Uncertainty in the analysis of urban water supply and distribution systems

George Tsakiris and Mike Spiliotis

ABSTRACT

Conventionally, the design of urban water supply and distribution systems is based on the assumption that all the involved parameters are known a priori and remain unaltered throughout the life cycle of the system. However, significant uncertainties do appear during the analysis and design of these systems, such as the equivalent pipe roughness and the actual internal diameters of the pipes. To study these uncertainties, the water supply and the looped water distribution systems are studied separately. For the water supply system, these uncertainties are incorporated in the analysis of the system, using the extension principle of the fuzzy sets and a new operation of the fuzzy subtraction. Based on the calculation of head losses for each branch of the system, the nodal heads are obtained as fuzzy numbers. In regard to the looped water distribution system, a methodology is developed and proposed, based on the extension principle and leading to several optimisation problems with respect to the branches of the system. The aim of the proposed methodology is to determine the α-cuts and finally produce the shape of the membership function of flows in the branches of the system. Both methodologies are illustrated by numerical examples.

Key words | extension principle, fuzzy algebra, fuzzy sets, pipe roughness, urban water supply

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INTRODUCTION AND BASIC NOTIONS

It is commonly accepted that the urban water supply system is a branched pipe system, whereas the distribution systems are looped pipe systems. During the design stage, it is customary to analyse both systems based on the assumption that all the involved parameters are known or can be calculated at the design stage, and remain unchanged throughout the life cycle of the facility.

However, most of the involved parameters are accompanied by uncertainties which cannot be neglected at the design stage. Indeed, the real conditions of 'running' water supply systems differ greatly from theoretical considerations for a variety of reasons, such as financial shortcuts (Kanakoudis 2004), the ravages of time, the uncertainty in the maximum water consumption, etc. Among the parameters exhibiting uncertainty are the pipe roughness, the internal pipe diameter and the required water flow through each branch of the system (Spiliotis & Tsakiris 2012; Tsakiris & Tsakiris 2012). To deal with these uncertainties, two cases are studied separately: the branched water supply system and the looped water distribution system.

A number of methodologies dealing with the uncertainties of these parameters exist in the related literature (Xu & Goulter 1998; Xu *et al.* 2003; Giustolisi *et al.* 2009; Alvisi & Franchini 2010). The uncertainties studied by Xu & Goulter (1998) and Xu *et al.* (2003) are restricted to small deviations from the mean, since they are based on the development of Taylor series around the mean values.

This paper deals with all these uncertainties using two innovative, 'low data demand' methodologies based on fuzzy sets and logic. In addition, the proposed methodologies can cover a wider range of values around the mean values. Before presenting the proposed methodologies, it is wise to briefly summarise some of the most basic principles of the analysis of the water supply and distribution systems.

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In the case of a branched pipe water system, since the flow at each branch can be considered known, the head at each node, H, can be determined directly from the head of the previous node by subtracting the head losses of the branch connecting the two nodes.

Let the nodes i, n and j be three consecutive nodes of the main branch of a system moving from upstream to downstream. The flow downstream of the node n, Q_{nj}, is equal to:

$$Q_{nj} = Q_{in} - q_n \tag{1}$$

where q_n is the outflow at the node n. The head at the node n is equal to:

$$H_n = H_0 - \sum_{0 \to n} h_f \tag{2}$$

where H_0 is the initial head (e.g., the elevation of the free water surface in a tank) and Σh_f is the sum of head losses calculated by equations such as the equations of Darcy–Weisbach or Hazen–Williams.

The uncertainty in the branched water supply systems has been recently studied also in Tsakiris & Spiliotis (2016).

Regarding the looped pipe systems, most often encountered in municipal water distribution systems, there is an extra complication since we cannot predict the direction of the flow. Furthermore, when examining the maximum flow at a branch, then larger values of the pipe roughness coefficient for a water path should be combined with smaller values of the pipe roughness coefficient for another water path. The unknown distribution of flows and the complicated interaction between the several variables of the looped pipe systems makes the fuzzy analysis in this case more difficult and complicated.

Based on the continuity equation at each node n and assigning a direction to each single branch, the first **Q**-Equations may be written as follows (Lansey & Mays 2000):

$$\begin{cases} \sum_{in}^{in \in I_1(1)} \pm Q_{in} = q_1 \\ \ldots \\ \sum_{in}^{in \in I_1(m)} \pm Q_{in} = q_m \\ \ldots \\ \sum_{in}^{in \in I_1(M-1)} \pm Q_{in} = q_{M-1} \end{cases} \tag{3}$$

where M is the total number of nodes of the network; I_1 is the set of all branches converging into the examined node; and q_m is the water demand concentrated at node m.

The conservation of energy principle for loops leads to the following L equations:

$$\begin{cases} \sum_{in}^{in \in I(1)} \pm R_{in} |Q_{in}|^{n-1} Q_{in} = 0 \\ . \\ . \\ . \\ \sum_{in}^{in \in I(L)} \pm R_{in} |Q_{in}|^{n-1} Q_{in} = 0 \end{cases} \tag{4}$$

where L is the number of the loops, and I is the set which contains all the branches of the examined loop; in addition, where R_{in} is the hydraulic resistance of the branch in, and its equation is dependent on the selected equation for determining the linear head losses.

This paper focuses on the uncertainty encountered in a looped water distribution system assuming that the internal diameters and the pipe roughness coefficients are not crisp numbers, as conventionally thought. In addition the authors extend the methodology of Revelli & Ridolfi (2002), who suggested a fuzzy approach for the analysis of a pipe system considering uncertain parameters of looped pipe system based on **Q**-equations. An innovative aspect of the modification is the use of α-cuts instead of the direct use of the membership functions of the corresponding fuzzy sets. In fact, the optimisation procedure is used in order to determine the fuzzy output quantities (i.e., flows) from a system with crisp **Q**-equations, while some variables are fuzzy numbers. In this article, the pipe roughness coefficients and the internal diameters are expressed as fuzzy quantities. In contrast, the method presented by Spiliotis & Tsakiris (2012) analysed the uncertainty based on fuzzy water consumption at the nodes.

The uncertainty on internal diameters stems from the fact that due to deposition of material in the pipes over the years, the effective diameters of the pipes are getting smaller.

Finally, in this article, a global objective function is proposed with the aim to reduce the fuzziness and simplify the computational process. The global objective function takes into account all the branches of the system.

FUZZY ANALYSIS

A fuzzy number is a fuzzy set satisfying the properties of convexity and normality. It is defined in the axis of real numbers and its membership function is a piecewise continuous function. A simple fuzzy number for representing the water demand at the node n is a fuzzy triangular number.

An extension of a fuzzy triangular number is the LR fuzzy number with membership function of a non-linear shape (Dubois & Prade 1978). A fuzzy number is of LR-type if there exist functions L (for left) and R (for right) with:

$$\mu_{\mathrm{A}}(x) = \begin{cases} L\left(\dfrac{x-\alpha_1}{\alpha_2-\alpha_1}\right) & \text{if } \alpha_1 \le x \le \alpha_2 \\ R\left(\dfrac{\alpha_3-x}{\alpha_3-\alpha_2}\right) & \text{if } \alpha_2 \le x \le \alpha_3 \\ 0 & \textit{otherwise} \end{cases}$$

where L and R are two non-decreasing shape functions which satisfy the following equations:

$$\begin{aligned} &L{:}[0,\ 1] \rightarrow [0,\ 1]\ \ \kappa\alpha\iota{:}R[0,\ 1] \rightarrow [0,\ 1] \\ &L(0) = R(0) = 0,\ \ L(1) = R(1) = 1 \end{aligned} \tag{5}$$

The α-cut set of the fuzzy number A (with 0 < α≤ 1) constitutes the basic concept of fuzzy sets theory which is used in order to move from the fuzzy to the crisp sets and it is defined as follows (Zimmermann 1991; Klir & Yuan 1995):

$$\tilde{A}_\alpha = \{x|\mu_{\tilde{A}}(x) \ge \alpha,\ \ x \in \Re\}. \tag{6}$$

One can notice that the α-cut set is a crisp set determined from the fuzzy set according to a selected value of the membership function, and reciprocally, a fuzzy set can be derived from a significant number of α-cut sets.

