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UNCERTAINTY AND THE FORMAL THEORY OF POLITICAL CAMPAIGNS*

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I. INTRODUCTION

Since the publication of Downs's seminal book two decades ago, the concepts and theories that he introduced have led to a large and expanding literature on the theory of electoral competition and voter decision making. While some of this work has abandoned the assumption that voters have perfect information about candidates' issue positions (see Shepsle, 1972), nearly all of it assumes that candidates know how the voters will respond when any given platform is offered. Yet many empirical works on congress and the presidency find that political decision makers are uncertain about the response of voters to their actions (see Bauer, Pool and Dexter, 1963; Kingdon, 1973), and claim that this fact is fundamental to understanding their behavior.

This paper explores the neglected issue of candidate uncertainty. It examines the strategic decision making of candidates in duopolistic elections in which they are uncertain about the outcomes of majority rule decisions in pairwise contests of alternative positions on political issues.

Part of the motivation for this paper lies in the relatively unrobust predictions of existing theoretical models of political campaigns. It is well known that the likelihood of cyclic majorities is very high if individual preferences are randomly distributed over all possible orderings of the relevant alternatives (see DeMeyer and Plott, Kramer). As a result, Downs predicted that without severe restrictions on the manner in which alternatives are put before the electorate, political outcomes should be unstable. The winning issue position and candidate in one election should be defeated in the next; further with certainty of defeat facing incumbents, the connection between actions after an election and campaign positions before the election could be expected to be slight.

Even if there are no cycles under majority rule, equally implausible predictions arise. If a single majority-rule winning position is known to exist, both candidates will take it, and voters will be indifferent as to who wins. Elections will end in ties and voters will have no incentive to vote.

Collective choice theory based upon rational individual behavior can be extended in two ways to make its predictions more in line with reality. The first, which we will not explore here, is to constrain each candidate's feasible set of election strategies in such a way that majority rule cycles are not exploited.[1]

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[1] For example, candidates might care about the positions they take for reasons other than the impact of positions on election outcomes, might have monopoly rights to some issue positions (e.g. party identification, incumbency, etc.), or might be

constrained in their selection of strategies by the past behavior of themselves or of other candidates of the same party.

A second possible motivation for stable yet different platform choices is that candidates may have conflicting beliefs about voter behavior. If information is costly enough that they cannot obtain new information too easily, strategic equilibria could exist which vanish in the absence of uncertainty.

In this paper, we assume that two candidates, engaged in a duopolistically competitive election, wage a campaign that can be represented as a two-person game. The strategies of the game are the elements of the feasible set of alternatives that can be put before the electorate. A candidate selects one of the set of alternatives, and the election consists of a majority rule choice by the electorate of one of the two alternatives offered by the candidates. Each alternative can be interpreted as a vector of positions on the issues in a campaign. Although in reality the set of feasible alternatives may differ between two candidates, in order to investigate the implications of uncertainty alone we assume throughout that any alternative may be offered by either candidate.

At least one candidate is assumed to be unsure of the majority rule outcome of at least some of the pairwise contests of alternatives. A candidate deals with this uncertainty by estimating the likely outcome of the uncertain contests, based upon whatever information he has about the distribution of opinion. If the

candidates differ with respect to the amount of information they have and the manner in which they process this information, the expectations of the candidates about the outcome of a particular pairwise contest can differ. In fact, if opinions are sufficiently different, both candidates might expect to win an election involving some particular choice of strategies by each candidate.

In the presence of uncertainty, candidates might wish to acquire additional knowledge about voter preferences before selecting a strategy. An obvious mechanism for obtaining this knowledge is the public opinion poll.—/ One issue examined in this paper is the desirability to a candidate of acquiring better information about the true response of an electorate to a particular strategy pair.

The key element in the following analysis is the observation that uncertainty about electoral behavior can alter the game

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—/Two generic types of polls are possible. A candidate could take a hypothetical poll by asking voters to state a preference for one of the two issue positions, without either position being taken by or identified with a particular candidate. Or a candidate could gauge the response of the electorate to an issue position by expounding a position publicly and then using the change in his standing in a popularity poll to estimate the effect of the position on his electability. In this paper we abstract for the differences in these polls.

theoretic structure of the campaign. A campaign in which outcomes are known with certainty must be a game of perfect opposition since payoffs must be symmetric (one person's win is another person's loss). But *ex ante*, two candidates can both expect to win if at least one is uncertain about the true social preference relation. In such a milieu, information can have a negative value, in the same sense that information about the true outcome of a football game before bets are placed can reduce the welfare of both of the participants in a wager. Both bettors on the outcome of a football game, if their bets are consistent with their own subjective probabilities over possible states of the world, believe *ex ante* that more information is more likely to confirm their own beliefs than those of the other person, and hence both would believe that more information would be likely to cause the other person to change his mind about entering the bet. Translated to campaign decision making, this means that under some circumstances, depending upon the structure of their subjective expectations about electoral behavior, both candidates can attach a negative value to certain kinds of information since both expect that this information would only cause the pre-poll equilibrium strategy selections to be upset in a manner that runs counter to the preceived self-interest of each.

II. THE MODEL AND SOME EXAMPLES

In this section we formalize what is meant by partial or incomplete information. In an electoral campaign each candidate

communicates a sequence of messages to the electorate and the opponent. Somehow these messages are decoded by the voters in order to determine how (or whether) to cast their votes on election day. Voters may, of course, ignore many of the messages and "misinterpret" some as well, but when the campaign concludes they either vote for some candidate or abstain. The electorate can be viewed in aggregate as a function mapping sequences of messages from the candidates as well as other information into a distribution of votes for the candidates. This function can be represented as a matrix, with the rows being the strategies available to one candidate, the columns being the strategies open to the second contestant, and the elements indicating whether candidate one won, tied or lost the election run on that strategy pair. We call such a function a majority dominance relation.

In this paper voters are assumed to respond only to the most recent message in the sequence. These messages may be thought of as "issue positions" or "platforms." Further, the electorate is assumed to respond in an *unbiased* manner to the announced platforms of candidates. That is, if two candidates switch platforms their vote totals switch.

Suppose that the candidates are not certain which of many possible majority dominance relations truly represents the electorate. Given a set of three platforms (x, y, z), a total of 27 outcome matrices may reflect the electorate's true response to all possible campaigns.

This situation may be represented as a game by regarding the matrix representation of each majority dominance relation as a symmetric, two-person, zero-sum game. Each element of such a matrix is +1, 0, or -1 according to whether the candidate playing the rows -- candiate A -- wins, ties or loses an election run on the corresponding pair of strategies.

