar{\theta}^2}E[(\theta-\bar{\theta})^2 h'']\right] < 0 .$$

4. Since θ is restricted to $[0, 1]$, the described shift is not strictly spread preserving.

5. A special case of some interest is $v(c_1, \theta) = \hat{v}(c_1)\theta$. Then $-\frac{E[v_{11}]c_1}{E[v_1]} = \frac{\hat{v}''(c_1)c_1}{\hat{v}'(c_1)}$, which is the standard definition of relative risk aversion, and $\frac{E[v_{12}]\bar{\theta}}{E[v_1]} = 1$. Hence, $\frac{da^*}{d\bar{\theta}} > 0$, $\frac{ds^*}{d\bar{\theta}} < 0$, and $\frac{d(a^*+s^*)}{d\bar{\theta}} > 0$.

6. Expected wealth is defined as the expected present discounted value of consumption, including expected bequests. By (3.1),

$$c_0^i + \frac{c_1^i}{R} + \frac{GE[B^i]}{R} = w + B^{i-1} .$$

In the case of a pay-as-you-go system, the definition includes <u>net</u> bequests, i.e. bequests minus the taxes paid by the next generation. By (2.1) and (2.2')

$$c_0^i + \frac{c_1^i}{R} + \frac{GE[B^i - a]}{R} = w + B^{i-1} - a .$$

7. Notice that if $h'''(B) = 0$, i.e. utility of bequests is quadratic, then $\frac{ds^*}{da} + 1 > 0$ for <u>all</u> a.

8. When c_1 increases, the variance would still increase since $\frac{dc_1}{da} < \frac{R}{\bar{\theta}}$, i.e., part of social security benefits is left for bequests. However, mean bequests, due to the reduction in savings, decreases. These two effects combine to increase the expected marginal utility of bequests. When $\frac{dc_1}{da} < 0$, the variance of B increases while the mean increases. Our result indicates that the former effect is dominant.

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