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PAPER

Uncertainty in Traffic Forecasts

Literature Review and New Results for the Netherlands

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Preface

The objectives of the project 'Uncertainty in traffic forecasts' that RAND Europe carried out for the Transport Research Centre of the Dutch Ministry of Transport, Public Works and Water Management were:

- To develop a methodology to estimate the amount of uncertainty in forecasting for new infrastructure (especially roads).
- To implement and test this methodology in two case-studies (using the Dutch National Model system LMS and the New Regional Models NRM respectively).

This report presents the outcomes of all phases of this project:

- Literature review for public projects;
- Literature review for public-private partnership (PPP) projects;
- Development of a method to quantify the uncertainty in traffic forecasts for the LMS and NRM;
- Outcomes from a large number (100) of model runs with the LMS to derive uncertainty margins around the mean traffic forecasts;
- Outcomes from a large number (100) of model runs with the NRM for the Dutch province of Noord-Brabant to derive uncertainty margins around the mean traffic forecasts.

This report was written for modellers with an interest in the uncertainty margins around the model forecasts and methods to quantify the uncertainty margins.

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Summary

Although thousand of papers on transport model forecasts can be found in journals, conference proceedings and reports, the literature on quantifying uncertainty in traffic forecasts is fairly limited. In this report we present an overview of the literature on uncertainty in transport modelling and outcomes of interviews with a number of experts. Furthermore we provide the outcomes of our analysis of uncertainty in traffic forecasts from the Dutch national model system (LMS) and the regional model for (NRM) Noord-Brabant.

We distinguish between input uncertainty (e.g. on the future incomes) and model uncertainty (including specification error and error due to using parameter estimates instead of the true values).

All methods encountered in the literature for quantifying the amount of **input uncertainty** use some form of repeated model simulation (sensitivity testing). Many of the studies investigated postulate statistical distributions for the input variables and then draw (usually at random, sometimes at specific percentiles) input values from these distributions. The resulting values are then used in model runs. Final outcomes for uncertainty are calculated from the variance over all the runs for the different input values. Most studies use univariate distributions for the input variables; correlation between inputs is ignored (unlike scenario studies that try to sketch consistent futures). More realistic estimates of uncertainty can be derived if one takes account of correlations between inputs (e.g. income and car ownership) by drawing from multivariate distributions, but this requires knowledge on the correlations.

In our analysis of uncertainty in traffic forecasts from the LMS and the NRM Noord-Brabant, we used existing time series as the key source of information on means, standard deviations and correlations of input variables, and applied these to get multivariate distributions for the model input variables.

For quantifying **model uncertainty** in transport forecasts, we found a wider diversity of methods than for input uncertainty. Some studies used analytic expressions for the variance of the endogenous variable that results from using parameter estimates for the influence of the exogenous variables. This can only be done if the model equations are relatively straightforward. For more complicated models, these expressions become very cumbersome and often only approximations (e.g. from Taylor series expansion) can be given. To obtain proper t-ratio's or standard errors for the model coefficients in situations with specification error (such as repeated measurements in panel and SP data), the related Jackknife and Bootstrap method are sometimes used. After having calculated the proper t-

ratios for these parameters, the new standard errors can either be used in an analytic calculation of the standard error (due to estimation) of the model outcomes, or be used as information on the statistical distribution of the parameters of the model, from which values can be drawn for model simulation runs, similarly to the method used for input uncertainty. Again, it is important to take account of the correlations (between the parameter estimates), either in the analytical equations or in sampling from a multivariate distribution.

For quantifying the model errors we used the Bootstrap method to correct for specification error and Monte Carlo simulation for the uncertainty due to estimation, for the tour frequency and mode-destination choice models in the LMS and NRM.

This method was used to quantify uncertainty due to input variables of the LMS (income, car ownership, car costs, labour force, population by age group, household size, number of students by type of education) and the model uncertainty in the tour frequency and mode and destination choice models. Short-term cyclical fluctuations in the input variables were removed by using 20-year moving averages; we are looking at long run impacts (for 2020). The method used also takes account of the correlation between the input variables. Sources of uncertainty that were not included are: uncertainty in the base matrices, uncertainty in the assignment procedures, uncertainty in the regional distribution of future input changes and uncertainty in the future distribution between part-time and full-time workers.

Summarising the main outcomes we find substantial, but not very large, uncertainty margins for the total number of tours and kilometres (by mode) in the study area of the LMS and NRM and for the vehicle flows on selected links. The uncertainty margins for differences between a project and a reference situation are proportionally not much larger, unless these differences are of a small magnitude. In many cases, there is greater variation in vehicle hours lost due to congestion (Q-hours) than in hours travelled. The contribution of input uncertainty (e.g. in future incomes, car ownership levels) to these errors is generally much larger than that of model uncertainty (e.g. coefficients estimated with some error margin).

A difference between the Monte Carlo simulation approach used here and a sensitivity analysis of traffic outcomes by running the model for a number of scenarios (consistent possible futures) is that the simulation approach can provide confidence intervals for the traffic outcomes where the scenario approach does not attach probabilities to the different runs. Both approaches can take account of correlations between input variables. Scenarios can be used however to study different ways of distributing given national totals over zones, and the input simulation can be used to generate specific scenarios (e.g. high, middle and low growth in factors explaining traffic growth), and so both methods could also be used in combination.

Samenvatting

Hoewel er duizenden artikelen en rapporten over prognoses met transportmodellen zijn geschreven, is er maar weinig literatuur over de onzekerheidsmarges in verkeersprognoses. In dit rapport wordt een overzicht van deze literatuur gegeven, aangevuld met de uitkomsten van interviews met enkele experts. Verder worden de uitkomsten gepresenteerd van de berekening van de bandbreedte van verkeersprognoses met het Landelijk Model Systeem (LMS) en het Nieuw Regionaal Model (NRM) Noord -Brabant.

Hierbij maken we een onderscheid tussen onzekerheid in de invoervariabelen (bijvoorbeeld over de toekomstige inkomens) en modelonzekerheid (deze betreft zowel specificatiefouten als fouten door het gebruik van geschatte parameterwaarden in plaats van de werkelijke waarden).

Alle methoden die we in de literatuur zijn tegengekomen over **invoeronzekerheid** maken gebruik van herhaalde modelsimulatie (gevoeligheidanalyse). In diverse studies gebeurt dit door het veronderstellen van bepaalde statistische verdelingen voor de invoervariabelen, waaruit dan waarden voor de invoervariabelen worden getrokken (doorgaans a-select, soms bepaalde percentielwaarden). De modellen worden vervolgens doorgerekend met deze waarden uit de Monte Carlo simulatie. De uiteindelijke uitkomsten voor wat betreft de onzekerheid van de prognoses worden bepaald op basis van de variantie van de modeluitkomsten voor alle runs met het model. De meeste studies gebruiken univariate verdelingen voor de invoervariabelen, en gaan zo voorbij aan de correlatie die kan bestaan tussen de invoervariabelen (dit in tegenstelling tot scenariostudies waar geprobeerd wordt om een consistent toekomstbeeld te schetsen). Het realiteitsgehalte van de onzekerheidsmarges kan verhoogd worden door samenhangen tussen de invoervariabelen (zoals die tussen inkomen en autobezit) mee te nemen door het gebruik van multivariate verdelingen, maar dit vereist kennis over de correlaties.

In onze analyses van de onzekerheidsmarges in de prognoses van LMS en NRM Noord-Brabant, hebben we bestaand tijdreeksmateriaal gebruikt als de belangrijkste bron van informatie over gemiddelde, standaardafwijking en correlaties van de invoervariabelen, en hebben zo multivariate verdelingen voor de invoervariabelen opgesteld.

Voor het kwantificeren van **modelonzekerheid** in de verkeersprognoses hebben we in de literatuur een breder scala aan methoden aangetroffen dan voor invoeronzekerheid. Sommige onderzoeken gebruiken analytische functies voor de variantie van de te verklaren variabele die het gevolg is van het gebruiken van parameterschattingen voor het effect van exogene variabelen. Dit is uitsluitend mogelijk als de vergelijkingen in het model relatief eenvoudig zijn. Voor complexere modellen worden de analytische functies snel te

ingewikkeld en vaak zijn slechts benaderingen (bijvoorbeeld via Taylor reeksen) te geven. Twee methoden die wel gebruikt worden om correcte standaardfouten en t-waarden voor modelparameters te bepalen als er sprake is van specificatiefouten (zoals het probleem van herhaalde metingen in stated preference en panel data) zijn de Jackknife en de Bootstrap. Nadat hiermee de juiste t-waarden voor de modelparameters zijn berekend, kunnen de standaardfouten voor de endogene variabelen berekend worden via hetzij de analytische methode, hetzij als informatie over de statistische verdeling van de modelparameters waaruit vervolgens trekkingen worden gedaan voor herhaalde modelsimulatie (net als bij de invoeronzekerheid). Ook hier is het van belang om rekening te houden met de correlaties (nu tussen de parameterschattingen).

In de berekening van de modelonzekerheid in LMS en NRM hebben we de Bootstrap methode gebruikt om te corrigeren voor specificatiefouten en Monte Carlo simulatie voor de onzekerheid door schatting van het model. Het gaat hierbij om de modellen voor het aantal reizen (tours) en voor de keuze van vervoerwijze en bestemming.

Voor LMS en NRM Noord-Brabant zijn de onzekerheid in de verkeersprognoses als gevolg van invoervariabelen (inkomen, autobezit, autokosten, werkgelegenheid, bevolking naar leeftijdsklasse, huishoudgrootte, aantal studentenplaatsen) en de modelonzekerheid gekwantificeerd. Korte termijn conjuncturele fluctuaties in de invoervariabelen zijn hierbij verwijderd door gebruik te maken van 20-jaars voortschrijdende gemiddelden: we zijn op zoek naar effecten op lange termijn (voor 2020). Ook de correlatie tussen deze invoervariabelen is meegenomen. Bronnen van onzekerheid die niet zijn opgenomen zijn: onzekerheid in de basismatrices, onzekerheid in de toedelingsprocedures, onzekerheid in de regionale verdeling van de invoervariabelen in de toekomst, en onzekerheid over de toekomstige aandelen van voltijds- en deeltijdswerkers.

Hieronder vatten we de belangrijkste uitkomsten samen. We vinden bandbreedtes voor het aantal reizen en kilometers (per vervoerwijze) in het studiegebied van LMS en NRM en intensiteiten op geselecteerde wegvakken die niet te verwaarlozen zijn, maar toch betrekkelijk klein zijn te noemen. De onzekerheidsmarges voor verschillen tussen de referentiesituatie en de projectsituatie zijn proportioneel niet veel groter, tenzij het om absoluut kleine verschillen gaat (dan is de relatieve onzekerheid groot). In veel gevallen is de variatie in voertuigverliesuren (Q-hours) groter dan in het aantal gereisde uren. De bijdrage van de invoeronzekerheid (b.v. toekomstige inkomens, autobezit) aan de totale onzekerheidsmarges is doorgaans veel groter dan die van modelonzekerheid.

Een verschil tussen de Monte Carlo simulatie methode die hier is gebruikt en een gevoeligheidsanalyse van de modelprognoses door het draaien van een aantal scenario's (consistente toekomstbeelden) is dat de simulatiemethode betrouwbaarheidsintervallen kan leveren, terwijl de scenario-methode geen kansen koppelt aan de verschillende toekomstbeelden. Beide methoden kunnen rekening houden met correlaties tussen de invoervariabelen. Scenario’s kunnen echter ook gebruikt worden voor het analyseren van verschillende manieren om een gegeven landelijk totaal over de zones te verdelen. De simulatiemethode kan ook weer gebruikt worden om specifieke scenario's te genereren (bijvoorbeeld laag, midden, hoog voor de factoren die de verkeersgroei bepalen). Zo kunnen beide methoden ook in combinatie gebruikt worden.

CHAPTER 1 Introduction

1.1 Objectives of the project

The National Model System LMS and the New Regional Models NRM are regularly used in The Netherlands to forecast the national and regional transport volumes and traffic flows on specific network links for a single or a limited number of scenarios. The same models are also used to give the likely impacts of transport infrastructure projects (e.g. new roads, wider roads, new railway lines) and transport policies (e.g. road pricing). All these predictions are point estimates, and, even when produced for several scenarios, do not give insight into the uncertainty margin that exists around these forecasts.

The objectives of the project 'Uncertainty in traffic forecasts' that RAND Europe is carrying out for the Transport Research Centre of the Dutch Ministry of Transport, Public Works and Water Management are:

- To develop a methodology to estimate the amount of uncertainty in forecasting for new infrastructure (especially roads).
- To implement and test this methodology in two case-studies (using the Dutch National Model System LMS and the New Regional Models NRM respectively).

Throughout this report, we distinguish between input uncertainty (e.g. on the future incomes) and model uncertainty (including specification error and error due to using parameter estimates instead of the true values).

1.2 Role and contents of this report

Chapter 2 of this report presents the main outcomes of two literature reviews on quantifying the amount of uncertainty in forecasting with transport models:

- Literature review for public projects;
- Literature review for public-private partnership (PPP) projects.

In these reviews the national and international literature on methods to quantify the amount of uncertainty around the forecasts from transport models was described and assessed. Most relevant papers do not distinguish between public and PPP projects. Therefore we combined both phases into a single literature review on how to quantify uncertainty in transport modelling.

In Chapter 3, the method developed for the treatment of input uncertainty as well as for model uncertainty is described. Chapter 4 contains outcomes on uncertainty margins for the LMS, and Chapter 5 for the NRM Noord-Brabant. Finally, in Chapter 6 the conclusions from the project as a whole are listed and recommendations are given.

A general discussion on uncertainty in policy modelling (not specifically dealing with transport issues and transport models) can be found in Appendix 1. This chapter is based primarily on Walker et al. (2003), and summarises RAND's experience on the issue of uncertainty.

Appendix 2 contains the short descriptions of the papers and reports on quantifying uncertainty in transport models, as identified in the review of the international literature. It also contains the outcomes of interviews with international experts.

In Appendix 3, the analytic method is used to derive the equations for the variance for model uncertainty in the LMS mode-destination and tour frequency models.

Detailed outcomes for the LMS and NRM on quantifying uncertainty in traffic forecasts are in Appendix 4.

CHAPTER 2 Integration and assessment of the literature

A web-based search for papers and reports was carried out, using a pre-defined list of keywords. Moreover, a number of national and international journals and conference proceedings were scanned, focussing on the most recent years (going back ten years) for relevant articles. In Appendix 2, summaries can be found of the papers and reports identified in the web-based search and the journals and proceedings. As much as possible, a common format was used to describe the papers and reports. In this format we were seeking answers to the following questions:

1. Which methods have been used for quantifying uncertainty in traffic forecasts?
2. What type of uncertainty has been studied (uncertainty due to model inputs, the model itself or both)?
3. For which variables is uncertainty studied (e.g. link flows, value of time)?
4. How is uncertainty expressed?
5. What is the order of magnitude of the uncertainty around the forecasts?

Additionally we interviewed a number of experts on the issue of uncertainty in traffic forecasting. The detailed outcomes of this are in also in Appendix 2.In our review of the international literature, we did not find a large number of publications on calculating uncertainty measures for transport model forecasts, presumably because this topic has not been studied frequently. Although thousand of papers on transport model forecasts can be found in journals, conference proceedings and reports, the literature on quantifying uncertainty in traffic forecasts is fairly limited.

We distinguish between input uncertainty (e.g. on the future incomes) and model uncertainty (including specification error and error due to using parameter estimates instead of the true values).

In Table 1 below the outcomes from the literature review are integrated.

Table 1. Summary and integration of the literature on uncertainty of traffic forecasts (pkm: passenger kilometres; vkm: vehicle kilometres; VMT: vehicle miles travelled).

Publication	Methods to quantify uncertainty	Type of uncertainty studied	Variables for which uncertainty is studied	How is uncertainty expressed	Order of magnitude of uncertainty
Armoogum, 2003	Jackknife and scenario analysis	Model and input uncertainty	Number of trips and pkm	Variance and percentage deviation from reference	Model uncertainty: for trips in 2030 variance is 27% of the mean (pkm: 6%).
Ben-Akiva and Lerman, 1985	Analytic formula for model uncertainty in multi-parameter model	Model uncertainty	Transport cost and time coefficients (as an example)	95% confidence interval	
Beser Hugosson, 2004	Bootstrap sampling, repeated estimation and model application	Parameter uncertainty	Total and OD demand by mode, link flows, train lines and value of time	95% confidence interval	95% confidence interval mostly between ±5% and ±10%
Boyce, 1999	Repeated model simulation, drawing from distributions for input variables	Model and input uncertainty (focus on inputs)	vkm	Standard errors and ratio of forecasts	

Publication	Methods to quantify uncertainty	Type of uncertainty studied	Variables for which uncertainty is studied	How is uncertainty expressed	Order of magnitude of uncertainty
Boyce and Bright, 2003	Repeated model simulation, drawing from distributions for input variables; Scenario analysis	Model and input uncertainty (focus on inputs)	Revenue from privately-financed project	Percentiles; private funders want to see 95-99% probability of no loss.	Only the worst scenario fell below the first percentile
Brundell-Freij, 1997	Repeated estimation on simulated datasets for different sample sizes	Model uncertainty	Parameters of modal split model, including costs, time and constants	t-ratios and confidence intervals for estimated coefficients	Even with 850 observations 5 out of 11 parameters are not significant.
Brundell-Freij, 2000	As above; Bootstrap analysis	Model uncertainty (specification, sampling, estimation)	Value of time	Standard error of the value of time	Standard error between 3 and 20% of in-vehicle value of time
Ecorys, 2003	Sensitivity analysis	Inputs, value of time	Revenues	Different revenue amounts	Revenues 1.2 or 3.9 mln depending on traffic growth
Eriksson, 2003	One-variable and multi-variable sensitivity analysis	Sensitivity to input variables	Road traffic emissions	Regression coefficients	

Publication	Methods to quantify uncertainty	Type of uncertainty studied	Variables for which uncertainty is studied	How is uncertainty expressed	Order of magnitude of uncertainty
Fowkes, 1995	Repeated estimation on simulated datasets	Model uncertainty (parameters)	Coefficients of modal split model, including costs, wait time and in-vehicle time; Willingness to pay	Standard deviation of estimated coefficients; Confidence interval around mode benefit	95% confidence interval for mode benefit ranges from 0 to twice the average value.
Garcia Ferrer et al., 2003	Deviation between different model forecasts and observed	Model uncertainty	Number of tickets and travel cards in public transport	Root mean squared error RMSE and mean absolute error MAE	RMSE varies between 0.04 and 0.27; MAE between 0.04 and 0.23.
De Jong, 1989	Analytic formula for sampling and estimation variance	Model uncertainty (sampling, parameters)	Number of households with a car; number of car km/year	Variation and standard error	Estimation standard error between 3 and 6% of predicted values
De Jong et al, 1998	Jackknife method and draws from multivariate normal distribution	Model uncertainty (specification, parameters)	Value of time	Standard error	Standard deviation between 6 and 24% of average values of time
Kroes, 1996	Repeated model runs for simulated inputs and parameters	Input uncertainty and model uncertainty (incl. model application)	Link flows and revenues	Standard error and other statistics	

Publication	Methods to quantify uncertainty	Type of uncertainty studied	Variables for which uncertainty is studied	How is uncertainty expressed	Order of magnitude of uncertainty
Lempert et al, 2003; Lempert, 2004	Very many repeated model runs for an ensembles of scenarios	Input uncertainty	No transport applications	Landscape of plausible futures	
Leurent, 1996	Repeated model runs for simulated inputs	Input uncertainty	Travel time; Daily number of cars on a link	Standard error	Standard deviation is about 10% of predicted flow
Lowe, et al., 1982	Random draws from distributions for inputs and model coefficients	Input uncertainty (focus) and model uncertainty	Link flows	Percentiles	Probability of 5% that flow will be less than 14,000 vehicles/day
Ministerie van Financiën en CPB, 2003	Add a risk paragraph in project assessment	No distinction made between model and input uncertainty	Financial outcomes of projects		
Research Results Digest, 2003	Jackknife method	Model uncertainty (parameters)	Number of pavement sections	Correlation coefficient, standard error	
Rodier and Johnston, 2002	Sensitivity analysis on number of input factors	Input uncertainty	Trips, VMT, vehicle hours delay, (emissions)	Percentage over- and under-prediction	0-70% under- or overprediction

Publication	Methods to quantify uncertainty	Type of uncertainty studied	Variables for which uncertainty is studied	How is uncertainty expressed	Order of magnitude of uncertainty
Rodier, 2003	Comparison of predicted (with predicted or observed inputs) versus observed	Model and input uncertainty	Trips, VMT, vehicle hours, vehicle hours delay,	Percentage over- and under-prediction	0-39% under- or overprediction
Schrijver et al., 2003	Random draws from inputs distributions	Input uncertainty	Travel time	Interval around mean travel time	
Zhao and Kockelman, 2001	Random draws for inputs and parameters in 4-stage model	Model and input uncertainty	Link flows	Standard error	Uncertainty propagates when going from trip generation to distribution and modal split, but is reduced in assignment.

This overview led to the following observations, conclusions and recommendations.

Methods: input uncertainty

All methods encountered in the literature for quantifying the amount of input uncertainty use some form of repeated model simulation (sensitivity testing). The same model is applied over and over again, with different inputs. A commonly used method for generating different inputs is scenario analysis. However, in scenario analysis no probabilities are attached to the various scenarios under study. This makes calculation of overall standard errors or related uncertainty measures for the model outcomes impossible. Many of the studies investigated postulate statistical distributions for the input variables and then draw (usually at random, sometimes at specific percentiles) input values from these distributions. The resulting values are then used in model runs. Final outcomes for uncertainty are calculated from the variance over all the runs for the different input values. This seems to be the standard approach to produce input uncertainty. Most studies use univariate distributions for the input variables; correlation between inputs is ignored (unlike scenario studies that try to sketch consistent futures). More realistic estimates of uncertainty can be derived if one takes account of correlations between inputs (e.g. income and car ownership) by drawing from multivariate distributions, but this requires knowledge on the correlations. For the quantification of uncertainty in traffic forecasts from LMS and NRM, we used existing time series as the key source of information on means, standard deviations and correlations of input variables, and applied these to get multivariate distributions for the model input variables. Lowe et al. (1982) used an experimental design (as in an SP survey) on the input variation, which can increase the efficiency of the process of running the model (not all combinations are needed).

Some studies use information on the observed outcomes and/or inputs (Garcia Ferrer, et al. 2003; Rodier, 2003). This can of course only be done *ex post*. When forecasting for a future year, as in this project (and other *ex ante* project evaluations), such comparisons cannot be made and we have to calculate uncertainty in traffic forecasts on the basis of information from the past.

Methods: model uncertainty

For quantifying model uncertainty in transport forecasts, we find a wider diversity of approaches than for input uncertainty. Some studies used analytic expressions for the variance of the endogenous variable that results from using parameter estimates for the influence of the exogenous variables. This can only be done if the model equations are relatively straightforward. For more complicated models, these expressions become very cumbersome and often only approximations (e.g. from Taylor series expansion) can be given. To obtain proper t-ratio's or standard errors for the model coefficients in situations with specification error (such as repeated measurements in panel and SP data), the Jackknife method and the related Bootstrap method are sometimes used. In the Jackknife, subsamples are created from an original sample by systematically omitting a small fraction of the data. The Bootstrap is applied by sampling at random from the original sample with replacement. After having calculated the proper t-ratios for these parameters, the new standard errors can either be used in an analytic calculation of the standard error (due to estimation) of the model outcomes, or be used as information on the statistical distribution of the parameters of the model, from which values can be drawn for model simulation runs, similarly to the method used for input uncertainty. Again, it is important to take account of the correlations (between the parameter estimates), either in the analytical equations or in sampling from a multivariate distribution.

Some studies had objectives which come very close to those of our project. Beser Hugosson (2004), Boyce (1999), Boyce and Bright (2003), Lowe et al. (1982) and Zhao and Koppelman (2001) all study the problem of how a given transport model can not only produce a central estimate of traffic volume or revenues, but also uncertainty margins around these. Beser Hugosson (2004), Lowe et al (1982) and Zhao and Kockelman also study the same objective variable as we do (link flows). All these studies, with the exception of Beser Hugosson (2004), use relatively simple aggregate transport models. Beser Hugosson (2004) used the Bootstrap method on disaggregate mode-destination models, but leaves out input uncertainty, trip frequency models and congestion feedbacks. Analytical methods to calculate the uncertainty were not used in this paper on the Swedish passenger transport model. The LMS and NRM consist for a large part of disaggregate random utility models (including tour frequency, mode-destination choice). The last four studies mentioned use Monte Carlo simulation for the inputs and the parameter values instead of analytical methods. Zhao and Kockelman explicitly study the problem of propagation of errors: when a number of modules are used sequentially, errors can become bigger (reinforcing initial deviations) or smaller (equilibrium mechanisms).

On the basis of the review of the literature we concluded that for quantifying the model errors we should use either the Jackknife/Bootstrap method to correct for specification error and Monte Carlo simulation for the uncertainty due to estimation, or the analytic method. The models for which we study uncertainty in this report are the tour frequency models and the mode-destination choice model (called NSES or RSES) in the LMS and NRM. We investigated whether analytic expressions can be used for the tour frequency models (which are relatively simple) and the mode-destination choice models. The analytic expressions can be found in Appendix 3. In the end we decided to use Monte Carlo simulation for the model uncertainty as well, principally because of the long run times required to evaluate the analytical expressions.

How is uncertainty expressed?

Uncertainty in forecasted values can be expressed in many ways, but measures that were often used in the literature are:

- The variance of the forecast;
- Its standard deviation (square root of the variance);
- Its 95% confidence interval [-1.96 times the standard deviation; +1.96 times the standard deviation];
- Percentiles of its distribution, e.g. the lowest 1% or 5% for revenue or vehicle flow forecasts.

All these measures will be provided in this project.

CHAPTER 3 Treatment of input and model uncertainty in the LMS and NRM runs

3.1 List of key autonomous driving forces of transport demand

First of all a list of most important autonomous variables influencing transport demand has been prepared (principally by going through the explanatory variables of the LMS/NRM tour frequency and mode-destination choice models, and the zonal targets in the QUAD procedure[1]). This list does not include the policy variables that can also be found in these models (such as public transport fares, parking costs, the speeds of the transport modes), that can be influenced by users of the models (government at different levels, public transport operators). The sensitivity of the link flows to such variables is usually handled through policy sensitivity runs: changing one policy variable at a time, relative to a reference case, or by comparing the outcomes for different policy packages.

Licence holding is not included, as it is so high now (among the eligible age groups) that for the future only very limited variation is possible. The total amount of freight traffic, international traffic, the correction for changes in working hour practices and additional holidays will not be varied in the simulations with LMS and NRM.

The list of the main autonomous forces for simulation of input uncertainty on transport demand (defined here as tour generation and mode-destination choice) is as follows:

- Household disposable income
- Car ownership
- Car cost per kilometre (only the fuel cost part, which is partly an autonomous and partly a policy variable, but not the toll and parking cost which are fully policy variables)
- Number of jobs (by sector), which serves as an attraction variable.
- Population by age group (or population and average age)
- Household size

[1] QUAD is a computer routine within the LMS that produces the joint distribution of socio-economic attributes of the households, given the total population and the marginal distributions for these attributes, from external sources.

- Occupation (employed or unemployed by gender) and education (number of students per type of education)
- Part-time versus full-time.

The collection of the time series data on these variables, the time series analysis carried out and the results are discussed below.

3.2 Method in general

Variables that determine the tour generation and the mode destination models were examined and data over a long period, 1960-2000, were gathered. For some variables corrections had to be made, to ensure consistency over time or with the LMS/NRM. Checks were made to see whether the available statistics correspond with the base year for the LMS (1995). For one variable (part-time working) no consistent data was available, in a few cases data was available only for 1970-2000. All variables are expressed in the form of annual (year-to-year) growth rates. Based on the time series for the variables, means, minima, maxima and standard deviations were calculated and a covariance matrix was set up. These statistics were used as a starting point for determining the multivariate statistical distribution from which draws were made to simulate for input uncertainty. The numbers drawn were then converted into LMS and NRM input variables and these models were run for these values. These inputs are either explanatory variables in the tour frequency and mode/destination choice models, or target variables for the totals in the QUAD procedure that determines the joint distribution in terms of socio-economic variables for each zone in the LMS. Every LMS run has a different set of values for model coefficients, explanatory variables and QUAD targets. All variables for which uncertainty is studied refer to national totals. In the LMS runs, the percentage changes in these national totals were applied to the zonal variables (e.g., all zones get richer by x %). This means that the relative distribution of these variables over the zones did not change. For NRM applications the same proportionality method was used.

3.3 Discussion per input variable

3.3.1 Disposable household income

The growth factor of disposable household income is input to the tour generation models (as well as NSES and QUAD and effectively car ownership). In applications of the model for future years the growth in Gross Domestic Product (in total, not per capita) is used as proxy for the growth of disposable household income. In this analysis GDP values of past years have been collected and corrected for inflation.

3.3.2 Car ownership

The CBS has statistics on the number of passenger cars owned in The Netherlands for 1960-2000. Annual growth factors were calculated for the changes in the number of cars.

3.3.3 Car costs

The car costs are built up out of two statistics:

- Prices of petrol, diesel, and LPG;

- Distribution of fuel usage by fuel type.

Fuel prices are partly a policy variable (fuel taxes), and partly an autonomous variable (determined to a great extent by the oil market). We do not want to include policy variables in the input variation, which is meant to be the variation of autonomous inputs. Nevertheless the fuel prices were included in the input variables, because we expect the fuel costs to be an important determinant of travel demand. So for this variable we do not only include autonomous uncertainty, but also policy uncertainty. This will probably make the resulting uncertainty margins somewhat larger than would be the case purely on the basis of autonomous changes (but there could also be a damping if fuel taxes are increased only when oil is relatively cheap).

Prices are available for the whole period. However the distribution over the three fuel types is only available back to 1983, and for the years 1980 and 1975. For the period before 1983 the distribution is interpolated and extrapolated using statistics from 1983, 1980 and 1975.

3.3.4 Labour force

Labour statistics (annual average total number of jobs of employees) are available for the period 1950-2000. Statistics by sector (for retail and total services without retail) are available for the period 1969-2000. For the period 1950-1996 the CBS Historical Labour Accounts are available and for the period 1997-2000 the CBS National Accounts are used. Growth factors have been derived from these data. We are using a combination of these two sources, for the period 1960-2000.

3.3.5 Age

The number of people by age group is known for the period 1960-2000. The midpoint of each age group (0-20, 20-40, 40-65, 65-80, 80+) is assumed to be the average age. People over 80 are assumed to have an average of 85. The average age of the total population is calculated for each year and growth factors have been derived.

3.3.6 Household size

The total population and the total number of households are available for 1960-2000. Household size is calculated based upon these numbers.

3.3.7 Occupation/education

Statistics on the labour force (male/female) were available only for 1970-2000. For the derivation of the covariance matrix, this no real obstacle since this period is still considered long enough and the Excel software used needs a minimum of two values for each variable. Time series of different length can be accommodated.

Education in the LMS is divided into three categories:

- low ('basisonderwijs')
- medium ('VO/MBO')
- high ('HBO/WO')

Statistics on the number of students in each of the categories are available for the whole period.

3.3.8 Part-time/full-time workers

We decided not to include this variable because data about the number of part-time/full-time jobs is only available for a few years (for 1987-2000 from the CBS National Accounts). Furthermore the definition used by the CBS (full time is >35 hours a week) is not the same as the definition used in the LMS (full time is >30 hours a week).

3.4 Overview of results

The following table gives a summary of the variables for which the covariance matrix has been set up. All variables are expressed as relative annual growth factors. The development over time can be seen in Figure 1.

Income (GDP in real terms) has grown by 3.4% per year in the period 1960-2000. The highest growth rate was +17% (in 1969) and the lowest –2.7% (1975). The range spanned by 1.96 times the standard deviation is no less than 7.6 percentage points on each side of the mean (15.2 percentage points in total for the 95% confidence interval).