In the case of a fuzzy triangular number, its α-cut, $\tilde{A}_\alpha$, is the following (crip) set:

$\tilde{A}_\alpha = (A^{\mathrm{L}}_{\alpha},\ \ A^{\mathrm{R}}_{\alpha})$, or equivalently:

$$\tilde{A}_\alpha = \left[\bar{A} - (1-\alpha)\cdot(\bar{A} - A^-),\ \ \ \bar{A} + (1-\alpha)\cdot(A^+ - \bar{A})\right]. \tag{7}$$

in which α is the selected level of the α-cut, $A^-, \bar{A},\ A^+$ the left-hand boundary, the central value *($\mu = 1$)*, and the right-hand side of the fuzzy set $\tilde{A}$.

The crisp set including all the elements with non-zero membership function is the 0-strong cut which is defined as follows (Buckley & Eslami 2002):

$$\tilde{A}_{0^+} = \{x|\mu_A(x) > 0,\ \ x \in \Re\}. \tag{8}$$

The following symbols are used for the zero-cut (Figure 1):

$$\tilde{A}_{0^+} = (A^-,\ \ A^+). \tag{9}$$

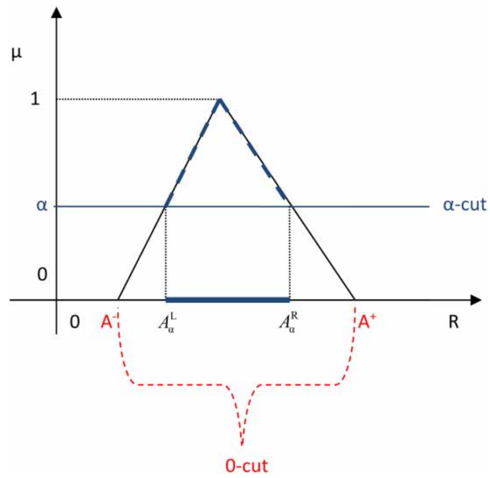

Figure 1 | Triangular fuzzy number and its α-cut and zero-cut.

For a fuzzy triangular number, it is sufficient to know only the left- and the right-hand boundary of its strong cut for describing its membership function. The crisp subset of X described by the 0-cut of a fuzzy set Ã is the support of Ã.

We can now extend the operation of the usual crisp functions, if the inputs are fuzzy sets, based on the extension principle, which is briefly presented below.

Let X be a Cartesian product of universe $X = X_1 \times X_2 \times \cdots \times X_n$ and $\tilde{A}_1, \tilde{A}_2,\ \ldots, \tilde{A}_n$ be defined in the universe sets $X_1, X_2,\ \ldots, X_n$, respectively. Let f be a (crisp) mapping from X to a universe Y, $y = f(x_1, x_2,\ \ldots, x_n)$. The mapping f for these particular input sets can now be defined as $\tilde{B} = \{(y, \mu_{\tilde{\mathrm{B}}}(y))|y = f(x_1, x_2,\ \ldots x_n), (x_1, x_2,\ \ldots, x_n) \in X\}$, in which the membership function of the image B̃ can be defined (Zimmermann 1991; Tsakiris & Spiliotis 2014) by:

$$\mu_{\tilde{B}}(y) = \sup_{(x_1, x_2, \ldots, x_n) \in f^{-1}(y)} \min\left(\mu_{A_n}(x_1),\ \ldots, \mu_{A_n}(x_n)\right) \tag{10}$$

where f^{-1} is the inverse image of *f*.

The implementation of the extension principle provides the opportunity to use a crisp function in which the variables are fuzzy numbers (Kechagias & Papadopoulos 2007; Tsakiris & Spiliotis 2014).

In most cases, it is preferable to use α-cuts in the fuzzy analysis. If *f* is a continuous function in the extension principle, the use of α-cuts can be also extended by determining the α-cuts of the function *f*, as follows (Buckley & Eslami 2002; Buckley *et al.* 2002):

$$\begin{cases} f^L\left(\tilde{A}_1, \tilde{A}_2, \tilde{A}_3\right)_\alpha = \min\left\{ f(x_1, x_2, x_3) \left| x_1 \in \tilde{A}_{1\alpha}, x_2 \in \tilde{A}_{2a}, x_3 \in \tilde{A}_{3a} \right.\right\}, \\ f^R\left(\tilde{A}_1, \tilde{A}_2, \tilde{A}_3\right)_\alpha = \max\left\{ f(x_1, x_2, x_3) \left| x_1 \in \tilde{A}_{1\alpha}, x_2 \in \tilde{A}_{2a}, x_3 \in \tilde{A}_{3a} \right.\right\}. \end{cases} \quad (11)$$

where $\tilde{A}_{1\alpha}, \tilde{A}_{2a}, \tilde{A}_{3a}$ are the α-cuts for the fuzzy parameters x_1, x_2 and x_3, respectively.

From the theorem of global existence for maxima and minima of functions with many variables, it is known that if the domain of a real function is closed and bounded and the real function is continuous, then the function will have its absolute minimum and maximum values at some points in the domain (Marsden & Tromba 2003; Tsakiris & Spiliotis 2014). Based on this theorem, it is evident that the α-cut for any real continuous function with real variables in this domain can be determined, given that the inputs are fuzzy triangular numbers.

FUZZY ARITHMETIC OPERATION AND A NEW FUZZY ALGEBRA

The arithmetic operations between fuzzy sets are defined by the means of the extension principle. However, in practice, since each fuzzy set and consequently each fuzzy number can be fully and uniquely represented by its α-cuts, the latter are closed intervals of real numbers for all the α-cuts. Therefore, we can apply the arithmetic operations on fuzzy numbers, in terms of arithmetic operations on their α-cuts (Kechagias & Papadopoulos 2007; Chrysafis & Papadopoulos 2009). The key property of the above methodology is as follows: Let A, B denote fuzzy numbers and let * denote any of the four basic arithmetic operations. Then, a fuzzy set in $\mathfrak{R}$, $A*B$ can be defined by determining its α-cuts as (Klir & Yuan 1995):

$${}^a(\mathrm{A}*\mathrm{B}) = {}^a\mathrm{A}*{}^\alpha\mathrm{B}, \quad \forall \alpha \in [0.1] \quad (12)$$

Between the binary arithmetic operations between the α-cuts, the interval arithmetic is applied. Here, from the fuzzy algebra we use the addition and the subtraction operations. Finally, in conjunction with the fuzzy decomposition theorem, the following equation holds for all the fuzzy sets of the fuzzy operation:

$$\mathrm{A}*\mathrm{B} = \cup_\alpha \ (\mathrm{A}*\mathrm{B}) \quad (13)$$

in which * means any algebraic operation.

In the case that the quantities of the problem are in the form of LR fuzzy numbers, based on the previous analysis, it can be easily proved that (e.g., Dubois & Prade 1978; Klir & Yuan 1995):

$$(\alpha_1, \ \alpha_2, \ \alpha_3)_{LR} + (\beta_1, \ \beta_2, \ \beta_3)_{LR} = (\alpha_1 + \beta_1, \ \alpha_2 + \beta_2, \ \alpha_3 + \beta_3)_{LR} \quad (14)$$

$$(\alpha_1, \ \alpha_2, \ \alpha_3)_{LR} - (\beta_1, \ \beta_2, \ \beta_3)_{LR} = (\alpha_1 - \beta_3, \ \alpha_2 - \beta_2, \ \alpha_3 - \beta_1)_{LR} \quad (15)$$

where the subscripts 1, 2, 3 indicate the left-hand boundary, the central value and the right-hand boundary of the LR fuzzy number, respectively.