Suppose that the candiates do not know which majority dominance relation is true, but uses the information available to them to produce a subjective probability distribution over all the possible dominance relations. Each candidate then selects an election strategy on these beliefs.

The reason for introducing alternative states of the world and probabilities over them, rather than hypothesizing payoff matrices directly, is to permit some generality in discussing the roll of information. In particular, this formulation permits better information about the outcome of one strategy pair to influence other elements of the payoff matrix in a systematic fashion, as will be shown in Section III.

The following example illustrates the candidate game resulting from our assumptions. Assume that the candidates, A and B, have subjective probabilities, P_A^k and P_B^k, over k = 4 possible states of the world as follows:

State of the World	I	II
P_A^k	$\frac{1}{4}$	$\frac{1}{2}$
P_B^k	$\frac{1}{4}$	0
Payoff Matrix	$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$

State of the World	III	IV
P_A^k	$\frac{1}{4}$	0
P_B^k	$\frac{1}{2}$	$\frac{1}{4}$
Payoff Matrix	$\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

Assuming both candidates are expected value maximizers, candidate A behaves as if he were playing the following game, which is constructed by summing the state of the world matrices by the P_A^k's, and denoted P_A.

$$P_A = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & 0 & -1 \\ -\frac{1}{2} & 1 & 0 \end{bmatrix}$$

By similar construction, B is playing:

$$P_B = \begin{bmatrix} 0 & 0 & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Essentially what has happened is that uncertainty has transformed the candidate game into a nonzero sum game. The elements of P_A and P_B, when incremented by one and divided by two, are respectively, A's and B's subjective probabilities that A will win. The unique pure strategy equilibrium point in this game is for candidate A to play platform (row) one and candidate B to play platform (column) three. The value of the game to A is $\frac{1}{2}$ and the value to B is $-\frac{1}{2}$ (i.e., both believe they are likely to win the election).

If both candidates know that the true state of the world is state I, the minimax strategy for each is to play each platform with probability $\frac{1}{3}$. The value of the game will then be zero for both candidates. The introduction of uncertainty has created a pure strategy equilibrium and has made the value of the game positive for both candidates. This example would seem to capture the essential idea that Downs had in claiming that uncertainty could create stability in electoral competition. Of course, it is easy to come up with a case in which the opposite phenomenon occurs. The true state of the world might be II, which has a pure strategy equilibrium, while both candidates might believe that all states are equally likely, in which case the equilibrium strategy is to play each strategy with probability $\frac{1}{3}$.

A few important principles may be drawn from this very simple model. First, disagreement between the candidates can create new equilibria. Second, new information that changes the beliefs of the candidates can upset equilibria. Third, equilibria can have the property that the candidates adopt different platforms. This last phenomenon cannot occur in a classical spatial model unless the electoral response function is biased or the candidates place value on features of an election other than who wins.

While it is of interest to note that uncertainty may generate stable platform choices, it is perhaps even more surprising to learn that uncertainty may induce candidates to collude with each other. The following applications of some simple strategic structures that can arise in two-person games illustrate this points.

A "Prisoner's Dilemma" Campaign

Suppose candidates A and B accord subjective probabilities over four possible states of the world as follows:

State of the World	I	II
P_A^k	.7	.2
P_B^k	.05	.05
	$\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

State of the World	III	IV
P_A^k	.05	.05
P_B^k	.7	.2

$$\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

The resulting expected payoff matrix, written in bimatrix form, is then:

$$P_{AB} = \begin{bmatrix} (0,\ 0) & (.95,\ .8) & (.6,\ -.9) \\ (-.95,\ -.8) & (0,\ 0) & (-.5,\ .5) \\ (-.6,\ -.9) & (.5,\ -.5) & (0,\ 0) \end{bmatrix}$$

Recall that A seeks positive payoffs and B negative. This game is a prisoner's dilemma with payoff (0, 0), corresponding to each candidate taking the first platform. Each would be better off if the candidates played platform pair (three, two). But if the candidates were to take this latter position and obtain (.5, -.5) each would have an incentive to switch to platform one.

This sort of strategic structure might underlie a tacit or explicit agreement between two candidates not to campaign on a certain issue. Both candidates agree in this case that platform one is the most popular platform among the voters. Candidate A's beliefs are such that he thinks that platform one is unbeatable with probability .8, there is a cycle with probability .2. Mr. B believes that with probability .95 platform one is unbeatable.

Precisely because they agree that platform one is popular, they would prefer to agree to restrict the campaign to platforms two and three over which their beliefs conflict.

Scholars and editorialists have sometimes observed that an issue about which the public holds strong beliefs (bussing or corruption in office, for example) failed to arise in a campaign. This phenomenon is usually ascribed to the presence or absence of the candidates' "public responsibility." But the prisoner's dilemma example indicates that the failure of a "dominant" issue to arise could be due to the same phenomenon that leads bettors not to wager on football games about which their expectations are identical. If so, "public responsibility" is really quite something else -- collusion to prevent a public policy that is a clear majority rule winner.

The implications of these phenomena for democratic theory depend, of course, on the ability of candidates to make and enforce agreements on campaign strategies. This ability depends, among other things, on the cost of information to the candidates, as is demonstrated in Section III. It also depends upon each candidate knowing the ranking by the opponent of alternative campaigns in the subset of possibilities that constitutes the prisoner's dilemma. In the absence of this knowledge, the candidates will not perceive the possibility of advantageous collusion.

A "Battle of the Sexes" Campaign

Another set of incentives can arise if the candidates have somewhat different beliefs. For example, assume that the candidates put all subjective probability on the following five states of the world:

$$
\begin{array}{cccccc}
P_A^k & \frac{1}{4} & \frac{3}{8} & \frac{3}{8} & 0 & 0 \\
P_B^k & \frac{1}{4} & 0 & 0 & \frac{1}{2} & \frac{1}{4} \\
 & \text{I} & \text{II} & \text{III} & \text{IV} & \text{V}
\end{array}
$$

$$
\begin{bmatrix} 0 & +1 & -1 \\ -1 & 0 & -1 \\ +1 & +1 & 0 \end{bmatrix}
\begin{bmatrix} 0 & -1 & +1 \\ +1 & 0 & -1 \\ -1 & +1 & 0 \end{bmatrix}
\begin{bmatrix} 0 & -1 & -1 \\ +1 & 0 & -1 \\ +1 & +1 & 0 \end{bmatrix}
\begin{bmatrix} 0 & +1 & +1 \\ -1 & 0 & +1 \\ -1 & -1 & 0 \end{bmatrix}
\begin{bmatrix} 0 & +1 & -1 \\ -1 & 0 & +1 \\ +1 & -1 & 0 \end{bmatrix}
$$