The largest relative increases (up to +22% per year) in the number of cars in The Netherlands took place in the sixties. After 1980, the annual growth rate for the number of cars has not exceeded 4%. The word 'saturation' might not be fully appropriate here, but there is a clear levelling off of the car ownership growth in the observation period.

Car costs (fuel costs) per kilometre has seen years with large increases (1974, 1980, 1981, 2000: over 30%) and decreases (1986: -43%).

For the number of jobs in the service sector and average age, no declines in a single year are observed in the period 1960-2000. Both have been increasing steadily. The number of jobs in the service sector and in retail have been growing at a faster pace (+2.4% on average) than all jobs (+1.7%). These variables follow a cyclical pattern.

For household size we see practically only reductions in this period (only for two single years there was an increase).

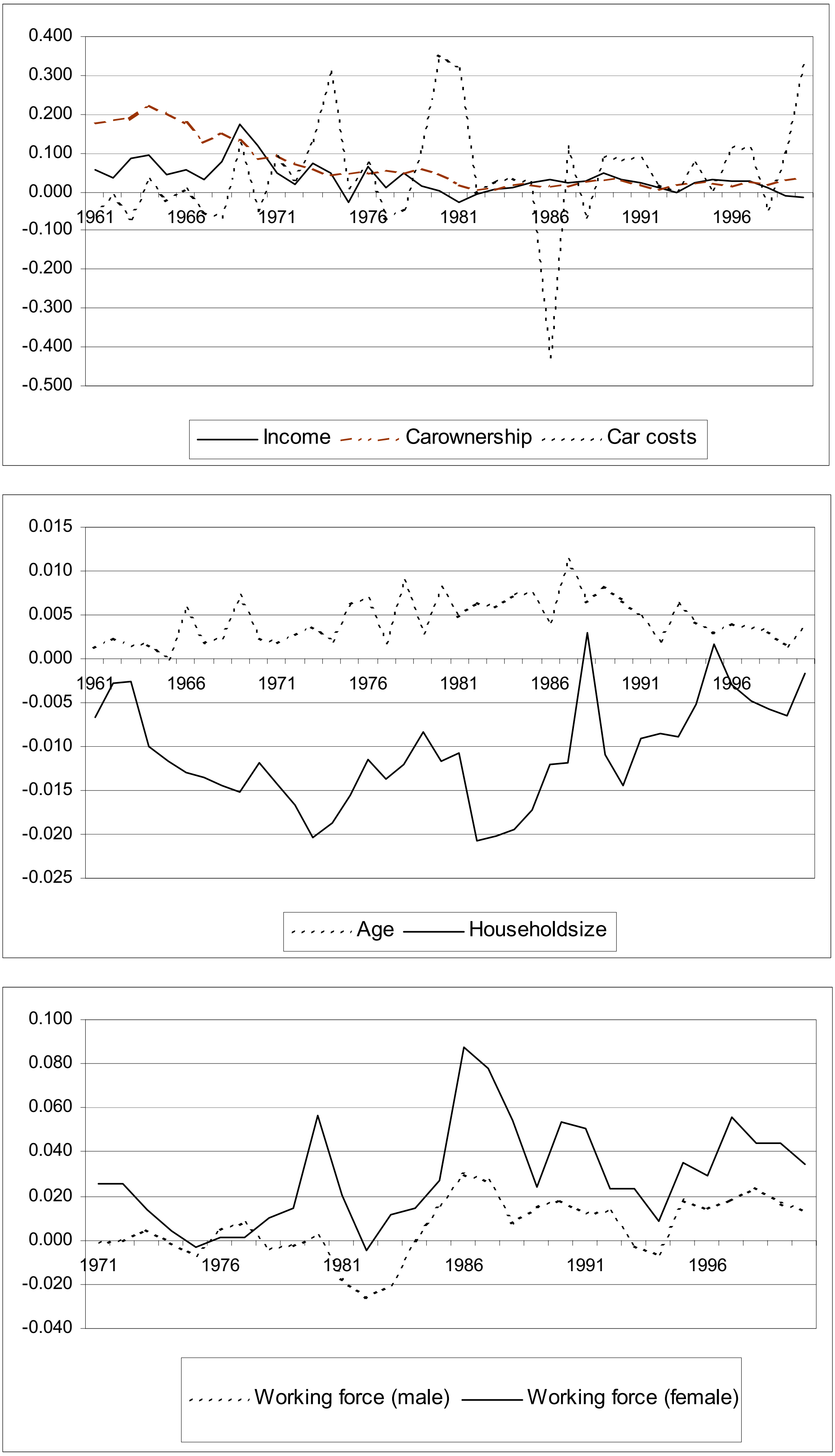
Female labour participation has been growing clearly faster (+2.9% per year on average) than male labour force participation (+0.6%). However, in some years, both went down. These are highly cyclical variables.

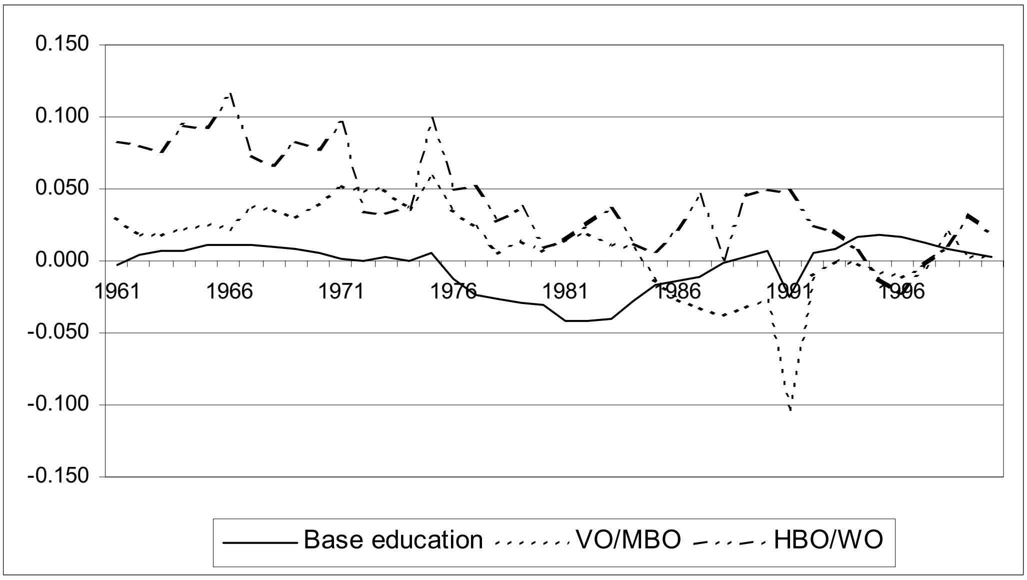
Table 2. Descriptive statistics for selected input variables (measured in annual growth rates)

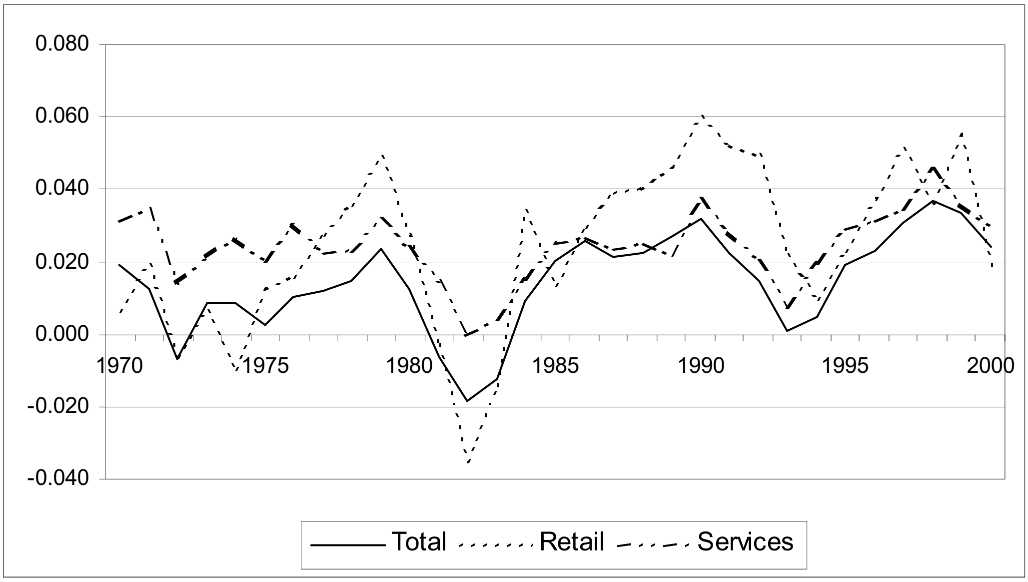
Variable	Mean	Minimum	Maximum	Standard Deviation	Period	Source
Income	1.034	0.973	1.172	0.039	1960-2000	CBS Statline
Car ownership	1.066	1.008	1.223	0.064	1960-2000	CBS Statline
Car costs	1.046	0.574	1.352	0.102	1960/1983-2000	CBS Statline[2]
Total jobs	1.017	0.9815	1.037	0.013	1960-2000	CBS Statline
Jobs retail	1.024	0.9651	1.060	0.023	1970-2000	CBS Statline
Jobs services	1.024	1.000	1.046	0.010	1970-2000	CBS Statline
Average age	1.004	1.000	1.011	0.003	1960-2000	CBS Statline
Household size	0.989	0.979	1.003	0.006	1960-2000	CBS Statline
Labour force (male)	1.006	0.974	1.030	0.014	1970-2000	CBS Statline
Labour force (female)	1.029	0.996	1.087	0.023	1970-2000	CBS Statline
Education (low)	0.996	0.958	1.018	0.017	1960-2000	CBS Statline
Education (medium)	1.010	0.897	1.059	0.030	1960-2000	CBS Statline
Education (high)	1.043	0.978	1.115	0.034	1960-2000	CBS Statline

[2] Additional information was used from CBS (1996) Auto's in Nederland; CBS and Kluwer Voertuigtechniek, Heerlen/Deventer, and from SEO (1991) De kosten van de auto en het openbaar vervoer vergeleken 1962-1990, SEO, Amsterdam.

Figure 1. Rates of change over time of key input variables







The number of students in higher education witnessed a much faster (though structurally declining) growth (+4.3%) than medium (+1.0%) and low (-0.4%) education, but for all three there is considerable variation.

In the long run however, periods of high growth and periods of low growth or even decline occur alternately (the business cycle). Therefore the income growth expectations for a 20-30 year horizon, which is likely to include a few of each, become smoothed. Other variables (number of jobs, number of cars, labour force) are also related to the business cycle. To express this phenomenon, we calculated 20-year moving averages (e.g. 1960-1979, 1961-1980, etc.). A time horizon of 20 years is not unusual for project evaluation. Often even longer periods (such as 30 years) are applied. The outcomes for these 20-year moving averages are given in Table 3.

The standard deviations of the 20-year moving averages are indeed much smaller than for the annual data of Table 2. The cyclical pattern has successfully been eliminated. Similarly the minimum and maximum values are much closer now to the mean values.

We propose to use the standard deviations (to make 95% confidence intervals) and/or the minimum and maximum values of the 20-year moving averages from Table 3 in the determination of the multivariate distribution from which the input values will be drawn. The idea is that the amount of variation in the input variables over the next 20 years is determined from all 20-year moving averages over the past 40 years. We also propose to use the multivariate (standard) normal distribution for this.

An exception is made for the car ownership developments. When specifying the multivariate distribution of the input variables, from which the draws will be made, we cannot accept all variation that is represented in Table 3 at face value. For some variables, and especially for car ownership, certain trends can be observed, and for forecasting the next decades, some growth rates observed in the past are more likely than others. For car ownership, the growth rates of the sixties cannot be expected to come back; some tendency towards saturation can be expected. We propose to use values between 1% (the lowest annual growth rate in the past 40 years) and 4% (the overall mean from the moving averages, and the highest annual growth in the past 10 years).

Table 3. Descriptive statistics for selected input variables (measured in 20-year moving averages)

Variable	Mean	Minimum	Maximum	Standard Deviation	Period	Source
Income	1.026	1.014	1.047	0.009	1960-2000	CBS Statline
Car ownership	1.041	1.021	1.083	0.019	1960-2000	CBS Statline
Car costs	1.043	1.025	1.078	0.018	1960/1983-2000	CBS Statline[3]
Total jobs	1.013	1.009	1.020	0.003	1960-2000	CBS Statline
Jobs retail	1.018	1.004	1.030	0.009	1970-2000	CBS Statline
Jobs services	1.018	1.009	1.028	0.007	1970-2000	CBS Statline
Average age	1.005	1.004	1.006	0.001	1960-2000	CBS Statline
Household size	0.987	0.985	0.990	0.002	1960-2000	CBS Statline
Labour force (male)	1.002	0.996	1.009	0.004	1970-2000	CBS Statline
Labour force (female)	1.022	1.007	1.037	0.011	1970-2000	CBS Statline
Education (low)	0.989	0.984	0.998	0.004	1960-2000	CBS Statline
Education (medium)	1.008	0.988	1.030	0.016	1960-2000	CBS Statline
Education (high)	1.035	1.018	1.058	0.012	1960-2000	CBS Statline

The correlations between the 20-year moving averages have also been calculated. These are often considerably higher than for the annual rates (e.g. a correlation coefficient between income and car ownership of 0.98).

To increase efficiency we used a specific type of Halton draws instead of random or conventional Halton draws (Hess et al., 2003), as described in the next section. Due to running time constraints, the maximum number of LMS runs to calculate the uncertainty margins is 100. This number was selected for the LMS runs. 100 NRM runs were performed as well.

[3] Additional information was used from CBS (1996) Auto's in Nederland; CBS and Kluwer Voertuigtechniek, Heerlen/Deventer, and from SEO (1991) De kosten van de auto en het openbaar vervoer vergeleken 1962-1990, SEO, Amsterdam.

3.5 Generating random draws from a multivariate normal distribution

In order to determine the input variables for the simulation with the National Model System and the New Regional Models, random draws from a multivariate normal distribution have to be drawn. Software is available (from the Internet; Alogit has no built-in facility of drawing from a multivariate normal distribution, although this could be programmed) to draw from a multivariate normal distribution. Built-in random number generators are used (depending on the programming language, in this case Delphi) to obtain multivariate random numbers using a Choleski decomposition (e.g. see Train, 2003). The Choleski decomposition is used here as a method to generate a multivariate normal distribution with correlation between the variables on the basis of uncorrelated univariate normal draws η. Multivariate random draws δ are then calculated using initial averages μ and the corresponding Choleski factor (matrix Λ). The Choleski factor expresses K correlated terms as arising from K independent components, with each component "loading" differently onto each term (Train, 2003). For any pattern of covariance, there is some set of loadings from independent components that reproduces that covariance. Formula 1 shows the functional form for two variables.

$$\begin{aligned} \delta_1 &= \mu_1 + \Lambda_{11} \times \eta_1 \\ \delta_2 &= \mu_2 + \Lambda_{21} \times \eta_1 + \Lambda_{22} \times \eta_2 \end{aligned} \qquad (1)$$

where

δ = the multivariate normal draw (vector)

μ = the initial average (vector)

Λ = the Choleski factor matrix

η = the random draw (vector) generated from a univariate normal distribution

The software freely available from the internet has been modified for the purpose of this project. However, the random number generator was replaced by Halton draws, which provides a better distribution (greater coverage, i.e. fewer empty spaces) over the 'random' space (see Hess, 2003).

In addition to drawing 'regular' Haltons, the methods proposed by Hess (2003) of 'shuffled'[4] Haltons and 'shifted[5] and shuffled' uniform vectors were implemented in the tool. Tests with one hundred and one thousand draws for each method were carried out. Knowing in advance the true mean of the input variables, the best method out of the three, could be selected, using sums of squared difference between the draws and the true values of the parameters. For 100 draws the shuffled Haltons gave the best result and the shifted and shuffled uniform vectors the worst. For 1000 draws the shifted and shuffled uniform vectors outperformed the other two methods.

[4] In a multi-dimensional shuffled Halton sequence, the one-dimensional Halton sequences are randomly shuffled (the word refers to reordering the sequence within a deck of cards) before being combined into multi-dimensional sequences. This avoids correlation over the dimensions.

[5] A shifted and shuffled uniform vector is an alternative to a Halton sequence. It uses evenly spaced points to cover the number space, and then combines the one-dimensional sequences into multi-dimensional as described for shuffled Haltons above.

In this project, where the first twenty draws are selected, the shuffled Haltons seemed the best method, and this was selected.

Twenty draws were made for the input variables which are used both for twenty LMS runs for the reference scenario and the twenty LMS runs with the new infrastructure project (paragraph 3.6). For the model coefficients again twenty draws were made, which were used in 40 runs (reference and project situation) as well (3.7). The first ten draws for input variables were further combined with the first ten draws for the model coefficients for reference and infrastructure scenario (twenty runs in total). This sums to a total of 100 LMS runs, 50 reference runs and 50 with the new infrastructure project. The 100 NRM runs are distributed in exactly the same way as the LMS runs.

3.6 Varying the input data for the LMS and the NRM

To analyse the effect of uncertainties in the input data on outcomes of the National Model System and the Regional Models twenty runs with different input were performed. The standard input was marked as the reference, and input was varied using random draws from a multivariate distribution described in the previous paragraph. Table 4 is the variance-covariance matrix used for the input variables. The correlations are given in Table 5 and the coefficients of variation in Table6.

Table 4. Covariance matrix for the input variables (20 years average).

	Income	Car ownership	Fuel price	Average age	Average hh size	Male labour f.	Female labour f.	Students low e.	Students medium e.	Students high e.	Employm. serv/gov	Employm. Retail	Total employm.
Income	0.000085												
Car ownership	0.000172	0.000356											
Fuel price	0.000105	0.000244	0.000311										
Average age	-0.000006	-0.000013	-0.000010	0.000001									
Average hh size	-0.000008	-0.000015	-0.000006	0.000000	0.000003								
Male labour force	-0.000029	-0.000059	-0.000044	0.000002	0.000005	0.000014							
Female labour force	-0.000094	-0.000190	-0.000130	0.000007	0.000013	0.000039	0.000122						
Students low education	0.000025	0.000055	0.000053	-0.000003	0.000002	-0.000005	-0.000022	0.000018					
Students medium education	0.000136	0.000276	0.000181	-0.000010	-0.000019	-0.000054	-0.000170	0.000032	0.000245				
Students high education	0.000106	0.000221	0.000140	-0.000007	-0.000013	-0.000041	-0.000125	0.000026	0.000181	0.000147			
Employment services/gov.	-0.000076	-0.000151	-0.000103	0.000005	0.000013	0.000033	0.000102	-0.000016	-0.000144	-0.000100	0.000092		
Employment retail	-0.000052	-0.000103	-0.000064	0.000003	0.000009	0.000023	0.000070	-0.000009	-0.000097	-0.000069	0.000062	0.000046	
Total employment	-0.000008	-0.000015	-0.000003	0.000000	0.000005	0.000008	0.000019	0.000006	-0.000026	-0.000017	0.000019	0.000014	0.000010

Table 5. Correlation matrix for the input variables (20 years average).

	Income	Car ownership	Fuel price	Average age	Average hh size	Male labour f.	Female labour f.	Students low e.	Students medium e.	Students high e.	Employm. serv/gov	Employm. Retail	Total employm.
Income	1.0000												
Car ownership	0.9844	1.0000											
Fuel price	0.6464	0.7312	1.0000										
Average age	-0.8940	-0.9278	-0.7905	1.0000									
Average hh size	-0.5083	-0.4632	-0.2009	0.2118	1.0000								
Male labour force	-0.8381	-0.8478	-0.6689	0.6895	0.7676	1.0000							
Female labour force	-0.9205	-0.9148	-0.6674	0.8032	0.7041	0.9576	1.0000						
Students low education	0.6374	0.6917	0.7098	-0.8891	0.2092	-0.3128	-0.4696	1.0000					
Students medium education	0.9417	0.9344	0.6542	-0.8134	-0.7254	-0.9390	-0.9824	0.4880	1.0000				
Students high education	0.9476	0.9624	0.6541	-0.8118	-0.6377	-0.9039	-0.9301	0.4993	0.9556	1.0000			
Employment services/gov.	-0.8569	-0.8330	-0.6089	0.7323	0.7735	0.9324	0.9639	-0.3954	-0.9617	-0.8560	1.0000		
Employment retail	-0.8339	-0.8076	-0.5386	0.6760	0.7578	0.9277	0.9399	-0.3143	-0.9206	-0.8464	0.9634	1.0000	
Total employment	-0.2910	-0.2521	-0.0589	-0.0150	0.9121	0.6829	0.5615	0.4429	-0.5310	-0.4443	0.6204	0.6570	1.0000

Table 6. Coefficients of variation (standard deviation divided by mean) for the input variables (20 years average).

Income	0.160
Car ownership	0.232
Fuel price	0.242
Average age	0.035
Average hh size	0.087
Male labour force	0.018
Female labour force	0.151
Students low education	0.084
Students medium education	0.070
Students high education	0.213
Employment services/gov.	0.119
Employment retail	0.123
Total employment	0.080

It was decided to use the covariance matrix derived from the 20-year moving average data only, which has the benefit of greater internal consistency.

Table 7 summarizes the growth changes for the relevant input variables between 1995 and 2020. The first column shows the indices for the reference run, following are the 20 randomized indices.

Table 7. Input variable changes between 1995 and 2020 for the 20 draws

	20ref_1995	BB01	BB02	BB03	BB04	BB05
Employment government/services	1.432	1.351	1.928	1.351	1.023	1.240
Employment Retail	1.221	1.204	1.885	1.030	0.773	0.893
Total employment	1.279	1.306	1.483	1.191	1.094	1.180
Students low education	1.067	1.158	1.117	1.019	1.008	1.002
Students medium education	1.209	1.389	0.619	1.533	2.027	1.645
Students high education	1.176	1.463	0.660	1.251	1.475	1.318
Male labour force	1.115	1.104	1.327	1.077	0.962	1.062
Female labour force	1.600	1.391	2.493	1.408	1.029	1.371
Average age	1.089	1.073	1.097	1.092	1.081	1.090
Size of household	0.957	0.981	1.043	0.906	0.884	0.892
Income	1.650	1.854	1.293	1.733	2.008	1.917
Car ownership	1.573	*1.986*	**1.282**	1.586	*2.069*	1.716
Fuel price	1.050	1.128	0.563	0.844	1.365	0.996

	20ref_1995	BB06	BB07	BB08	BB09	BB10
Employment government/services	1.432	1.361	1.645	1.304	1.727	1.620
Employment Retail	1.221	1.156	1.529	1.222	1.564	1.373
Total employment	1.279	1.428	1.438	1.200	1.256	1.325
Students low education	1.067	**1.319**	1.180	0.946	0.942	1.089
Students medium education	1.209	1.501	0.823	1.167	0.722	1.208
Students high education	1.176	1.582	0.878	1.239	0.748	1.287
Male labour force	1.115	1.110	1.225	1.110	1.215	1.140
Female labour force	1.600	1.451	2.019	1.722	2.087	1.698
Average age	1.089	1.059	1.086	1.104	1.115	1.090
Size of household	0.957	0.999	**1.034**	0.929	0.963	0.961
Income	1.650	2.100	1.353	1.570	*1.133*	1.673
Car ownership	1.573	***1.969***	**1.282**	1.407	**1.282**	1.701
Fuel price	1.050	1.902	1.430	0.873	0.621	1.346

	20ref_1995	BB11	BB12	BB13	BB14	BB15
Employment government/services	1.432	1.339	1.609	1.609	**1.023**	1.269
Employment Retail	1.221	1.191	1.514	1.562	**0.754**	1.030
Total employment	1.279	1.237	1.383	1.357	1.139	1.317
Students low education	1.067	0.977	1.090	1.043	1.123	1.234
Students medium education	1.209	1.268	0.922	0.895	**2.605**	1.773
Students high education	1.176	1.292	1.042	1.039	2.043	1.583
Male labour force	1.115	1.124	1.267	1.191	**0.926**	1.080
Female labour force	1.600	1.595	1.999	1.963	0.958	1.291
Average age	1.089	1.101	1.095	1.095	1.063	1.061
Size of household	0.957	0.933	0.992	0.993	**0.879**	0.953
Income	1.650	1.785	1.467	1.525	2.508	2.187
Car ownership	1.573	1.604	**1.282**	1.315	***2.079***	***2.011***
Fuel price	1.050	0.770	0.634	0.992	1.785	1.074

	20ref_1995	BB16	BB17	BB18	BB19	BB20
Employment government/services	1.432	1.741	1.093	1.431	1.388	1.205
Employment Retail	1.221	1.687	0.798	1.246	1.223	1.009
Total employment	1.279	1.337	1.209	1.365	1.378	1.114
Students low education	1.067	1.007	1.181	1.144	1.204	0.944
Students medium education	1.209	0.687	2.167	1.179	1.129	1.260
Students high education	1.176	0.749	1.557	1.223	1.070	0.995
Male labour force	1.115	1.204	0.987	1.165	1.174	1.079
Female labour force	1.600	2.334	1.078	1.763	1.618	1.459
Average age	1.089	1.110	1.064	1.085	1.072	1.102
Size of household	0.957	1.010	0.918	0.977	0.995	0.913
Income	1.650	1.199	2.262	1.709	1.627	1.419
Car ownership	1.573	**1.282**	***2.039***	1.683	1.576	**1.282**
Fuel price	1.050	0.706	1.544	1.168	0.931	1.036

Because we do not want to rely on (extreme) outliers, the annual parameter change was restricted to the 95%-interval, i.e. draws that were outside the bandwidth of two times the standard deviation were cut off at this value. For car ownership the annual changes were restricted to the interval between +1% and +4% per year (see section 3.6). The **bold** figures in Table 7 show in which scenario and for which variable this (reaching the 95% cut-off points or the +1% or +4% per year boundaries for car ownership) occurred. It turns out that the car ownership boundaries are attained in 50% of all cases. Nevertheless the range investigated for car ownership growth (the lowest annual growth rate in the past 40 years and the highest annual growth in the past 10 years) seems adequate, given that there is evidence for some saturation.

The figures shown in *italic* are adjusted due to model restrictions: the total number of cars in 2020 exceeded the limits of the car ownership that the current LMS can handle (there are 20 differrent car ownership models within the LMS; the limit here is the number of cars of the car ownership model that can give the highest car ownership). It is recommended that these restrictions be removed in the future (e.g. by increasing the number of car ownership models used to more than 20). In this report we stay within the limits of the current LMS, since we are investigating the uncertainty margins of the existing LMS. The maximum number of cars given by the model was taken instead. In one case, the income growth was too low, relative to the number of licenses and cars owned. A minimum annual growth of 0.5% was assumed.

The values for the 2020 reference were used as average input to the random generator that produced the 20 output values. The average of these twenty output values are summarized in Table8.

Table 8. Reference values in 2020 and average values of the 20 draws

Variable	**Employ. Gov/Services**	**Employ. Retail**	**Total employ.**	**Students l.e.**	**Students m.e**	**Students h.e.**	
2020 ref	1.4319	1.2209	1.2790	1.0674	1.2091	1.1756	
Average 20 draws	1.4128	1.2321	1.2868	1.0864	1.3259	1.2248	
Variable	**Male l.f.**	**Female l.f.**	**Average age**	**Size of hh**	**Income**	**Car ownership**	**Fuel price**
2020 ref	1.1154	1.5998	1.0886	0.9572	1.6500	1.5727	1.0500
Average 20 draws	1.1264	1.6364	1.0867	0.9577	1.7162	1.7596	1.0854

When the number of draws is infinite the average should be the same as the reference values. Using only 20 draws, there are slight departures from the reference values. Car ownership differs most from the reference value, and also income, and the number of students for medium and high education show smaller deviations. The other variables are quite close.

The following subparagraphs discuss the adjusted variables in more detail. The focus of this discussion is on the expected effect of the variable in the (different parts of the) model. All other variables in the National Model System remain unchanged.

3.6.1 Employment (government/services, retail and total)

Employment in the LMS/NRM serves as an attraction variable in the mode/destination choice model (NSES). The new employment statistics for each zone are calculated by adjusting the national reference growth factor with the national new growth factor. A national correction for employment (by category) has no effect on the zonal distribution. However, when in the three categories of the LMS/NRM different growth rates are used, a shift in attraction will occur. This will have an impact on the number of kilometres and the modal split for each purpose.

3.6.2 Number of students

The number of students influences the model on two levels. First, the number of tours for education (both children and higher education) are related to the number of students. In the mode/destination choice model, the number of students that the schools can handle is an attraction variable. Again, the zonal totals are calculated in the same way as employment, so only a shift in children/higher education purpose is to be expected.

3.6.3 Male and female labour force

The labour force has a significant impact on the number of tours for the purposes ‘home-work’, ‘home-based business’ and ‘non-home-based business’. The tour frequency model of the LMS reacts to the level of the labour force. A zonal correction was made based on the national growth factor.

3.6.4 Average age

An increase of the age of the population will lead to more licence holders and potential car owners. This will impact the modal shift. Also, the number of tours for the various purposes will change, for example between ‘children education’ and ‘higher education’. The LMS works with four age bands. An average age per two age bands is assumed to calculate the national average age. Then, the national correction is applied and zonal ages are adjusted accordingly.

3.6.5 Size of household

A change in the size of household will have an impact on the number of cars. The trend of declining household size will lead to an increase in car ownership and hence an increase in car use in the modal split.

A household size correction is made on a national level and disaggregated to zonal values.

3.6.6 Income

Income influences the number of licences and the number of tours. The change of household type to other income bands will alter the mode/destination model as well. An increase in income will lead to a change in destination choice, in general these destinations will be further away.

The LMS has a specific tool that calculates zonal income bands (five for every household category).

3.6.7 Car ownership

Income and car ownership are highly correlated. The national car ownership is set as a target for the car ownership models in the National Model System. The level of car ownership is a characteristic of the different household- and person types in the model, and has an impact on the mode/destination choice for each type.

3.6.8 Fuel price

The level of the fuel price has a direct impact on the kilometer costs for car driver. Other modes than car will benefit directly from an increase in the fuel price. The fuel price is an explanatory variable in the mode/destination choice model.

3.7 Treatment of model uncertainty

We assumed that the input uncertainty and the model uncertainty are two independent sources of error. Consequently we were able to provide both the share of the output uncertainty that is due to inputs and the share that is due to model uncertainty.

The LMS and NRM consists of a series of submodels. In calculating the uncertainty around the link flows we focussed on the tour frequency models and the mode-destination choice models. We did not include specification and estimation error in the licence holding and car ownership models (but treated the future year national car ownership total as one of the input variables to be varied, as described above). Similarly, we treated the parameters in the time of day choice models and the assignment (e.g. the speed-flow curves) as having been determined without error. Watling's studies on assignment uncertainty (see the literature review in this project) have shown that in general the error in the OD matrices is the dominant source of total link flow error. Applying his methods in the context of LMS/NRM would be very difficult. These methods try to calculate the derivative of the link flow errors from the derivatives of the errors in the matrices (through a process called 'linearisation', also see the literature review), which should be considered as the leading edge in traffic analysis. Moreover, he uses stochastic assignment. These methods cannot be used unchanged in assignment procedures that have a discrete nature, such as the non-stochastic assignment in LMS and NRM.

The general approach in this project therefore is that we study variations in the OD matrices (due to input variables and the tour frequency and mode-destination choice models) and assign these using the same assignment procedures, without introducing extra variation due uncertainties (e.g. through Monte Carlo draws) in the departure and route choice functions. In this project we study the impact on the predicted flows for a set of selected links (3-4 links in one direction). The assignment mechanism itself can change the amount of uncertainty (e.g. reduce it, as in Zhao and Kockelman, 2001), because larger transport demand leads to more congestion and this increases travel times, which reduces demand for specific routes, periods and modes, etc. This also implies that we had to run the full assignment procedure (with the normal number of iterations[6]). The

[6] Recent LMS and NRM runs in other projects (Deltametropool, tollcases) have shown that at the normal number of iterations within the assignment and between NSES and assignment, convergence may not yet have been achieved, and counter-intuitive differences between various runs can occur for small input changes and at the link level. However, it would have been infeasible in terms of computer run time to do more than the standard number of iterations since we

result of the first iteration were stored separately, to see the effect without the congestion feedback mechanism. Freight matrices for road transport were added to the OD matrices for cars that were varied in this project, but these freight matrices did not vary (fixed background vehicle loads).