However, in some physical problems the standard fuzzy subtraction leads to ambiguous results with irrational large spread. To cure this problem, especially for fuzzy triangular numbers, Gani proposed a modification of the standard fuzzy subtraction as follows (Gani 2012):

$$(\alpha_1, \ \alpha_2, \ \alpha_3)_{TR} - (\beta_1, \ \beta_2, \ \beta_3)_{TR} = (\alpha_1 - \beta_1, \ \alpha_2 - \beta_2, \ \alpha_3 - \beta_3)_{TR}$$

provided that:

$$\frac{(\alpha_3 - \alpha_1)}{2} \geq \frac{(\beta_3 - \beta_1)}{2} \quad (16)$$

In this paper, the definition of Gani is adopted to calculate the flow of each branch of the water pipe system, moving from upstream to downstream direction. The total flow and the outflow at each branch are selected to be fuzzy triangular number and, hence, the flow at branches will also be fuzzy triangular numbers. This method results in a more realistic representation of the flow through the branches and mitigates the fuzziness of the flow and the head losses and the resulting nodal heads.

METHODOLOGY FOR WATER SUPPLY SYSTEMS

The basic idea is that for every branch of the water conveyance system, we calculate the head losses based on the extension principle and, thus, we take into account all the associated uncertainties. The effective internal diameter, the equivalent roughness and the discharge are considered as fuzzy triangular numbers based on the experience of the professionals. Therefore, based on our experience, the left- and the right-hand sides of the zero-cut for a fuzzy number can be estimated. In addition, in contrast to the interval or the grey systems approach (e.g., Alvisi & Franchini 2010), a central value can be supposed. To understand the concept of central value, the used fuzzy number can be named as 'around the central value'. On the contrary, in the case of the probabilistic approach, a historical sample is required and not only the extremes and the central value. By taking into account all the α-cuts of the parameters an optimisation problem should be solved. Further, according to the extension principle (selection of several α-cuts), the membership function of the head losses can be determined.

Another complication in the analysis is the manipulation of the fuzziness considering the flow at each branch. In the examined case, the application of the fuzzy arithmetic for the flow of each branch leads to rather irrational results. To solve this problem, we suppose that the worst scenarios related to the water consumption at the nodes (or at the secondary branches) occur simultaneously. Therefore, considering the pessimistic scenario, we start from the upstream node which is a water source node (e.g., a tank) with the maximum flow. However, since the water consumption is concentrated at the node, passing from upstream to downstream, we use the maximum outflow at the current node and not the minimum flow as in the conventional fuzzy subtraction. The opposite occurs in the case of the optimistic scenario. Thus, we could use the new fuzzy subtraction of Gani (2012) to calculate the flow at each branch:

$$\left(Q_{n\to j}^{-},\ \overline{Q_{n\to j}},\ Q_{n\to j}^{+}\right)_{TR} = (Q_{i\to n}^{-},\ \overline{Q_{i\to n}},\ Q_{i\to n}^{+})_{TR} - (q_n^{-}, \bar{q}_n, q_n^{+})_{TR} = (Q_{i\to n}^{-} - q_n^{-},\ \overline{Q_{i\to n}} - \bar{q}_n,\ Q_{i\to n}^{+} - q_n^{+})_{TR} \tag{17}$$

On the contrary, for the head losses the conventional fuzzy arithmetic is used:

$$(H_n^{-},\ \overline{H_n},\ H_n^{+})_{LR} = (H_i^{-},\ \overline{H_i},\ H_i^{+})_{LR} - \left(h_{f,in}^{-}, \overline{h_{f,in}}, h_{f,in}^{+}\right)_{LR} = \left(H_i^{-} - h_{f,in}^{+},\ \overline{H_i} - \overline{h_{f,in}},\ H_i^{+} - h_{f,in}^{-}\right)_{LR} \tag{18}$$

As mentioned previously, the head losses at each branch are calculated based on the extension principle for a representative number of α-cuts. Thus, the following equations are used to determine the bounds of each fuzzy cut and for each branch. In this study, the Darcy–Weisbach equation for the calculation of head losses is adopted and the Swamee and Jain approximate equation (Swamee & Jain 1976; Tsakiris & Spiliotis 2014) for the estimation of the friction factor (instead of the Colebrook and White equation, Colebrook & White 1937):

$$\begin{cases} h_{f,in}^{L}\left(\widetilde{k_{in}},\ \widetilde{D_{in}},\ \widetilde{Q_{in}}\right)_{\alpha} = \min\left\{ h_{f,in}(x_1,x_2,x_3) = \dfrac{8L_{in}}{g\pi^2 x_2^5} \dfrac{0.25}{\left[\log\left(\dfrac{5.74}{(4x_3/(\nu\cdot\pi\cdot x_2))^{0.9}} + \dfrac{x_1/x_2}{3.7}\right)\right]^2} x_3^2 \,\middle|\, x_1 \in \widetilde{k_{in\alpha}}, x_2 \in \widetilde{D_{in\alpha}}, x_3 \in \widetilde{Q_{in\alpha}} \right\}, \\ h_{f,in}^{R}\left(\widetilde{k_{in}},\ \widetilde{D_{in}},\ \widetilde{Q_{in}}\right)_{\alpha} = \max\left\{ h_{f,in}(x_1,x_2,x_3) = \dfrac{8L_{in}}{g\pi^2 x_2^5} \dfrac{0.25}{\left[\log\left(\dfrac{5.74}{(4x_3/(\nu\cdot\pi\cdot x_2))^{0.9}} + \dfrac{x_1/x_2}{3.7}\right)\right]^2} x_3^2 \,\middle|\, x_1 \in \widetilde{k_{in\alpha}}, x_2 \in \widetilde{D_{in\alpha}}, x_3 \in \widetilde{Q_{in\alpha}} \right\}. \end{cases} \tag{19}$$

In Equation (19), ν = kinematic viscosity of water (m^2/s); k_{in} = pipe roughness coefficient (m); D_{in} = internal diameter of the branch *in* (m); L_{in} = length of the branch *in* (m).

It should be clarified that the minor losses are included in the linear losses by following an indirect way according to the Greek experience, that is, by increasing the pipe roughness. This practice can be found also in other countries, especially when we deal with water distribution networks.

Since the head losses of each branch have a non-linear mode, non-linear membership functions for the nodal heads are expected.

METHODOLOGY FOR WATER DISTRIBUTION SYSTEMS

Here, the **Q**-equations are used to deal with the uncertainties encountered in the looped water distribution systems. To simplify the calculation procedure, instead of the Darcy–Weisbach equation, the Hazen–Williams equation is used to calculate the head losses:

$$h_{f,in} = R_{in} Q_{in}^{n}, \quad n = 1.852 \tag{20}$$

in which the hydraulic resistance R_{in} is:

$$R_{in} = \frac{10.7\, L_{in}}{C_{in}^{1.852} D_{in}^{4.87}} \tag{21}$$

C_{in} is the Hazen–Williams roughness coefficient dependent mainly on the pipe material and L_{in}, D_{in} are the length and the internal diameter of the branch *i-n*, respectively.

In this methodology, both the pipe roughness coefficients and the internal diameters are considered as fuzzy quantities. Based on the α-cut concept, we establish the corresponding bound constraints, i.e., for pipe roughness coefficients:

$$x_{2,in} \in \left[C_{in_a}^{L},\ C_{in_a}^{R} \right] \tag{22}$$

where $x_{2,in}$ is a decision variable for the pipe roughness coefficient of the branch *in*, and $C_{in\,a}^{L}$, $C_{in\,a}^{R}$ the left- and the right-hand bounds of the α-cut.

As mentioned previously, the use of the α-cut instead of the membership function of the corresponding fuzzy sets is adopted. The analysis is based on the crisp **Q**-equations, whereas the fuzzy inputs are the α-cut sets for the pipe roughness coefficients and the internal diameters. However, it should be stressed that for each optimisation problem, the decision variable $x_{1,in}$, which corresponds to the flow at branch *in,* takes only crisp values.

Revelli & Ridolfi (2002) proposed the use of an optimisation procedure to determine the minimum and the maximum water flow at each branch for each α-cut:

$$\begin{cases} \min x_{1,in} \quad (or \max x_{1,in}) \\ \sum\limits_{in}^{in \in I(l)} \pm \dfrac{10.7 L_{in}}{x_{2,in}{}^{1.852} x_{3,in}{}^{4.87}} \left| x_{1,in} \right|^{0.852} x_{1,in} = 0, \quad l = 1, \ldots, L \\ \sum\limits_{in}^{in \in I_1(m)} \pm x_{1,in} = q_m, \quad m = 1, \ldots, M \\ x_{2,in} \in \widetilde{C_{in_a}},\ x_{3,in} \in \widetilde{D_{in_a}},\ x_{1,in}(crisp) \end{cases} \tag{23}$$

Therefore, the same problem is solved for a number of α-cuts to find the maximum water flow at branch *in*. Similarly, in order to find the lower boundary of the water flow at branch *in* we use the same constraints, but with different objective function aiming at the minimum water flow at branch *in*. The process is repeated for each branch by changing only the objective function for each α-cut.