The expected payoff matrix is then the following:

$$
P_{AB} = \begin{bmatrix} (\ 0,\ 0) & (-\frac{1}{2},\ +1) & (-\frac{1}{4},\ 0) \\ (+\frac{1}{2},\ -1) & (\ 0,\ 0) & (\ -1,\ +\frac{1}{2}) \\ (+\frac{1}{4},\ 0) & (\ +1,\ -\frac{1}{2}) & (\ 0,\ 0) \end{bmatrix}
$$

This game has two pure strategy equilibria, strategy pairs (two, one) and (three, two), with payoffs $(\frac{1}{2}, -1)$ and $(1, -\frac{1}{2})$. Candidate B prefers the first, and candidate A the second, producing the battle of the sexes structure discussed in Luce and Raiffa. The important feature of this expected payoff matrix is disagreement over which equilibrium should be achieved could lead to strategies and threats that produce disequilibrium outcomes.

The information structure that produces this result has two elements: candidates differ in their assessment of the likely outcome of two strategy pairs, and the strategy pair about which the first candidate is more certain is the one about which the second candidate is less certain. In this case, candidate B is certain that if the electorate is faced with a choice between platforms one and two, one will prevail. Candidate A is less certain about the outcome of this contest, but believes the outcome is likely to be the opposite of B's belief. Similarly, A is certain that platform three would beat platform two in a pairwise contest, while candidate B is less sure of the outcome of such a contest, but disagrees with A. Each would prefer to fight the campaign on platforms which appear to guarantee victory rather than take a chance.

III. POLLS AND INFORMATION

Candidates who are uncertain about the behavior of voters on election day may find it worthwhile to seek information about the preferences of the electorate. This section examines some conditions under which information would be valuable to candidates. We restrict ourselves to a particular class of information-gathering processes which we call, suggestively, polls.

We define a <u>poll</u> as information about the true value of the cells of the subjective probability matrices P_A and P_B defined in Section II. A poll is <u>private</u> if only one of the candidates learns the result. If both candidates learn of the true value, the poll is called <u>public</u>. We assume that the outcomes of polls are certain so that if a public poll is taken, the corresponding elements of P_A and P_B are replaced by 1 or -1. This hypothesis may, of course, be relaxed. In particular, the outcome of the poll could be a probability of victory anywhere from zero to one, depending upon the nature and timing of the poll. Considering outcomes other than 1 or -1 greatly complicates the examples considered in this section, but does not change the essence of the results.

Candidates are assumed to revise their subjective probabilities in a Bayesian manner. That is, if a poll is taken on a particular element of the matrix of outcomes of strategy pairs, each candidate revises his probability estimates over the possible majority dominance relation in a manner that assigns all probability to the subset of majority dominance relationships consistent with the poll and that increases the probability assigned to each relationship in that set by the same proportion.[/]

[/]Poll outcomes between 1 and -1 would assign probabilities such that the expected outcome of the polled cell equaled the poll result, with probabilities assoicated with matrices having entries of a given sign in the polled cell incremented or decremented proportionately.

The following example illustrates the effects of a poll on the subjective probability matrices and the equilibrium pair of strategies. This example is based upon the first example presented in Section II, in which, before a poll is to be taken, the only

pure strategy equilibrium is for A to play platform one and B to play three, with payoff vector $(\frac{1}{2}, -\frac{1}{2})$. Suppose that a poll is to be taken on element (one, two). The following table gives the revised subjective probabilities, strategies and outcomes for both candidates, given that the true value of (one, two) is either +1 or -1.

<u>True Value of (1, 2)</u>

Poll result	1				-1			
State of the World	I	II	III	IV	I	II	III	IV
P_A^k	0	$\frac{2}{3}$	$\frac{1}{3}$	0	1	0	0	0
P_B^k	0	0	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$
Equilibrium strategy of A	(1, 0, 0)				(0, 0, 1)			
Equilibrium strategy of B	(0, 0, 1)				(0, 1, 0)			
Payoff to A	$\frac{1}{3}$				1			
Payoff to B	-1				0			

In this case after the poll A's expected return either drops to $\frac{1}{3}$ or rises to 1, depending on the outcome of the poll. A's <u>ex ante</u> expectation of the change in the value of the game due to the poll is the sum of these outcomes weighted by the subjective probabilities over the possible results of the poll, e.g. $\frac{3}{4}(-\frac{1}{6})+\frac{1}{4}(\frac{1}{2}) = 0$. That is, A is indifferent between having a poll and not having one. In this particular example, B is also indifferent about a public poll on (one, two).

A rather surprising result occurs if a public poll is proposed on (one, three). Going through the same operations as above, the *ex ante* evaluations of this poll are negative for both candidates. In particular, the expected payoff of the game to candidate A drops by $\frac{7}{20}$ if a poll is taken on (one, three) while Mr. B is forced to surrender an additional $\frac{5}{12}$.[1] In this case, both candidates are better off *ex ante* if they can prevent a public poll on this element. As it turns out, this "ignorance is bliss" phenomenon occurs under fairly general circumstances. Section IV establishes sufficient conditions for both candidates to prefer preventing a public opinion poll on a given strategy pair. For the present we restrict our attention to some illustrative examples that provide a little insight into the effect on campaign strategy of increasing the amount of information held by candidates in an uncertain election.

The prisoner's dilemma example in Section II was constructed to show that uncertainty may induce candidates to behave in rather curious and, one might say, unfortunate ways. We now consider the possibility that, rather than forming their strategies on the basis of limited information, one or the other of the candidates might choose to purchase additional information or that a third party -- a polling agency -- might introduce new information to the candidates.

[1] The poll result +1 leads to a mixed strategy by both candidates.

The impact of a public poll on cell (three, two), the collusive outcome of the prisoner's dilemma, depends upon whether the candidates are colluding. If the candidates collude their expected pre-poll payoff is $(\frac{1}{2}, -\frac{1}{2})$. If a public poll is taken on (three, two) and Bayesian updating takes place, the expected post-poll payoff is $(\frac{1}{4}, -\frac{3}{4})$ so that, given pre-poll collusion, candidate A would oppose a poll while candidate B would favor it. If, on the other hand, the candidates failed to collude, both players would favor a public poll on (three, two).