We re-estimated all tour frequency models (both the 0/1+ and the stop/go models) and one of the mode-destination models: the one for commuting. This slightly increased most of the standard deviations (decreased the t-ratios) of the tour frequency models, to reflect the additional uncertainty due to misspecification of the model (e.g. in the functional form, the independence and homoskedasticity assumptions on the error distribution, but also including misspecification due to omitted variables). The correlations between the parameter estimates were taken from the Bootstrap estimation as well. We found no systematic differences between the Bootstrap estimates of the commuting mode-destination model and the original estimates for this model (there were differences, usually small, in both directions). Therefore we used the standard deviations and correlations of the original estimation runs for the mode-destination models for all purposes other than commuting.

3.8 Varying the model coefficients for the LMS and NRM

For two types of models within the LMS and NRM the coefficients were changed: the tour frequency models and the mode destination models. An assumption was made that the coefficients of the two models are uncorrelated. Although this might not be true in reality, the two models were estimated separately in practice as well.

For each of the 0/1+ and stop-repeat models and the eleven purposes in the tour frequency model coefficients were estimated (for a more detailed description, see the LMS 7.0 documentation). Twenty-two covariance matrices were derived based on the original estimations.

The mode/destination model runs for eight purposes use different coefficients for each of these purposes. Eight covariance matrices were derived based on the original estimations.

The procedure followed to set up 20 draws for the coefficients is practically the same as for the input variables. Using the bootstrap estimations of the tour frequency models and the commuting mode-destination model and the original estimates for the other mode-destination models, variance-covariance matrices for the model coefficients were produced. After this, the tool described in paragraph 3.5 was used to generate 20 random draws from a multivariate normal distribution using this variance-covariance matrix.

In contrast to the 20 draws for the input variables, outliers of the coefficient draws were not corrected. Coefficients that have a fixed value (of one), such as structural coefficients for multinomial logit models and coefficients for the basic size variables, have not been changed.

The covariance matrices and the coefficient files that results from the draws are not shown in this report.

The growth module within the NRM used the same coefficient values for tour frequencies and mode-destination choice as the LMS. But it also has additional scaling coefficients to balance overall national behaviour with market shares observed in the region. In the simulations for the

had to perform 100 LMS and 100 NRM runs in this project. Also in this project, we are interested in the behaviour of the model under standard settings.

NRM only the coefficients that correspond to the LMS were varied, not the region-specific coefficients.

CHAPTER 4 Case study 1: the LMS

4.1 Description of the project studied

The project for which uncertainty in traffic forecasts is studied using the LMS is the extension of the A16 motorway in the Rotterdam area (see Figure 1). Since the focus of this project is on road link flows, we have selected a road project as a case study. It concerns a major new road. At present, the A16 enters the Rotterdam area from the South (Breda, Dordrecht) and continues until the Terbregseplein, where it meets the A20. The extension would continue north of the A20, and after a few kilometres it would go west until it meets the A13. This new road has three new links in the LMS, with several access links. It would form an alternative for several existing routes, but especially for the A20 between the Kleinpolderplein and the Terbregseplein.

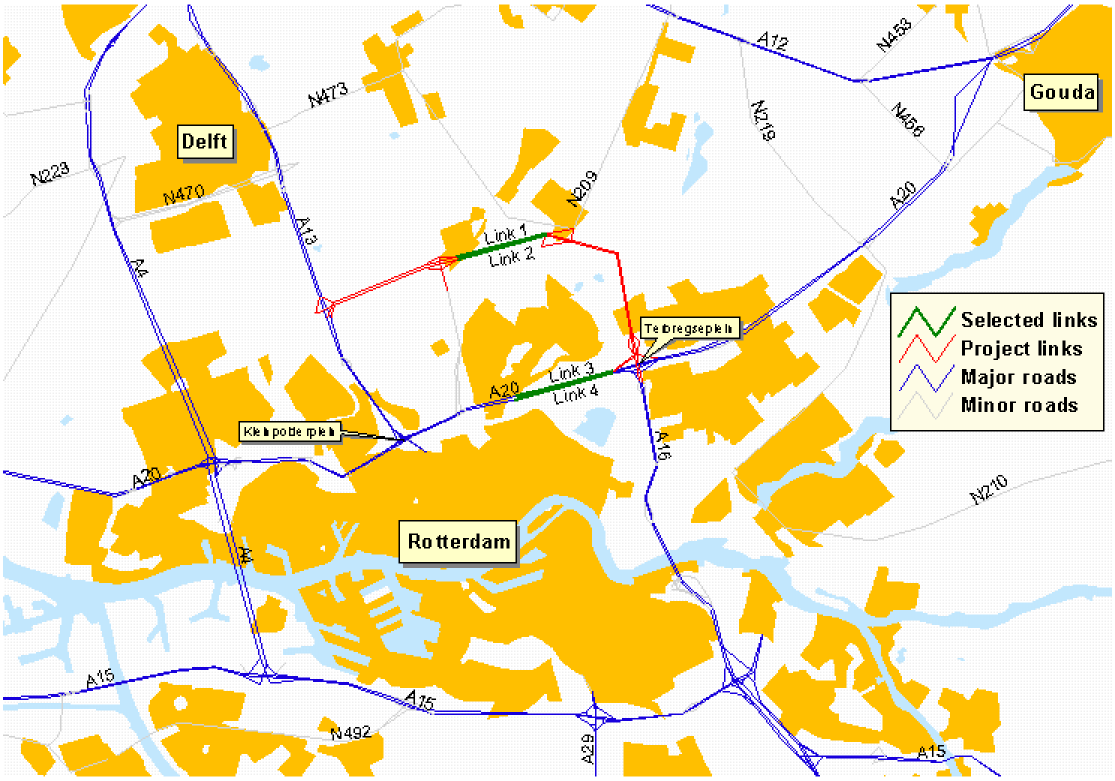
The uncertainty of the LMS forecasts is studied both at the national level and at link level:

- National level: number of tours and passenger kilometres, by mode and purpose, irrespective of whether this takes place on the network or not. This is output of the NSES mode-destination component of the LMS. Most of the NSES output that is studied in this report concerns the equilibrium situation after the demand-supply equilibration (congestion feedback: when there is congestion, new travel times are calculated on the networks and re-inserted into the travel demand models).
- Link level: traffic flow (in passenger car equivalent units), travel times (hours) and vehicle hours lost on a number of selected links (see below).

A small number of one-directional links was selected from the road network (the numbers correspond to the link numbers in Figure 2):

1. The northern part of the new extension of the A16 from the east (LMS node 44477) to west (node 44484).
2. The northern part of the new extension of the A16 from the west (node 44488) to the east (node 44485).
3. The A20 from the Terbregseplein (node 5060) to the Kleinpolderplein (node 4736), direction Gouda-Rotterdam.
4. The A20 from the Kleinpolderplein (node 4729) to the Terbregseplein (LMS node 5062), direction Rotterdam-Gouda.
5. The A2 near Abcoude (in the Amsterdam area) from node 9204 to node 9350, direction Amsterdam-Utrecht (not in Figure 2), as a sort of ‘control group’. This link will not be influenced by the extension of the A16.

Figure 2. The road network in the Rotterdam area in 2020 (in red the project situation).



Both results for the Reference scenario 2020 (the LMS reference run called 2020.RF5) and for the situation in 2020 with the project (extension of the A16) were studied. We also looked at differences (e.g. in link flows) between project run and the Reference 2020 (pairwise comparison: differences were calculated for runs that have identical inputs in all regards, but for the new road).

In Total, 100 LMS runs were carried out in this project:

- 50 runs for the Reference 2020;
- 50 runs for the Reference 2020 with the extended A16 project.

Each set of 50 runs consists of

1. 20 runs for variation in the model input variables;
2. 20 runs for variation in the model coefficients;
3. 10 runs with variation in both model input variables and coefficients.

In a number of tables and figures in this report (including those in Appendix 4), these numbers 1, 2 and 3 are used to indicate these subsets of LMS runs. The sets for the Reference 2020 and for the project situation contain exactly the same variation in input variables and coefficients. Each time, the same run was done with and without the project.

4.2 Uncertainty in outputs at the national level (NSES)

The detailed outcomes from NSES for the national level for the Reference situation 2020 and the project situation are in Appendix 4. In Table 9 and 10 these outcomes are summarised.

For the Reference 2020, on average 11.7 mln tours are made by car drivers on a working day (all purposes; see Appendix 4). The standard deviation from the input variation runs yield a standard deviation around this mean of 1.4 mln (11.7% of the mean, also see Table 6). On the basis of the Normal distribution, the 95% confidence interval around the mean would be [9.1 mln, 14.4 mln]. When using the observed distribution from the set of model runs (percentiles), we obtain a 95% confidence interval [9.7 mln, 13.5 mln] for the input variation.

In Table 9 are results for the Reference 2020 simulation with the LMS tour frequency and mode destination models, at the national level by mode for the sum of all purposes.

Table 9. Standard deviation for input uncertainty and model uncertainty of tours for the reference situation.

	Standard deviation for input uncertainty in tours (% of mean)	Standard deviation for model uncertainty in tours (% of mean)	Standard deviation for input and model uncertainty in tours (% of mean)
Car driver	11.7	0.8	12.1
Car passenger	6.3	0.9	6.0
Train	15.3	2.4	16.2
BTM	12.2	1.4	12.1
Slow modes	4.1	0.5	4.3
Total	1.8	0.6	1.9

In assessing these input error margins, one should keep in mind the amount of variation in the input variables that was introduced in these LMS runs. The average income increase of 65% between 1995 and the Reference 2020 for instance was varied from a 13% increase to a 110% increase.

The standard deviations for input variation at this level are between 4 and 16% (by mode). For the total number of tours over all modes, the standard deviation is even below 2%. The total across all modes does not include the distribution over modes from the mode destination models, and therefore it can be more forecast more precisely.

The standard deviations that result from model uncertainty are clearly smaller than for input uncertainty. This happens for all of the modes and for the total over modes. For instance for car drivers this standard deviation is only 0.8% of the mean for tours, and for all modes together it is 0.6%. Uncertainty in the input variables such as income and car ownership clearly dominates the uncertainty that is due to the uncertainty in the model coefficients.

Table 10 presents the same results for the Reference scenario, but now for passenger kilometres.

Table 10. Standard deviation for input uncertainty and model uncertainty of passenger kilometres for the reference situation.

	Standard deviation for input uncertainty in passenger km (% of mean)	Standard deviation for model uncertainty in passenger km (% of mean)	Standard deviation for input and model uncertainty in passenger km (% of mean)
Car driver	8.3	0.7	8.3
Car passenger	10.2	3.9	11.0
Train	14.4	2.5	15.2
BTM	10.4	2.1	10.0
Slow modes	4.7	0.5	5.0
Total	4.4	0.9	4.5

For car driver kilometres, the standard deviation due to input uncertainty is 8.3% of the mean. The 95% confidence interval from the Normal is [287 mln, 399 mln] and from the percentiles [287 mln, 377 mln]. For car drivers, the relative input uncertainty for passenger kilometres is smaller than for tours (compare Tables 9 and 10). For car passenger and the slow modes however, the reverse in true.

It is an interesting outcome that the errors in the kilometres are of the same order of magnitude as the errors in the numbers of tours, while for policy simulations that change travel times and costs, the kilometres are mostly more volatile than the tours (e.g. greater time and cost elasticities for kilometres). Since this happens for all modes, the explanation cannot (only) be the effect of congestion (that would dampen the kilometrage shifts). We conclude that runs that change time and cost affect destination choice more than mode choice, and tour frequencies not at all. The changes in the input variables performed here (of which income and car ownership are the most important) affect tour frequencies more than other choices, and therefore lead to broadly similar effects in terms of tours and kilometres.

In most cases the standard deviations for input and model uncertainty are slightly higher than those for input uncertainty alone, but in some cases the standard deviations for both sources of uncertainty are the same or just below those for input uncertainty. This is probably an artefact of having only performed a limited number of LMS runs for the combination of the two sources of uncertainty. But, as for tours, the input uncertainty is substantially greater than the model uncertainty.

For the situation with the project (see Appendix 4), the variation in tours and kilometres of the same order of magnitude as for the Reference Situation. Again, the uncertainty due to input variation dominates the output variation.

Figure 3 and Figure 4 depict the outcomes as discussed above for all purposes graphically. Again, we can observe that the variation in tours and in passenger kilometres is of similar magnitude, that it affects all modes and that the input variation (labels (1)) is a much more important source of variation in tours and passenger kilometres than model uncertainty.

In Figure 5 to Figure 9 the number of tours and passenger kilometres are given for each mode, by travel purpose (from the same NSES runs as the Tables and Figures above). All purposes have substantial variation in tours and kilometres. For tours, the variation is similar for all travel purposes. For passenger kilometres (especially as car driver), commuting and other travel are clearly more uncertain than non-home-based business travel, shopping and education.

Figure 3. Total tours by kilometers (NSES) for the reference situation

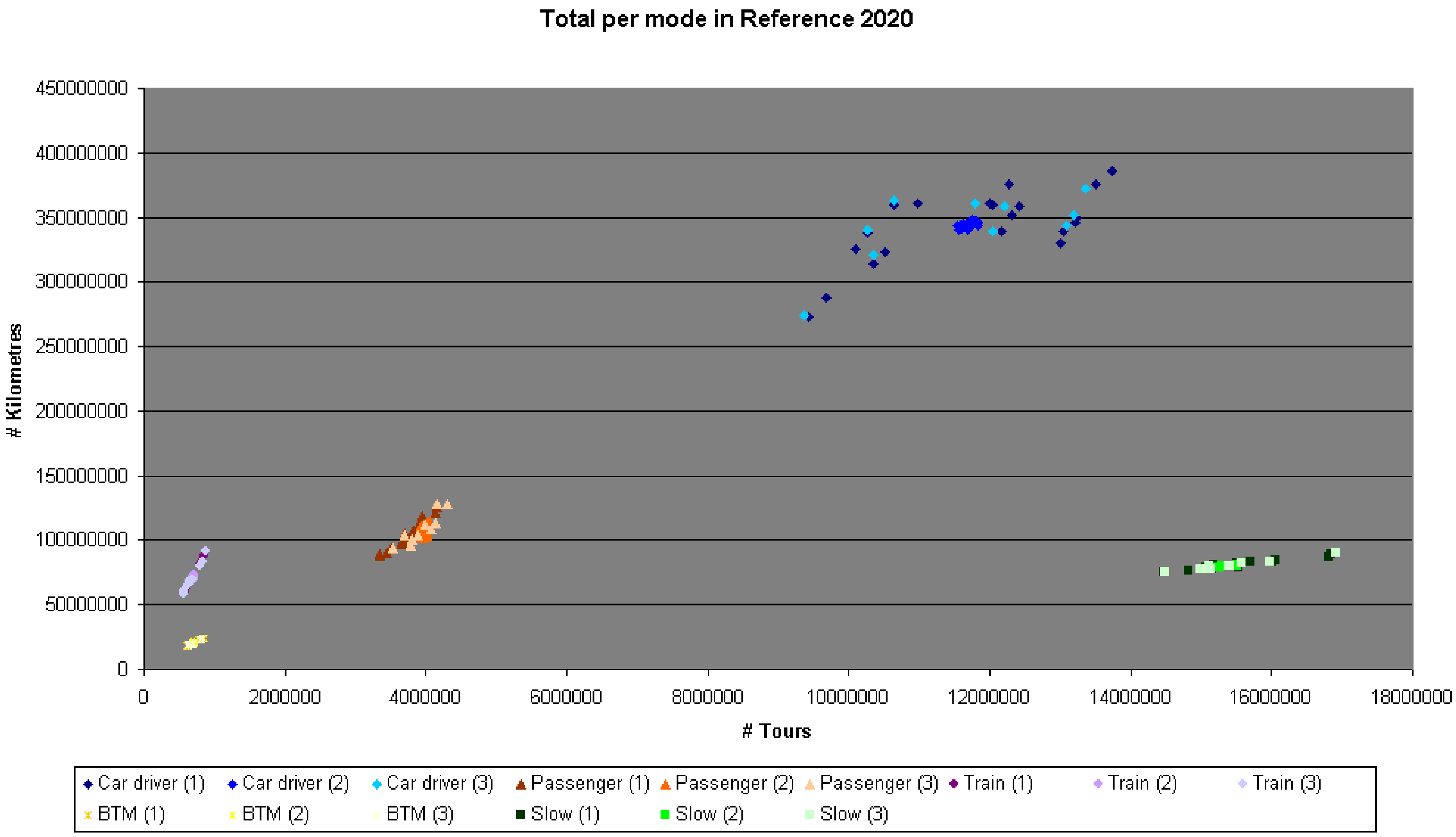


Figure 4. Total tours by kilometers (NSES) for the project situation

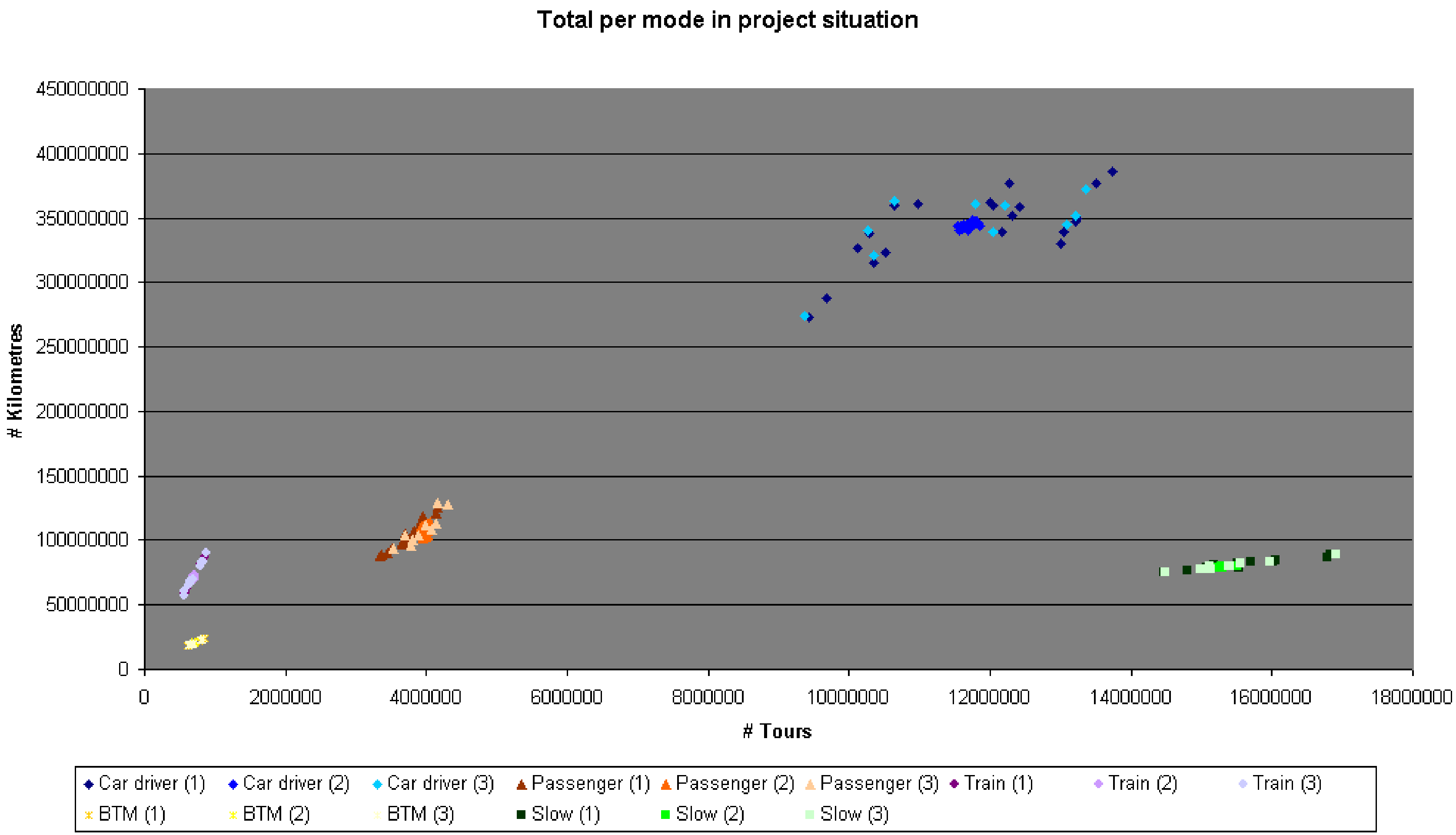


Figure 5. Number of tours by number of kilometers (NSES) for car driver for the reference scenario

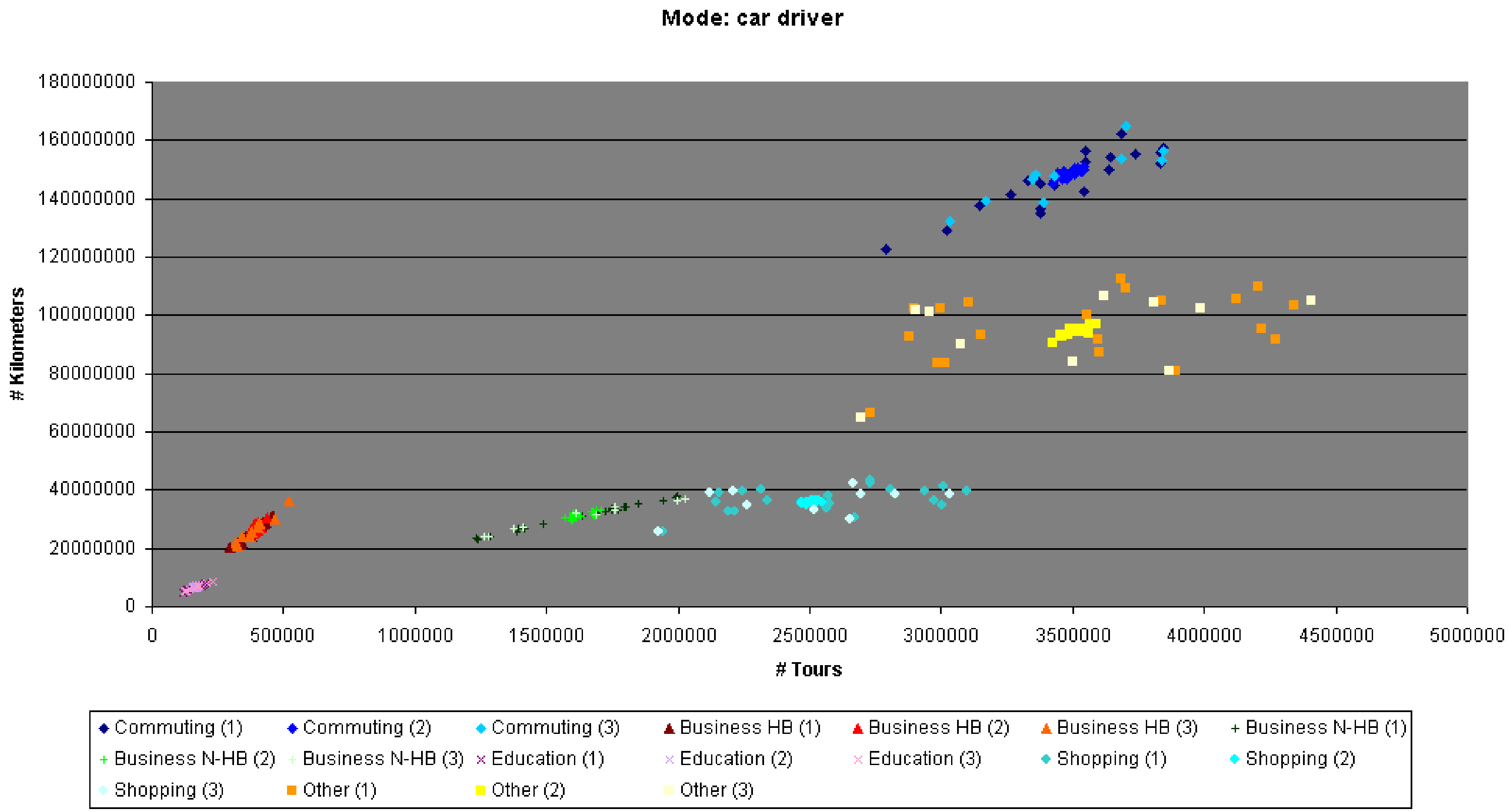


Figure 6. Number of tours by number of kilometers (NSES) for car passenger for the reference scenario

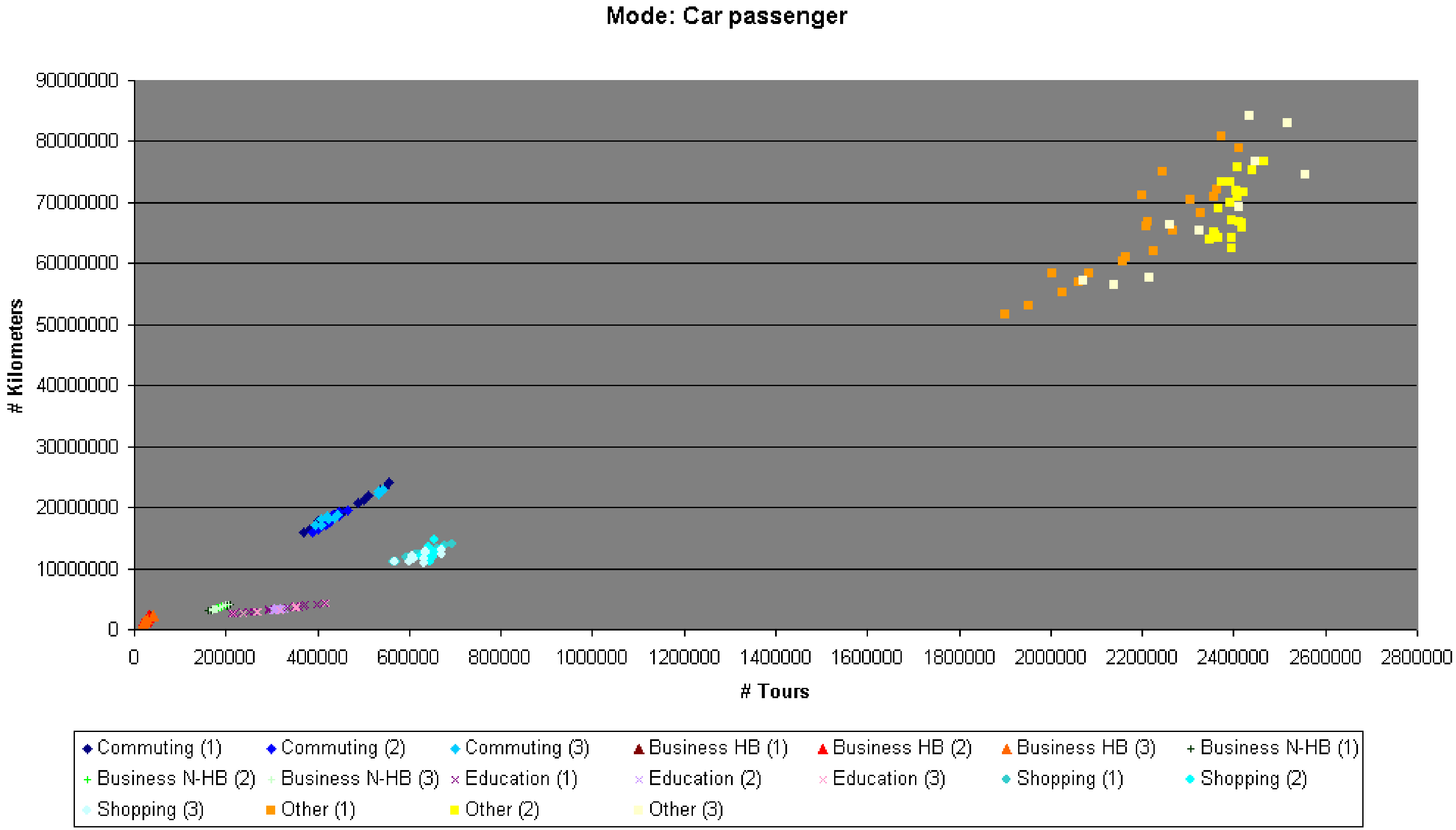


Figure 7. Number of tours by number of kilometers (NSES) for train for the reference scenario

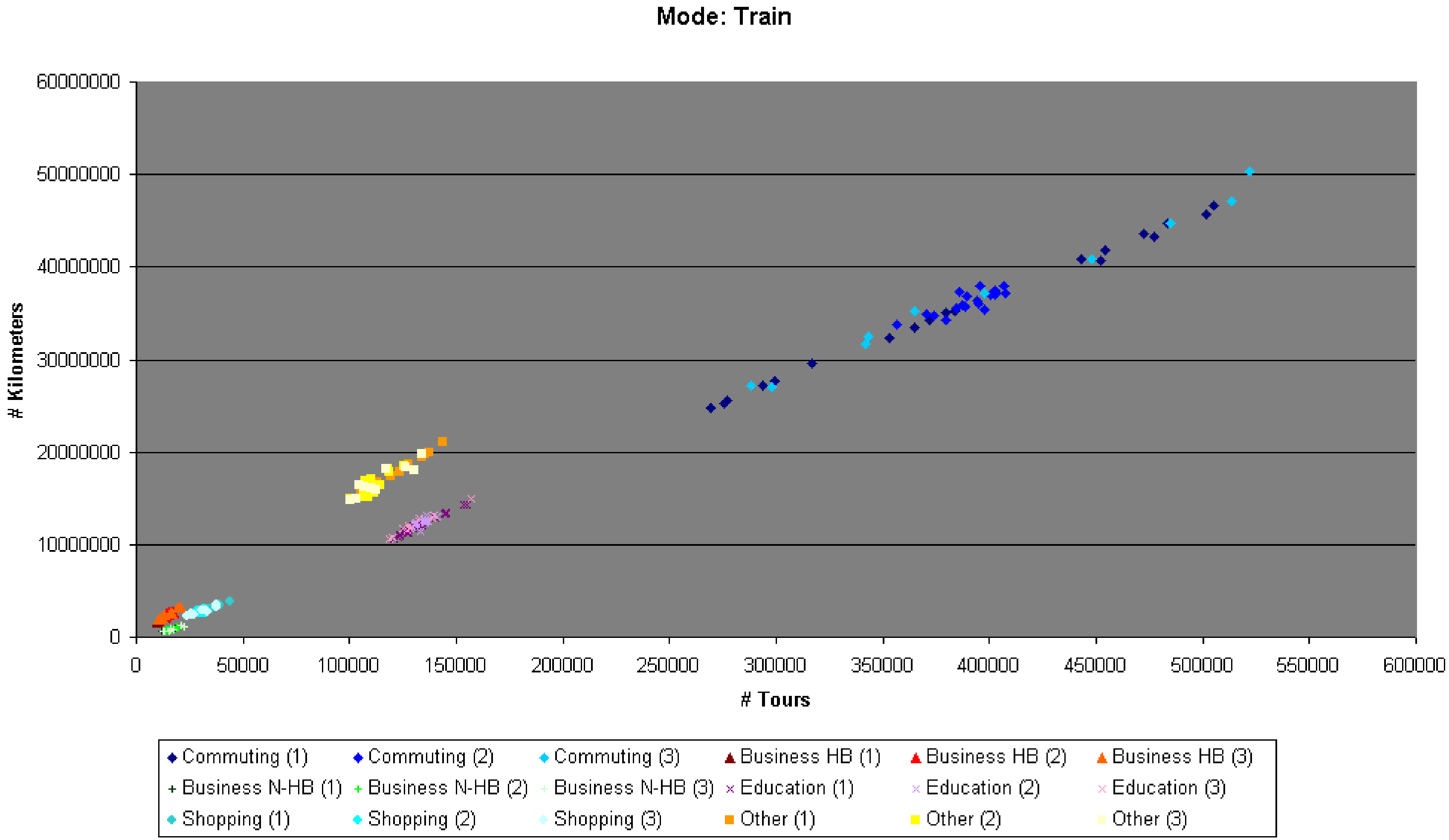


Figure 8. Number of tours by number of kilometers (NSES) for BTM for the reference scenario