In contrast with the water supply system, the analysis of looped distribution systems involves all the branches of the system simultaneously, since it is not possible to deal with the head losses of each branch separately.

Revelli & Ridolfi (2002) proposed the use of fuzzy algebra in order to estimate the head losses at each branch based on the flows obtained from Equation (23). However, it seems more reasonable to perform a different optimisation procedure for determining the α-cuts of the head losses. Following this modification of Revelli and Ridolfi's approach, the objective function can be linked directly to the head losses, as follows:

$$\begin{cases} \min \left| \dfrac{10.7L_{in}}{x_{2,in}{}^{1.852} x_{3,in}{}^{4.87}} \left|x_{1,in}\right|^{0.852} x_{1,in} \right| \ (or \max) \\ \sum\limits_{in}^{in \in I(l)} \pm \dfrac{10.7L_{in}}{x_{2,in}{}^{1.852} x_{3,in}{}^{4.87}} \left|x_{1,in}\right|^{0.852} x_{1,in} = 0, \quad l = 1, \ldots, L \\ \sum\limits_{in}^{in \in I_1(m)} \pm x_{1,in} = q_m, \quad m = 1, \ldots, M \\ x_{2,in} \in \widetilde{C_{in_\alpha}}, x_{3,in} \in \widetilde{D_{in_\alpha}}, \ x_{1,in}(crisp) \end{cases} \tag{24}$$

By comparing the solutions of the above two procedures, we can observe that they do not reach the same results. This can also be seen in the numerical example of this article.

In addition, in this work, a new method is proposed to produce a simultaneous estimation of all hydraulic variables and their fuzziness. The aim of this method is to reduce the fuzziness and simplify the computational process based on a global objective function; that is, to use a function which involves all the branches of the system. Therefore, an objective function which comprises the weighted sum of the absolute values of the head losses is proposed. The selection of the weights is based on the principle that each weight is directly proportional to the corresponding flow:

$$\begin{cases} \min \sum\limits_{in=1}^{L+M-1} \left(\dfrac{\left|x_{1,in}\right|}{\left(\sum\limits_{in=1}^{L+M-1} \left|x_{1,in}\right| \right)} \right) \cdot \left| \dfrac{10.7L_{in}}{x_{2,in}{}^{1.852} x_{3,in}{}^{4.87}} \left|x_{1,in}\right|^{0.852} x_{1,in} \right| (or \max) \\ \sum\limits_{in}^{in \in I(l)} \pm \dfrac{10.7L_{in}}{x_{2,in}{}^{1.852} x_{3,in}{}^{4.87}} \left|x_{1,in}\right|^{0.852} x_{1,in} = 0, \ l = 1, \ldots, L \\ \sum\limits_{in}^{in \in I_1(m)} \pm x_{1,in} = q_m, \ m = 1, \ldots, M \\ x_{2,in} \in \widetilde{C_{in_\alpha}}, x_{3,in} \in \widetilde{D_{in_\alpha}}, \ x_{1,in}(crisp) \end{cases} \tag{25a}$$

where $\left(\dfrac{\left|x_{1,in}\right|}{\left(\sum_{in=1}^{L+M-1} x_{1,in} \right)} \right) = w_{in}$ is the weight. It holds:

$$\sum_{in=1}^{L+M-1} \left(\frac{\left|x_{1,in}\right|}{\left(\sum\limits_{in=1}^{L+M-1} x_{1,in} \right)} \right) = 1. \tag{25b}$$

However, if the user wishes to see all the fuzzy boundaries of each variable, then a double optimisation problem for several α-cuts for each variable should be formulated and performed.

Furthermore, considering the simultaneous worst (or ideal) case for the head losses, we can deal with the uncertainty of the flows at nodes; that is, the methodology can incorporate the case of variable demand at nodes. Indeed, it is reasonable to suppose that the maximum values of flow (worst case) occur almost simultaneously as the minimum discharges too. It is obvious that the larger the total water consumption the larger the head losses become and, thus, the upper bounds of the examined α-cut for all the water consumptions at nodes can be examined to determine the maximum weighted sum of the head losses. Therefore, the proposed approach can include the fuzziness of the water flows at nodes by changing the continuity equation at nodes. Then, the problem is formulated in Equation (26): where $q^L_{m\alpha}$, $q^R_{m\alpha}$ is the left- and right-hand bounds of the α-cut for the m^{th} water consumption at node m.

We should confess that by using the global objective function, we do not capture the entire range of fuzziness as in the previous two methods. However, the global

$$\begin{cases} \min \sum\limits_{in=1}^{L+M-1} \left(\dfrac{\left|x_{1,in}\right|}{\left(\sum\limits_{in=1}^{L+M-1} \left|x_{1,in}\right| \right)} \right) \cdot \left| \dfrac{10.7L_{in}}{x_{2,in}{}^{1.852} x_{3,in}{}^{4.87}} \left|x_{1,in}\right|^{0.852} x_{1,in} \right| \\ \sum\limits_{in}^{in \in I(l)} \pm \dfrac{10.7L_{in}}{x_{2,in}{}^{1.852} x_{3,in}{}^{4.87}} \left|x_{1,in}\right|^{0.852} x_{1,in} = 0, \ l = 1, \ldots, L \\ \sum\limits_{in}^{in \in I_1(m)} \pm x_{1,in} = q^L_{m\alpha}, \ m = 1, \ldots, M \\ x_{2,in} \in \widetilde{C_{in_\alpha}}, x_{3,in} \in \widetilde{D_{in_\alpha}}, \ x_{1,in}(crisp) \end{cases} \& \begin{cases} \max \sum\limits_{in=1}^{L+M-1} \left(\dfrac{\left|x_{1,in}\right|}{\left(\sum\limits_{in=1}^{L+M-1} \left|x_{1,in}\right| \right)} \right) \cdot \left| \dfrac{10.7L_{in}}{x_{2,in}{}^{1.852} x_{3,in}{}^{4.87}} \left|x_{1,in}\right|^{0.852} x_{1,in} \right| \\ \sum\limits_{in}^{in \in I(l)} \pm \dfrac{10.7L_{in}}{x_{2,in}{}^{1.852} x_{3,in}{}^{4.87}} \left|x_{1,in}\right|^{0.852} x_{1,in} = 0, \ l = 1, \ldots, L \\ \sum\limits_{in}^{in \in I_1(m)} \pm x_{1,in} = q^R_{m\alpha}, \ m = 1, \ldots, M \\ x_{2,in} \in \widetilde{C_{in_\alpha}}, x_{3,in} \in \widetilde{D_{in_\alpha}}, \ x_{1,in}(crisp) \end{cases} \tag{26}$$

optimisation method leads to the reasonable conclusion that the most unfavourable values for the pipe roughness, internal diameters and water consumptions will take place almost simultaneously.

CASE STUDY FOR THE WATER SUPPLY SYSTEM

In this section, a simple water supply system with third-generation PE pipes is studied for illustrating the use of the proposed methodology (Figure 2). The internal diameter, the length and the flow of each branch are depicted in Table 1. We assume that the fuzzy numbers representing the pipe roughness, the internal diameter and the flow can be described as follows: (a) pipe roughness coefficient, k: 0.45, 0.50, 0.60 (mm); (b) internal diameter, D: $D-4$, D, $D+2$ (mm), if $D > 200$ (mm), $D-2$, D, $D+1$ (mm), if $D \leq 200$ (mm); (c) design flow, Q: $0.95Q$, Q, $1.05Q$ (L/s). The first and the third values are the left-hand and the right-hand boundary of the 0-cut, whereas the second value denotes the central value of each fuzzy set. Fuzzy triangular numbers are adopted for all the above parameters.