The effects of a private poll are less clear because of the possibility of strategic behavior with respect to the selective release of private poll results. If Mr. B takes a private poll on (three, two), regardless of the outcome the best strategy is to choose platform one. But candidate A, still being in the dark about the outcome of the poll, will interpret B's choice as an unwillingness to collude. In the absence of information, he may switch to platform one. But if candidate B thinks that A will react in this manner, and if before the poll the collusive outcome is chosen, B will find the poll information valueless.

Another possibility can also arise. If the result of a public poll on (three, two) is -1, the post-poll equilibrium is for A to choose platform three and B to play platform one, with payoff (1, -1). Mr. B could take a private poll and, if the outcome is

-1, publicize it and play platform one while, if it is +1, suppress the information and switch to platform one. In this case, assuming that A believes the information that B distributes, he will choose platform three when B announces the poll result and the choice of platform one, but will play one if B picks platform one without announcing the result of the poll. If B adopts this strategy his expected payoff to a private poll is $-\frac{3}{4}$, just what he would have obtained had a public poll occurred.

The "battle of the sexes" example illustrates still another strategic aspect of information. One possibility is for one candidate to threaten the other with the possibility of buying more information as a device to force the strategy pair he prefers. In this example, candidate B can threaten to undertake a poll on the pairwise contest, platform two versus platform one, the results of which are as follows.

Poll on (two, one) = - (one, two)

Result = -1 $\Longrightarrow$

$$\begin{array}{cccccc} & \mathrm{I} & \mathrm{II} & \mathrm{III} & \mathrm{IV} & \mathrm{V} \\ P_A^k & 1 & 0 & 0 & 0 & 0 \\ P_B^k & \frac{1}{4} & 0 & 0 & \frac{1}{2} & \frac{1}{4} \end{array}$$

$$P_{AB} = \begin{bmatrix} (\ 0,\ \ 0) & (+1,\ +1) & (-1,\ \ 0) \\ (-1,\ -1) & (\ 0,\ \ 0) & (-1,\ +\frac{1}{2}) \\ (+1,\ \ 0) & (+1,\ -\frac{1}{2}) & (\ 0,\ \ 0) \end{bmatrix}$$

Equilibrium (three, two), payoff $(+1, -\frac{1}{2})$

Result = +1 $\Longrightarrow$

$$\begin{array}{cccccc} & \mathrm{I} & \mathrm{II} & \mathrm{III} & \mathrm{IV} & \mathrm{V} \\ P_A^k & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ P_B^k & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{array}$$

$$P_{AB} = \begin{bmatrix} (\ 0,\ \ 0) & (-1,\ -1) & (\ 0,\ \ 0) \\ (+1,\ +1) & (\ 0,\ \ 0) & (-1,\ -1) \\ (\ 0,\ \ 0) & (+1,\ +1) & (\ 0,\ \ 0) \end{bmatrix}$$

Equilibrium (three, three), payoff (0, 0)

Expected Payoff to A with Poll = $\frac{1}{4}(+1) + \frac{3}{4}(0) = \frac{1}{4}$

Expected Payoff to B with Poll = $1(-\frac{1}{2}) + 0(0) = -\frac{1}{2}$

If A refuses to play B's preferred platform pair (two versus one), steadfastly sticking to platform three, B expects to lose nothing by polling on (two, one); however, A would stand to suffer a substantial loss. The worst A can do without a poll is platform pair (two, one) with payoff $+\frac{1}{2}$, which is preferable to the expected result of the threatened poll.

In this example, A has no effective information threat comparable to B's. Although the calculations are not presented here, it is easily shown that polls on the other two off-diagonal strategy pairs produce ambiguous results that do no more damage to B than to A. Likewise, A has no effective counterthreat to B's threat to poll on (two, one). Thus, in this example, the strategic threat to gather new information resolves the ambiguity in the battle of the sexes campaign, creating a stable equilibrium.

IV. THE FORMAL MODEL

We assume that candidates select campaign strategies from a finite set of X alternatives, X = {a, b, c, ..., y, z} and that each of a finite number, n, of voters has preferences on X that weakly order the alternatives. Let n(xPy) be the number of voters preferring strategy x to strategy y. We define xMy if and only if n(xPy) > n(yPx), and call M the majority dominance relation. Numerous majority dominance relations are possible, of course, over any set of alternatives. Any particular majority dominance relation can be represented as a symmetric, two-person, zero-sum game with a payoff matrix defined as follows:

$$A(x, y) = \begin{cases} 1 \text{ if } n(xPy) > n(yPx) \\ 0 \text{ if } n(xPy) = n(yPx) \\ -1 \text{ if } n(yPx) > n(xPy) \end{cases}$$

Each candidate has a matrix of expected payoffs, P_A for candidate A and P_B for candidate B, which gives the expected payoff for each combination of platforms. Candidates are assumed to select Nash equilibria in choosing strategies. The interpretation of the payoff matrices that was given in Section II was that:

$$\sum_{k \in K} P_A^k M^k = P_A$$

where K contains the indexes over states of the world, M^k is a

skew-symmetric matrix representing the majority dominance relation in a possible state of the world, and P_A^k is A's subjective probability that state k will occur. We assume that the elements of M^k are zero on the diagonal and +1 or -1 off the diagonal. As long as each element in P_A is less than or equal to 1 in absolute value, a probability distribution $\{P_A^k\}$ exists such that P_A can be expressed as in the above equation, so that no loss of generality is entailed by preventing off diagonal elements of M^k from being zero.[/]

[/]Proved in an unpublished appendix, available from the authors on request.

The first theorem illustrates how uncertainty can lead to pure equilibrium strategies even if both candidates agree that the majority dominance relation contains a cycle.

<u>Theorem 1</u>: Suppose in a three-platform election each candidate assigns subjective probabilities to majority dominance relations such that the expected payoff matrix of each is cyclic. The campaign has a pure strategy equilibrium involving two of the cycling elements if, and only if, the candidates disagree about which order of the cycle is more likely.

The formal proof of Theorem 1 follows from two observations: (1) cycles occur when off-diagonal elements of the payoff matrix alternate in sign, and (2) pure strategy equilibria require that the payoff for the equilibrium campaign exceed zero for both candidates, since either candidate can guarantee a zero payoff by playing the same strategy as the opponent. It can easily be shown that these requirements yield the proof of the theorem.

For a strategy pair satisfying Theorem 1 to be an equilibrium in the larger campaign depends upon the structure of subjective probabilities over outcomes of contests involving other strategies and, consequently, of the entire nature of subjective payoff expectation. One interesting case is the following.