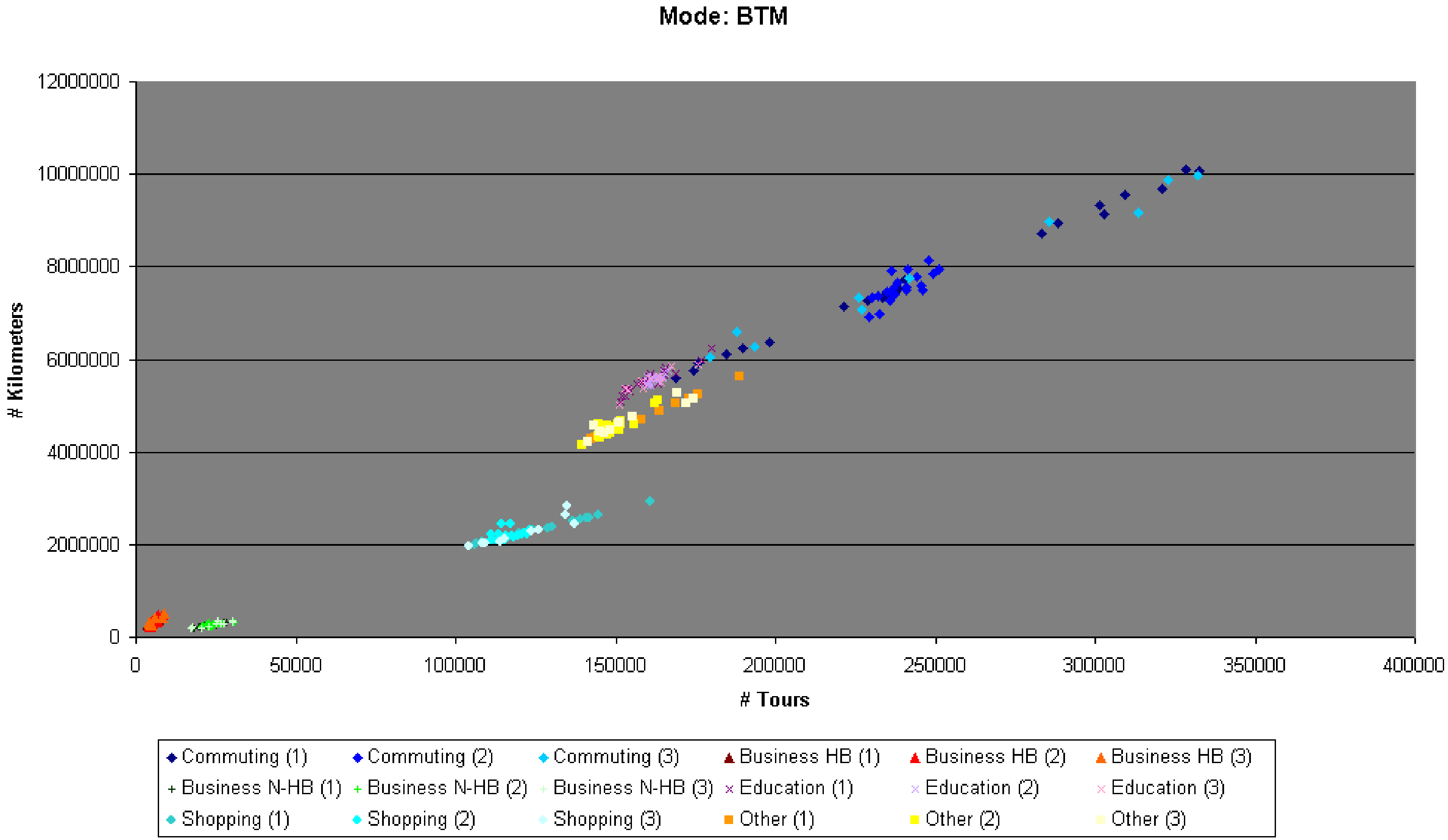
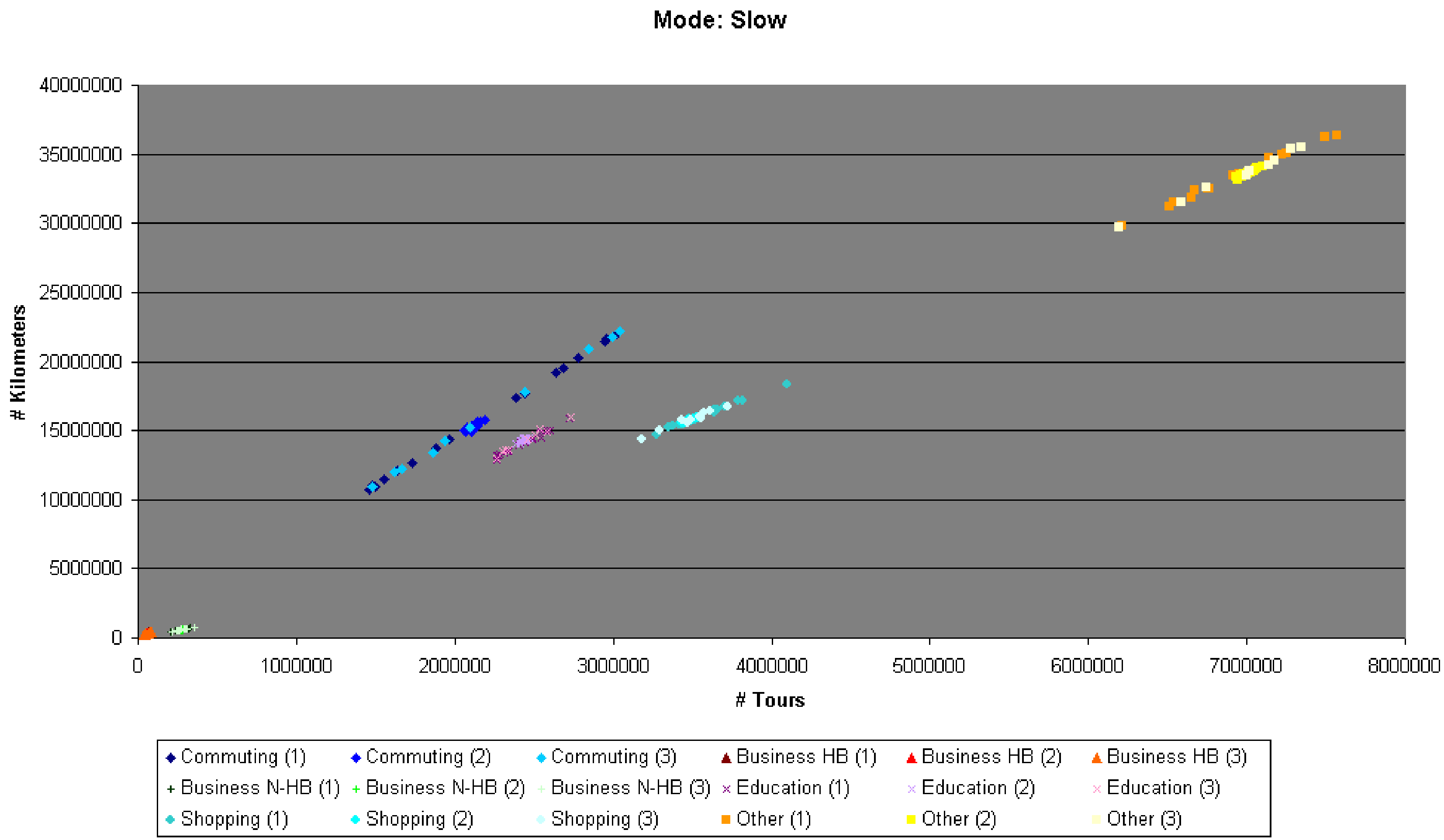


Figure 9. Number of tours by number of kilometers (NSES) for car driver for the reference scenario



We also compared the differences between the project run and the corresponding Reference 2020 run, from NSES at the national level. We found that the variation in the difference between two runs, both in terms of tours and kilometres, is much larger than the variation in the total number of tours or kilometres in either the project or the reference 2020 run. For instance the change caused by the project in car driver commuting tours varies between +1000 to +3500 tours. This is a large degree of variation relative to the mean change in tours, but at a low level: the variation is between 0.05% and 0.1% of all car driver commuting tours in the country. For car driver kilometres for commuting, the result of the project can be more kilometres (mode and destination choice effect) but also less kilometres (route choice effect: for some tours, the new motorway provides a shorter route). Here the maximum change (240,000 km) is 0.17% of the total number of kilometres for this mode and purpose.

For home-based business travel, the route choice effect is often more important than the mode-destination choice effect. For education, shopping and other travel, the mode-destination effect (leading to more car driver kilometres) dominates.

4.3 Uncertainty in outputs before congestion feedback

We also had a look at the simulation results at the national level for the Reference 2020 from NSES, without congestion feedback, to see whether the congestion feedback leads to a damping of the variation on tours and kilometres or to a propagation of errors (the detailed results are in Appendix 4).

We can conclude that the uncertainty in the number of tours is the same with and without congestion feedback, and that with congestion feedback the variation in kilometres is slightly smaller (after including the feedback, the standard deviation for input uncertainty goes from 9.6% to 8.3%; standard deviation for model uncertainty from 0.8% to 0.7%; standard deviation for combined uncertainty from 9.4% to 8.3%).

4.4 Uncertainty in outputs at selected links

Reference 2020

In Tables 11-13 are the key outcomes for the vehicle flow (in passenger car equivalent units), the number of hours travelled and the vehicle hours lost (Q-hours) at the selected links in the Reference situation (in the Reference we only have three links, in the situation with the project we have five). All these variables refer to a full 24-hours day, not just to the peak hours.

Table 11. Standard deviation for input uncertainty and model uncertainty of vehicle flow for the reference situation.

	Standard deviation for input uncertainty (% of mean)	Standard deviation for model uncertainty (% of mean)	Standard deviation for input and model uncertainty (% of mean)
A20 Rotterdam-Gouda	4.1	0.3	4.3
A20 Gouda-Rotterdam	4.6	0.6	4.7
A2 Amsterdam-Utrecht	8.3	1.3	8.3

Table 12. Standard deviation for input uncertainty and model uncertainty of hours travelled for the reference situation.

	Standard deviation for input uncertainty (% of mean)	Standard deviation for model uncertainty (% of mean)	Standard deviation for input and model uncertainty (% of mean)
A20 Rotterdam-Gouda	4.5	0.5	4.6
A20 Gouda-Rotterdam	13.2	2.7	13.8
A2 Amsterdam-Utrecht	8.8	1.5	8.0

Table 13. Standard deviation for input uncertainty and model uncertainty of vehicle hours lost for the reference situation.

	Standard deviation for input uncertainty (% of mean)	Standard deviation for model uncertainty (% of mean)	Standard deviation for input and model uncertainty (% of mean)
A20 Rotterdam-Gouda	22.3	16.9	12.5
A20 Gouda-Rotterdam	16.9	4.8	22.8
A2 Amsterdam-Utrecht	54.4	29.8	61.1

For input variation, the standard deviations of the link flows (Table 11) are between 4.1% and 8.3% of the means. For model uncertainty these are between 0.3% and 1.3%. Consequently the variation in link flows is even smaller than the variation in total tours and kilometres. This is probably due to the equilibrium nature of the assignment stage, which dampens the variation somewhat. Again input uncertainty clearly dominates model uncertainty. The variation of the combined runs (inputs and model uncertainty) are the same or larger than for input uncertainty only.

The input uncertainty in the number of hours (Table 12) varies between 4.5% and 13.2%, and the model uncertainty for hours travelled is between 0.5% and 2.7%. The variation in hours per link is the same or slightly greater than the variation in link flows.

The variation in vehicle hours lost due to congestion, or Q-hours, (Table 13) is sometimes very large (much larger than the variation in flows and hours), especially where the absolute congestion levels are low. The standard deviation ranges from 13% of the mean for both types of uncertainty for the A20 Rotterdam-Gouda to more than 50% of the mean for the A2. For both of these links, the absolute number of Q-hours in the reference situation were already low. Q-hours are a volatile measure with a large degree of associated uncertainty. Again the input uncertainty is more important than the model uncertainty. On the A20, between the Kleinpolderplein and the Terbregseplein, the congestion (measured as Q-hours) is much more severe in the Gouda-Rotterdam direction than in the Rotterdam-Gouda direction (on an average working day). This happens because of the queues on links before The Kleinpolderplein leading to the A20 in the direction of Gouda: the traffic has been blocked earlier on. For the traffic in the opposite direction the blocking before the selected link does not occur at such a high level.

The project situation

For the project situation, we used the same sets of random numbers for input and model uncertainty as in the reference situation. The results in terms of uncertainty margins with the project for hours and flows for the 'old' routes (see Appendix 4 for the detailed outcomes) are mostly similar to the Reference 2020. The variation for the new link is relatively high: the standard deviation for input uncertainty is 12.7% and 15.9% (two directions) for hours (see Table 15) and 11.9% and 14.8% for the flow (see Table 14). For model uncertainty it is 1.3% and 0.8% for hours and 1.2% and 6.4% for flows. The extension of the A16 leads to a clear reduction in hours travelled on the A20-Gouda-Rotterdam, which is visible in all runs.

Table 14. Standard deviation for input uncertainty and model uncertainty of vehicle flow for the project situation.

	Standard deviation for input uncertainty (% of mean)	Standard deviation for model uncertainty (% of mean)	Standard deviation for input and model uncertainty (% of mean)
A20 Rotterdam-Gouda	3.9	0.6	4.0
A20 Gouda-Rotterdam	4.8	0.5	4.5
A2 Amsterdam-Utrecht	8.4	1.3	8.2
New link A16 A20-A13	11.9	1.2	12.3
New link A16 A13-A20	14.8	6.4	15.7

Table 15. Standard deviation for input uncertainty and model uncertainty of hours travelled for the project situation.

	Standard deviation for input uncertainty (% of mean)	Standard deviation for model uncertainty (% of mean)	Standard deviation for input and model uncertainty (% of mean)
A20 Rotterdam-Gouda	4.1	0.6	4.2
A20 Gouda-Rotterdam	5.0	1.0	4.2
A2 Amsterdam-Utrecht	8.9	1.5	8.8
New link A16 A20-A13	12.7	1.3	13.0
New link A16 A13-A20	15.9	0.8	16.8

The Q-hours in the project situation are very different from those in the Reference 2020. On the A20 in the direction Rotterdam-Gouda, there were not so many Q-hours in the reference 2020, and this is reduced to almost zero with the extended A16. The variation around the mean is relatively large here, but this is because most runs yield zero Q-hours on this link and an occasional run gives a small positive number (3) of Q-hours. On the A20 Gouda-Rotterdam, the Q-hours are reduced from 1481 on average to 276. The variation around the new value is not very large: a standard deviation of 6.7% of the mean for input uncertainty, 7.2% for model uncertainty (high relative to the input uncertainty) and 10.4% for both sources of uncertainty together. On the selected link on the A2, the Q-hours remain low, (as expected, the project does not lead to changes in congestion here) and the new extension of the A16 remains uncongested.

Differences between the project situation and the Reference 2020

We also studied the uncertainty in the differences between the project run and the corresponding Reference 2020 run for the selected links (only for the 'old' links, since there was no new link in the Reference). The standard deviations for the flows differences for the A20 Rotterdam-Gouda are 11.7% for input uncertainty, 5.9% for model uncertainty and 12.5% for the combined uncertainty. For the A20 Gouda-Rotterdam, the standard deviations of the flows are 8.0%, 4.9% and 7.2% respectively. For the flows, hours and Q-hours differences on the A2, the relative variation is much larger, because the absolute differences are so small.

The standard deviation of the variation in differences in hours for the A20 Rotterdam-Gouda is 11.3% (input), 4.7% (model) and 11.6% (both), which is very similar to the outcomes for differences in flow. For the A20 Gouda-Rotterdam the standard deviation of the flow differences are a larger fraction of the means (22.7%, 5.9%, 25.2%).

On the A20 Rotterdam-Gouda, the standard deviation of Q-hour differences is 22.6% of the mean for input variation, 17.0% for model uncertainty and 12.5% (lower than the individual components) for the combined uncertainty. For the reverse direction these percentages are 25.7%, 6.5% and 28.6%. Q-hour differences are more volatile (larger uncertainty) than flows and hours.

4.5 Conclusions

Both the input variables and the model coefficients of the LMS tour frequency and mode-destination models were varied. The resulting standard deviation for uncertainty due to input error for total car tours is 11% of the mean, and for total car kilometres it is 8% of the mean. The model uncertainty is much smaller: the standard deviation is 0.7% of the mean for both car tours and kilometres. The standard deviation for both sources of uncertainty together is 12% of the mean for car tours and 8% for car kilometres.

For other modes the standard deviations for tours and kilometres are between 4% and 15% of the mean for input uncertainty and between 4% and 16% for combined uncertainty. The model errors again are much smaller than the input errors. The uncertainty margins for the different travel purposes are rather similar to those for all purposes together.

The relative uncertainty around the difference in total car tours or kilometres (with and without a road project) from the NSES mode-destination models is much larger than the above shares, but this concerns small amounts of traffic (at the national scale).

At the level of selected links of the road network, the standard deviations of the link flows are between 4% and 9% for input uncertainty, and around 1% for model uncertainty. For the number of hours travelled, the standard deviations are between 4 and 13% for input variation and 1-3% for model uncertainty. Q-hours (number of hours lost due to congestion) can have a much larger uncertainty, especially when the absolute numbers of Q-hours are low.

The standard deviations for the differences in link flows for links competing with the new road between the situation with and without the road project are 8-12% for input uncertainty, 5-6% for model uncertainty and 7-13% for combined uncertainty. Again the Q-hour differences are very uncertain, and the difference in hours travelled are in between.

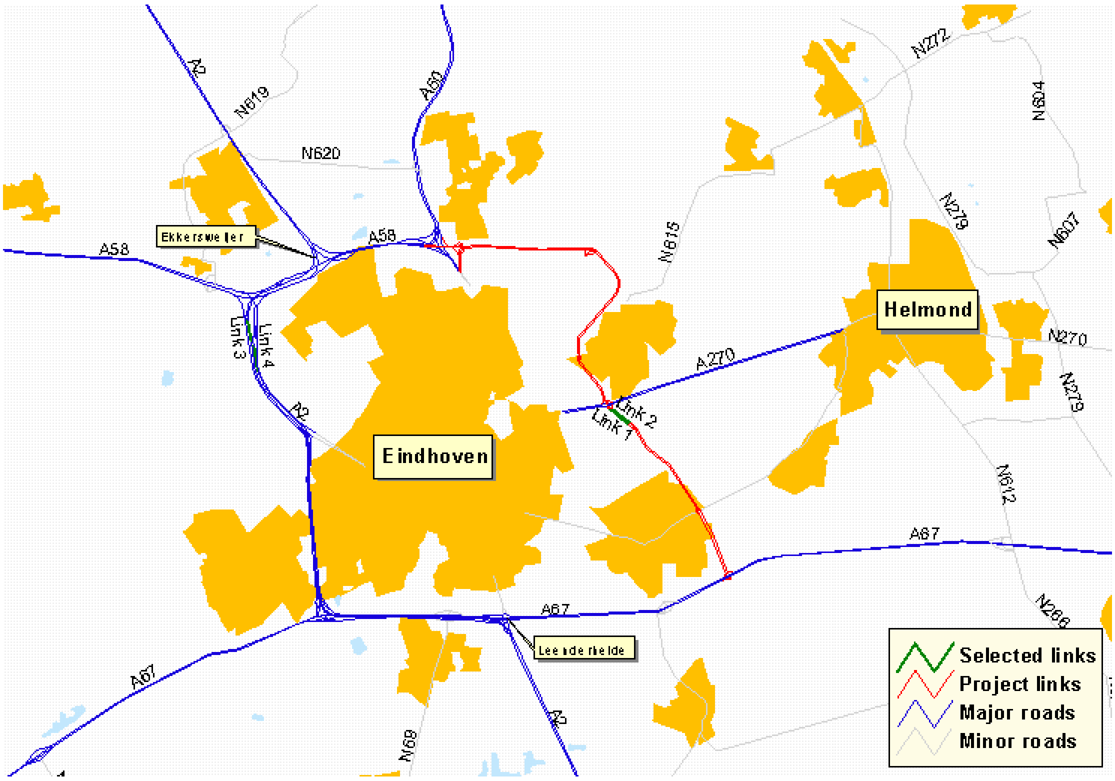
With regards to the evaluation of the project (the A16 extension in this example): the flow on this new link is predicted with a substantial level of uncertainty: the link flows can be up to 29% higher or lower than in the most likely case. This means that for cost-benefit analysis of the project, relatively large variations in the benefits need to be evaluated to account for uncertainty in the inputs and models.

CHAPTER 5 Case study 2: the NRM

5.1 Description of the project studied

For the case study on quantifying uncertainty for NRM predictions, we used the NRM Noord-Brabant (with permission from the Regional Directorate, who own this model system, and co-operated in providing the appropriate input files). As for the LMS application, we selected a road project, in this case the Eindhoven eastern ringroad ('Oostelijke Randweg'), that would complete the beltway around the city of Eindhoven.

Figure 10. The road network in the Eindhoven area in 2020 (in red the project situation).



The selected links for this case study are (the link numbers correspond to those in the Figure):

1. Project link: Eindhoven eastern ringroad in the direction north to south (from node 32375 to 32376).

2. Project link: Eindhoven eastern ringroad in the direction south to north (from node 32403 to 32404).
3. Competing link: Eindhoven A2-West in the direction north to south (from node 2106 to 3015).
4. Competing link: Eindhoven A2-West in the direction south to north (from node 3011 to 3016).
5. Control link: A58, between Bergen op Zoom and Roosendaal from west to east (not in the Figure) from node 13794 to 13804).

50 NRM runs have been carried out for the project situation for 2020 and 50 for the Reference Scenario 2020. Each set of 50 runs consists of

1. 20 runs for variation in the model input variables;
2. 20 runs for variation in the model coefficients;
3. 10 runs with variation in both model input variables and coefficients.

5.2 Uncertainty in outputs at the regional level

In Table 16 the outcomes from the mode-destination models within the NRM (called RSES) are given in terms of tours at the level of the region (study area) as a whole. For the mode car driver, the standard deviation for input uncertainty is 9.1% of the mean (in the LMS runs it was 11.7%). The standard deviation for model uncertainty is 1.0% of the mean (this was 0.8% in the LMS). The combination of input and model uncertainty leads in this NRM application to a standard deviation of 9.2% of the mean (LMS: 12.1%). The outcomes for tours with the other modes are also similar to those of the LMS. Again, the relative magnitude of the uncertainty due to model inputs (such as income and car ownership) is clearly larger than the uncertainty in the model coefficients.

Table16. Standard deviation for input uncertainty and model uncertainty of tours for the reference situation.

	Standard deviation for input uncertainty in tours (% of mean)	Standard deviation for model uncertainty in tours (% of mean)	Standard deviation for input and model uncertainty in tours (% of mean)
Car driver	9.1%	1.0%	9.2%
Car passenger	6.8%	3.4%	5.7%
Train	13.1%	8.3%	10.2%
BTM	8.8%	8.5%	9.4%
Slow modes	3.9%	1.3%	4.6%
Total	1.7%	0.6%	1.8%

The RSES outcomes for passenger kilometres are in Table 17. The standard deviation for car driver for both sources of uncertainty together now is 6.9% of the mean (LMS: 8.3%). Here too we find that in general input uncertainty is more important than model uncertainty. The errors in the forecasts for passenger kilometres are of the same magnitude as for tours.

Table 17. Standard deviation for input uncertainty and model uncertainty of passenger kilometres for the reference situation.

	Standard deviation for input uncertainty in passenger km (% of mean)	Standard deviation for model uncertainty in passenger km (% of mean)	Standard deviation for input and model uncertainty in passenger km (% of mean)
Car driver	7.1%	0.7%	6.9%
Car passenger	11.4%	6.4%	10.3%
Train	13.1%	9.2%	10.4%
BTM	8.5%	8.8%	9.8%
Slow modes	4.8%	1.2%	6.0%
Total	4.9%	1.7%	5.1%

The variation in tours and kilometers in the project situation (see Appendix 4) is similar to that is the Reference Situation presented above. This can also be seen by comparing Figures 11 and 12. These figures also illustrate that input errors are usually larger than model errors. Figure 13 depicts the Reference Situation, for car drivers only, by travel purpose. In terms of tours, the relative variation is similar for all travel purposes. For car driver kilometres, commuting and travel for other purposes have a larger variation (are more uncertain) than non-home-based business travel, shopping and education.

5.3 Uncertainty in outputs before congestion feedback

The results without congestion feedback for the Reference Situation can be found in Appendix 4. For the variation in tours, the congestion feedback does not lead to significant changes. The congestion feedback however reduces the variation in passenger kilometres somewhat (as in the LMS): for car driver the input uncertainty is reduced from 8.9% to 7.1%, the model uncertainty from 0.9% to 0.7% and the combined uncertainty from 8.6% to 6.9%.

5.4 Uncertainty in outputs at selected links

Reference 2020

The outputs at the link level consist of vehicle flows, hours travelled and vehicle hours lost. All outputs here refer to a full 24-hours day. The vehicle flows are expressed in passenger car equivalent units. In Table 18, we can see that for the selected links that exist in the Reference Situation, the standard deviation for input uncertainty in the vehicle flows is between 0.9% and 3.7% of the mean. This is noticeably smaller than for the LMS, where the range was from 4.1% to 8.3%. This might be due to the finer network and zoning in the NRM. Also, the NRM study area is smaller and the traffic to/from the rest of The Netherlands is kept constant. Again the model uncertainty is considerably smaller than the input uncertainty. The variation in link flows for the NRM is smaller than for total study area tours and kilometres (as was found for the LMS), probably a result of the equilibrium properties of the assignment.

Figure 11. Total tours by kilometers (RSES) for the Reference Situation

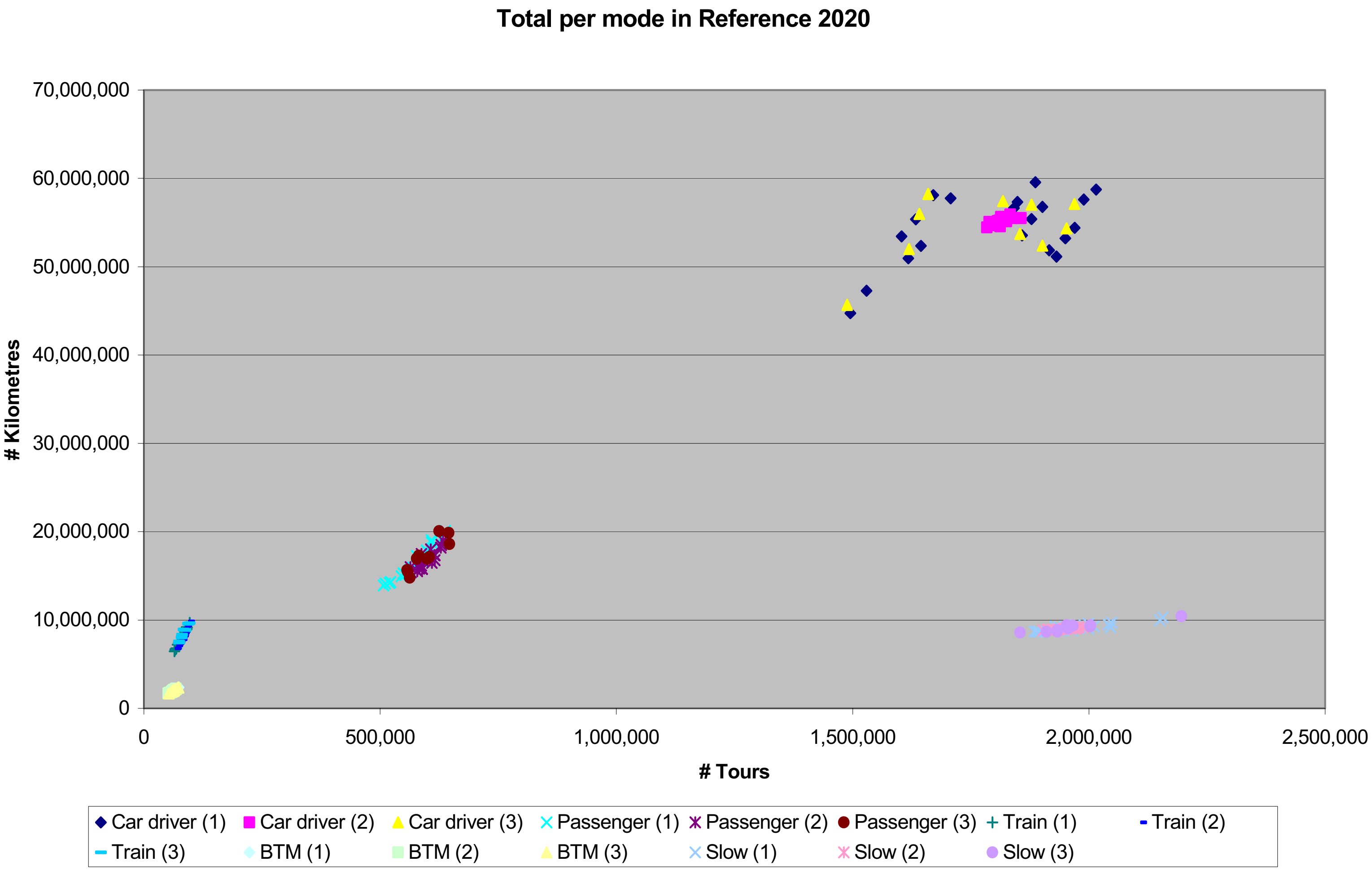


Figure 12. Total tours by kilometers (RSES) for the project situation

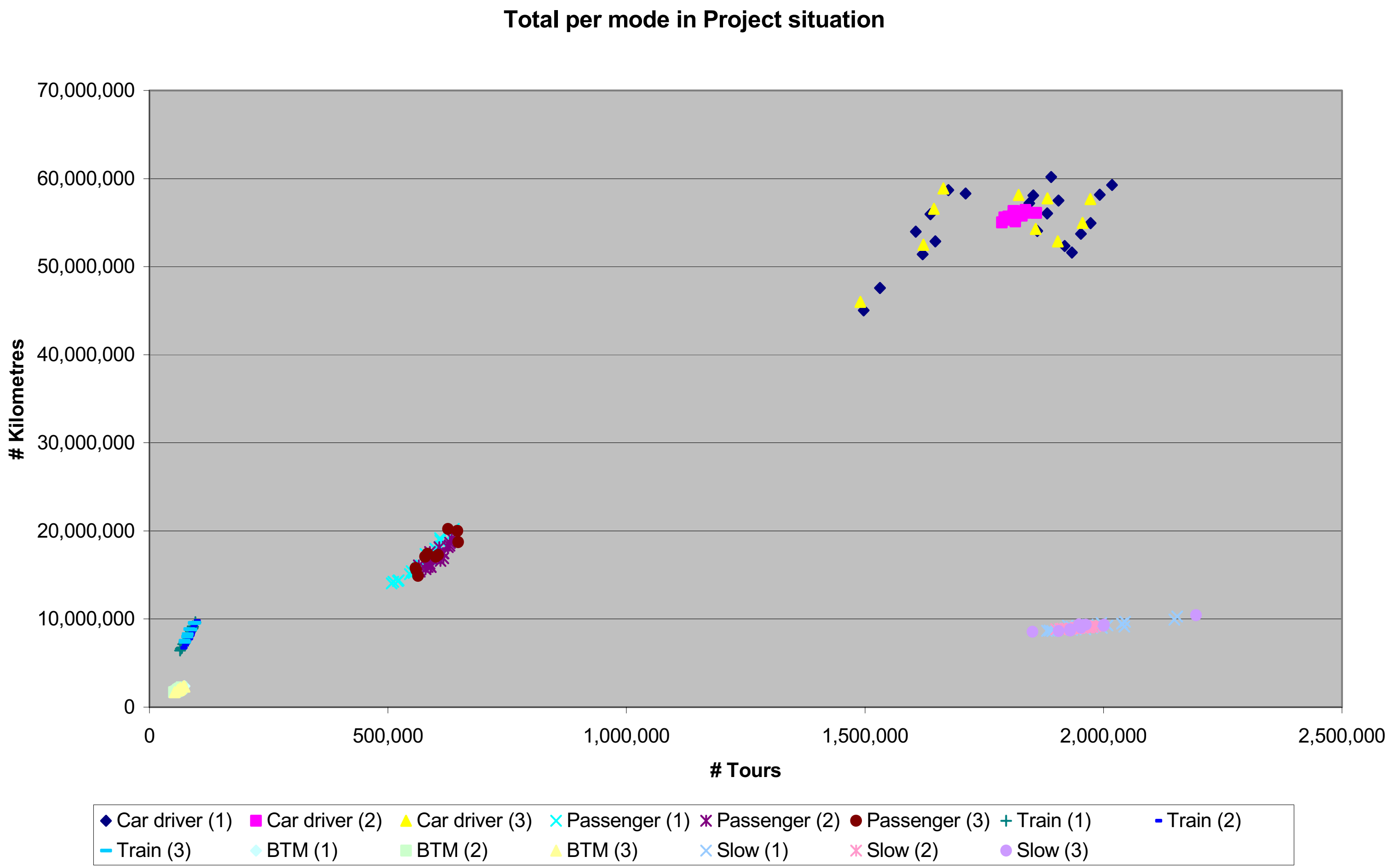


Figure 13. Number of tours by kilometers (RSES) for car driver for the Reference Situation