In brief, the nodal heads can be calculated following the steps below:

(1) The fuzzy parameters (e.g., k, D) of the water supply system are expressed as fuzzy triangular numbers. In the case of the fuzzy flow at the main branches of the water supply system, the authors apply the modified fuzzy subtraction as explained above (Equation (16), Figure 3).
(2) The head losses through each branch are calculated individually based on the extension principle (Equation (18)) for a significant number of α-cuts.
(3) Going from upstream to downstream and by following the water flow direction, the hydraulic head at each node is calculated based on the conventional fuzzy subtraction of Equation (16). The results are shown in Figure 4.

It is observed that by applying the proposed method for increased fuzziness of water flow, e.g., higher than ±5% of the initial flow, the membership function of the head at the nodes loses its linear shape.

It can also be concluded that the pipe roughness is a critical and sensitive parameter in relation to the fuzziness of the final results. To show the importance of the uncertainty of pipe roughness towards the fuzziness of the nodal heads, the nodal heads are calculated keeping only the uncertainty of the pipe roughness. The results of this calculation are presented in Figure 5 for comparison purposes.

Finally, a critical point in this analysis is associated with the initially selected diameters. In the case where smaller diameters are selected and therefore high velocities produced, the fuzziness of the final results is significantly increased.

NUMERICAL EXAMPLE FOR THE LOOPED DISTRIBUTION SYSTEMS

The system, presented in Figure 6, consists of two loops, five branches and four nodes. The examined system is the system presented by Revelli & Ridolfi (2002), slightly modified. Larger diameters have been selected to fulfil the constraints of maximum allowable velocity. The Hazen–Williams equation is used for the calculation of

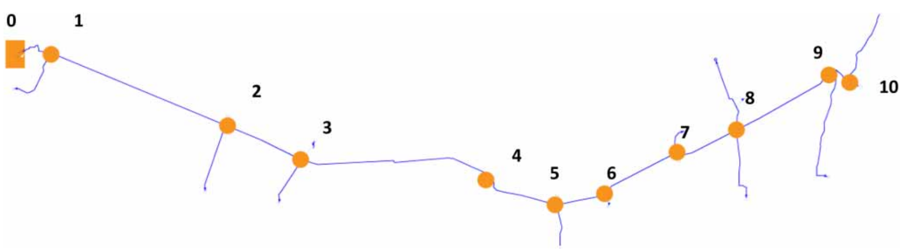

Figure 2 | The water conveyance system of the example.

Table 1 | Data of the water conveyance system (central values)

Main branches	Length, *L* (m)	Internal diameter, *D* (mm)	Flow, *Q* (L/s)
0–1 (A1)	1,446.0	312.8	34.1
1–2 (A3)	8,445.7	312.8	33.32
2–3 (A5)	3,677.3	312.8	31.73
3–4 (A8)	8,666.4	277.6	27.83
4–5 (A10)	3,422.2	277.6	24.95
5–6 (A12)	2,136.0	246.8	23.28
6–7 (A14)	3,841.4	246.8	20.90
7–8 (A16)	2,929.6	220.4	15.17
8–9 (A21)	5,158.7	141.0	5.17
9–10 (A23)	717.3	123.4	3.38

head losses. Furthermore, the uncertainty in this example refers not only to the pipe roughness coefficients (as in the original paper), but also to the internal pipe diameters. Indicatively, Figure 7 presents the fuzzy inputs for the pipe roughness coefficient and the internal diameter of the branch 1–3.

Following the Revelli and Ridolfi methodology, the separate solution for the maximum or minimum flow at each branch for several α-cuts, provides the corresponding α-cut of the flow at each branch and finally the shape of each membership function. In Figure 8, the membership function of the flow of the branch 1–2 is presented. This process must be repeated for every

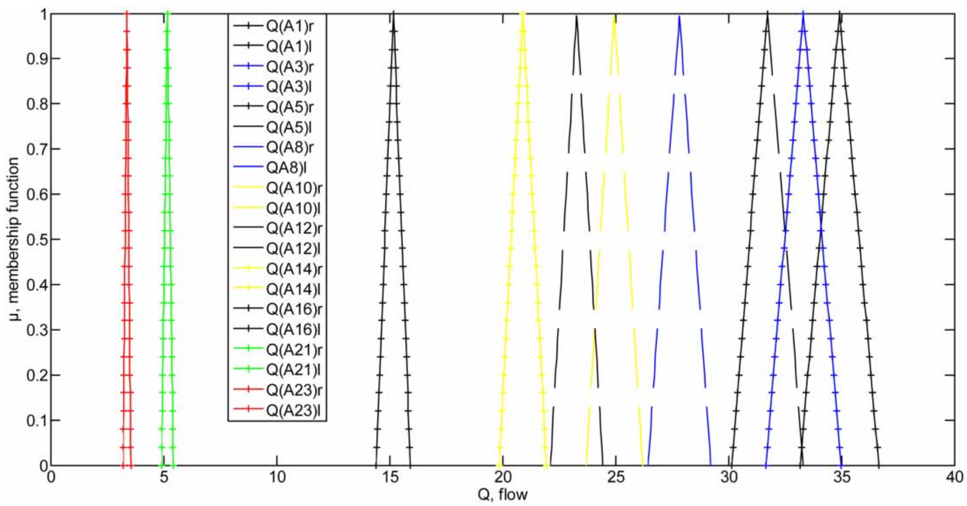

Figure 3 | Flow at each branch, based on the proposed fuzzy subtraction.

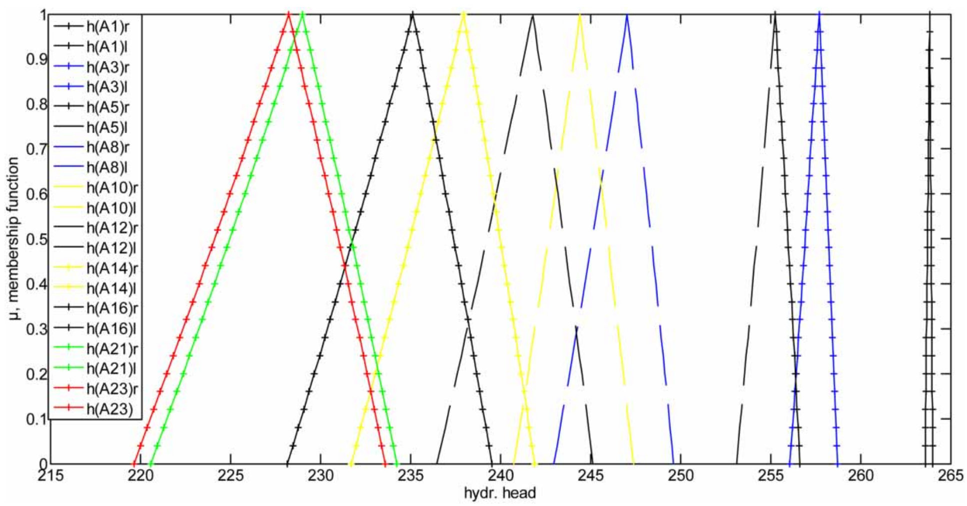

Figure 4 | Head at each node (output of the proposed methodology).

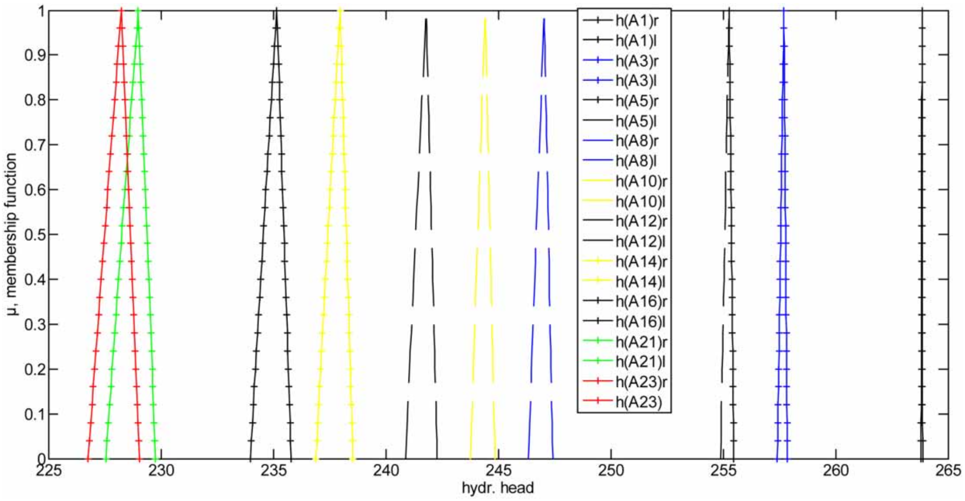

Figure 5 | Head at each node with uncertainty only on the pipe roughness.