Corollary 1.1. In an $n > 3$ platform election, if three issues are known to cycle, the conditions of Theorem 1 obtain for cycling issues, and the candidates agree qualitatively on the outcome of all campaigns other than those involving the cycling elements, then strategy pairs satisfying the conditions of Theorem 1 are the only ones that give both candidates the expectation of winning.

Proof. Let (a, b, c) be the cycling platforms. Since for all strategies $(r, s) \notin (a, b, c)$ and $r \neq s$ the candidates agree that either r P s or s P r, each pair must be associated with an expectation of losing on the part of one candidate.

Q.E.D.

The strategy pair of Corollary 1.1 will not be a pure strategy equilibrium, however, unless the structure of the game is such that neither candiate, given the choice of strategy by his opponent, has a strategy of greater expected payoff (e.g., a

prisoner's dilemma situation). If, for example, both candidates agree that strategy $r \notin (a, b, c)$ defeats all three cycling strategies, and if one candidate chooses his element of (x, y) satisfying Theorem 1, the other can guarantee a win by playing r rather than his element of (x, y). Only collusion between candidates can then result in a campaign run on platform pair (x, y).

Define a full transitive subordering of a cyclic ordering over a set as being a transitive ordering of all the elements of the set that is consistent with the cyclic ordering. For example, the cycle xPyPzPx contains three full transitive suborderings:

xPyPz, yPzPx and zPxPy.

Also, define the inverse of a transitive ordering as being the same ordering but with the direction of preferences reversed, e.g. the inverse of xPyPz is zPyPx. Finally, define an n-element cycle as a cycle of the form $a_1Pa_2P \ldots Pa_nPa_1$ for $n \geq 3$.

If the conditions of Theorem 1 are satisfied, the three-dimensional game has three pure strategy equilibria, one for each pairwise contest. The resulting game resembles the "Battle of the Sexes" structure discussed in the preceeding section. We offer without proof the following obvious corollary:

Corollary 1.2: A sufficient condition for the three-dimensional, two candidate campaign of Theorem 1 to have the structure of the "Battle of the Sexes" game is that both candidates believe that the payoff matrix is cyclic, that the candidates disagree about the order of the cycle, and that

$$\exists i,\ j \ \epsilon\ (1,\ 2,\ 3) \ni : P_A(R_i) > P_B(R_{i+3}) \iff P_A(R_j) < P_B(R_{j+3}).$$

The last condition of the theorem requires that, although the candidates disagree whether particular transitive orderings (R_i and R_j) or their inverses $(R_{i+3},\ R_{j+3})$ are more likely, the ranking of transitive orderings by A according to the likelihood that each represents the time state of the world is not the same as the ranking by B of the inverse orderings. If the conditions of Corollary 1.2 are generalized to include positive subject probabilities for cycling majority dominance relations, the corollary remains true if i and j are allowed to vary over the three transitive orderings within a cycle and the cycle itself.

Just as the certainty that a cycle exists does not preclude a pure strategy equilibrium involving the cycling elements, the certainty that no cycle exists does not preclude the possibility that the payoff matrix facing the candidates contains a cycle.

<u>Theorem 2</u>. A candidate who believes that cycles are not possible will face an n-element cycle in his subjective payoff matrix for the campaign if, and only if, every full transitive subordering of the n-element cycle is regarded as more likely than its inverse ordering.

<u>Proof</u>. If P is a complete, inflexive relation, every $n > 3$ element cycle contains a three element subcycle, so that the theorem need only be proved for the three element case. Suppose a, b, and c are

the three strategies. Candidate A believes that only transitive majority dominance relations are possible, and for three elements these are:

R_1: aPbPc

R_2: bPcPa

R_3: cPaPb

R_4: cPbPa

R_5: aPcPb

R_6: bPaPc

Note that R_1, R_2 and R_3 are the full transitive suborderings of cycle C_1: aPbPcPa; R_4, R_5 and R_6 are the full transitive suborderings of cycle C_2: aPcPbPa; R_4, R_5 and R_6 are the inverse orderings of R_1, R_2 and R_3; and C_1 and C_2 are the only three-element cycles that are possible.

Using the notation introduced above, candidate A, with subjective probabilities P_A^i over the R_i, faces a payoff matrix:

$$P_A = \begin{bmatrix} a & 0 & P_A^1-P_A^2+P_A^3-P_A^4+P_A^5-P_A^6 & P_A^1-P_A^2-P_A^3-P_A^4+P_A^5+P_A^6 \\ b & -P_A^1+P_A^2-P_A^3+P_A^4-P_A^5-P_A^6 & 0 & P_A^1+P_A^2-P_A^3-P_A^4-P_A^5+P_A^6 \\ c & -P_A^1+P_A^2+P_A^3+P_A^4-P_A^5+P_A^6 & -P_A^1-P_A^2+P_A^3+P_A^4+P_A^5-P_A^6 & 0 \end{bmatrix}$$

P_A contains a cycle over {a, b, c} if and only if the off-diagonal elements of P_A have alternating sign, or:

(1) $P_A^1 - P_A^2 + P_A^3 - P_A^4 + P_A^5 - P_A^6 > 0 \iff$

(2) $P_A^1 - P_A^2 - P_A^3 - P_A^4 + P_A^5 + P_A^6 < 0 \iff$

(3) $P_A^1 + P_A^2 - P_A^3 - P_A^4 - P_A^5 + P_A^6 > 0.$

Pairwise combinations of these inequalities produces the following equivalent relations:

(4) $P_A^3 - P_A^6 > 0 \iff$

(5) $P_A^1 - P_A^4 > 0 \iff$

(6) $P_A^2 - P_A^5 > 0.$

Similarly, linear combinations of (4), (5) and (6) can be constructed that yield (1), (2) and (3). Hence P_A cycles on {a, b, c} if and only if (4), (5) and (6) are all true or all false.

Q.E.D.

The conditions of Theorem 2 are strong, as examination of the R_i demonstrates. For example, assume that Candidate A believes R_1 is the most likely of the six orderings listed above. For a cycle to come about in his payoff matrix, he must also believe $P_A^3 > P_A^6$. Now R_6 differs from R_1 only by interchanging the first two elements, while R_3 moves the last-place finisher in R_1 to first place. Thus, the conditions of the theorem require that A believe that if he is wrong about R_1, it is more likely to be because the last place element really ranks first while the others retain their relative ordering than because he was wrong about the ordering of adjacent elements in the ordering R_1 -- i.e., it is less likely that b is preferred to a than it is that c, the third place element in the most likely relation, is preferred to both a and b.