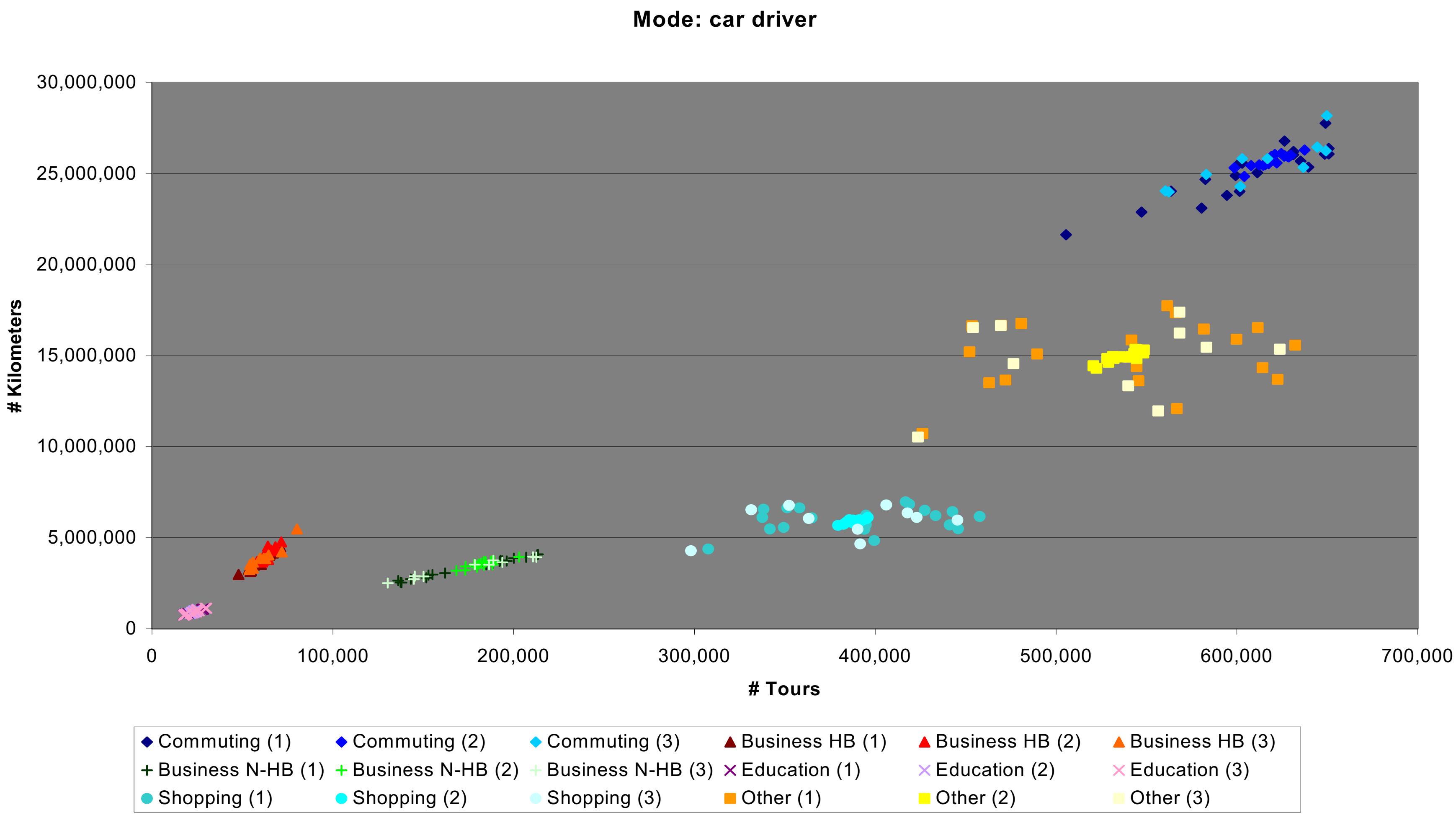


Table 18. Standard deviation for input uncertainty and model uncertainty of vehicle flow for the reference situation.

	Standard deviation for input uncertainty (% of mean)	Standard deviation for model uncertainty (% of mean)	Standard deviation for input and model uncertainty (% of mean)
A2 West Eindhoven N-Z	1.8	0.3	1.7
A2 West Eindhoven Z-N	0.9	0.5	0.7
A58 Bergen op Zoom - Roosendaal	3.7	0.3	3.9

Table 19. Standard deviation for input uncertainty and model uncertainty of hours travelled for the reference situation.

	Standard deviation for input uncertainty (% of mean)	Standard deviation for model uncertainty (% of mean)	Standard deviation for input and model uncertainty (% of mean)
A2 West Eindhoven N-Z	5.9	3.3	5.3
A2 West Eindhoven Z-N	1.2	0.7	1.3
A58 Bergen op Zoom - Roosendaal	8.0	1.2	8.0

Table 20. Standard deviation for input uncertainty and model uncertainty of vehicle hours lost for the reference situation.

	Standard deviation for input uncertainty (% of mean)	Standard deviation for model uncertainty (% of mean)	Standard deviation for input and model uncertainty (% of mean)
A2 West Eindhoven N-Z	6.1	3.7	5.4
A2 West Eindhoven Z-N	28.3	47.9	28.7
A58 Bergen op Zoom - Roosendaal	50.1	11.0	48.2

The variation in hours travelled (Table 19) is clearly larger than for the vehicle flows (and more similar to the percentages found for the LMS). The variation in vehicle hours lost (Table 20) is much larger than the variation in vehicle flows or hours travelled. For the LMS we also found that the predictions for hours lost were more unstable than all other predictions studied.

Project situation

Results for vehicle flows, hours travelled and vehicle hours lost can be found in Appendix 4. These outcomes for the 'old' routes are similar to those for the Reference Situation. In the LMS application, the variation in the vehicle flows on the new link was relatively high, but in this NRM application it is within the range of the variation for the 'old' routes: 2.0-2.6% (depending on the direction) for input uncertainty, 0.2-0.4% for model uncertainty and 2.1-2.7% for combined uncertainty. In the project situation, the vehicle flows on the new eastern ringroad are nearly equal to those of the western ringroad (A2) of Eindhoven: 44,000-45,000 vehicles per day per direction. Vehicle flows and hours travelled are reduced on the A2 west of Eindhoven, especially in the direction north-south, in all model runs.

At the same time, Q-hours (vehicle hours lost) are reduced on the A2 west of Eindhoven in all runs. For the A2 in the direction north to south it is reduced on average by a factor of more than 3. The variation in the Q-hours for the A2 from north to south in the project situation is not very large: 7.9% for input uncertainty, 4.6% for model uncertainty and 9.2% for the combined uncertainty. For the A2 direction south to north the congestion in terms of vehicle hours lost is reduced to 0 in all model runs after the opening of the eastern ringroad (no variation whatsoever). For the 'control' link, congestion is almost the same in the project situation as in the Reference Situation.

Differences between the project situation and the reference 2020

The standard deviations for the differences in the vehicle flows between the project and the Reference situation are 13.3%. 8.5% and 11.2% (input, model and combined uncertainty respectively) for the A2 north-south. For the direction south-north, the absolute differences are smaller and the variation is larger: 33.8%, 22.6% and 55.8% respectively. For the direction north-south the variation for hours travelled and vehicle hours lost is also not very large, unlike for the opposite direction (again due to the small absolute differences between the situation with and without the project). But unlike the results for the LMS, the relative variation in the difference in the number of hours travelled and in Q-hours does not exceed the relative variation in the vehicle flow difference. For the control link, that is not substantially affected by the new project, the differences between the situation with and without the project are indeed very small, but the relative variation is large.

5.5 Conclusions

100 model runs were carried out with the New Regional Model (NRM) Noord-Brabant to quantify the effect of uncertainty in the inputs and of uncertainty in the model coefficients of the tour frequency and mode-destination models. For the uncertainty due to model inputs we found a standard deviation for total car tours in the study area of 9% of the mean. For total car kilometres this was 7% of the mean. These values are rather similar (just below) to what was found for the LMS. As in the LMS, the uncertainty due to model uncertainty is much smaller: for total car tours we obtained a standard deviation of 1.0% of the mean and for car kilometres of 0.7% of the mean. The combined model and input uncertainty is 9.2% for total car tours and 6.9% for total car kilometres in the study area.

The standard deviations in tours and kilometres for other modes are between 2% and 13% of their means for input uncertainty and between 2% and 11% for combined input and model uncertainty. Again, the input errors are generally much larger than the model errors. This also holds for the different travel purposes separately.

Half of the model runs were for the Reference Situation 2020, the other half for the situation with a specific road project (Eindhoven eastern ringroad). The conclusions on uncertainty for each of those two situations were very similar. Also the results for tours with and without congestion feedback were quite similar. The congestion feedback reduces the variation in total car kilometres in the study area somewhat.

A number of links of the road network were studied in more detail. The standard deviations for input uncertainty in vehicle flows on those links are between 1% and 4% (clearly smaller than in the LMS). For model uncertainty we found standard deviations up to 0.5% of the mean, and for combined uncertainty between 1% and 4% of the mean values. The error margins for the number of hours travelled are somewhat larger: between 1% and 8% for input uncertainty and between 1% and 3% for model uncertainty. The variation in the number of hours lost due to congestion (Q-hours) can be considerably larger (up to 50%), especially for links where congestion is low (small absolute number of Q-hours).

For the differences in the link flows between the situation with and without the road project the standard deviation for input uncertainty is between 13% and 34% for the links most affected by the project. For these links, model uncertainty varies between 9% and 23% and combined uncertainty between 11% and 56%. For a link not substantially affected by the project, the relative standard deviation of the difference with and without the project can be even larger, but this concerns very limited absolute numbers of vehicles. The variations in the differences in the number of hours travelled and Q-hours here do not exceed those for the vehicle flows.

With regards to the evaluation of the project (the Eindhoven eastern ringroad in this example): the link flows on the new link are predicted with a very small level of uncertainty: the link flows can be up to 5.5% higher or lower than in the most likely case. The impact of this project on travel times (reference compared to project) on competing links however has a considerably larger uncertainty margin.

CHAPTER 6 Conclusions

This report presents the outcomes of all phases of the project ‘Uncertainty in traffic forecasts’ that RAND Europe undertook for the Transport Research Centre of the Dutch Ministry of Transport, Public Works and Water Management, namely:

- A literature review for public projects and public-private partnership (PPP) projects;
- Development of a method to quantify the uncertainty in traffic forecasts for the Dutch National Model system LMS and the New Regional Models NRM;
- Outcomes from a large number (100) of model runs with the LMS to derive uncertainty margins around the mean traffic forecasts;
- Outcomes from a large number (100) of model runs with the NRM for the Dutch province of Noord-Brabant to derive uncertainty margins around the mean traffic forecasts.

Review of the literature

We found that the literature on quantifying uncertainty in traffic forecasts is fairly limited. We distinguished between input uncertainty (e.g. on the future incomes and car ownership levels) and model uncertainty (including specification error and error due to using parameter estimates instead of the true values).

For quantifying the amount of **input uncertainty** all contributions that we found in the literature use some form of repeated model simulation (sensitivity testing). Usually statistical distributions are postulated for the input variables and then random draws are made from these distributions. This generates input values that are used subsequently in model runs. The uncertainty is calculated from the variance over all the runs for the different input values. Most studies apply univariate distributions for the input variables (ignoring correlation between inputs).

Several methods have been found in the litarature for quantifying **model uncertainty** in transport forecasts. A few studies used analytic expressions for the variance of the endogenous variable that results from using parameter estimates for the influence of the exogenous variables. For complicated models, these expressions become very cumbersome. The Jackknife and Bootstrap method can be used to obtain proper t-ratio’s or standard errors for the model coefficients in situations with specification error (such as repeated measurements in panel and SP data). These proper standard errors of the parameters can either be used in the analytic calculation of the standard error of the model outcomes, or be used as information on the statistical distributions from which values can be drawn for model simulation runs. Again, it is important to take account of the correlations (between the parameter estimates).

Development of a method for LMS and NRM

In our analysis of uncertainty in traffic forecasts from the Dutch national model system (LMS) and the regional model for (NRM) Noord-Brabant, we used existing time series as the key source of information on means, standard deviations and correlations of input variables, and applied these to get multivariate distributions for the model input variables, to account for correlation between the input variables.

Analytic methods to quantify the model uncertainty were considered and the analytic expressions were worked out, but the evaluation of these expressions would take too much computer time. For quantifying the model errors we used the Bootstrap method to correct for specification error and Monte Carlo simulation for the uncertainty due to estimation, for the tour frequency and mode-destination choice models in the LMS and NRM.

Outcomes for LMS and NRM

Both the input variables and the model coefficients of the LMS and NRM tour frequency and mode-destination models were varied. The resulting standard deviation for uncertainty due to input error for total car tours in the LMS is 11% of the mean, and for total car kilometres it is 8% of the mean. For the NRM application these relative standard deviations are slightly lower: 9% and 7% for car tours and car kilometres in the study area. The model uncertainty is much smaller: the standard deviation is 0.7% of the mean for both car tours and kilometres in the LMS and 1% for car tours and 0.7% for car kilometres in the NRM. The standard deviation for both sources of uncertainty together is 12% of the mean for car tours and 8% for car kilometres in the LMS and 9% and 7% for the NRM.

For other modes the standard deviations for tours and kilometres are between 2% and 15% of the mean for input uncertainty and between 2% and 16% for combined uncertainty. The 95% confidence intervals in Figure 14 (for the LMS) were calculated as 1.96 times the standard deviation. These are the uncertainty margins for the combined variance. The margins are smallest for car driver and the slow modes and largest for train and bus/tram/metro (BTM).

In Figure 15 we can see again that the model errors (the second bar for each mode) again are much smaller than the input errors (the first bar for each mode). The third bar for each mode is for the combined error.

Figure 16 and 17 give the same information for the NRM.

Half of the model runs were for the Reference Situation 2020, the other half for the situation with a specific road project (extension of the A16 near Rotterdam for the LMS application, the Eindhoven eastern ring road for NRM). The conclusions on uncertainty for each of those two situations were very similar. Also the results for tours with and without congestion feedback were quite similar. The congestion feedback reduces the variation in total car kilometres in the study area somewhat.

At the level of selected links of the road network, the standard deviations of the link vehicle flows are between 4% and 9% for input uncertainty, and around 1% for model uncertainty in the LMS. For the NRM the variation in the link flows is smaller: between 1% and 4% for inputs and for combined uncertainty and up to 0.5% for model uncertainty. This could be a reflection of the finer network and zoning system in the NRM. For the number of hours travelled, the standard deviations are somewhat larger. Q-hours (number of hours lost due to congestion) can have a much larger uncertainty, especially when the absolute numbers of Q-hours are low.

Figure 14. 95% confidence interval for kilometres by mode in the LMS study area (the mean per mode is made equal to 100)

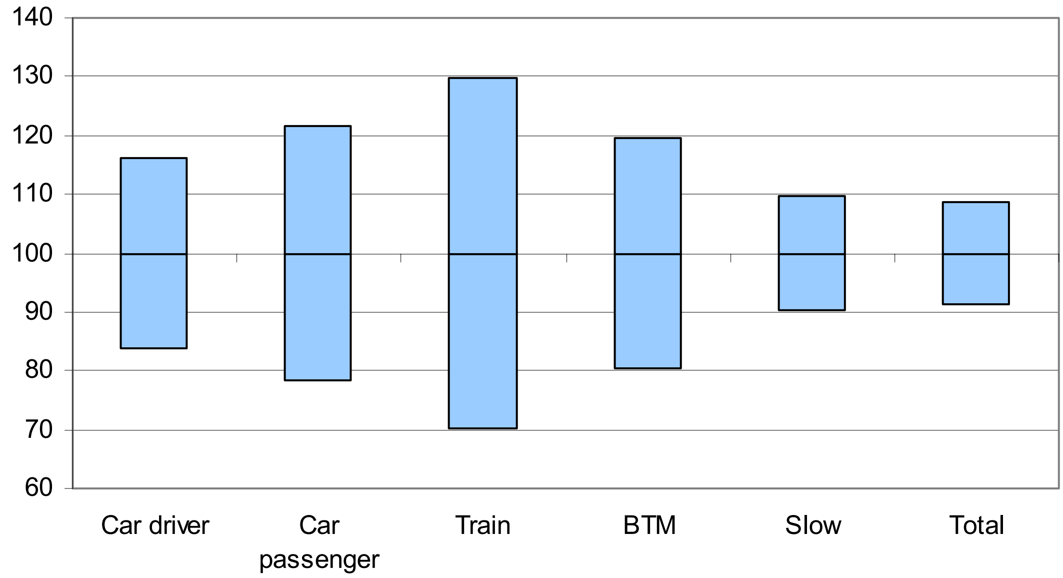


Figure 15. 95% confidence interval for kilometres by mode in the LMS study area (mean value=100): 1 = input error; 2 = model error; 3 = combined error.

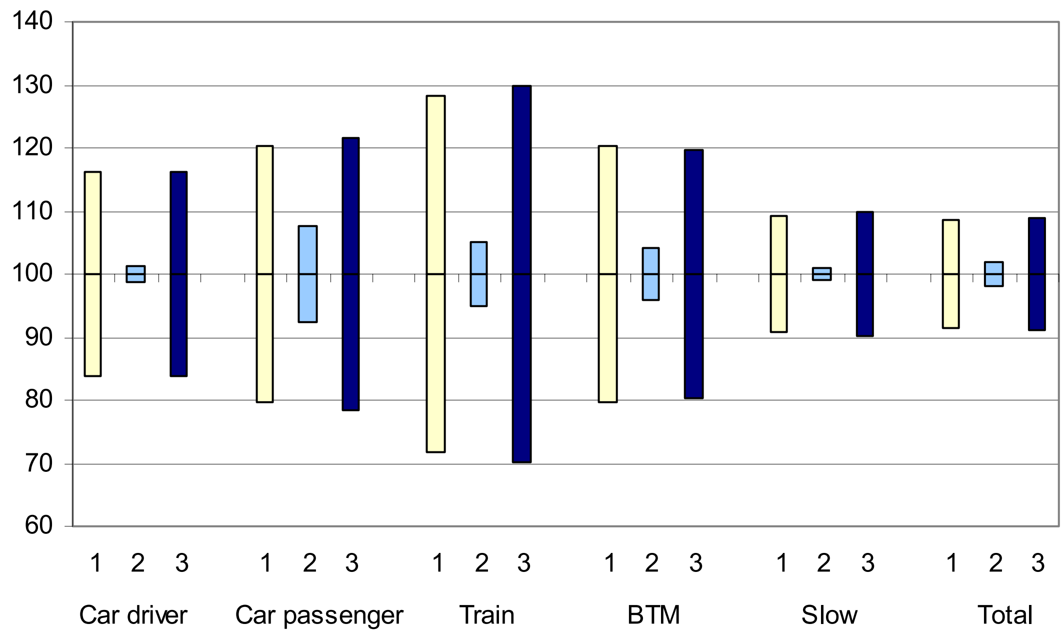


Figure 16. 95% confidence interval for kilometres by mode in the NRM study area (the mean per mode is made equal to 100)

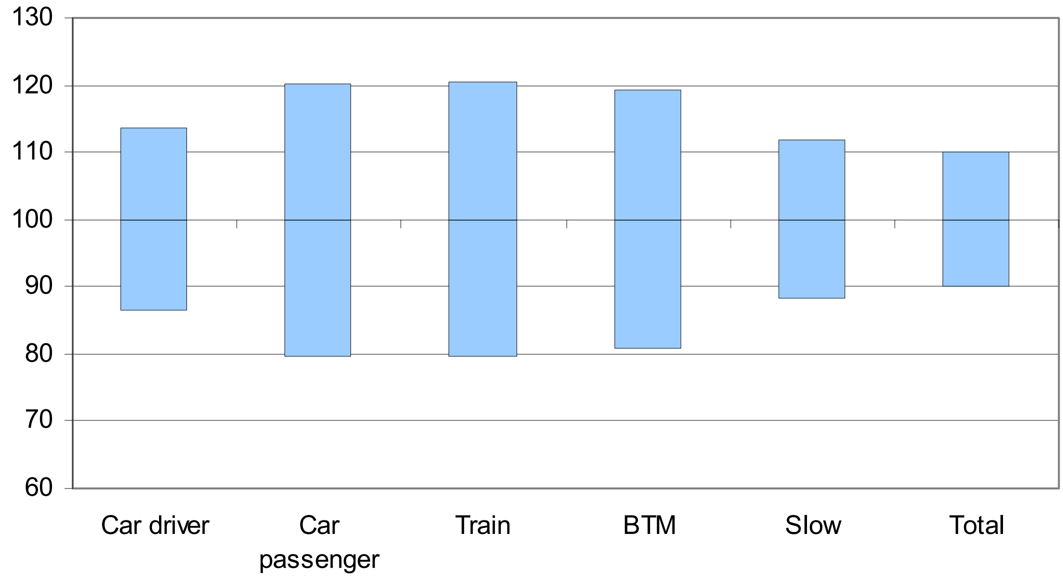
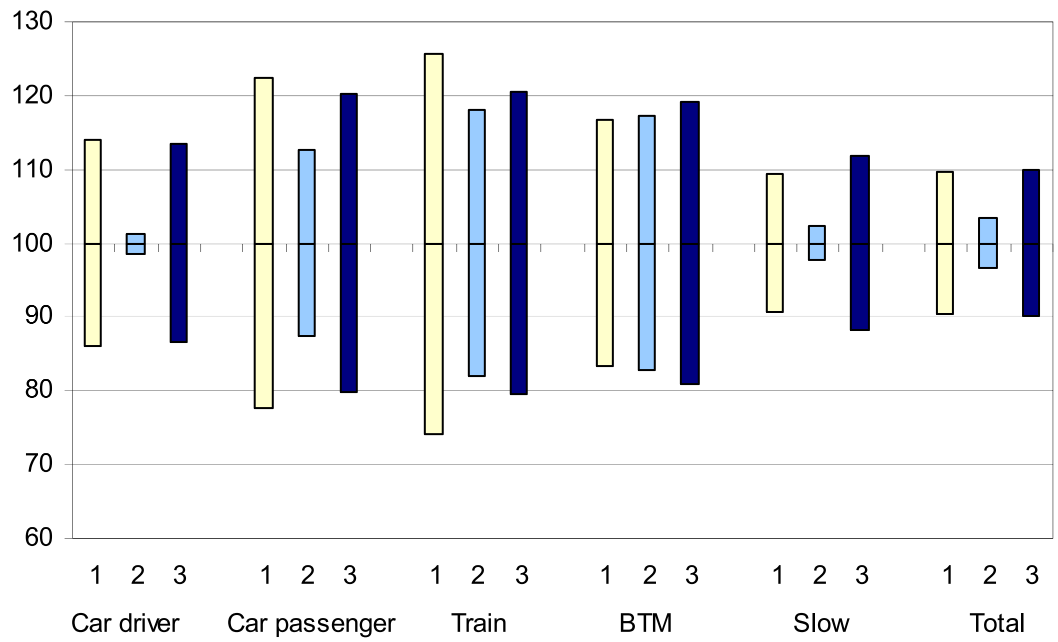


Figure 17. 95% confidence interval for kilometres by mode in the NRM study area (mean value=100): 1 = input error; 2 = model error; 3 = combined error.



The standard deviations for the differences in link flows for links competing with the new road between the situation with and without the road project are 8-12% for input uncertainty, 5-6% for model uncertainty and 7-13% for combined uncertainty in the LMS. For the NRM the variation in these differences in flows is larger: up to 56% (but the larger percentages refer to rather small absolute differences). For the LMS the Q-hour differences are very uncertain, and the difference in hours travelled are in between. For the NRM, the variation in the differences in hours travelled and Q-hours does not exceed the variation in the link flow differences.

Summarising the main outcomes we find substantial, but not very large, uncertainty margins for the total number of tours and kilometres (by mode) in the study area of the LMS and NRM and for the vehicle flows on selected links. The uncertainty margins for differences between a project and a reference situation are not much larger, unless these differences are of a small magnitude. In many cases, there is greater variation in Q-hours than in hours travelled. The contribution of input uncertainty (e.g. in future incomes, car ownership levels) to these errors is generally much larger than that of model uncertainty (e.g. coefficients estimated with some error margin).

These outcomes for uncertainty in traffic forecasts include variation in most of the input variables for the LMS and NRM travel frequency and mode-destination choice models, as well as the error in these models. Sources of variation that were not included are:

- Uncertainty in the base matrices, that are combined with model outcomes for a base year and a future year to obtain forecasts for the future year[7].
- Errors in the licence holding and car ownership models (note however that errors in the total number of cars were included in the input variation).
- Errors in the assignment and time-of-day procedures. These models are used in the LMS and NRM runs carried out (for different demand forecasts from the tour frequency and mode-destination models) but without varying their parameters.
- Uncertainty due to a different distribution over zones. In our simulations we applied the same proportional change for some variable in each zone.
- Uncertainty about the distribution of workers over part-time and full-time workers.
- Because in our method for quantifying uncertainty we relied on the **long-run equilibrium** models LMS and NRM, we were not able to present the **time path** of the uncertainty estimates, but only final 2020 outcomes. Nevertheless, especially for PPP projects, the returns in the first years and the uncertainty attached to these are often very important. This would require **dynamic** models.

The distribution over zones can to some degree be incorporated in scenario studies, where different zonal distributions can be postulated. Scenario studies however do not include probabilities for the variables and future states that they describe and can therefore not be used to calculate uncertainty margins. Our study overlaps to some degree with a scenario approach in that both methods try to include correlations between attributes that characterise the future state. We went beyond scenarios by using a specific probabilistic approach so that we could produce quantitative uncertainty estimates. On the other hand a scenario approach could complement the approach used here, because it offers a way to include varying assumptions on the zonal distribution (e.g. of incomes). Conversely, the probabilistic simulation approach using information from past time series on input variables (including correlations) could also be used in the generation of scenarios, by selecting a limited number of settings for the input variables from the simulations (e.g. one in the middle, one where drivers of demand for travel take on low values and one where the drivers take high values).

[7] The LMS base matrices were estimated on multiple data sources using formal maximum likelihood methods. This means that standard deviations for model uncertainty should be available and could be used in simulation methods to include uncertainty from the base matrices.

The method for quantifying uncertainty that was developed in this project can be used in the assessment of proposed transport projects where the LMS or NRM are used to provide the traffic demand changes. But since the method is very computer-intensive (requiring 100 model runs: 20 for input error, 20 for model error and 10 for combined error, all in the reference, and the same for the project situation; a smaller number of runs would no longer be acceptable), this will only be feasible for the evaluation of major transport projects. For other projects, the quantitative outcomes for the applications presented in this report can provide guidance.

Appendix 1. Uncertainty in policy models

This appendix summarizes and characterizes the various types of uncertainty that are encountered in policy modelling. It is based primarily on a recent paper by Walker, et al. (2003), which distinguished three dimensions of uncertainty:

1. the location of uncertainty – where the uncertainty manifests itself within the model complex;
2. the level of uncertainty – where the uncertainty manifests itself along the spectrum between deterministic knowledge and total ignorance;
3. the nature of uncertainty – whether the uncertainty is due to the imperfection of our knowledge or is due to the inherent variability of the phenomena being described.

The types of uncertainty within each of the dimensions are described in detail below.

The location of uncertainty (identified by the logic of the model formulation)

One of the steps of the policy analysis process involves, for each alternative policy being considered, estimating the consequences that are likely to follow if that policy were to be implemented, where the consequences are measured in terms of the specified outcomes of interest. This step usually involves using a model or set of models representing the system and is usually performed for each scenario. Among the inputs to the model are the changes to the system represented by the policy variables and the scenario variables; among the outputs are the outcomes of interest. The role of the system model in policy analysis can, therefore, be represented as shown in Fig.1.

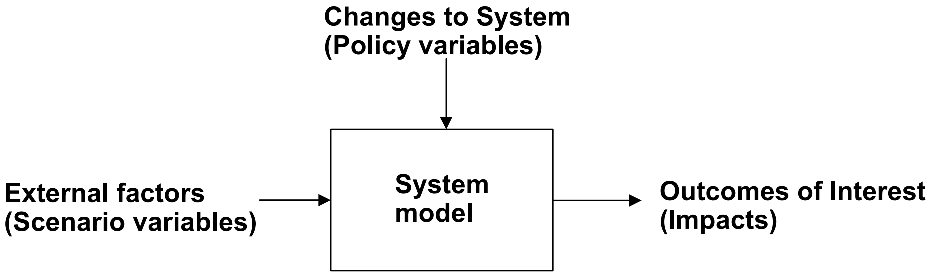


Fig. 1 – The role of the system model in policy analysis

Walker, et al. (2003) have defined uncertainty as being *any deviation from the unachievable ideal of completely deterministic knowledge of the relevant system*. They discriminate among three dimensions of uncertainty: location, level, and nature. In terms of the simple depiction of Fig. 1, one can identify three primary locations of uncertainty related to the outcomes of interest produced by a system model (i.e. *output uncertainty*): 1) uncertainty about the model inputs (i.e. *input uncertainty*), 2) uncertainty about the model structure, the equations and the underlying assumptions (i.e. *model uncertainty*), and 3) uncertainty about parameter values (i.e. *parameter uncertainty*).[8] These three locations of uncertainty are shown in Fig. 2 and are discussed in more detail below.

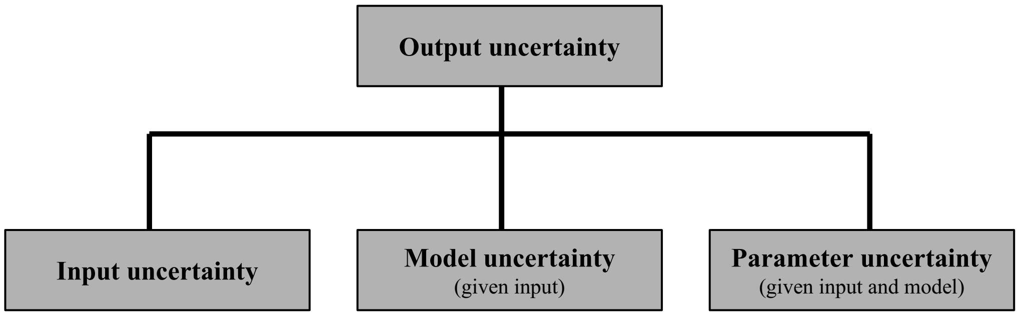


Fig. 2 –Typology for the location of uncertainty

Input uncertainty

Input is associated primarily with data that describe the reference (base case) system and the external driving forces that have an influence on the system and its performance. The "input" location, therefore, includes two sub-categories:

1. Uncertainty about the external driving forces that produce changes within the system (the relevant scenario variables and policy variables) and the magnitude of the forces (the values of the scenario and policy variables). The external forces driving system change (FDSCs) that are not under the control of the policymakers are of particular importance

[8] In the main text, parameter uncertainty and model uncertainty were merged into one category, also called 'model uncertainty'.

to policy analyses, especially if they affect the outcomes of interest. Not only is there great uncertainty in the FDSCs and their magnitudes, there is also great uncertainty in the system response to these forces. This is one of the factors that may lead to significant model structure uncertainty.