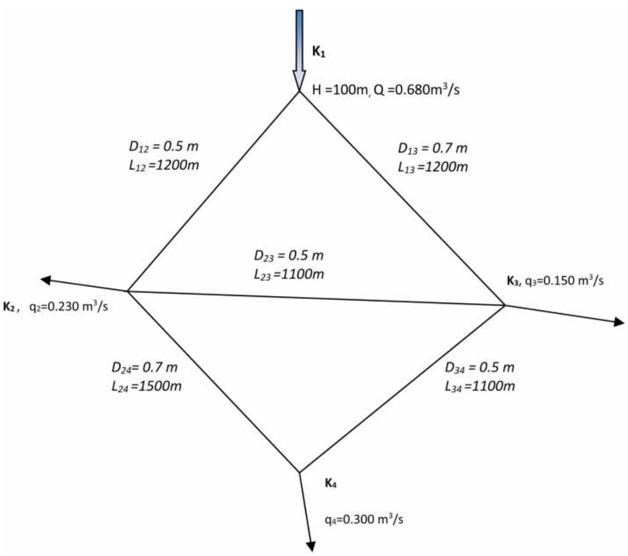

Figure 6 | The looped distribution system of the example.

branch and simultaneously for a discrete number of α-cuts.

It should be noticed that even if the inputs contain fuzzy variables (e.g., the α-cuts of the pipe roughness coefficients and the internal diameters), each optimisation problem leads only to crisp values. The synthesis of all of these solutions produce the final shape of the membership function of the decision variables. For instance, the following optimisation problem must be solved to determine the left-hand boundary of the flow at branch 1–2 with respect to the 0-cut:

$$\begin{cases} \min x_{1,12} \\ \sum_{in}^{in\in I(l)} \pm \dfrac{10.7L}{x_{2,in}{}^{1.852} x_{3,in}{}^{4.87}} \left|x_{1,in}\right|^{0.852} x_{1,in} = 0, \quad l = 1, \ldots, L \\ \sum_{in}^{in\in I_1(m)} \pm x_{1,in} = q_m, \quad m = 1, \ldots, \mathrm{M} \\ x_{2,in} \in [130, \quad 140], x_{3,in} \in [0.696, 0.702], \ x_{1,in}(crisp) \end{cases} \tag{27}$$

Thus, the left-hand boundary of the flow at branch 1–2 is equal to $x_{1,12}\left(= Q^L_{1-2\,0}\right) = 0.2183$. Based on the optimisation results, branches 1–3, 3–4, 3–2 have Hazen–Williams coefficients near the lower boundary, whereas the branches 1–2 and 2–4 lead to Hazen–Williams coefficients near to the upper boundary to facilitate the water path $(1 \to 2 \to 4)$. The same behaviour is obtained considering the values of the internal diameters.

Indicatively, Figure 9 presents the head losses at branch 1–2 based on the minimisation and maximisation of flow at branch 1–2, whereas Figure 10 presents the same varieties based on the minimisation and maximisation of the head losses at branch 1–2. By comparing the two solutions it is obvious that they are not the same. Equation (24) leads to a larger support of the head loss fuzzy number than Equation (23). This is clearly due to the fact that in Equation

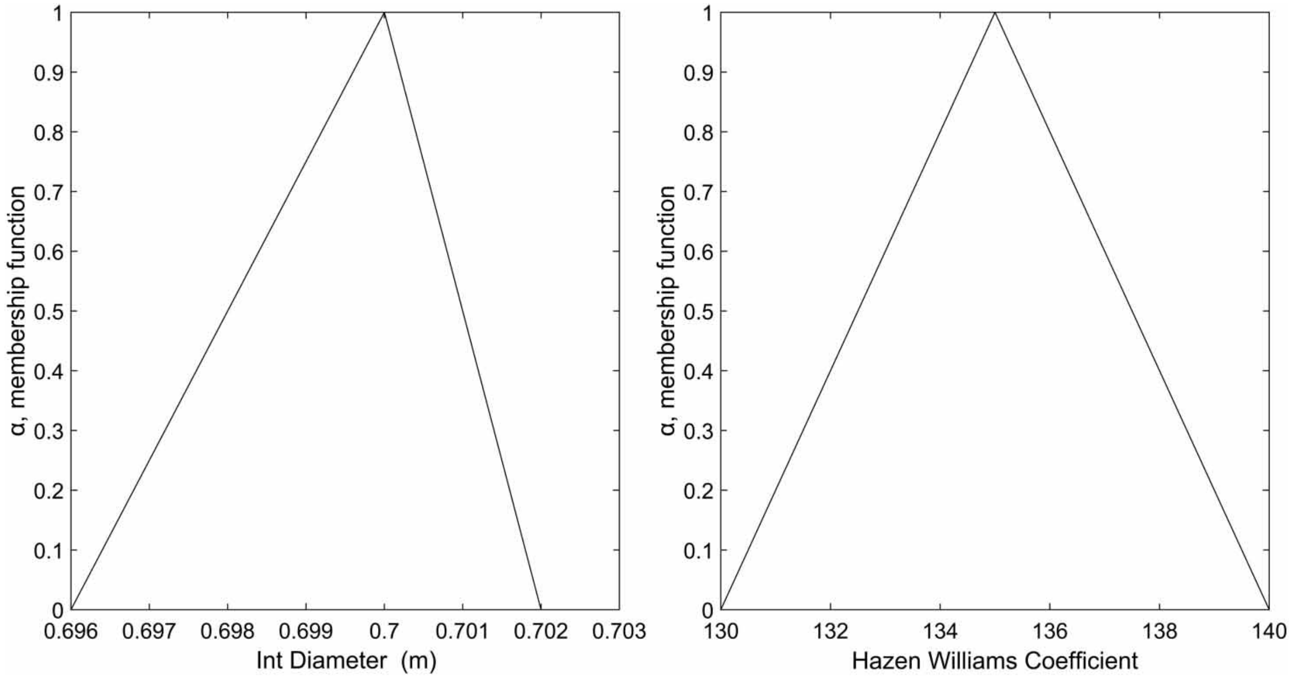

Figure 7 | Fuzzy inputs for the internal diameter and pipe roughness coefficient for the branch 1–3.

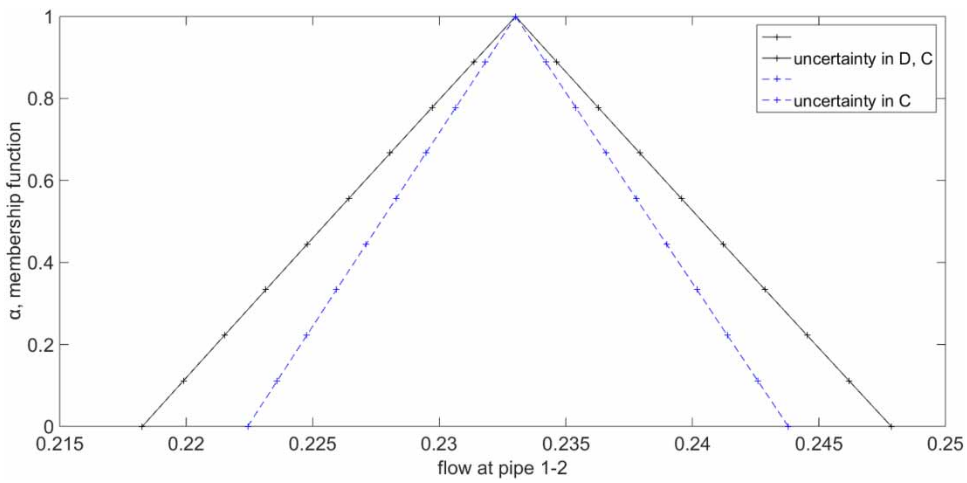

Figure 8 | Fuzzy output (flow at branch 1-2) based on the minimisation and maximisation of flow at branch 1–2.