Theorems 1 and 2 demonstrate that there is no direct connection between whether candidates behave as if the majority dominance relation is known to be cyclic and whether they believe it is cyclic. If a cycle results because the conditions of Theorem 2 are met, Theorem 1 may still be satisfied. A pure strategy equilibrium is still possible if the candidates do not agree which cycle has the property that its transitive suborderings are all more likely than their inverses.

The remainder of this section explores the process by which candidates gather information about the majority dominance relationship and alter their strategies accordingly.

A <u>poll</u> supplies information about the true value of the polled cell, P_A^{ij} and P_B^{ij} of the matrices P_A, P_B, and by symmetry, the obverse elements of the payoff matrix. According to assumption, after a poll P_A^{ij} and P_B^{ij} are replaced by 1 or -1, depending upon which certain outcome transpires.

After a poll, candidates update their subjective probability distributions in a Bayesian manner. If a poll is taken on the $ij^{\underline{th}}$ element and candidate A learns that the true value of this element is $x \in \{1,-1\}$, then for all k such that $M_{ij}^k = x$, if $P_A^{ij} = x$ is subjectively possible:

$$P_A^{k*} = \frac{P_A^k}{\sum_{\ell \in L} P_A^{\ell}}$$

where $L = \{k | M_{ij}^k = x\}$. If $P_A^{ij} = x$ is not subjectively possible, then $P_A^{k_1^*} = P_A^{k_2^*}$ for all $k_1, k_2 \in L$ and $\sum_{\ell \in L} P_A^{\ell} = 1$. In both cases $P_A^{k*} = 0$ for all k not in L.

First we prove a lemma that establishes a straightforward mechanism for calculating updated probabilities.

Lemma 1. If the candidates A and B have probability distributions $\{P_A^k\}$, $\{P_B^k\}$ such that strategy (i, j), $i \neq j$, in the game (P_A, P_B) has payoffs $(\alpha, -\beta)$, if a poll is taken on the ij$^{\underline{th}}$ element with outcome $x \in \{1, -1\}$, and if subjectively both 1 and -1 are possible, the revised probabilities of the two candidates may be computed as follows:

$$P_A^{k*} = \frac{2}{1 + x\alpha} P_A^k \qquad \forall k \in L = \{k | M_{ij}^k = x\}$$

$$P_A^{k*} = 0 \qquad \forall k \notin L$$

$$P_A^{k*} = \frac{2}{1 - x\beta} P_B^k \qquad \forall k \in L$$

$$P_B^{k*} = 0 \qquad \forall k \notin L$$

Proof. Since candidates update probabilities in a Bayesian fashion,

for all $k \in L$, $P_A^{k*} = \dfrac{P_A^k}{\sum_{\ell \in L} P_A^{\ell}}$, and $P_A^{k*} = 0 \qquad \forall k \notin L$

Now $$\alpha = \sum_{k \in L} P_A^k x - \sum_{k \notin L} P_A^k x$$

And since $x^2 = 1$

$$\sum_{k \in L} P_A^k - \sum_{k \notin L} P_A^k = x\alpha$$

By adding $$\sum_{k \in L} P_A^k + \sum_{k \notin L} P_A^k = 1$$

we obtain $$2 \sum_{k \in L} P_A^k = 1 + x\alpha$$

or $$\sum_{k \in L} P_A^k = \frac{1 + x\alpha}{2}$$

which, when substituted into the formula for P_A^{k*}, produces the hypothesized result. The same argument obtains for candidate B.

Q.E.D.

For convenience let t_{ij} denote the true outcome of the contest between i and j. For example, $t_{ij} = 1$ means that platform i beats platform j. We continue to assume that $t_{ij} = 1$ or -1 for all $i \neq j$ and that $t_{ii} = 0$ for all i.

Theorem 3.—/ If (i, j) is a pure strategy equilibrium with payoff $(\alpha, -\beta)$, a public poll on (i, j) will increase the value of the campaign to candidate A only if there are post-poll equilibria (r, s) and (t, u), depending on the poll outcome, that have the property that Prob A$(t_{rs} = -1, t_{ij} = 1)$ + Prob A$(t_{tu} = -1, t_{ij} = -1)$ $< \dfrac{1 - \alpha}{2}$, where Prob A(z) means A's subjective, ex ante probability that z will transpire.

_/Theorem 3 and its corollary are stated in terms of one candidate, but owing to the symmetry assured between the two candidates, they are obviously true for the other as well.

Proof. Let $L(i, j) = \{k | M_{ij}^k = 1\}$

$$\overline{L}(i, j) = \{k | M_{ij}^k = -1\}$$

From the way probabilities are updated by the candidates, the result of the poll will be to tell the candidates that the true state of the world is in $L(i, j)$ or $\overline{L}(i, j)$. Let (r, s) be a post-poll strategy equilibrium if $t_{ij} = 1$. Then

$$\sum_{k \epsilon L(i,j)} P_A^{k*} M_{rs}^k \geq \sum_{k \epsilon L(i,j)} P_A^{k*} M_{\tilde{r}s}^k \qquad \forall \tilde{r}$$

and

$$\sum_{k \epsilon L(i,j)} P_B^{k*} M_{rs}^k \leq \sum_{k \epsilon L(i,j)} P_B^{k*} M_{r\tilde{s}}^k \qquad \forall \tilde{s} .$$

By Lemma 1

$$\sum_{k \epsilon L(i,j)} P_A^{k*} M_{rs}^k = \frac{2}{1 + \alpha} \sum_{k \epsilon L(i,j)} P_A^k M_{rs}^k$$

Now

$$\sum_{k \epsilon L(i,j) \cap L(r,s)} P_A^k = \frac{1 + \alpha}{2} - \delta$$

for some $\delta \geq 0$ since, by Lemma 1, the sum over $k \epsilon L(i, j)$ is $\frac{1 + \alpha}{2}$.

Hence $\sum_{k \epsilon L(i,j) \cap \overline{L}(r,s)} P_A^k = \delta = \text{Prob } A(t_{rs} = -1, t_{ij} = +1)$.

so that

$$\sum_{k \epsilon L(i,j)} P_A^k M_{rs}^k = \frac{1 + \alpha}{2} - 2\delta .$$

Repeating the same argument, if $t_{ij} = -1$,

$$\sum_{k \epsilon L(i,j)} P_A^k M_{tu}^k = \frac{1 - \alpha}{2} - 2\varepsilon$$

where $\varepsilon = \text{Prob } A(t_{tu} = -1, t_{ij} = -1)$.