2. Uncertainty about the system data that 'drive' the model and typically quantify relevant features of the reference system and its behaviour (e.g. land-use maps, data on infrastructure (roads, houses)). Uncertainty about system data is generated by a lack of knowledge of the properties (including both the deterministic and the stochastic properties) of the underlying system and deficiencies in the description of the variability that can be an inherent feature of some of the phenomena under observation. Uncertainties due to variability are discussed in the 'nature' dimension below.

Model uncertainty

There are two major categories of uncertainty within this location of uncertainty: (1) model structure uncertainty, and (2) model technical uncertainty.

Model structure uncertainty arises from a lack of sufficient understanding of the system (past, present, or future) that is the subject of the policy analysis, including the behaviour of the system and the interrelationships among its elements. Uncertainty about the structure of the system that we are trying to model implies that any one of many model formulations might be a plausible representation of the system, or that none of the proposed system models is an adequate representation of the real system. We may be uncertain about the current behaviour of a system, the future evolution of the system, or both. Model structure uncertainty involves uncertainty associated with the relationships between inputs and variables, among variables, and between variables and output, and pertains to the system boundary, functional forms, definitions of variables and parameters, equations, assumptions and mathematical algorithms.

Model technical uncertainty is the uncertainty generated by software or hardware errors, i.e. hidden flaws in the technical equipment. Software errors arise from bugs in software, design errors in algorithms and typing errors in model source code. Hardware errors arise from bugs, such as the bug in the early version of the Pentium processor, which gave rise to numerical error in a broad range of floating-point calculations performed on the processor (van der Sluijs, 1997).

Parameter uncertainty

Parameters are constants in the model, supposedly invariant within the chosen context and scenario. There are the following types of parameters:

- *Exact parameters,* which are universal constants, such as the mathematical constants π and e.
- *Fixed parameters,* which are parameters that are so well determined by previous investigations that they can be considered exact, such as the acceleration of gravity (g) at a particular location in earth.

- *A priori chosen parameters,* which are parameters that may be difficult to identify by calibration and are chosen to be fixed to a certain value that is considered invariant. However, the values of such parameters are associated with uncertainty that must be estimated on the basis of *a priori* experience.
- *Calibrated parameters*, which are parameters that are essentially unknown from previous investigations or that cannot be transferred from previous investigations due to lack of similarity of circumstances. They must be determined by calibration, which is performed by comparison of model outcomes for historical data series regarding both input and outcome. The parameters are generally chosen to minimise the difference between model outcomes and measured data on the same outcomes.

There is a relationship between model structure uncertainty and calibrated parameter uncertainty. A simple model with few parameters that does not simulate reality well may be calibrated with data obtained for both input and output under well-known conditions. In this case, model structure uncertainty will most likely dominate the result. In the case of a more complicated model with many parameters, the parameters may be manipulated to fit the calibration data beautifully, but the result may be dominated by parameter uncertainty. This would happen if the calibration data did not contain sufficient information to allow for the calibration of some parameters with an adequate degree of certainty. This could be revealed by attempting to validate the model using a different set of data. There is in principle an optimum combination of model complexity and number of parameters as a function of the data available for calibration and the information contained in the data set used for calibration. Increased model complexity with an increased number of parameters to be calibrated may in fact increase the uncertainty of the model outcomes for a given set of calibration data. This has been described in detail (see Harremoës and Madsen, 1999). The calibration data must contain variations of information fit to deal with all parameters chosen for calibration. Otherwise the parameter estimates become very uncertain and the model outcomes become uncertain accordingly. Finally, even when the parameters are well calibrated, a residual uncertainty will often remain, and is usually treated as a parameter in itself.

Calibrated parameter uncertainties result from an inability to assess exactly the parametric values from test or calibration data due to limited numbers of observations and the statistical imprecision attendant thereto. These include data uncertainties deriving from measurement errors, inconsistency of data, data handling and transcription errors, and poor representativeness of sampling schemes due to time and space limitations. In the physical sciences, this type of uncertainty is often referred to as statistical uncertainty (or, sometimes, simply "uncertainty").

The most obvious example of statistical uncertainty is the *measurement uncertainty* associated with all data. Measurement uncertainty stems from the fact that measurements can practically never precisely represent the "true" value of that which is being measured. Measurement uncertainty in data can be due to *sampling error*, or *inaccuracy* or *imprecision* in the measurements. S*ampling error* is the error associated with the degree to which the sample is representative. The location, the time and the circumstances at which the sample has been taken may not be completely representative of those of the "true" value. *Inaccuracy* is the deviation from the "true" value; i.e., it refers to how close a measured value is to the value considered "true". *Imprecision* reflects variation of measurements around a mean value, which may or may not be the "true" value because of sampling error or inaccuracy. This is in fact a measure of the reproducibility of the result. These terms belong to a well-established vocabulary that can be found in most textbooks on physical and chemical experimentation. A good primer on measurement uncertainty is (Kimothi, 2002).

"Statistical uncertainty" may also relate to uncertainty in measuring the probabilities in a stochastic model (see section on variability below).

Model outcome uncertainty

This is the accumulated uncertainty caused by the uncertainties in all of the above locations that are propagated through the model and are reflected in the resulting estimates of the outcomes of interest. One of the best treatments of model outcome uncertainty is given in (Morgan and Henrion, 1990, Chapter 8: "The propagation and analysis of uncertainty"). Model outcome uncertainty is sometimes called *prediction error,* since it is the discrepancy between the true value of an outcome and the model's predicted value. If the true values are known (which is rare, even for scientific models), a formal validation exercise can be carried out to compare the true and predicted values in order to establish the prediction error. However, practically all policy analysis models are used to extrapolate beyond known situations to estimate outcomes for situations that do not yet exist. For example, the model may be used to explore how a policy would perform in the future or in several different futures. In this case, in order for the model to be useful in practice, it is necessary to (1) build the credibility of the model with its users and with consumers of its results (see, for example, (Bankes, 1993)), and (2) describe the uncertainty in the model outcomes using a typology of uncertainties (e.g., that presented in (Walker, et al., 2003))

Figure 3 summarizes the information about the location of uncertainty in system modeling that was presented above.

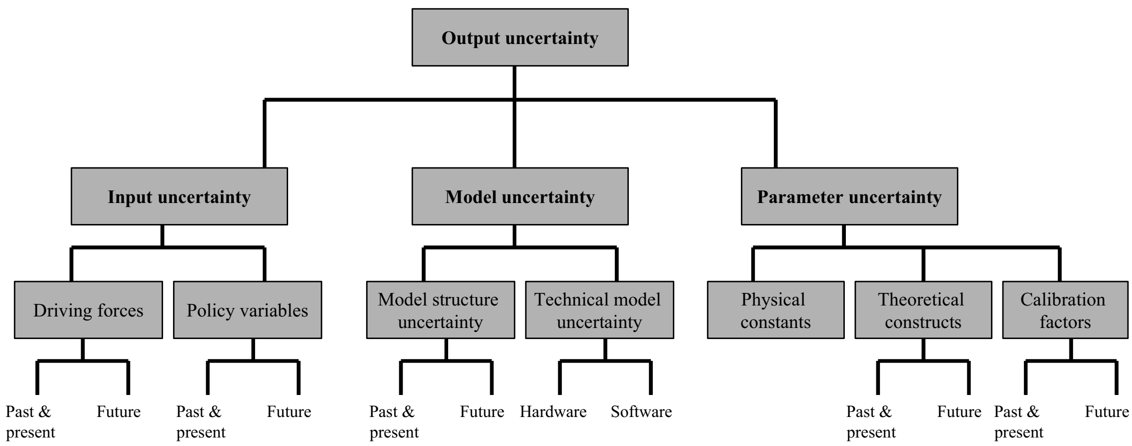


Fig. 3 – More detailed typology of location of uncertainty in modelling exercise

Levels of uncertainty: A Progression from "Know" to "No-Know"

Contrary to the common perception, an entire spectrum of different levels of knowledge exists, ranging from the unachievable ideal of complete deterministic understanding at one end of the scale to total ignorance at the other. In many cases, decisions must be taken when there is not only a lack of certainty about the future situation or about the outcomes from policy changes, but also when some of the possible changes themselves remain unknown. Here, decisionmaking is faced with the continual prospect of surprise. It is in this grey area between the well known and what is not known that the degree of uncertainty and ignorance ought to affect the approach to decisionmaking. The ultimate goal of decisionmaking in the face of uncertainty should be to reduce the undesired impacts from surprises, rather than hoping or expecting to eliminate them (Dewar, 2002). Many different approaches are used in practice to cope with uncertainty. It is useful to try to match the approach to the level of uncertainty. For example, Schlesinger (1966) distinguishes between Captain Cook's tour planning for circumnavigating the globe and Lewis and Clark's tour planning for exploring the previously unexplored western United States. In Cook's case, the future was sufficiently certain that one could chart a straight course years in advance. By contrast, Lewis and Clark's planning "acknowledges that many alternative course of action and forks in the road will appear, but their precise character and timing cannot be anticipated." Thus, very uncertain situations call for robust plans (which will succeed in a variety of situations) (Lempert and Schlesinger, 2000) or adaptive plans (which can be easily modified to fit the situations encountered) (Walker, Cave, and Rahman, 2001). For example, in the case of applying the precautionary principle, the level of uncertainty and ignorance should be accounted for by deciding on an appropriate level of proof as the basis for decisions to act or not act, if there is potential for large-scale and/or irreversible harm from an activity or a chemical (EEA, 2001).

To distinguish between the various levels of uncertainty, Walker et al. (2003) employed the following terminology: determinism, statistical uncertainty, scenario uncertainty, recognised ignorance and total ignorance. This is illustrated in Figure 4.

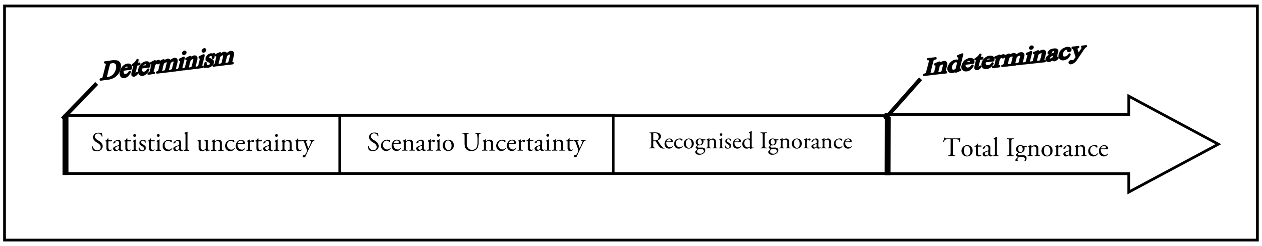


Fig. 4 – The progressive transition between determinism and total ignorance

Determinism is the ideal situation in which we know everything precisely. It is not attainable, but acts as a limiting characteristic at one end of the spectrum.

Statistical uncertainty is any uncertainty that can be described adequately in statistical terms. Statistical uncertainty can apply to any location in the model, even to model structure uncertainties, as long as the deviation from the true value can be characterised statistically.

Statistical uncertainty is what is usually referred to as "uncertainty" in the natural sciences. An exclusive focus on statistical uncertainty, however, implicitly assumes that the functional relationships in the given model are reasonably good descriptions of the phenomena being simulated, and the data used to calibrate the model are representative of circumstances to which the model will be applied. If this is not the case, deeper forms of uncertainty supersede statistical uncertainty, and statistical uncertainty should not be accorded as much attention as other levels of uncertainty in the uncertainty analysis.

The most obvious example of statistical uncertainty is the *measurement uncertainty* associated with all data. Measurement uncertainty stems from the fact that measurements can practically never precisely represent the "true" value of that which is being measured. Measurement uncertainty in data can be due to *sampling error*, or *inaccuracy* or *imprecision* in the measurements.

"Statistical uncertainty" may also relate to uncertainty in measuring the probabilities in a stochastic model (see section on variability below).

Scenario uncertainty. The use of scenarios is one approach used in policy analysis to deal with uncertainty related to the external environment of a system (usually its future environment) and its effects on the system (see, for example, RAND Europe, 1997 and van der Heijden, 1993). A scenario is a plausible description of how the system and/or its driving forces may develop in the future. To be plausible, it should be based on a coherent and internally consistent set of assumptions about key relationships and driving forces (e.g., technology changes, prices). Scenarios do not forecast what will happen in the future; rather they indicate what might happen (i.e., they are plausible futures). Because the use of scenarios implies making assumptions that in most cases are not verifiable, the use of scenarios is associated with uncertainty at a level beyond statistical uncertainty.

Contrary to statistical uncertainty, where the functional relationships are well described and a statistical expression of the uncertainty present can be formulated, scenario uncertainty implies that there is a range of possible outcomes, but the mechanisms leading to these outcomes are not well understood and it is, therefore, not possible to formulate the probability of any one particular outcome occurring. There is a demarcation in the transition from statistical uncertainty to scenario uncertainty at the point where a change occurs from a consistent continuum of outcomes expressed stochastically to a range of discrete possibilities, where choices must be made with respect to the options to analyse without allocation of likelihood.

Scenario uncertainty can manifest itself in various ways – for example, (a) as a range in the outcomes of an analysis due to different underlying assumptions, (b) as uncertainty about which changes and developments (e.g. in driving forces or in system characteristics) are relevant for the outcomes of interest, or (c) as uncertainty about the levels of these relevant changes.

Recognised ignorance is fundamental uncertainty about the mechanisms and functional relationships being studied. We know neither the functional relationships nor the statistical properties and the scientific basis for developing scenarios is weak.

Uncertainty due to ignorance can further be divided into *reducible ignorance* and *irreducible* *ignorance*. Reducible ignorance may be resolved by conducting further research, which implies that it might be possible to somehow achieve a better understanding. Irreducible ignorance applies when neither research nor development can provide sufficient knowledge about the essential relationships. Irreducible ignorance is also called *indeterminacy*.

Total ignorance is the other extreme from determinism on the scale of uncertainty, which implies a deep level of uncertainty, to the extent that we do not even know that we do not know. In Figure 4, the continuing arrow at this end of the scale is used to indicate that we have no way of knowing the full extent of our ignorance.

The nature of uncertainty: Inherent variability or lack of knowledge?

An important feature of the nature of uncertainty is the distinction between two extremes:

- **Epistemic uncertainty**: The uncertainty due to the imperfection of our knowledge, which may be reduced by more research and empirical efforts.
- **Variability uncertainty**: The uncertainty due to inherent variability, which is especially applicable in human and natural systems and concerning social, economic, and technological developments.

Assessing the nature of uncertainty may help to understand how specific uncertainties can be addressed. In the case of epistemic uncertainty, additional research may improve the quality of our knowledge and thereby improve the quality of the output. However, in the case of variability uncertainty, additional research may not yield an improvement in the quality of the output.

Although the terminology used may differ, the above distinction in the nature of uncertainty is well recognised in the literature about uncertainty. For example, the terms *epistemic* or *epistemological* uncertainty (from the Greek word for "knowledge") have been used to refer to imperfection of knowledge, while the term *aleatory* uncertainty (from the Latin, meaning "gambler" or "die caster"), derived from physical science, has been used to describe uncertainty due to variability. An overview of terms used to characterise the nature of uncertainty is given in (Baecher and Christian, 2001). They stipulate that it is not always easy to clearly distinguish between these categories of uncertainty; it often remains a matter of convenience and judgement linked up to features of the problem under study as well as to the current state of knowledge or ignorance.

Epistemic Uncertainty. This form of uncertainty is related to many aspects of modelling and policy analysis – e.g., limited and inaccurate data, measurement error, incomplete knowledge, limited understanding, imperfect models, subjective judgement, ambiguities, etc. With their NUSAP method, Funtowicz and Ravetz (1990) have introduced the concept of pedigree to systematically assess the imperfection in the knowledge base, thereby providing an indication of the degree to which uncertainty may be reducible. Pedigree conveys an evaluative account of the production process of information, and indicates different aspects of the underpinning of the numbers and scientific status of the knowledge used. Assessment of pedigree involves qualitative expert judgement. It should be noted that pedigree and degree of reducibility of uncertainty do not necessarily correspond to each other in a one-to-one fashion: increasing the pedigree by more research may either reduce or increase uncertainty. The latter can be the case if, for instance, unforeseen complexities are revealed by the research. Examples of pedigree analysis can be found in (Van der Sluijs et al., 2002) and on the website www.nusap.net.

Related to the NUSAP method are methods being developed to rate the strength of scientific evidence that are grouped under the heading of "evidence-based practice" (see, for example, (Research Triangle Institute, 2002)). These methods, which are primarily used in the health care field, are designed to protect against the use of study results in individual and policy-level health care decisions that contain selection, measurement, and confounding biases.

Variability Uncertainty. Many empirical quantities (measurable properties of the real-world systems being modelled) vary over space or time in a manner that is beyond control, simply due to the nature of the phenomena involved. *Variability uncertainty* is defined here as the inherent uncertainty or randomness induced by variation associated with external input data, input functions, parameters, and certain model structures.

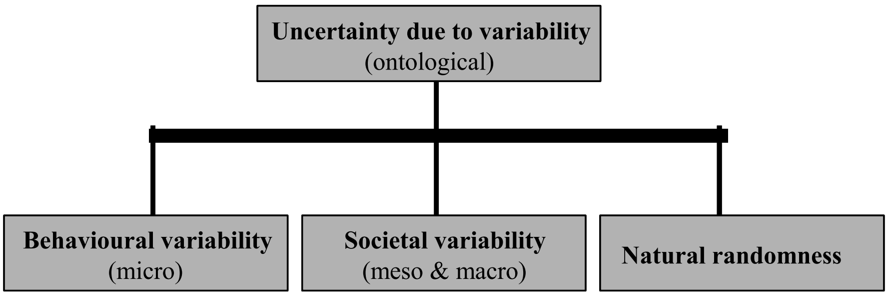


Fig. 5 – Detailed typology of sources of variability uncertainty

Different *sources* of variability uncertainty can be distinguished. As shown in Fig. 5,[9] van Asselt divides these into three categories:

- Inherent randomness of nature: the chaotic and unpredictable nature of natural processes -- see also (Morgan and Henrion, 1990);
- Human behaviour (behavioural variability): 'non-rational' behaviour, discrepancies between what people say and what they actually do (cognitive dissonance), or deviations of 'standard' behavioural patterns (micro-level behaviour);
- Social, economic, and cultural dynamics (societal variability): the chaotic and unpredictable nature of societal processes (macro-level behaviour). The need to consider societal and institutional processes as a major contributor to uncertainty due to variability can be inferred from various papers of Funtowicz, Ravetz, and de Marchi (see, for example (de Marchi, et al. 1993; de Marchi 1995)).

Another source of variability uncertainty is:

- Technological surprise: New developments or breakthroughs in technology or unexpected consequences ('side-effects') of technologies.

These sources may contribute to variability uncertainty, but it may be difficult to identify precisely what is reducible through investigations and research, and what is irreducible because it

[9] See (Van Asselt, 2000) or (Van Asselt and Rotmans, 2002).

is an inherent property of the phenomena of concern. However, it is important to make an assessment, because the information may be essential to the political process.

Models may use frequency distributions to represent variability uncertainty in case the property falls into the level of statistical uncertainty. From (Morgan and Henrion, 1990): "It is possible to have a high degree of certainty about a frequency distribution. For example, it is not hard to imagine obtaining the statistics on the weights of all newborns in Washington, D.C. during 2000 and compiling a precise frequency distribution for the weight of newborn infants in Washington, D.C. during 2000. On the other hand, one may be quite uncertain about a frequency distribution, for example, the frequency distribution for newborn infants in Washington, D.C. during 2020". Uncertainty about a frequency distribution may be represented by probability distributions about its various parameters, such as its mean, standard deviation, or median.

A common mistake is failure to distinguish between the uncertainty inherent in sampling from a known frequency distribution (variability uncertainty), and the uncertainty that arises from incomplete scientific or technical knowledge (epistemic uncertainty). For example, in throwing a fair coin, one knows that the outcome will be heads ½ the time, but one cannot predict what specific value the next throw will have (variability uncertainty). In case that the coin is not fair, there will also be epistemic uncertainty, concerning the frequency of the heads.

Similarly, input functions can exhibit variability that can be described as a mathematical relationship with an associated uncertainty. Such functions may be considered part of the model structure or separate as an external input function. An example is seasonal variation, which can be described functionally (van Asselt, 2000) or the variation in time and space of extreme rainfall, giving rise to flooding (Mikkelsen et al., 1999). The location of this form of variability is in either the model structure or in input data. Input data can exhibit variability with an associated uncertainty. As with all locations of uncertainty, the uncertainty associated with variability of input data or model structure can fall into all four levels of uncertainty: statistical uncertainty, scenario uncertainty, recognised ignorance, or total ignorance. If the model is used for extrapolation (e.g. projection into the future), the uncertainty associated with variability is also due to the application of the model to circumstances different from those associated with the experience upon which the model and data were developed.

Trading off Aleatory and Epistemic Uncertainty (from (Baecher and Christian, 2000))

In some cases, dividing uncertainty between aleatory and epistemic components is an active choice by the modeller and not an innate property of nature. A simple example from water management demonstrates this balancing. Consider the variation of soil properties (e.g., soil strength) along the length of a dike. Say that the strengths increase gradually as a function of location. In making a limiting equilibrium calculation of dike stability and the risk of dike failure, one needs to assess an average strength and then add to it some measure of variation about the average. In the simplest case, one could use a fixed spatial average of soil strength (trend order = 0) and add to it a relatively large variance about the average (relatively large epistemic uncertainty). Alternately, one could fit a linear spatial trend (trend order = 1) to soil strength and add to it a reduced variance about the trend. Indeed, as the order of the trend approaches the number of data points, the added variance about the trend would tend to zero (relatively large aleatory uncertainty). On the other hand, the statistical confidence with which one can estimate the trend parameters becomes ever smaller – that is, the statistical uncertainty in the trend parameters becomes every larger – as the order of the trend curve increases. As the order of the curve approaches the number of data points, the number of remaining degrees of freedom approaches zero, and the parameter uncertainty increases without limit. (Conceptually, there may be some optimum point in the middle.)

Trading off Aleatory and Epistemic Uncertainty (from (National Research Council, 2000))

Although the distinction between natural variability and knowledge uncertainty is both convenient and important, it is at the same time hypothetical. The division of uncertainty into a component related to natural variability and a component related to knowledge uncertainty is attributable to the model developed by the analyst. Consider flood frequency. In the future – at least in principle – the sophistication of atmospheric models might improve sufficiently such that flood time series could be modelled and forecast with great accuracy. All the uncertainty currently ascribed to natural variation might become knowledge uncertainty in the modelling, and thus reflect incomplete knowledge rather than randomness. Modelling assumptions may cause "natural randomness" to become knowledge uncertainties, and vice versa.

Analysts should be clear about which variables they treat as natural variability, which they treat as knowledge uncertainty, and why and how they make this distinction. Differences in the effects of these sources of uncertainty on risk calculations can be large. For example, variations in stream flow, treated as natural variability, average out in a calculation from one year to the next (high flows in one year balance against low flows in another). In contrast, uncertainty in the mean annual flow parameter, treated as knowledge uncertainty, introduces a systematic effect into a calculation. If the mean flow is overestimated in one year, it is overestimated in every year of the calculation.

Appendix 2. Summaries of literature on uncertainty in traffic forecasts

Measuring the impact of uncertainty in travel demand modelling with a demographic approach, J. Armoogum, paper presented at the European Transport Conference, 2003.

Objective of the study described in this paper

The objective of this paper is to discuss three sources of uncertainty, when predicting travel demand, in an age-cohort model:

1. calibration errors
2. the impact of behaviour of the future generations
3. the impact of errors for demographic projections

Here, the age-cohort approach can be treated as a model of analysis of variance, where a reference generation is set and other cohorts are measured against the difference from the reference cohort. The model explains the number of trips and the number of kilometres travelled from individual characteristics like age, gender, motorization and residential zone. The model was calibrated based on 4 years of census data.

Method used to calculate uncertainty of model forecasts

Calibration errors are measured using the Jackknife technique. The Jackknife method corrects for all kinds of specification error (correlation between observations of the same respondent, but also for instance hetero-skedasticity). It belongs to the group of re-sampling methods, which also includes the Bootstrap method. The basic idea of the re-sampling methods is that of taking samples within the sample of observations available, and calculate statistics for each re-sample, and from those also average statistics for the original sample. The Jackknife method re-samples from the original sample by omitting one or a small number of observations each time. In Cirillo et al (1996), the Jackknife method is recommended as a solution to the repeated measurements problem.

To measure the uncertainty of the impact of behaviour of future generations and the impact of errors for demographic projections three scenario's were built and simulated in the model.

Type of uncertainty studied

The type of uncertainty in this study deals with uncertainty within the model (calibration errors), as well as uncertainty within the input variables (behavioural uncertainty and demographic uncertainty).

Variables for which uncertainty is studied

This study focuses on the prediction of the number of trips and the number of kilometres travelled.

How is uncertainty expressed?

The uncertainty of the calibration errors is expressed by the variance of the variables, i.e. the number of trips and the number of kilometres travelled. The deviation from a reference scenario (in percentages) is used to express the uncertainty of the input variables.

What is the order of magnitude of the uncertainty around the forecast?

The impacts of uncertainty of calibration of the model were tested using the Jackknife technique. Here, four subdatasets are created by eliminating 1 year of census data each time. The variance and confidence intervals can be calculated of the projections of the four estimations. For the year 2000 the variance of number of trips is 0.37 (average 3.48 trips) and for 2030 0.92 (average 3.45 trips). The variance rises as the projection horizon augments. The same conclusion can be drawn for the number of kilometres travelled; the variance is 0.5 (average 16.7 km) for 2000 and 1.1 (average 18.8 km) for 2030. Because the model predicts the number of kilometres travelled better, the confidence interval is narrower.

The change in behaviour of future generations is measured by running the model for three scenarios: behaviour is stable (no change between new and old generations), behaviour is based upon the last three generations, and behaviour is based on the last two generations. The trip frequency is 3.73 when it is based on the last three generations and 3.93 based on the last two generations compared to stable behaviour in 2030 For distance travelled this is 19.8 km resp. 19.0 km. Again, distance travelled performs better than trips frequency. The impact due to the change of behaviour remains within the confidence intervals estimated earlier.

The third analysis focuses on the impact of errors in demographic projections. Four scenarios with different migration quotients (probability to move between type of residence: city, inner suburbs, outer suburbs) were set up. For 2030, the number of trips were underestimated by 3% for a scenario where the migration quotient was lowered by 0.001 for each age group, and overestimated by 3% when the migration quotient was increased 0.001 or set to 0. Practically the same results were found for travelled kilometres, -4% respectively 3%.

Confidence region for Several Parameters Simultaneously, M.E. Ben-Akiva & S.R. Lerman, Discrete Choice Analysis, Chapter 6, page 162-165, The MIT Press, 1985.

Objective of the study described in this paper

This is not a paper, but a paragraph in Chapter 6 from the book ‘Discrete Choice Analysis’. The aim of this paragraph is a theoretical discussion how to setup a confidence region for more than one variable.

In a discrete choice (transport) model coefficients can be evaluated using confidence intervals. These intervals are normally set up for individual parameters. When coefficients are estimated simultaneously, it would be more useful to study the confidence region that is spanned by the number of estimated coefficients, provided that the variables are not highly correlated.

Method used to calculate uncertainty of model forecasts

An asymptotic confidence region (interval) for one coefficient, where 1- ? is the confidence interval, can be defined as:

$$\Pr\left[-t_{\alpha/2} \le \left(\hat{\beta} - \beta\right)' \frac{1}{Var_{\beta}} \left(\hat{\beta} - \beta\right) \le t_{\alpha/2}\right] = 1 - \alpha$$

For two or more parameters a similar formula can be set up (under the same conditions) where ? now represents a vector of coefficients and ? is the corresponding covariance matrix:

$$\Pr\left[-t_{\alpha/2} \le \left(\hat{\beta} - \beta\right)' \frac{1}{\sum_{\hat{\beta}}} \left(\hat{\beta} - \beta\right) \le t_{\alpha/2}\right] = 1 - \alpha$$

For two coefficients the confidence region will have the form of an ellipse. For k-order confidence regions it will be a k-dimensional ellipse.

Type of uncertainty studied

The type of uncertainty studied relates to uncertainty within the model, especially the uncertainty of several parameters simultaneously.

Variables for which uncertainty is studied

In the chapter of Ben-Akiva and Lerman an example is given within a transport context for transport costs and transport time. The confidence region (ellipse) lies almost fully in the negative quadrant, meaning that both the transport cost coefficient and the transport time coefficient are, according to expectation, negative.

How is uncertainty expressed?

The uncertainty is expressed in confidence regions.

What is the order of magnitude of the uncertainty around the forecast?

No forecasts were carried out in this paragraph.

Quantifying uncertainties in the SAMPERS long distance forecasting model system, M. Beser Hugosson, paper presented at the World Conference on Transport Research, 2004.

Objective of the study described in this paper

The objective of the paper is to quantify the size of the standard errors in the forecasts (of the Swedish long distance model within the national passenger transport model SAMPERS), accruing from uncertainties in the sampling process.

Method used to calculate uncertainty of model forecasts

To calculate the error that is caused by the fact that the model parameters are estimated on a sample of the population only, the bootstrap method is used. This means that a large number (more than 800) of bootstrap samples is drawn from the original estimation data (with replacement). For every bootstrap sample, the mode-destination model is re-estimated using Alogit. This gives a new vector of parameter estimates for each bootstrap sample. Then, for this bootstrap sample, the demand by mode is computed at these parameters and assigned to the networks using EMME/2. Both for the demand by mode and for specific link flows, 95% confidence intervals around the mean are then calculated on the basis of the outcomes for all the bootstrap samples together. The frequency models are not included in this exercise, and therefore the uncertainty values obtained might be somewhat underestimated. On the other hand, a congestion feedback from assignment to demand is not used, which, when used, could have dampened the fluctuations.

Type of uncertainty studied

The author distinguishes between uncertainty in the forecasts due to scenario variables (income, demography, etc) and uncertainty due to the estimated model. The latter consists of a component related to model specification and uncertainty due to the fact that the model has been estimated on a sample. In this paper, only uncertainty due to the estimation of parameters on a sample of the population in studied.

Variables for which uncertainty is studied

The uncertainty is studied for total demand by mode, car demand on specific OD relations, specific link flows, specific train lines and values of time.

How is uncertainty expressed?

Three different ways to calculate the 95% confidence interval around the mean are used:

- Based on the standard error found and assuming a normal distribution (then the 95% confidence interval is $x \pm 1.96\sigma$, with x as the mean and σ as the standard deviation from the bootstrapping).
- The pivot method using the empirical distribution.
- Directly on percentiles of the empirical distribution.

The distribution of the forecasts turned out to be quite close to the normal, so the first (simple) approach provides a good approximation.

What is the order of magnitude of the uncertainty around the forecast?

On the basis of the uncertainty due to using parameter estimates, the following confidence intervals are found.The 95% confidence interval for total demand by mode varies between ±8.5% (car) to ±13.3% (one of the train modes). For OD demand by car it varies between ±6.5% and ±14% and for the link flows between ±8.4% and ±10.8%.

Risk analysis for privately funded transport schemes, A.M. Boyce, paper presented at the European Transport Conference, 1999.

Objective of the study described in this paper

The objective of the paper is to explore the causes of uncertainty in traffic forecasts and their significance.