(24) minimum and maximum head losses are explicitly searched for, but it could also be due to the combined effect of fuzzified roughness and internal diameter.

Another interesting point of view is that in the case of Equation (23), if we consider fuzziness only in pipe roughness, then, the flow at pipes 1–2 will have the same central value but with less uncertainty (without dramatic reduction) compared with the case where uncertainty exists both regarding the pipe roughness and the internal diameters (Figure 8).

Further, the proposed global objective function of the weighted sum of the absolute values of the head losses is

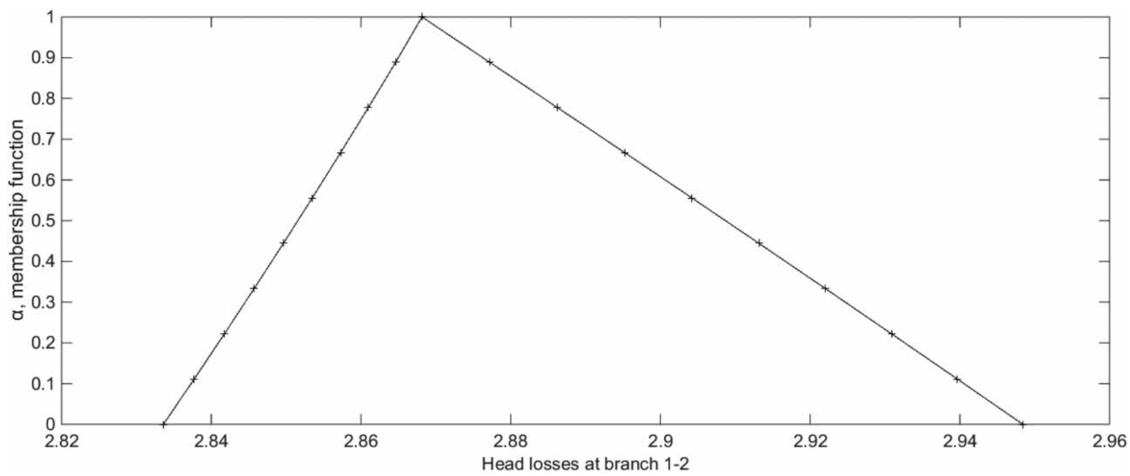

Figure 9 | Fuzzy outputs (head losses at branch 1–2) based on the minimisation and maximisation of flow at branch 1–2.

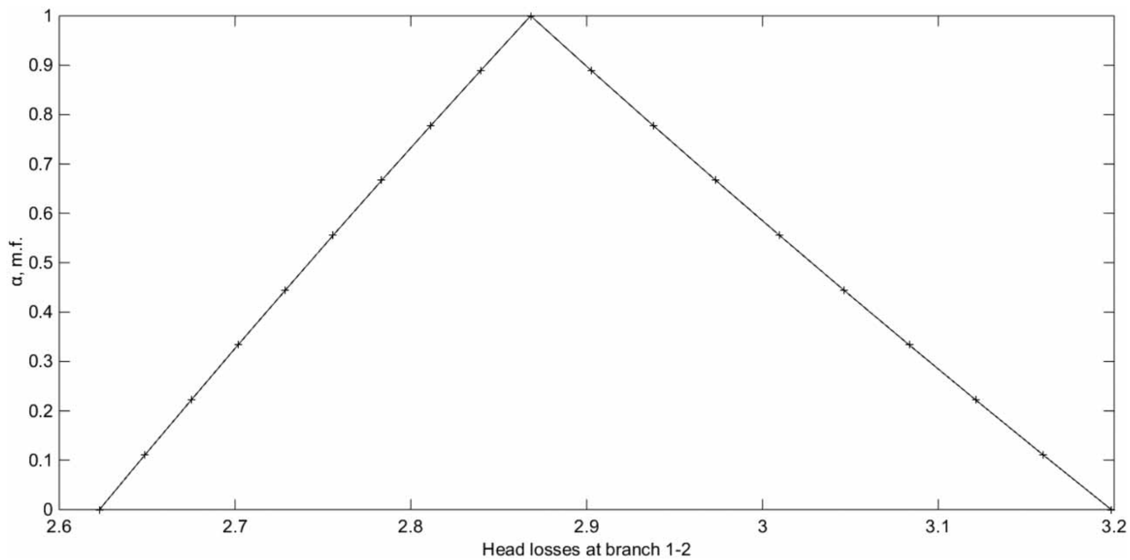

Figure 10 | Fuzzy output (head losses at branch 1–2) based on the minimisation and maximisation of the head losses at branch 1–2.

examined, by considering crisp numbers for the water consumption at the nodes. As mentioned earlier, the selection of each weight is proportional to the magnitude of flow at each branch and, hence, the objective function becomes:

$$\sum_{in=1}^{5}\left(\frac{|x_{1,in}|}{\left(\sum_{in=1}^{L+M-1}|x_{1,in}|\right)}\right)\cdot\left|\frac{10.7L_{in}}{x_{2,in}{}^{1.852}x_{3,in}{}^{4.87}}\,|x_{1,in}|^{0.852}x_{1,in}\right| \quad (28)$$

The results from the application of this methodology are presented in Figure 11. As can be seen, the proposed objective function leads to a more condensed scenario with lower uncertainty.

In contrast with the individual maximisation of the flow at each branch, by using the proposed global objective function, we derive a solution which consists of the left-hand boundary of both the pipe roughness coefficients and the internal diameters.

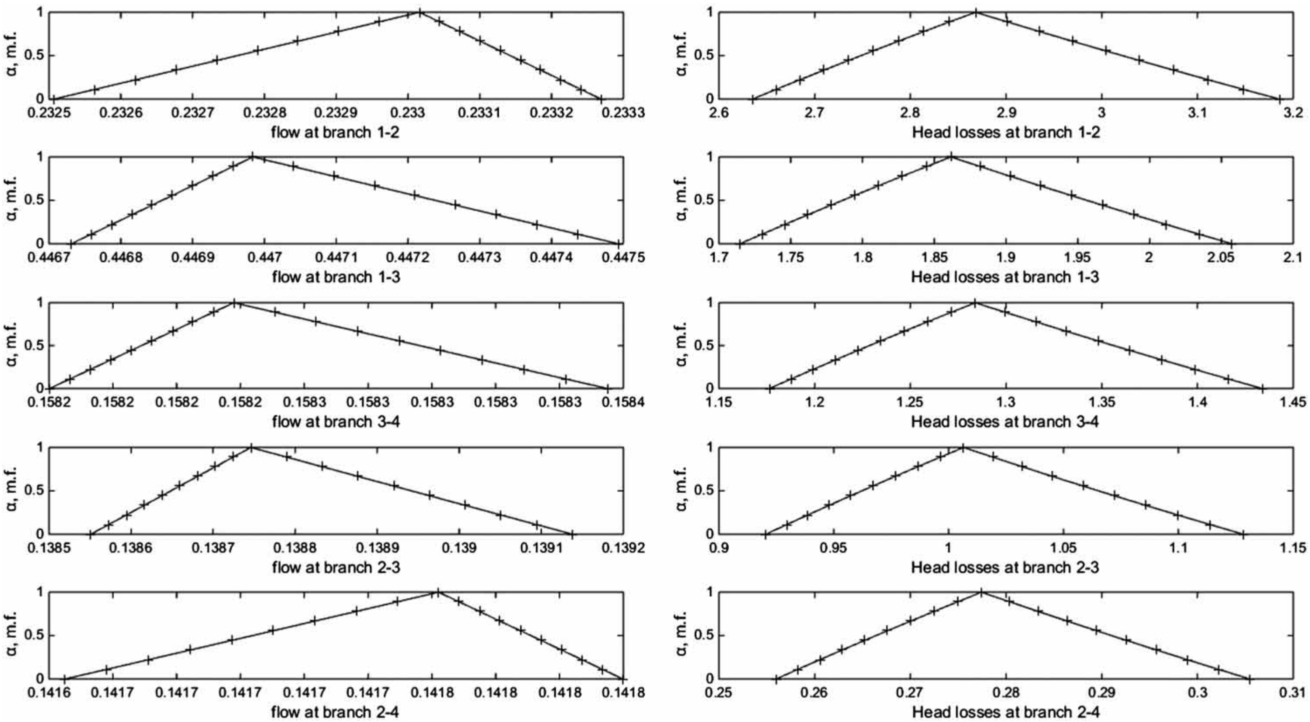

Figure 11 | Fuzzy output based on the minimisation and maximisation of the proposed global objective function by considering crisp numbers for the water consumption at the nodes.