Therefore, if the poll is taken, A expects to receive

$$\frac{1 + \alpha}{2} - 2\delta + \frac{1 - \alpha}{2} - 2\varepsilon = 1 - 2\delta - 2\varepsilon .$$

If this quantity exceeds α, his expected reutrn without a poll, then

$$\delta + \varepsilon < \frac{1 - \alpha}{2} .$$

Q.E.D.

Corollary 3.1. If (i, j) is a pure strategy equilibrium with payoff $(\alpha, -\beta)$, then a poll on (m, n) will increase the value of the campaign to candidate A only if there are post-poll equilibria (r, s) and (t, u) that have the property

$$\text{Prob } A(t_{rs} = -1, t_{mn} = 1) = \text{Prob } A(t_{tu} = -1, t_{mn} = -1) < \frac{1 - \alpha}{2} .$$

Proof. First, note that

$$\sum_{k \epsilon L(m,n)} P_A^k = \frac{1 + \alpha}{2} + \gamma$$

for some $\gamma \in \left(-\frac{1 + \alpha}{2}, \frac{1 - \alpha}{2}\right)$.

Proceeding as in Theorem 1, since

$$\sum_{k \epsilon L(m,n) \cap L(r,s)} P_A^k = \frac{1 + \alpha}{2} + \gamma - \delta ,$$

for some $\delta \geq 0$, then

$$(7) \qquad \sum_{k \in L(m,n)} P_A^k M_{rs}^k = \frac{1+\alpha}{2} + \gamma - 2\delta \ .$$

Similarly,

$$\sum_{k \in \overline{L}(m,n)} P_A^k = 1 - \frac{1+\alpha}{2} + \gamma = \frac{1-\alpha}{2} - \gamma$$

so that $$\sum_{k \in \overline{L}(m,n) \cap L(t,u)} P_A^k = \frac{1-\alpha}{2} - \gamma - \varepsilon$$

and hence

$$(8) \qquad \sum_{k \in \overline{L}(m,n)} P_A^k M_{tu}^k = \frac{1-\alpha}{2} - \gamma - 2\varepsilon \ .$$

Hence the <u>ex ante</u> expected payoff to A if a poll is to be taken is just as in Theorem 3, since the γ's cancel in the sum of (7) and (8).

Q.E.D.

Theorem 3 and its corollary indicate that, in a situation in which there are pre- and post-poll strategy equilibria, if candidate A expects to be made better off by a poll, certain joint probabilities must be "small." Since $\frac{1-\alpha}{2}$ is A's subjective probability that A will lose if no poll is taken, the theorem and its corollary reduce to an algorithm for computing the desirability of a poll by partitioning the state-of-the-world matrices according to which will be relevant under each poll outcome, calculating the outcome under each circumstance assuming an equilibrium strategy is adopted, and computing the sum of these outcomes weighted by the subjective, <u>ex ante</u> probability that the corresponding poll result will transpire. Thus, Prob $A(t_{rs} = -1, t_{mn} = 1)$ is the weighted sum of elements t_{rs} in the subset of matrices for which

t_{mn} is +1, weighted by the <u>ex ante</u> probabilities that each matrix is the true state of the world, and Prob $A(t_{tu} = -1, t_{mn} = -1)$ is a similar calculation from the subset of matrices for which $t_{mn} = -1$.

We can give another result that does not depend on there being a post-poll pure strategy equilibrium.

<u>Theorem 4</u>. If (i, j) is a pre-poll pure strategy equilibrium with payoff $(\alpha, -\beta)$, a poll on (i, j) can increase the expected value of the campaign to both candidates only if

1. $\exists$ strategies (r, s) and (t, u) which, if adopted when $t_{ij} = 1$ or -1, respectively, have a total expected payoff exceeding $\alpha + \beta$.

2. Either

   (A) Prob $A(t_{rs} = -1, t_{ij} = 1)$ + Prob $A(t_{tu} = -1, t_{ij} = -1) < \frac{1-\alpha}{2}$

   or

   (B) Prob $B(t_{rs} = 1, t_{ij} = 1)$ + Prob $B(t_{tu} = 1, t_{ij} = -1) < \frac{1-\beta}{2}$

and

3. The sum of (A) and (B) is less than $1 \frac{\alpha+\beta}{2}$.

<u>Proof</u>. #1 Is obvious.

#2 Consider the two candidates jointly maximizing payoffs. The maximum joint payoff that can be obtained with a poll is

$$V^* = \underset{\tilde{r},\tilde{s}}{\mathrm{Max}}\left[P_A(t_{ij} = 1) \sum_{k\in L(i,j)} M_{\tilde{r}\tilde{s}}^{k} P_A^{k*} - P_B(t_{ij} = 1) \sum_{k\in L(i,j)} M_{\tilde{r}\tilde{s}}^{k} P_B^{k*}\right]$$

$$+ \underset{\tilde{t},\tilde{u}}{\mathrm{Max}}\left[P_A(t_{ij} = -1) \sum_{k\in \overline{L}(i,j)} M_{\tilde{t}\tilde{u}}^{k} P_A^{k**} - P_B(t_{ij} = -1) \sum_{k\in \overline{L}(i,j)} M_{\tilde{t}\tilde{u}}^{k} P_B^{k**}\right]$$

$$= \underset{\tilde{r},\tilde{s}}{\mathrm{Max}} \sum_{L(i,j)} (P_A^k - P_B^k) M_{\tilde{r}\tilde{s}}^k + \underset{\tilde{t},\tilde{u}}{\mathrm{Max}} \sum_{\overline{L}(i,j)} (P_A^k - P_B^k) M_{\tilde{t}\tilde{u}}^k .$$

Now $V^* > \alpha + \beta \Longrightarrow$ payoff to A exceeds α or the payoff to B exceeds $-\beta$. Applying Theorem 3, if the expected payoff to A exceeds α, (A) must hold. The same argument holds for B, establishing #2.

#3 Let (r, s) and (t, u) maximize the expression for V^*. The payoff to A is

$$V_A^* = \sum_{k\in L(i,j)} P_A^k M_{rs}^k + \sum_{\overline{L}(i,j)} P_A^k M_{tu}^k = 1 - 2\delta - 2\varepsilon$$

by Theorem 3.

By the same argument the payoff to B is

$$V_B^* = -(1 - 2\gamma - 2\rho)$$

$$V^* = V_A^* - V_B^* > \alpha + \beta \Longrightarrow$$

$$2 - 2\delta - 2\varepsilon - 2\gamma - 2\rho > \alpha + \beta$$

$$\text{or} \quad \delta + \varepsilon + \gamma + \rho > \frac{\alpha + \beta}{2} .$$

The symbols on the left are simply the joint probabilities in #3 and thus the theorem is proved.