Method used to calculate uncertainty of model forecasts

Uncertainty in this paper is measured based on the risk analysis approach, which combines data and forecasting uncertainties and produces a central estimate and an overall range of uncertainty.

Sensitivity functions for key parameters are derived based on a standard aggregate traffic model. The key parameters are then varied according to their corresponding distribution. Based on the sensitivity functions and distribution functions, simulation results can be obtained from a spreadsheet varying (a range of) input parameters.

Type of uncertainty studied

In this risk analysis uncertainty due to the input of the model (and the specification of the model itself) is studied. The following input variables are determined as key parameters when modelling traffic:

Base year factors	Forecast year factors
Accuracy of Base Flows	GDP Growth
Model Specification	Car Ownership/GDP Elasticity
Values of Time	Value of Time growth
Model Estimates of Journey Time	Distribution of Growth
Matrix Estimation Techniques	Generated Traffic
Motorway Bonus	Modelling of Increased Congestion
Route Choice Methodology	Strategic Effects

For each key parameter a distribution is chosen. These distributions vary around the expected value of each parameter. The issue of correlation between the key parameters is not studied.

Variables for which uncertainty is studied

The uncertainty is studied for the change in vehicle-kilometres modelled as a result of a privately-financed project.

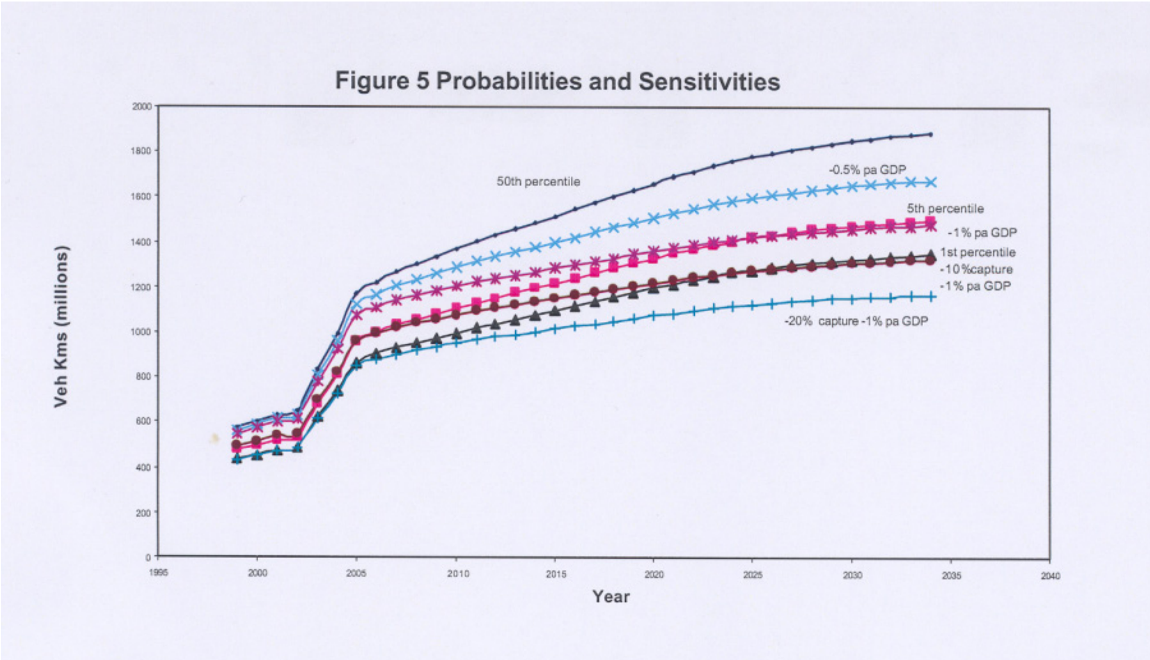
How is uncertainty expressed?

From the distribution of each input variable values are drawn. Within this study a Monte Carlo simulation process (10,000 iterations) is used to calculate the central estimate (or forecast) of the number of vehicle kilometres modelled . The statistics that can be generated are minimum and maximum values, the mean, standard errors and percentiles.

What is the order of magnitude of the uncertainty around the forecast?

The following table shows the difference between the central forecast (from the Monte Carlo simulation) and the expected forecast (i.e. using the mean of each input variable). When the model is non-linear, the expected forecast will theoretically never give a good forecast.

Year	Central forecast (expected model forecast 1996=100)	Expected model forecast (1996=100)	Central forecast/ expected model forecast
1996	99.152	100	99.15
2000	121.88	120.64	101.03
2010	193.04	186.07	103.75
2020	257.81	251.62	102.46



From: Boyce (1999)

Reducing or managing the forecasting risk in privately-financed projects, A.M. Boyce and M.J. Bright, paper presented at the European Transport Conference, 2003

Objective of the study described in this paper

The objective of this paper is to convince investors and lenders that better tools should be used in order to reduce or manage the risk in privately-financed projects. The probability of a certain revenue should be estimated with more care.

Method used to calculate uncertainty of model forecasts

The uncertainty of the model forecasts is calculated by a probability distribution around the median traffic and revenue using formal Risk Analysis, (see Risk Analysis for Privately Funded Transport Schemes). Correlations (existing or expected) between input factors (e.g. higher incomes usually coincide with higher values of time) are taken into account.

An alternative method to Risk Analysis is scenario testing, where different scenarios are set up and fed into the model. The authors state that in scenario-analysis there are no probabilities attached to the scenarios and there can be many implicit correlations between the variables.

Type of uncertainty studied

The uncertainty of each individual parameter is studied and its effect on the potential variability in the forecasts. A distinction is made between factors that influence the base year and factors influencing the forecast year, see the table below. These parameters deal with uncertainty due to the model as well as input for the model.

Base year factors	Forecast year factors
Quality of the base data	GDP
Matrix development	GDP/car ownership elasticities
Model specification	growth in value of time
Estimates of journey time savings	effect of other road schemes
Value of time estimates	induced traffic
Assignment/route choice techniques	

Variables for which uncertainty is studied

The uncertainty is studied for revenue of a privately-financed project, where revenue is closely related to the number of vehicle-kilometres modeled as a result of the project.

How is uncertainty expressed?

Uncertainty is given in percentiles (50th, 5th and 1st percentile). Private funders are especially interested in not losing money with 95-99% certainty.

What is the order of magnitude of the uncertainty around the forecast?

Four scenario's were tested varying GDP (-0.5% GDP, -1.0% GDP) and traffic capture (-10% and –20% capture in combination with –0.5% GDP). Each scenario was evaluated against the base scenario. Only the worst scenario (-20% capture and –1% GDP) fell below the 1st percentile, i.e. indicating a high risk investment.

How good is an estimated logit model? Estimation accuracy analysed by Monte Carlo simulations, Karin Brundell-Freij, paper presented at the European Transport Conference, 1997.

Objective of the study described in this paper

The objective of the study is to discuss the kind of errors related to model imperfections and sample size. The impact of the errors on individual assessment as well as prediction of policy effects are studied here as well.

Method used to calculate uncertainty of model forecasts

First, a base mode choice model was estimated on an existing dataset (n=850) of commuters in Sweden. To calculate the uncertainty a Monte Carlo simulation was set up (500 simulations). For each simulation a sub-sample from the observed dataset was drawn (N=850, 210, 85 and 40) and a ‘synthetic’ choice was derived, where choices were made based upon deterministic utilities varied by a random component following a Gumbel distribution. A model was then estimated using this ‘synthetic’ sample.

Type of uncertainty studied

The type of uncertainty modelled is uncertainty due to the model: specification error and estimation accuracy and over-specification.

Variables for which uncertainty is studied

The model that is studied is a modal split model (car, carpool, train and bus), where cost, time and alternative specific constants are the explanatory variables.

How is uncertainty expressed?

Estimation errors are presented in a standardized way:

$$\frac{(\alpha_i^* - \alpha_i)}{\sqrt{\mathrm{var}(\alpha_i^*)}}$$

where α_i^* is the i’th estimated parameter from the Monte Carlo simulations, and α_i is the base model parameter (on the observed data) that had been used to generate the choice. A confidence interval can be set up for this measure. Any coefficient that falls outside this interval will then be biased.

To evaluate estimation accuracy the variance in the observed simulations is compared to the estimated asymptotic variance (that is commonly used in logit models, but has desirable properties only in large samples) at each sample size. For large enough samples the asymptotic variance should be close to the observed.

Over-specification (defined here as including a variable that does not in reality influence choice) can be measured by the quasi-T-test. Normally, a variable will be included if the ordinary T-test is met, where the test statistic is T-distributed at infinity. For smaller sample size the T-test should be adjusted: the quasi-T-test.

What is the order of magnitude of the uncertainty around the forecast?

For N=850 five of eleven parameters fall outside the confidence interval set up earlier. For smaller samples all parameters are biased.

The difference between observed variance and estimated asymptotic variance is close for N=850, however there is systematic underestimation of the asymptotic variance. For N=85 estimated variance is 10-15% below the observed variance.

In this paper the effective critical region (quasi-T-test) for the cost coefficient has been calculated. The 'standard' t-test (at p=0.05) has limits of –1.96 to 1.96. The critical region for this parameter is –1.72 and 1.58 for N=210 and –1.69 and 1.42 for N=42, indicating that the model might be overspecified if the standard t-test is used.

Sampling, specification and estimation as sources of inaccuracy in complex transport models – some examples analysed by Monte Carlo simulations and Bootstrap, Karin Brundell-Freij, paper presented at European Transport Conference, 2000.

Objective of the study described in this paper

The objective of the study is to discuss inaccuracy and bias when developing a transport model caused by model formulation, sampling and the estimation method, thus encompassing the reliability that is commonly studied by looking at the standard errors of the parameter estimates.

'Inaccuracy' and 'uncertainty' are similar expressions within model estimation, i.e. how reliable are the estimates.

Method used to calculate uncertainty of model forecasts

Two methods are used to calculate uncertainty:

- Monte Carlo simulation: random numbers are generated following a predefined distribution and the function is evaluated for those random numbers. Repeating this process (a large number of times) will lead to a discrete distribution of observed function values. In this application of Monte Carlo simulation, the synthetic choice observations are generated according to the deterministic utilities defined by the base model (that was estimated on observed data for 845 commuters in Sweden), with Gumbel distributed error terms added. The model is then estimated based on these synthetic observations.
- Bootstrap: Repeated subsamples are drawn from the original sample (with replacement: each observation can occur more than once in a subsample). The model is then estimated for each subsample.

Type of uncertainty studied

Three types of uncertainty (inaccuracy) are studied in this paper:

- model formulation as a source of uncertainty
- sampling as a source of uncertainty
- estimation as a source of uncertainty

The input data for the analyses in this paper remain constant.

Variables for which uncertainty is studied

In this study, an indirect parameter, value-of-time, measures the quality of the model (estimations). Value-of-time is often used in behaviourial interpretation and applications, more than the direct parameters are.

How is uncertainty expressed?

The author distinguishes between 32 different model specifications, with different sets of mode-specific and personal variables included in the model. From this, a base model was selected based on predefined criteria (as commonly used: correct signs, proper t-values, etc,) using the full set of observed data. Both Monte Carlo and Bootstrap methods create new (sub-)samples and the procedure for the selection of the preferred model specification was rerun each time (100 repetitions). Uncertainty is expressed in the VOT and its standard error based on the hundred iterations.

After each model selection, the Monte Carlo and Bootstrap methods were re-run with the new model formulation. Uncertainty can then be expressed in different VOT estimates and their corresponding standard errors.

This procedure gives the impacts of uncertainty through the selection of the preferred model specification and within the selected model.

What is the order of magnitude of the uncertainty around the forecast?

As a first indication of uncertainty due to model formulation, the Monte Carlo simulations only selected the originally (on the observed full data) preferred base model specification again 42 out of the 100 runs; for the Bootstrap simulations this was 62 out of the 100.

The following table displays the VOT for in-vehicle and waiting time for males in this study, for the base model, the Monte Carlo simulations, the Monte Carlo simulation for the selected models, the Bootstrap simulations and the Bootstrap simulations for the selected models[10].

	Base model	Monte Carlo (selection)	Monte Carlo (selected)	Bootstrap (selection)	Bootstrap (selected)
VOT in-vehicle	78.96	79.86	123.17	80.62	86.82
s.e.	15.47	1.98	23.06	2.32	2.79
VOT wait	51.71	40.41	40.42	25.69	44.63
s.e.	16.92	1.03	1.80	1.39	1.63

The results obtained from the table indicate a bias-by-selection. When the model specification is changed, the estimates of VOT in-vehicle and VOT –waiting are larger. The analyses in this study showed that substantial bias and increased variability may be introduced by a restrictive search for model specification.

[10] 'Selection' means: model specification similar to the base model specification (estimation on the full observed dataset). 'Selected' means: model specification might differ from the base model, as more or less variables become significant.

Doorstroomroute A4, Ecorys, 2003.

Objective of the study described in this paper

The objective of this report is to present the cost-benefit assessment results of a specific road infrastructure project in The Netherlands (concerning the A4 Motorway and tolling). We included this report in this literature review of uncertainty in traffic forecasts, because it is a good example of the state of practice (or even 'best practice') of transport project evaluation in The Netherlands and the way uncertainty in forecasts is handled for major projects.

Method used to calculate uncertainty of model forecasts

Two different assumptions were used in the cost-benefit analysis for the annual traffic growth rate after 2010: low (1%) and high (2%). These are the two base-cases. Also different calculations have been modeled out (called 'sensitivity analyses' in the Ecorys report), for each of the two base-cases:

- different toll rates
- different road maintenance costs
- different interest rates for discounting the future
- different length of the new toll road.

The toll rates and the length of the toll road affect the traffic intensities and could be called 'policy variants', as have been simulated in many other studies. This is another matter as trying to calculate the margin around the predicted traffic intensities that is due to uncertain exogenous inputs and model errors. Such calculations are missing in this report. What is included in the Ecorys report is the use of a value of time distribution instead of a fixed value by travel purpose, but this too cannot be regarded as calculating an uncertainty margin around the traffic forecasts. For the future values of time, three different assumptions (unchanged, +1% per year, +2% per year) are tested. The report thus contains a substantial number of variants for exogenous variables and behavioural parameters instead of a single 'central' forecast, and as such can be regarded as an example of best practice. Nevertheless this is a pretty standard form of sensitivity analysis (per single variable), not a systematic treatment of uncertainty.

Type of uncertainty studied

In this report sensitivity analyses are presented that focus on the input variables (income, policy variables such as the toll levels) and include only the value of time as behavioural model parameter.

Variables for which uncertainty is studied

The output variable that is studied are the (social) revenues from the project and the benefit-costs ratio.

How is uncertainty expressed?

Sensitivity is expressed in terms of different revenue amounts (without assigning a probability to the likelihood of these outcomes).

What is the order of magnitude of the uncertainty around the forecast?

The total discounted lifetime revenues with low traffic growth after 2010 are 1.2 billion Euros and with high traffic growth after 2010 3.9 billion Euros.

Sensitivity analysis methods for road traffic emission models, Olle Eriksson, paper, 2003.

This paper summarizes the findings of three related papers from the same author:

- A new program for fast emission calculations based on the COPERT III model (2003);
- Basic sensitivity analysis methods for road traffic emission models, applied on the COPERT III model (2003);
- Global sensitivity analysis methods using response surface descriptions for road traffic emission models, applied on the COPERT III model (2003).

In this summary, the findings of the overall paper will be extended with detailed descriptions and results of the three related papers, especially the second and third.

Objective of the study described in this paper

The objective of this paper is to discuss derivative–like sensitivity methods for emissions from road traffic.

Method used to calculate uncertainty of model forecasts

The first method used analyses the sensitivity of the margin means (average emission values) to one explanatory (input) variable at a time (OAT: one at a time). Different values are selected for all input variables called a grid. When more values than normal are selected, this is called a fine grid. For all possible combinations of input variables, emissions are calculated. To analyse the sensitivity of one variable, emissions are averaged over all cells (margin mean) except the variable studied, and the mean values are regressed onto the selected levels of the studied variable using the following formula:

$$E[y_{k=x}] = \beta_0 + \beta_1 x (+\beta_2 x^2)$$

where Y is the average emission level, and X is the corresponding covariate level (explanatory variable), for each value k that the covariate can take, keeping all other influences constant.

Three sensitivity measures are then calculated:

- a straight line, where the sensitivity measure is β_1
- a quadratic line, where the sensitivity measure is β_1 + 2 β_2 *A with A a predetermined representative point, because of the non-linearity
- a straight line with a more refined grid for variable k, where the sensitivity measure is β_1

The second method is to analyze global sensitivity, i.e. the sensitivity of all input variables at once. The Response Surface Method (RSM) is used in this paper. The response function (shape of the surface) is unknown and has to be estimated. The observations vary around the surface by a random component, with expected value 0.

$$E[y] = \gamma_i + \delta_j + \beta_0 + \beta_1 x_1 + \beta_3 x_3 + \beta_4 x_4 + \ldots (+\beta_2 x_2^{\,2})$$

where the x's are the variables to be analysed and γ_i is the factor for legislation class I, and δ_j is the factor for engine capacity class j.

Two sensitivity measures are calculated here:

- Straight plane, where the sensitivities are equal to the slopes
- Quadratic plane with a more refined grid, where the sensitivity is equal to $\beta_1 + 2\beta_2$ *A with A a predetermined representative point, because of the non-linearity

When all x's are uncorrelated, the two methods (OAT and RSM) are theoretically equivalent for the first sensitivity measure.

Type of uncertainty studied

In this paper the objective is to study sensitivity rather than uncertainty. The sensitivity analyses focus on the input variables, not on the model parameters.

Variables for which uncertainty is studied

The study focuses on road traffic emissions from vehicles by legislation class and engine capacity class.

How is uncertainty expressed?

Sensitivity is expressed in estimated coefficients from a regression model. The uncertainty is captured in the distribution of the random component.

What is the order of magnitude of the uncertainty around the forecast?

As there is no uncertainty modeled, there is no indication of magnitude. The five sensitivity measures proposed are compared based on the COPERT III model and the conclusion is that there is not much difference in the outcomes. The recommended method here for sensitivity analysis is the RSM method without quadratic terms as it is the most natural, and computational easiest measure.

The influence of modelling error on the shapes of estimated demand functions, Tony Fowkes, paper presented at the European Transport Conference, 1995.

Objective of the study described in this paper

The objective of the paper is to investigate the influence of modelling error on conclusions drawn regarding traveller benefits and 'willingness to pay'.

Method used to calculate uncertainty of model forecasts

The uncertainty is calculated using simulations, where random errors were drawn from a double exponential (Weibull) distribution. A Stated Preference survey was designed where artificial respondents were given the choice between two modes (one existing, one new mode) while varying costs, in-vehicle time and walk/wait time. A total of 900 simulated responses (9 SP-questions, 100 respondents) were determined. The simulations were repeated 50 times, resulting in 50 sets of 900 responses. The uncertainty was calculated by the average and standard deviations based on 50 estimations using the 50 simulated datasets.

Type of uncertainty studied

The accuracy of the estimated coefficients is studied here, where synthetic utility functions are set up and coefficients are estimated. The uncertainty is measured by comparing the average standard error of coefficients with the standard error of all estimated coefficients. The effect of uncertainty on traveller benefits is analysed as well.

Variables for which uncertainty is studied

A simple two-modes model has been set up for this study, where one is a newly introduced mode. The modal split can be influenced by the uncertainty in the estimated coefficients. Benefits of the introduction of the new mode are estimated as well.

How is uncertainty expressed?

Uncertainty is expressed by the standard deviation of the 50 estimated coefficients and the standard deviation of the standard deviation belonging to the coefficients in these 50 estimations.

The standard deviation of mode benefits reflects the uncertainty in the value of a new mode.

What is the order of magnitude of the uncertainty around the forecast?

The following table gives the standard error based on: (1) the 50 estimates of the cost, wait and in-vehicle time coefficients and (2) the mean standard errors of the 50 estimates of the coefficients.

Coefficient	Standard error 1	Standard error 2
Cost	0.001908	0.001778
Wait	0.017261	0.017612
Time	0.009075	0.011322

A statistical test reveals that there is no significant difference between the two standard errors.

Depending on the cost of the new mode, the benefits can be estimated. With the cost of the new mode at 200, the mode benefit per person is 71 pence per person with a confidence level of 95%.

However, the confidence interval is coincidentally plus or minus 71 pence as well (i.e. twice the standard error of 35.5), or [0;142]. To reduce the bandwidth of the estimated benefits much larger sample sizes are needed than the 900 presented here.

Forecasting public transportation in large cities, A. Garcia Ferrer et al., paper, Dept. Analisis Economica: Economia Cuantitativa, 2003.

Objective of the study described in this paper

In this paper the problem of forecasting the demand for a large number of bus and metro tickets is studied in the Madrid Metropolitan Area. A dynamic transfer function causal model and a dynamic harmonic regression model (DHR) are used to produce forecasts. Forecasts accuracy is measured with traditional root mean squared error (RMSE) and mean absolute error (MAE).

Method used to calculate uncertainty of model forecasts

Uncertainty of accuracy of the model forecasts is measured with RMSE and MAE:

$$RMSE_i(h) = \sqrt{\frac{1}{N}\sum_{t=T}^{T+N} e_{i,t+h}^2}$$

$$MAE_i(h) = \frac{1}{N}\sum_{t=T}^{T+N} \left| e_{i,t+h} \right|$$

where N is the number of forecasts, T is the number of observations used in the estimation and e is the forecast error defined as true value minus forecast produced with model I (causal model or DHR) and h is the forecast horizon (h indicates a certain step size in terms of time t). The initial estimation sample ends in December 1999. 1, 6, 12 and 24 months ahead forecasts are made. Each time a data point was added and new predictions were made until the last sample data point was available (November 2002 for a 1-month forecast, June 2002 for a 6-month forecast, December 2001 for a 12-month forecast and December 2000 for a 24-month forecast).

Type of uncertainty studied

The type of uncertainty that this paper deals with is uncertainty due to the model, not that due to the inputs to the model.

Variables for which uncertainty is studied

The model forecasts the number of tickets and travel cards for a certain forecasting horizon. The accuracy of the predicted number of tickets and travel cards is studied here.

How is uncertainty expressed?

The uncertainty is expressed in RMSE and MAE with values between 0 and 1.

What is the order of magnitude of the uncertainty around the forecast?

The following table gives an example of the RMSE and MAE, where the number of metro tickets is predicted for 1 month, 6 months, 12 months and 24 months.

Single metro ticket			
Causal model		DHR model	
1 month			
RMSE	0.037	RMSE	0.043
MAE	0.031	MAE	0.035
6 months			
RMSE	0.051	RMSE	0.066
MAE	0.046	MAE	0.054
12 months			
RMSE	0.088	RMSE	0.100
MAE	0.078	MAE	0.085
24 months			
RMSE	0.174	RMSE	0.265
MAE	0.156	MAE	0.233

The RMSE measure gives higher values of prediction error than the MAE measure. However, the conclusions based on each of these measures is the same. The causal model outperforms the DHR model or each forecasting horizon. The prediction error increases is the horizon increases, which is to be expected.

Some joint models of car ownership and car use, Gerard de Jong, Ph.D. Thesis, Faculty of Economic Science and Econometrics, University of Amsterdam, 1989.

Objective of the study described in this paper

This study was carried out to develop models that simultaneously determine car ownership and use. In this Ph.D. thesis these models were also used to predict for 2010 and 2020 and for policy simulation. For two models, the so-called 'statistical model' and the 'indirect utility model', predictions of car ownership and use for the unweighted estimation sample were produced.

Method used to calculate uncertainty of model forecasts

The sampling variation in the number of car-owning households in the sample is calculated as:

$$\operatorname{var}\left(\hat{m}\right) = \sum \hat{P}_i\left(1 - \hat{P}_i\right) \tag{1}$$

where m is the expected number of car-owning households in the sample and the P's give the estimated probabilities of car ownership for a household I (which is standard for a binomial distribution).

This variance only reflects the probability characteristics of the model used. Another variance denoted $\operatorname{var}_e(m)$ is the result of using parameter estimates instead if the true values:

$$\operatorname{var}_e\left(\hat{m}\right) = \left[\sum \frac{\partial \hat{P}_i}{\partial \hat{\theta}}\right]\left[\hat{V}\right]\left[\sum \frac{\partial \hat{P}_i}{\partial \hat{\theta}}\right] \tag{2}$$

where the first term between brackets is a vector of length J (the number of parameters) and the second term between brackets is the estimated variance-covariance matrix of the parameters.

Equation (2) was also used to estimate the standard error of annual car kilometrage per household, due to using parameter estimates instead of the true values.

Type of uncertainty studied

The type of uncertainty that this thesis deals with is uncertainty due to the model, not that due to the inputs to the model. Within model error, both sampling error and the complete variance-covariance structure of the model parameters are taken into account.

Variables for which uncertainty is studied

This thesis investigated the uncertainty in the sample prediction for the number of households with a car as well as the sample prediction for the number of kilometers per year per car.

How is uncertainty expressed?

The measures of uncertainty are the standard error of the number of car-owning households in the sample and the standard error in the annual car kilometrage.

What is the order of magnitude of the uncertainty around the forecast?

The sampling standard error for car ownership was only 1% of the predicted car ownership in both models. The standard error due to using parameter estimates instead of true values in both models is 3%. For annual car kilometrage the standard errors due to using parameter estimates are between 4 and 6% of the predicted kilometrage.

Preparatory research into updating values of travel time in The Netherlands, Gerard de Jong, Dick Ettema and Hugh Gunn (Hague Consulting Group) and Francis Cheung and Henk Kleijn (Transport Research Centre), Paper presented at WCTR conference, Antwerp, 1998.

Objective of the study described in this paper

This study was carried out in 1997 to prepare for a new value of time study in passenger transport in The Netherlands to update the values from the 1988 study. The specification of the model and the sample size that is required for the new model were investigated on the basis of the 1988 data. The reason that uncertainty of the value of travel time is studied here stems from the desire to calculate the sample size that is required for the update study to produce values of time that will not differ from the 1988 estimates by more than 10% (only taking into account stochastic factors). The update study was carried out in 1997/1998, using the calculated sample sizes from this preparatory study (except for business travel, where a very large and costly sample would have been required). The resulting differences between the values of time in 1988 and 1997/1998 can safely be interpreted as real changes instead of random differences.

Method used to calculate uncertainty of model forecasts

First the Jackknife method was applied on the 1988 models to correct for specification bias. The t-ratios after applying the Jackknife method could have been used to calculate the variance, but this would lead to very messy equations, since the variance of a ratio (time parameter divided by cost parameter) is a rather complex function of the variances of the parameters and their covariance (an this only by a Taylor series approximation). Furthermore, both the time parameter and the cost parameter were multiplicative functions of various parameters. Instead of the analytical method, Monte Carlo simulation was used, based on the Jackknife variances of the parameter estimates. Also, the correlation structure from the original estimates was used. Together this produced a multivariate normal distribution from which the parameter values were drawn many (1,000) times. For each draw (complete set of parameters), the VOT and its variance were calculated. Then we averaged over all 1,000 draws, to get an average VOT (by purpose and income group; by mode and income group and by purpose and mode) and average standard deviation of the VOT.

Type of uncertainty studied

The type of uncertainty that this paper deals with is uncertainty due to the model, not that due to the inputs to the model. Within model error, both specification error (through use of the Jackknife) and the complete variance-covariance structure of the model parameters are taken into account.

Variables for which uncertainty is studied

This paper investigated the uncertainty in the value of travel time (the marginal utility of travel time divided by the marginal utility of travel costs).

How is uncertainty expressed?

The measure of uncertainty is the standard deviation of the value of time.

What is the order of magnitude of the uncertainty around the forecast?

For commuting, the standard deviation on the 1988 data was between 6 and 11% (depending on the travel mode) of the average value of time. A 95% confidence interval would therefore be not wider than [-22%, +22%]. For business the standard deviation was between 16 and 24% of the average value of time and for other between 4 and 7% of the average value of time. The widest 95% confidence intervals for business and other travel would be [-48%, +48%] and [-14%, +14%].

Some Issues in Traffic Forecasting for Privately Financed Infrastructure, Eric P. Kroes, paper presented at the European Transport Conference, 1996.

Objective of the study described in this paper

The objective of the paper was to explore the specific methodological requirements of privately financed infrastructure with respect to traffic forecasting, including in particular issues like accuracy, uncertainty and time profile of the forecasts.

Method used to calculate uncertainty of model forecasts

The paper does not provide a specific worked-out method, but describes the element that is of particular importance for privately financed infrastructure: uncertainty. The two main sources are: (1) uncertainty in the model procedure, and (2) uncertainty in the model inputs. It describes a pragmatic 5 step procedure based upon simulation to estimate the combined effect of a series of model input variables.

Type of uncertainty studied

The following input variables are mentioned as key parameters when creating traffic forecasts:

- Quality of the input data, including:
    - Existing and future travel behaviour, based upon SP and RP, including VOT;
    - Existing and future transport system performance, including accurate travel times;
    - Existing and future traveling population.
- Quality of the model system, including:
    - Detailed description of mode choice, destination choice, route choice, timing (and season) of travel and frequency of travel;
    - Complete and comprehensive causal model structure;
    - Account for relevant taste variation;
    - Detailed network simulation with toll roads.
- Quality of the application system, including:
    - Speed of computation;
    - Quality of display and editing software (GIS capabilities).

Variables for which uncertainty is studied

The paper is focused on specific link flow volumes, and revenues generated by means of tolls.

How is uncertainty expressed?

The paper contains a description of a general method (that has later been worked out and applied in a case study in Lyon).

First the key input variables influencing the outcome are identified (say 10 variables). Then for each individual variable 3 anchor points are specified: a low estimate, a medium (most likely) estimate and a high estimate. Subjective probabilities of occurrence are attached to each of these points (these are experts' judgements for say the 10^{th}, 50^{th} and 90^{th} percentile for each input variable). Then the available transport model system is applied to provide, for each input variable separately and with all other variables set at their medium level, the expected vehicle flows and revenues for the different levels (univariate simulations). This requires 3x10=30 runs. Then using the results of these univariate simulations, the combined flow and revenue effect is estimated for all possible combinations of the levels (3^{10}=59,049 in total), assuming independence between the variables. Finally the result is plotted in a cumulative graph, taking account of the associated probabilities. This enables an analysis of the statistics of the distribution of the outcomes.

What is the order of magnitude of the uncertainty around the forecast?

The paper does not provide numerical results of an application of the method.

Shaping the Next 100 Years: New Methods for Quantitative, Long-Term Policy Analysis, Lempert, R.J., S.W. Popper, and S.C. Bankes (2003), RAND, MR-1626-RP

The Robust Decisionmaking (RDM) Approach, Lempert, R.J. (2004); presentation given at the RAND Europe Seminar, Leiden, May 10, 2004.

A recent development in the area of scenario methods is the use of *assumption-based planning*, also called *exploratory analysis* or *robust decisionmaking. On May 10 2004 a RAND* Europe Sponsored Seminar was held in Leiden where Robert Lempert presented his Robust Decision Making Approach to a number of invited experts. In this approach fast strategic computer models are being used to simulate very large numbers of *ensembles of scenarios*. In order to direct and evaluate these computer based experiments and to present their results RAND has developed specific software: *Computer Assisted Reasoning (CAR*). CAR is not only used to explore the spectrum ("*landscape"*) of end results, but also and particularly to highlight which assumptions or actions are most crucial (in that they have a large influence on the final results). These insights can then be used to design a more robust policy strategy. Normally the analysis is carried out in an iterative procedure, where each test is followed by a counter-test and the process zooms in continually on the most crucial variables. This approach is particularly useful for taking very complex decisions, in situations with a large amount of uncertainty ("*deep uncertainty"*) and where there are many hedging actions possible to counter unwanted effects. Lempert contrasts his RDM approach with what he calls the more traditional "*predict-then-act*" approach, which he thinks is more appropriate for situations where there is limited uncertainty and limited possibilities for hedging actions.

