# Uncertainty in Pin-on-Disk Wear Volume Measurements Using Surface Scanning Techniques 

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#### Abstract

The uncertainty of wear volumes measured using surface scanning techniques is often neglected or assumed to be equivalent to the instrument error. A method is proposed that accounts for the number of wear volume scans, the variations in those scans, and the geometry of the experimental system as an improved measure of uncertainty. It demonstrates that the uncertainty in volume is directly correlated to the number of scans taken. A nonuniform wear track was used to validate the method, and the minimal and optimal number of scans was found.


Keywords Uncertainty • Profilometry • Wear track . Wear volume

Surface scanning techniques can be extremely precise in analyzing wear tracks to compute wear volumes; however, in many instances it is impractical to analyze an entire wear track using atomic force microscopy, interferometry, or other techniques [1, 2]. For this reason, tribologists commonly measure subsections of the wear track at evenly spaced intervals to compute wear volume, which leaves room for errors due to the variations in the wear track [3-6]. These errors are far more significant than the uncertainties of the instruments themselves. A useful measurement of uncertainty must consider the number of scans, variation of those scans, and the geometry of the experimental system.

[^0]The number of scans ( $N$ ) required to accurately measure and predict the volume loss $(V)$ of the wear track can be determined using uncertainty analysis. The volume loss of a wear track of nominal radius $R$ with individually scanned cross-sectional areas $\left(A_{i}\right)$ subdivided into $N$ sections of $\theta_{i}$, where $\theta_{i}$ is held constant for each scan, is shown in Fig. 1. This volume loss is approximated in Eq. 1.

$$
\begin{align*}
V & \cong \sum_{i=1}^{N} A_{i} R \theta_{i}=R \theta_{i} \sum_{i=1}^{N} A_{i}=R \frac{2 \pi}{N} \sum_{i=1}^{N} A_{i} \\
& =2 \pi R\left(\frac{\sum_{i=1}^{N} A_{i}}{N}\right) . \tag{1}
\end{align*}
$$

The uncertainty of the volume loss $\left(u_{\mathrm{V}}\right)$ is determined using the law of propagation of uncertainty as shown in Eq. 2:

$$
\begin{align*}
u_{\mathrm{V}}^{2} & =\left(\frac{\delta V}{\delta A_{i}}\right)^{2} u_{A_{i}}^{2}+\left(\frac{\delta V}{\delta R}\right)^{2} u_{\mathrm{R}}^{2} \\
& \cong \sum_{i=1}^{N} u_{A_{i}}^{2}\left(R \theta_{i}\right)^{2}+\left(\sum_{i=1}^{N} A_{i} \theta_{i}\right)^{2} u_{\mathrm{R}}^{2} \tag{2}
\end{align*}
$$

Assuming the uncertainty in the area $\left(u_{\mathrm{A}}\right)$ is equal for every scan, the summation of $u_{\mathrm{A}}$ is defined in Eq. 3 .
$\sum_{i=1}^{N} u_{A_{i}}=u_{A} N$.
This definition is applied to Eq. 2 and multiplied by unity $\left(\frac{N}{N}\right)$ yielding Eq. 4.
$u_{\mathrm{V}}^{2} \cong R^{2} \theta_{i}^{2} u_{\mathrm{A}}^{2} N\left(\frac{N}{N}\right)+\theta_{i}^{2}\left(\sum_{i=1}^{N} A_{i}\right)^{2} u_{\mathrm{R}}^{2}\left(\frac{N}{N}\right)^{2}$.

Fig. 1 Schematic representation of experimental wear track with geometric nomenclature defined. The magnified view shows a 2D cross-sectional area between markers 5 and 6


Fig. 2 a Area removed versus circumferential location for wear track shown in Fig. 1. b Three example iterations of volume loss values of the wear track shown with error bars representative of a one standard deviation uncertainty in the volume. The true volume loss is labeled

The number of scans multiplied by the portion of the wear track analyzed is equal to the full revolution of $2 \pi$, which is shown in Eq. 5.
$\sum_{i=1}^{N} \theta_{i}=\theta_{i} N=2 \pi$.
Equation 4 may now be rewritten as Eq. 6 .
$u_{\mathrm{V}} \cong \frac{(2 \pi R)^{2}}{N} u_{\mathrm{A}}^{2}+\frac{\left(2 \pi R u_{\mathrm{R}}\right)^{2}}{R^{2} N^{2}}\left(\sum_{i=1}^{N} A_{i}\right)^{2}$.
$V_{\mathrm{T}}$ and the simplified uncertainty from Eq. 9 bounds the volumes. c The percent of volume loss values in range out of 1000 simulated scenarios is shown versus the number of scans

The definition of the standard deviation of the area can be manipulated to express the summation of the area in terms of the mean area $(\bar{A})$, standard deviation of the area $\left(\sigma_{A}\right)$, and number of scans $(N)$.

$$
\begin{equation*}
\sigma_{A}^{2}=\frac{\left(\sum_{i=1}^{N} A_{i}\right)^{2}}{N}-\bar{A}^{2} \Rightarrow\left(\sum_{i=1}^{N} A_{i}\right)^{2}=N\left(\sigma_{A}^{2}+\bar{A}^{2}\right) \tag{7}
\end{equation*}
$$

The uncertainty in the area measurement due to instrument error is much smaller than the standard deviation of the measured areas; therefore, the standard
deviation $\left(\sigma_{A}\right)$ will be used in place of the uncertainty in area $\left(u_{\mathrm{A}}\right)$ to account for maximum error. The final expression for approximating uncertainty in volume is shown in Eq. 8.
$u_{\mathrm{V}} \cong \frac{2 \pi R}{\sqrt{N}} \sqrt{\sigma_{A}^{2}+\frac{u_{\mathrm{R}}^{2}}{R^{2}}\left(\sigma_{A}^{2}+\bar{A}^{2}\right)}$.
The uncertainty in the volume loss is dependent upon the wear track radius, the variation in scanned area measurements, and the total number of scans used. The largest contributing factor to the uncertainty value is the number of scans. The standard deviation of the area scans and the uncertainty in the volume loss decrease as the number of scans used increases.

In many instances $u_{\mathrm{R}}^{2} \ll R^{2}$ which leads to a simplified expression for volume uncertainty shown in Eq. 9.
$u_{\mathrm{V}} \cong \frac{2 \pi R}{\sqrt{N}} \sigma_{A}$.
Data from a non-uniform wear track pin-on-disk experiment was used to validate this method. A total of 100 scans were taken to image the entire wear track using a Veeco Wyko NT9100 scanning white light interferometer with the area data shown in Fig. 2a. From the data, sets of $N=2,3,4,5,7,8,10,25$, and 50 scans were simulated to compute wear volume for 1000 scenarios. Three scenarios for each $N$ are shown in Fig. 2b.

The difference between the predicted and true volume, based on scanning the entire track, can be compared to the uncertainty in the volume and used to establish a percentage of times the data was predicted to be in range of the true value. The percent of values in range for the thousand scenarios of each $N$ are shown in Fig. 2c.

As the number of scans increased, the estimated volume loss was closer to the true value and the associated uncertainty decreased. This is always valid, though increases in accuracy become negligible well before $N=100$. For this extremely non-uniform wear track, four scans accurately estimated the volume loss $68 \%$ of the time, but with a large uncertainty value; Fig. 2 b and c show eight scans yielded a better estimate. This method can be used in conjunction with other methods that compute wear rates and their uncertainties [7], and add validity to these types of measurements in scientific pursuits.

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