Theorem 4 supplies us with some conditions which, if violated, imply that at least one candidate will not be made better off by a public poll on the (i, j)$^{\underline{th}}$ element, where (i, j) is the pre-poll pure strategy equilibrium. We do not give theorems for more general situations here but it seems that similar results could be easily obtained. The conditions in Theorem 4 amount to saying that if the candidates "disagree enough" so that α and β are near 1, at least one will not want to have a public poll.

The reason that this phenomenon occurs seems clear. If candidates A and B have two different prior distributions on the states of the world, the candidates think that a public poll will reveal more information to the opponent than to themselves and thereby will induce the opponent to change strategy from the present poor choice to one based on better information.

While candidates frequently express hostility to public opinion polls, most candidates for major office spend a good deal of money generating information on public preferences through privately commissioned polls. In his 1968 campaign, Richard Nixon employed the Opinion Research Corporation to conduct "opinion panels" in fourteen states in addition to several "spot" surveys during the election to test public reaction to new issues. When all the costs were totaled the Nixon polling effort in 1968 allegedly cost more than $200,000. It would be strange, indeed, to find that major candidates would spend such sums on opinion research if they felt that it would not be beneficial.

<u>Theorem 5</u>. A private poll cannot reduce the expected value of the campaign to the candidate who receives the information.

Proof. Let $\overline{x}$ and $\overline{y}$ be the optimal mixed strategies of A and B without a poll. Then $\overline{x}P_A\overline{y} \geq x'P_A\overline{y}$ for all z such that $x \geq 0$ and $\Sigma x_i = 1$. The expected payoff to A is $V_A = \overline{x}'P_A\overline{y}$. Suppose a poll is to be taken on the $ij^{\underline{th}}$ element. Then the value of the election to A is

$$V_A^* = P_A(t_{ij} = 1)x^{*\prime}P_A^*\overline{y} + P_A(t_{ij} = -1)x^{**\prime}P_A^{**}\overline{y}$$

where P_A^* is the revised payoff matrix if $t_{ij} = 1$ and P_A^{**} is the revised payoff matrix if $t_{ij} = -1$. x^* and x^{**} are A's optimal strategies for the two situations. Using Lemma 1 we can write V_A^* as follows:

$$V_A^* = x^{*\prime}(\sum_{k \in L(i,j)} P_A^k M^k)\overline{y} + x^{**\prime}(\sum_{k \in \overline{L}(i,j)} P_A^k M^k)\overline{y}$$

Note that $x^* = \overline{x}$, $x^{**} = \overline{x}$ is a feasible strategy after the poll and, if it is substituted in the above expression, $V_A^* = V_A$. Thus, $V_A^* \geq V_A$.

Q.E.D.

Now private information never hurts a candidate, but, since such information is costly, many candidates may not find it worthwhile to purchase it. One potential cost is that the poll will not stay private and, therefore, may reduce the value of the election to the candidate that purchased it. Not surprisingly Kingdon found that candidates for lower (and less valuable) offices relied on private polls to a lesser extent than candidates for more prestigious positions. All of the politicians he studied relied on less expensive information, even though it was frequently known to provide biased

estimates of various probabilities. The candidates felt, nevertheless, that they could "correct" these biases and come up with reasonable guesses about the relevant features of the election. We do not investigate the circumstances in which cheap but inaccurate or biased information will be purchased by a candidate in preference to polls. The methods used here could be applied to study that situation.

V. DISCUSSION

We have explicated a simple model of electoral competition with candidate uncertainty that illustrates the strategic complexities that are introduced by lack of information. The model indicates that uncertainty with respect to voter preferences profoundly affects electoral strategies. Uncertainty can produce pure strategy equilibria in which candidates adopt different platforms, rather than the identical pure and mixed strategies of traditional theory. Furthermore, under some plausible condition candidates will prefer not to increase the amount of information available on voter responses to campaign strategies.

Perhaps the most interesting result is that if both candidates are sure that a given platform would defeat all alternative platforms and if they "disagree enough" in their beliefs about the likely outcomes of other possible contests, they may find it in their interest to conspire not to campaign on the dominant platform. The example we provided of this phenomenon runs counter in spirit to much of the work in the theory of electoral competition. After all, the theorems of "median dominance" encourage the belief that a "reasonable" position will generally prevail in the sense that if one alternative defeats all others, both candidates will adopt is as a platform. While in our model there is no necessary connection between candidate beliefs and the "true" majority dominance relation, if both candidates agree that there is a dominant platform, that belief probably (but not necessarily) had some basis in reality. But, in just that sort of case, the candidates may find it desirable to collude against the will of the majority.

It seems quite possible that candidate uncertainty explains why in some campaigns politicians do not campaign on certain issues, even though they regard those issues to be popular within the electorate. If so, several potentially testable propositions emerge that might fruitfully be examined. In the example of a prisoner's dilemma, some sort of cooperation between candidates is necessary in order to achieve an outcome that is more desirable than the pre-poll equilibrium. The ability of candidates to agree to coordinate their strategies should depend on 1) how difficult it is for the candidates to police defections from the collusive strategy; 2) the cost of information (a candidate has an incentive to collect information which favorably alters the strategic structure of the game); 3) the presence of public polls that reveal information on public choices over which the candidates disagree; and 4) whether a "dominant issue" exists and is recognized by the candidates. We are aware of no empirical research into collusion in two-party

competition but it seems that if such a phenomenon were widespread, and some researchers seem to think it is, the model introduced here may explain it.

The model presented in this paper is obviously no more than a rudimentary first step towards explaining and predicitng candidate behavior in uncertain situations. Our extremely simplified conceptual model of a campaign does not incorporate many important features of an actual election. We have not, for example, dealt with the problems of conveying issue positions, hypothetical or real, to the electorate or the subjects of the poll. The positions of candidates are assumed to be instantly promulgated at no cost to the candidates. No cost is associated with switching actual positions, so that a candidate is allowed to move around as much as he wants in the course of a campaign. And no difficulty arises in deriving useful information from hypothetical poll questions.

Finally, while this paper is partly addressed to the question of how candidates value public opinion polls, it does not provide a complete answer. One reason that some candidates do not like polls has been expressed by former California Assemblyman Walter Karabian. Karabian introduced a bill designed to limit the use of public polls in elections because he felt that a poll showing that a candidate was far behind would limit his ability to attract support. Our model of the campaign is too simplified to shed light on this issue.

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