As mentioned before, in the case of variable demand, in order to determine each bound of the α-cuts the following two optimisation problems must be solved based on the the proposed global objective function:

$$\begin{cases} \min \sum_{in=1}^{5} \left(\dfrac{|x_{1,in}|}{\left(\sum_{in=1}^{L+M-1} |x_{1,in}| \right)} \right) \cdot \left| \dfrac{10.7 L_{in}}{x_{2,in}{}^{1.852} x_{3,in}{}^{4.87}} |x_{1,in}|^{0.852} x_{1,in} \right| \\ \left| \begin{array}{l} \sum_{in}^{in \in I(l)} \pm \dfrac{10.7 L_{in}}{x_{2,in}{}^{1.852} x_{3,in}{}^{4.87}} |x_{1,in}|^{0.852} x_{1,in} = 0, \; l = 1,2 \\ \sum_{in}^{in \in I_1(m)} \pm x_{1,in} = q^{L}_{m\alpha}, \; m = 1, \ldots, 4 \\ x_{2,in} \in \widetilde{C_{in_\alpha}}, x_{3,in} \in \widetilde{D_{in_\alpha}}, \; x_{1,in} (crisp) \end{array} \right. \end{cases} \& \begin{cases} \max \sum_{in=1}^{5} \left(\dfrac{|x_{1,in}|}{\left(\sum_{in=1}^{L+M-1} |x_{1,in}| \right)} \right) \cdot \left| \dfrac{10.7 L_{in}}{x_{2,in}{}^{1.852} x_{3,in}{}^{4.87}} |x_{1,in}|^{0.852} x_{1,in} \right| \\ \left| \begin{array}{l} \sum_{i}^{i \in I(l)} \pm \dfrac{10.7 L_{in}}{x_{2,in}{}^{1.852} x_{3,in}{}^{4.87}} |x_{1,in}|^{0.852} x_{1,in} = 0, \; l = 1,2 \\ \sum_{in}^{i \in I_1(m)} \pm x_{1,in} = q^{R}_{m\alpha}, \; m = 1, \ldots, 4 \\ x_{2,in} \in \widetilde{C_{in_\alpha}}, x_{3,in} \in \widetilde{D_{in_\alpha}}, \; x_{1,in} (crisp) \end{array} \right. \end{cases} \quad (29)$$

Subsequently, the proposed global objective function is dealt with in order to cover the case of variable demand at nodes. The water consumption at nodes is expressed as fuzzy triangular numbers. A symmetrical semi-width of 10% around the central value is assumed. The results are presented in Figure 12. Since in this case the authors consider fuzzy consumption at nodes, the fuzziness (i.e., the length of the 0-cuts for the water flow at branches) is increased (Figure 12).

By comparing the fuzziness produced from the examples of both the water supply and the looped distribution systems, it can be deduced that the water supply system produces higher output fuzziness due to anticipated significantly greater pipe lengths involved. Also, the proposed methodology of the global optimisation of the

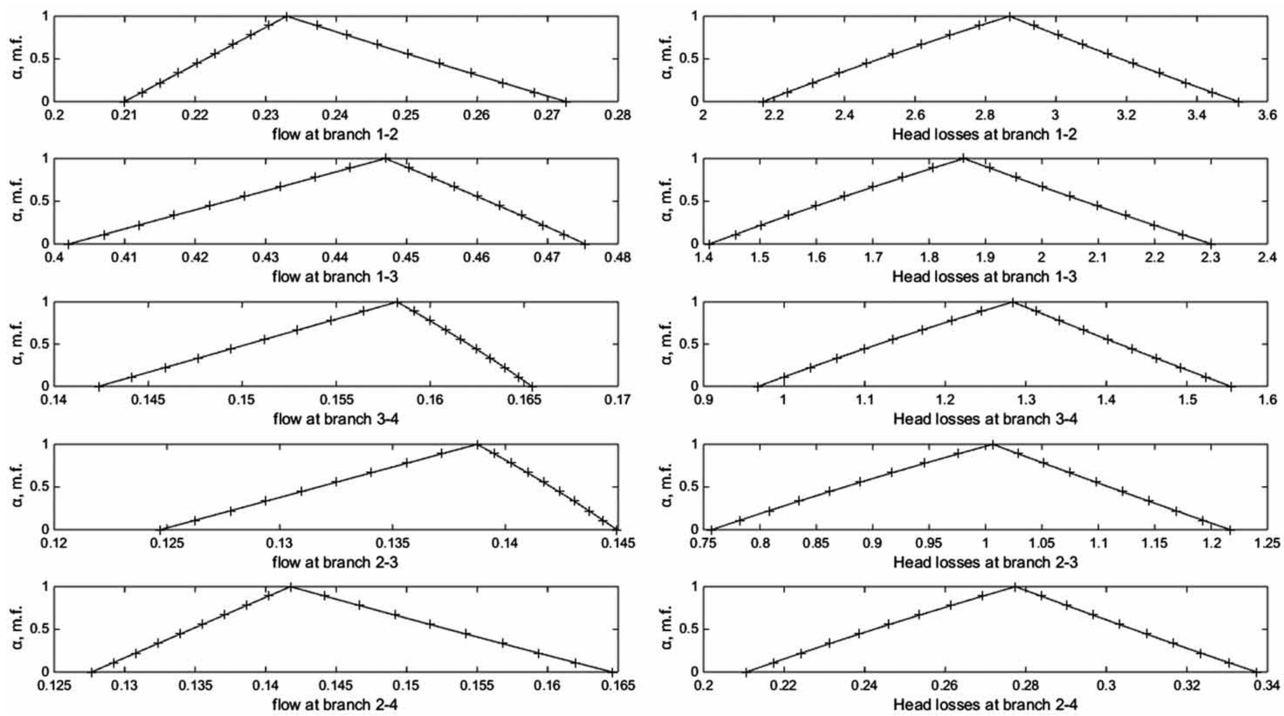

Figure 12 | Fuzzy output based on the minimisation and maximisation of the proposed global objective function and by considering fuzzy water consumptions at nodes.

weighted sum of the head losses at branches, leads to a more condensed subset of fuzziness, convenient for operational use.

CONCLUDING REMARKS

In this paper, an attempt is made to study the branched water supply and looped water distribution systems. The uncertainty of the variables involved (as the internal diameter of the pipes, the water consumption and the pipe roughness) is taken into account. In this context, innovative methodologies are developed and proposed for each type of system, separately. The proposed methodologies use a small number of data in comparison to existing ones, such as the probabilistic methods, found in the scientific literature.

The methodology developed for the analysis of branched water supply systems comprises the extension principle of fuzzy sets and new operations of fuzzy algebra. Based on the results, it can be concluded that this methodology leads to useful recommendations for the design of these systems, provided that the water velocities are kept at a medium or low level. It is also concluded that the pipe roughness coefficient is a critical parameter in the analysis, imposing significant fuzziness on the head losses at the branches and on the nodal heads.

For the analysis of the looped distribution systems under uncertainty, two methodologies are developed. In both methodologies, the fuzzy inputs are from the pipe roughness coefficients and the internal diameters. The first methodology is based on a modification of Revelli and Ridolfi's approach by determining the α-cuts and finally the membership functions of the flow at the branches. This is achieved by following an optimisation approach. The second methodology refers to a global optimisation function involving all branches simultaneously, which results in identifying a condensed subset of fuzziness which is, however, convenient for operational use.

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