The RDM approach is clearly related to what is being pursued in the Bandbreedte project, but the focus is different. In fact three important differences can be mentioned:

1. The RDM approach is focused primarily on the development of policy strategies; the bandbreedteproject, on the contrary, is focused primarily on the quantification of the uncertainties surrounding the estimated effects of pre-specified policy measures.
2. In the RDM approach time-dynamics are very important, particularly in relation to the timing of decisionmaking and taking hedging actions; in the bandbreedteproject the uncertainty of the end-result, in the target-year, is much more important.

The RDM approach is based upon the simulation of very large numbers (up to millions) of possible futures, and for obvious reasons the run times of the computer models used has to remain limited; this forces the use of fairly global models, with little detail; in the bandbreedteproject the LMS and NRM are really in a central position, with all the associated detail, and it is obvious that run time is clearly a limiting factor.

For these reasons the use of the RDM approach is not really an option for the bandbreedte project. Of course elements of the methodology may be of use anyway.

An analysis of modelling error, with application to a traffic assignment model with continuously distributed values of time, Leurent, M., paper presented at the ETC, 1996.

Objective of the study described in this paper

The objective of this paper is to describe and analyse the error of a model an to provide detection criteria and corrective treatments. Five types of error are identified in this article as potential sources of error:

- design errors
- formulation errors
- algorithm errors
- estimation errors
- exogenous errors

The focus in this study lies on exogenous errors and the propagation of exogenous errors in a simulation model.

Method used to calculate uncertainty of model forecasts

Exogenous errors influence the estimated coefficient(s) in a model. Uncertainty is calculated by taking standard errors of the estimated coefficient(s) and the standard errors of the input data.

Type of uncertainty studied

The uncertainty studied here is uncertainty due to exogenous errors, i.e. input errors and propagated errors, i.e. exogenous errors propagated through the simulation model.

Variables for which uncertainty is studied

A linear function for travel time per km (reciprocal of speed) is assumed and the coefficients are estimated using standard Ordinary Least Squares (OLS). Travel time (per km) is assumed to depend solely on a constant and the flow on a specific link (in the absence of any queuing effect).

The impact of the propagated uncertainty on the number of cars per day is analysed.

How is uncertainty expressed?

Uncertainty is first expressed as the sum of squared deviations in travel time (per km) from their mean. A measure for the relative uncertainty is expressed by dividing the standard error by the mean:

$$\frac{\sigma_x}{\overline{x}}$$

What is the order of magnitude of the uncertainty around the forecast?

The travel time function is estimated first with only a constant and secondly by also including a slope-coefficient. The average travel time estimated with only a constant is 0.549 with a standard deviation of 0.052. After including the slope coefficient the standard deviation drops to 0.046, indicating less uncertainty.

To evaluate the propagation of errors, three input errors were defined, error in the measurement of journey time, error in the measurement of O-D volumes, and error in the distribution of the Value-of-Time. A simulation model was run predicting the number of cars per day. 1220 cars/day on average passed the link, with a standard deviation of 123 cars, resulting in 10.1% as uncertainty measure. This uncertainty was mainly due to demand volumes (83%) and to a lesser degree journey times (14%) and the distribution of the Value-of-Time (3%).

Uncertainties in Highway Appraisal: the development of systematic sensitivity testing, S. Lowe, D. Morrell and G. Copley, paper presented at the PTRC SAM in Warwick, 1982.

Objective of the study described in this paper

This paper describes a fully-developed systematic sensitivity testing procedure and the application of this procedure to a real highway scheme. Predictions of flows on a new bypass road and two existing links are made based on zonal and socio-economic factors. The main sources of uncertainty are identified and suggestions on how this can be incorporated in standard highway appraisal practice are made.

Method used to calculate uncertainty of model forecasts

Uncertainty is calculated by drawing randomly from the distributions belonging to the appropriate factors. The paper does not say how these distributions were derived. This was repeated ten times to obtain means and standard deviations for the factors. They also use an experimental design for the simulations that can provide the information for establishing the main and interaction effects of 17 factors.

Type of uncertainty studied

The type of uncertainty studied here is mainly due to the input to the model: zonal characteristics, fuel price and GDP growth. Two model coefficients are studied as well, the route choice coefficient in the base year and the route choice elasticity. The link speeds were varied as well.

Variables for which uncertainty is studied

The sensitivity analysis was carried out at a local level, where the impact of a new by-pass route on the traffic flows of three links (including the new by-pass) was studied.

How is uncertainty expressed?

The uncertainty is expressed for each individual factor when a factor was changed to the 10^{th} or 90^{th} percentile of the appropriate distribution by the average change in number of vehicles per day on the links. The standard deviation of the effect is also given.

Given the probability distribution of the individual effects, an overall cumulative distribution of the total link flow can be built.

What is the order of magnitude of the uncertainty around the forecast?

The magnitude of uncertainty depends on the standard deviation of the factors that determine the number of vehicles passing a link each day. From the overall cumulative distribution a probability of 5% can be derived, that the flow on the new by-pass will be less than 14.000 vehicles per day.

The probability of the flow on the existing High Street being less than 16.500 vehicles is 2.5% when only the study area effects are considered. When the random effects (link speed and base year trip matrix), which have high standard deviation, are added this probability rises to 27.5%.

The most important determinants of uncertainty in the link flow predictions were found to be factors at the level of the study area as a whole, especially income growth, fuel prices and the factor for consistency with national forecasts. The influence of the spatial distribution was negligible and the interaction effects were not of great importance either. The key determinants of uncertainty can be accounted for in a simplified procedure that requires eight model runs.

Risk valuation of public investment projects, Dutch Ministry of Finance/Central Planning Bureau, 2003.

Objective of the study described in this report

The objective of this study is to recommend how one should evaluate risk in future public (or public-private) investment projects. This report is more a policy guideline than an analysis of risk in model forecasts. This summary only states the final conclusions and recommendations.

In this report risk is defined as uncertainty about the (financial) outcomes of a project. The distribution around the expected outcome of a project gives an indication of the uncertainty. When a (bell-shaped) normal distribution is assumed, with an expected value of zero, no losses on a project are expected. Because individuals have a general risk aversion, the incorporation of uncertainty in a project will skew the distribution, which might lead to a negative expected value on the outcomes.

Two types of risks are distinguished:

- **macro economic risk**: each project will have a general risk-premium and a project-specific multiplier, which adds a risk-premium to the general risk-premium.
- **diversifiable risk**: costs and benefits are not structurally, but stochastically related with benefits from other sources: risks can be spread. There are, however, two diversifiable risks that can have a negative effect on the outcome of a project:

  Concentrated risks, when (a group of) individuals carry more risks than others. **Asymmetrical risks**, where financial implications are (mostly negatively) skewed, e.g. decreasing marginal value of income.

Each cost-benefit analysis that is carried out before the start of a project should be aware of the implications project risks have on financing. A CBA should contain a paragraph on risk, where the following subjects are addressed:

- macro economic risks
- diversifiable risks
- scenario analyses
- sensitivity analyses

Jackknife Testing – An experimental Approach to Refine Model Calibration and Validation, Research Results Digest, National Cooperative Highway Research Program, 2003.

Objective of the study described in this paper

The objective of the study in this paper is to illustrate the jackknifing procedure for calibration and validation using simulations of rutting performance (road surface degradation) based on measured data from in-service pavement sections at the Minnesota Road Research Project.

Method used to calculate uncertainty of model forecasts

Prediction accuracy of the models was evaluated using four types of measures:

- calibration accuracy
- n-1 Jackknifing
- n-2 Jackknifing
- split-sample jackknifing

The n-1 Jack-knife procedure uses n-1 observations to calibrate the model and the withheld observation is used to predict. This is repeated for the entire dataset. Goodness-of-fit measures can be set up for the errors made in the predictions. The same procedure applies for n-2 Jackknifing, where 2 observations are withheld each time. Split-sample jackknifing divides the sample into two equal samples, one to calibrate and one to predict.

Type of uncertainty studied

The uncertainty studied here deals with the accuracy of the estimated parameters in the model due to repeated measurements in the data collection.

Variables for which uncertainty is studied

Uncertainty is studied for the number of pavement sections in a region, i.e. the number of data collection points.

How is uncertainty expressed?

Uncertainty is expressed in a correlation coefficient and a standard error ratio.

What is the order of magnitude of the uncertainty around the forecast?

Four different goodness-of-fit measures are compared. The n-1 and n-2 Jackknifing procedures indicate that the standard errors are underestimated by the original model estimations. The underestimation increases when the sample size decreases. The split-sample jackknifing is a poor measure of prediction accuracy, except possibly for large samples.

Uncertain socioeconomic projections used in travel demand and emissions models: could plausible errors result in air quality nonconformity?, Caroline J. Rodier and Robert A. Johnston, *Transportation Research Part A*, Volume 36, 613-631, 2002.

Objective of the study described in this paper

The objective of the study is to conduct a sensitivity analysis for the effect of plausible errors in population, employment, fuel price, and income projections on travel demand and emissions as predicted by models for the Sacramento region.

Method used to calculate uncertainty of model forecasts

The error in the annual population growth rates is analysed using the following measures:

Algebraic percentage point error:

$$ALPE_i = (P_i - A_i) * 100_i$$

where P is the projected annual growth rate, A is the actual population annual growth rate and I is the number of counties in the Californian region.

Mean algebraic percentage error:

$$MALPE = \frac{\sum_{i=1}^{n} ALPE_i}{n}$$

Standard deviation of the ALPE

$$S.D.(ALPE_i) = \sqrt{\frac{\sum_{i=1}^{n} (ALPE_i - MALPE)^2}{n-1}}$$

Sensitivity analyses have been carried out for a variety of combinations of socioeconomic factors in population and employment, household income and fuel price.

Type of uncertainty studied

In this study, the focus is on errors (uncertainty) in the socioeconomic projections, i.e. model input.

Variables for which uncertainty is studied

Uncertainty is studied for population attributes. Sensitivity analyses are conducted for percentage change of trips, vehicle-miles-travelled and vehicle-hours-delay (and vehicle emissions).

How is uncertainty expressed?

Uncertainty for the population is expressed in the standard deviation of the algebraic percentage error. The sensitivity analyses give an indication of the volatility of model outputs when model inputs are varied.

What is the order of magnitude of the uncertainty around the forecast?

For a five year interval the ALPE of projected annual growth rates (from the California Department of Finance) and the observed annual growth rates (from census counts) is calculated.

The following table summarizes the MALPE and the standard deviation of the MALPE for the 59 counties in the Sacramento region (corrected for outliers):

Projection year	MALPE	S.D. MALPE
1960	-0.28	1.30
1965	0.95	1.09
1970	-0.75	1.13
1975	-0.42	0.76
1980	0.31	1.01
1985	-0.30	0.88
1990	0.66	0.73
Average (all data)	0.05	1.14

The range of the standard deviation in projected county annual growth rates is 0.73-1.30%, where the average is approximately 1%.

Below are the percentage change results of the sensitivity analyses performed for the metropolitan transportation organization in the Sacramento region for the projection year 2005. Each scenario consists of only one change in modelling input, the reference scenario is no change in any of the socioeconomic inputs. Population and employment are changed simultaneously.

% Change in…	TRIPS	VMT	VHD
Population and Employment			
-2.0	-16.7	-12.0	-39.2
-1.5	-12.8	-9.1	-30.3
-1.0	-8.7	-6.2	-21.4
-0.5	-4.4	-3.1	-11.9
0.0	0.0	0.0	0.0
0.5	4.6	3.2	12.7
1.0	9.4	6.6	29.2
1.5	14.4	10.0	47.7
2.0	19.7	13.6	68.8
Household income			
0.0	0.0	0.0	0.0
10.0	0.0	0.1	0.5
20.0	0.0	0.2	1.1

30.0	0.1	0.3	1.4
32.8	0.1	0.3	1.8
Fuel price			
-7.11	0.0	0.1	0.5
-3.50	0.0	0.0	0.4
0.0	0.0	0.0	0.0
3.50	0.0	-0.1	-0.1
8.78	0.0	-0.1	-0.4

The predictions appear to be rather insensitive to uncertainties in household income and fuel price. Errors in projections of population and employment generate relatively large changes in travel. As is expected VHD (vehicle hours delay) are much more volatile than trips and vehicle miles travelled.

Verifying the Accuracy of Regional Models Used in Transportation and Air Quality Planning, Caroline Rodier, publication of the Mineta Transportation Institute, 2003.

Objective of the study described in this paper

This study describes three simulations in order to test model accuracy, the effects of errors in socioeconomic/land use projections and induced travel. This summary focuses on the first two model forecasts. A historical forecasting case study in the Sacramento, California region is performed using the original version of the Sacramento regional travel demand model.

Method used to calculate uncertainty of model forecasts

Model predictions of 2000 with the original 1991 version of the travel demand model were performed in 2003, called historical forecasting. Observed data in 2000 can than be evaluated against the 'historical' predictions.

The uncertainty (accuracy) of the model forecasts is measured in the following ways:

Model accuracy:

Model Error = (Forecast1_2000-Observed_2000)/Observed_2000 * 100

where Forecast1_2000 is the model prediction based on observed socioeconomic and land use data.

Projection accuracy:

Model & Projection Error = (Forecast2_2000-Observed_2000)/Observed_2000 * 100

where Forecast2_2000 is the model prediction based on 2000 socioeconomic and land use data predicted for 2000 in 1991.

Projection error = Model & Projection error – Model Error

Type of uncertainty studied

The type of uncertainty studied is here is uncertainty within the model as well as uncertainty due to model input.

Variables for which uncertainty is studied

Number of trips by purpose, the modal split and vehicle miles travelled, vehicle hours travelled and vehicle hours delay are the variables for which uncertainty is studied.

How is uncertainty expressed?

Uncertainty is expressed in percentages of under- or overestimation from the model values compared with observed values (model error and projection error).

What is the order of magnitude of the uncertainty around the forecast?

The following table summarizes the model and projection errors, when forecasts are compared with observed values from 2000 by trip generation, mode choice and daily vehicle travel.

In percentages	Model error	Model & prediction error	Prediction error
Trip generation			
Home-Work	0.6	8.0	7.4
Home-Shop	25.3	36.9	11.6
Home-Other	-8.5	-0.8	7.6
Work-Other	-10.5	-4.5	6.0
Other-Other	-19.1	-10.3	8.8
Home-School	-9.1	-1.6	7.5
Total	-6.1	2.0	8.1
Mode choice			
Drive-alone	7.7	6.7	-1.0
Shared-Ride 2	-2.9	-2.7	0.2
Shared-Ride 3+	-17.5	-17.0	0.5
Transit-Walk	-14.8	-5.9	8.8
Transit-Drive	-8.8	26.2	35.1
Walk	35.3	38.8	3.5
Bicycle	13.0	13.4	0.3
Daily Vehicle Travel			
Vehicle Miles Travelled	5.7	11.8	6.1
Vehicle Hours Travelled	4.2	12.8	8.6
Vehicle Hours Delay	17.1	38.4	21.3

For total trip generation model, error is slightly smaller than prediction error. Overall model and prediction error is only 2.0%, because the underestimation in model error is offset by the overestimation in prediction error.

In mode choice, the model error seems larger when the range of model error (-17.5 to 35.3 percent) is compared with prediction error (-1.0 to 35.1 percent).

Daily vehicle travel is overestimated due to model error as well as prediction error. The sum of these errors overestimates VMT by 11.8%, VHT by 12.8% and VHD by 38.4%, which indicates that VHD is much more volatile than VMT (as is expected).

Het voorspellen van betrouwbaarheid van reistijd met een vervoerprognosemodel: SMARA, Schrijver, J.M., Meeuwissen, A.M.H. en Hilbers, H., paper presented at the CVS, 2003.

Objective of the study described in this paper

The objective of the study is to describe a model that calculates bandwidth's for travel times within the Netherlands. The SMARA model(Strategic Model for Analyzing the Reliability of Accessibility) uses Monte Carlo simulations to construct reliability intervals of travel times.

Method used to calculate uncertainty of model forecasts

Draws from a given distribution are taken for each factor that influences the modelled traffic demand and the capacity of the infrastructure (weather, season, events, accident or road works)[11]. Correlation between these factors is not discussed. In this study 400 draws are taken to construct a reliable bandwidth.

Type of uncertainty studied

This paper investigates the uncertainty due to the inputs to the model, both demand driven inputs and inputs from the supply side.

Variables for which uncertainty is studied

The travel times between 250,000 O-D relations are studied in this paper.

How is uncertainty expressed?

The uncertainty will be expressed in bandwidth around the mean travel time, when the SMARA model is finished.

What is the order of magnitude of the uncertainty around the forecast?

No statistics were given at the time the paper was submitted.

[11] SMARA also uses inputs from the Dutch national model system for traffic and transport (LMS).

The propagation of uncertainty through travel demand models: an exploratory analysis, Yong Zhao and Kara Maria Kockelman, paper submitted to *Annals of Regional Science*, 2001.

Objective of the study described in this paper

The study's objective is to compare point estimates under input variation. Monte Carlo simulation and sensitivity analysis are used to investigate error propagation in a traditional four-stage transport model applied to the Dallas-Fort Worth metro region.

Method used to calculate uncertainty of model forecasts

A traditional four-stage transport model has been set up to forecast link flows in the DFW-region. The standard deviation of the coefficients in each stage has been fixed to 0.30 of the mean of the coefficients and a lognormal distribution of the coefficients is assumed. One hundred different sets of input and parameter values were set up resulting in one hundred model forecasts.

Type of uncertainty studied

In this study, uncertainty due to model input and model parameters is analysed. The exogenous inputs are varied, while the possible erroneous data that is processed within the four-stage model is varied as a result of the different values for the model coefficients.

Variables for which uncertainty is studied

The effect of uncertainty on the link flows in the DFW-network is studied here, in particular two selected links.

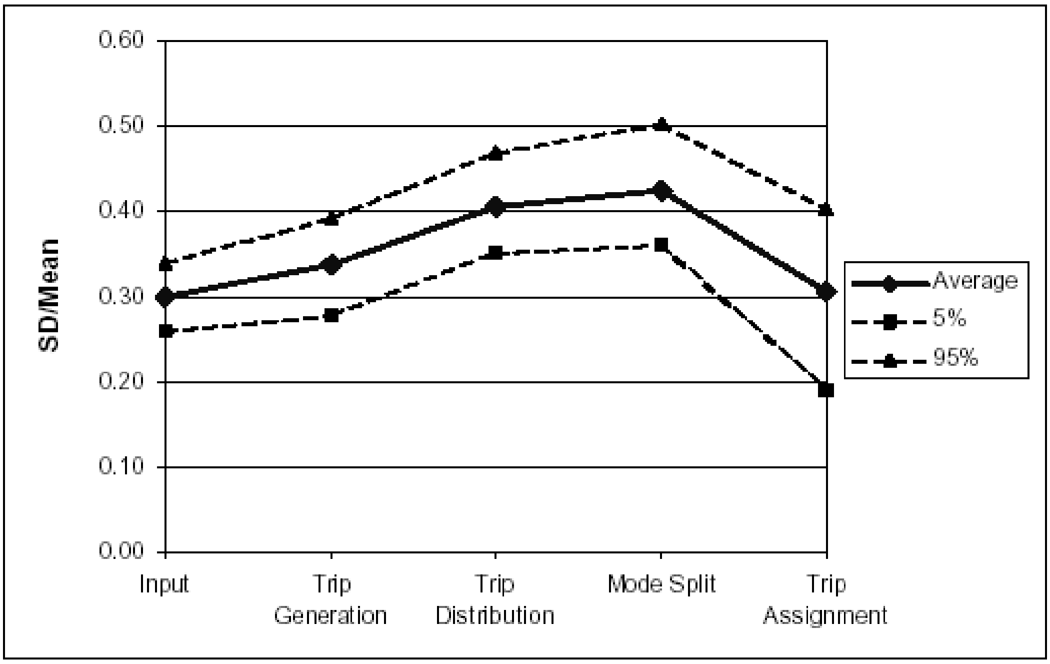
How is uncertainty expressed?

Uncertainty is expressed in standard deviation divided by the mean value, which is fixed at the starting point of the model at 0.30. Different input/parameter uncertainty levels are tested, i.e. 0.1, 0.3 and 0.5.

What is the order of magnitude of the uncertainty around the forecast?

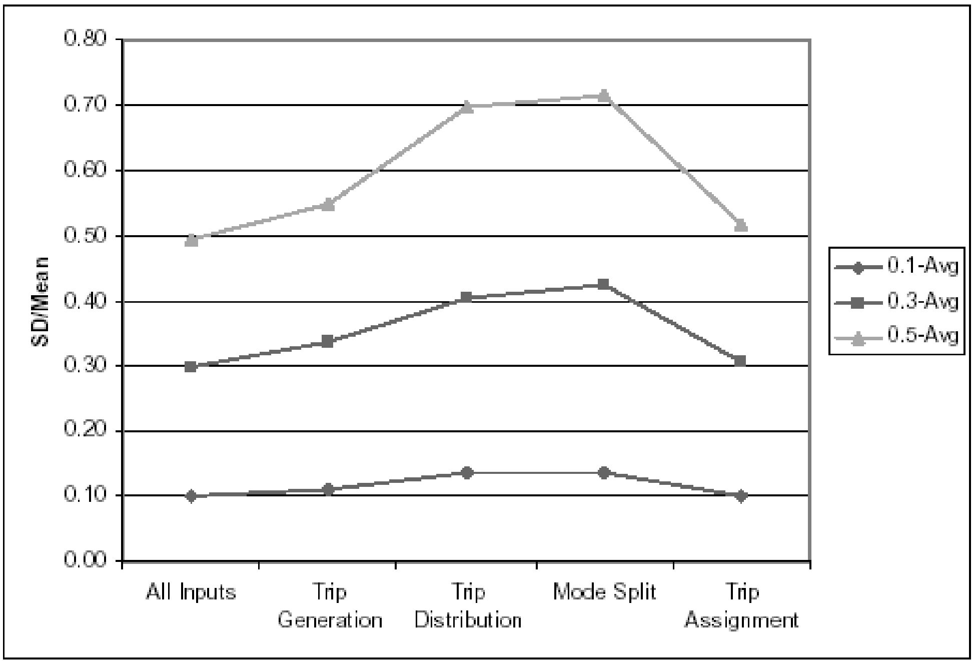
All the input uncertainties are set to the same value of 0.30, however due to the actual simulation data drawn, the 5% and 95%-values of demographic input uncertainty are 0.2592 and 0.3397, respectively.

The figures below (from Zhao and Kockelman, 2000) show the propagation of uncertainty in the four-stage transport model. For the given value of 0.30 an increasing average of uncertainty in the first three steps is apparent. The 5% and 95% bound expands indicating an increasing variability of uncertainty. Because the assignment model is a user-equilibrium assignment, the compounded uncertainty is reduced, although not lower than the input uncertainty.



Uncertainty propagation through 4-step model.

The input uncertainty values have been varied to 0.10 and 0.50. The pattern of propagated uncertainty is similar, only the absolute uncertainty values increase.



Uncertainty propagation through 4-step model with different input/parameter uncertainty levels.

Notes of Telephone Interview (19th April, 2004) of Geoff Copley – FaberMaunsell – UK by Eric Kroes

Geoff indicated that he had not been involved for a long time in exploring and quantifying the inaccuracy of, or uncertainty associated with model forecasts. The last time he worked in this area was in the early 1980's when he worked with Ashley and Lowe. Ashley had applied Monte Carlo techniques to carry out sensitivity analysis, Geoff's main contribution had been the development of a systematic type of analysis, involving the use of an experimental design to specify the simulation runs. This had served to reduce the number of simulation model runs necessary to estimate all main effects and relevant interactions of the uncertainty associated with the different model input elements. Following the work reported in 1982 the Department of Transport had decided they were no longer interested in pursuing this, as they felt the work was too model intensive. More recently this view has been abandoned, as the Green Book in the UK prescribes the use of an @risk analysis for forecasts. @risk is software designed specifically to quantify the uncertainty associated with a combination of variables, which are specified in terms of their individual variances and their correlations with other variables. He referred to Alan Boyce for further information about @risk.

Notes of Telephone Interview (21th April, 2004) with Alan Boyce of FaberMaunsell by Eric Kroes

Alan explained that he had been working in this area particularly in the context of privately financed infrastructure, where he had been retained by banks and other parties to assess the uncertainties associated with traffic forecasts, particularly looking at toll roads. He had worked in this area a lot with Martin Bright, the co-author of their 2003 ETC paper.

Alan explained that the process of estimating the uncertainties in traffic forecasts for toll roads consisted of two distinct parts:

Part 1 is to provide the best possible estimate of how much traffic would use the road now, what would be the capture rate, taking account of the values of time, the toll level and the congestion on other roads. Then look at the potential uncertainties associated with the input. Parameters such as VOT can be very uncertain. The effect of this uncertainty can be addressed by varying the VOT and reassigning the traffic model to obtain estimates of how the traffic volume is affected: an upside case and a downside case should be run to establish the sensitivity of the forecast flows to changes in VOT.

Part 2 is to make an assessment of what is most likely to happen in the future: make your best assessment of future demand growth, any development patterns which will affect demand, and the future VOT and run the forecasting model to estimate the saved travel time and obtain the most likely future situation. Then undertake sensitivity tests for the forecast variables to establish the extent to which variability in these values results in variability yin the forecast flows. Use @risk standard software (see www.@risk.com). What this does is to calculate (by means of Monte Carlo simulation) the combined probability distribution of a number of input variables and their impact on the forecasts. For each input variable a probability distribution needs to be specified, which can be e.g. a triangular distribution (lowest point, most likely point, highest point) or a normal distribution (mean and standard deviation). Correlations between input variables can also be specified. @risk then calculates the combined probability density distribution.

Alan said he usually starts with the standardized traffic flow (index 100) and looks at the percentages variability of the base year factors, also looking at how they might be correlated. Key factors are clearly the VOT, the travel time savings and the accuracy of the OD matrix (obviously the larger the survey is on which the matrix was based, the smaller the relative variance). Then the combined probabilities are determined, by using an additive or multiplicative procedure. @risk does this, typically by randomly selecting values from each distribution for 10.000 iterations. This gives the final distribution for the base year. When all the variables are independent, one variable with a wide distribution tends to affect the final result most; correlated variables tend to compensate each other.

Then the future year distribution is obtained, by factoring in the effects of GDP growth and the relation between GDP growth and traffic volumes (through car ownership). The forecast of GDP growth is obviously very uncertain on a year to year basis, but the average growth over a 10 year period is much more stable, and normally distributed. Another element of uncertainty is the growth in VOT. In the past a proportional growth to GDP was assumed, but nowadays half the GDP growth is used to account for the fact that the new car owners will tend to have less than average VOT.

By combining the distributions for the base year and for the forecast year the total forecast distribution is obtained, for the first year (10.000 draws). Then the combined (= forecast and base) relative (= to a central forecast) distribution is taken, and multiplied with the central model forecast, to obtain the uncertainty margins for each year modelled. Typically the uncertainty distribution gets wider over time, but slowly, because there is already a lot of variability in the base year, and there is a lot of compensation. If a new toll road is already open, the base year uncertainty is very small, and all future year traffic volumes are more accurate (but now the distribution may get wider more rapidly). This process is then repeated for year 2, 3, etc. up to a target year. Depending upon the amount of time available, a simple model (GDP growth and demand elasticities) or a more comprehensive forecasting model may be used to provide the key points of the distribution. Typically an application is based upon a mix of simple and detailed model simulations (also called: sensitivity tests) to generate the inputs for obtaining the combined distribution using @risk. The results are presented in the form of percentiles.

Alan has a lot of practical experience in the area, and would be willing to assist in further elaboration or to participate in a workshop if desired (provided his costs are covered, of course).

Notes of personal interview with David Watling – ITS Leeds, on assignment errors by Andrew Daly

The LMS, OGM and most other traffic forecasting systems contain an assignment phase, which, like other model components, produces forecasts which contain error. While assignment errors are not the central focus of this project, they are an essential component of the overall model error and an overview of modelling error would not be complete without considering assignment error.

To set this study in the context of theoretical analyses of assignment error, it is necessary to note the objective of the LMS and OGM to model a representative working day. That is, we are not interested in day-to-day or seasonal variation but in estimating average conditions. Of course average conditions are affected by variation, particularly because of the non-linearity of the functions involved, so that average travel time is higher than travel time with average flow, but the objective is to supply single representative values rather than to assess variations.

Components of assignment error

As with other sub-models, assignment error can be divided into two components:

- that caused by error in the inputs provided to that procedure and
- that caused by error in the assignment procedure itself.

Input error can in turn be considered as having two components: error in the matrix to be assigned and error in the description of the network, i.e. in the description of the links or, for public transport assignment, in the lines operated. Matrix error is essentially the focus of the main part of the *Bandbreedte* project and its causes will not be discussed in further detail here. However, an important point to note is that matrix error will contain important correlations, whether between cells in a matrix to be assigned or between matrices for different scenarios. These correlations will need to be considered in calculating assignment error as they will affect the calculation of errors in assignment outputs (link flows, travel times etc.) either at the level of a scenario, where flows from different matrix cells are added in calculating the flow on a link, or between scenarios, where correlated flows on links or travel times are compared across the scenarios. Expert opinion is that matrix error is likely to be more important than network description error in influencing the accuracy of assignment outputs.

Errors in the description of the network can be considered in three components:

- errors in the topological description of the network;
- errors in the length of the link; and
- errors in link capacities.

It might be thought that errors in the topological description of the network would be better described as 'mistakes' than as errors, i.e. coding error etc., and of course this is partly true. However, it also has to be recognised that modelled networks inevitably represent abstractions of the true network. Zones are represented as single points, with a limited number of connector links; minor roads are omitted from the network; public transport lines are simplified; etc.. Thus in addition to coding error – which ought to be eliminated as far as possible – there is also an irreducible topological error remaining in networks, which can probably be treated as constant across all the scenarios.

Errors in the length of links can reasonably be assumed to be small relative to the other errors in the model. However, the introduction of new links in future scenarios will inevitably introduce further error of this type so that there will be a slight difference between scenarios arising from this error component.

Errors in link capacities, however, are quite complex. An error in the number of lanes should be treated as a coding mistake, but there are a number of other sources of error in link capacity:

- error in the speed-flow curves representing the average speeds for specific vehicle types on links with given flow volumes;
- error, or perhaps more accurately, local variation, introduced by specific local physical circumstances and road layout, meaning that a 'link type' description is only an approximation;
- variation at local or national level in traffic management practices;
- variation in the mix of traffic on the road, above what might be indicated by purpose and vehicle type (car, LGV, HGV) splits; for example, commuters would have better knowledge of the road and therefore perhaps a higher driving speed than drivers for other purposes;
- variation in average driver and vehicle performance, particularly between base and forecast years.

Obtaining information on the magnitude of these errors and how they might change over time is quite challenging, but some information may be available from UK sources.

Error in the assignment procedure can be further considered in two components: failure of the computational process to deliver the model as specified and inaccuracy in the model itself.

The assignment procedure is modelled as an equilibrium in which each network user chooses the path which offers minimum travel cost. This equilibrium can be approximated by a computationally intensive process which has to be terminated arbitrarily to meet exogenous run time constraints. The extent of error remaining because of failure to achieve the true equilibrium can itself be approximated.[12] The use of a fixed number of iterations will generally mean that there is more convergence error in highly congested than in less congested scenarios, so that it may be preferable to use a fixed convergence criterion to improve consistency between scenarios.

Within the assignment model, a number of issues arise for which solutions are adopted that may give rise to error:

- most fundamentally, the assignment process may not be an equilibrium process but some more complicated day-to-day evolution that never achieves equilibrium – how this should be modelled has been substantially clarified in theory[13]; this theory could be used

[12] The error indicated here is error *within* the assignment process. In supply-demand equilibrium models, such as LMS and OGM, there is a further error arising from equilibration *between* the assignment and the demand model (matrix calculation). This supply-demand equilibration error can also be calculated approximately.

[13] See Watling, D. and Hazelton, M. (2003) The dynamics and equilibria of day-to-day assignment models, *Networks and Spatial Economics*, 3, pp 349-370, which also gives a list of references to earlier research. Further

to develop computer software but there is still a lack of empirical evidence on which quantitative forecasts could be based;

- error in the algorithms used (e.g. in QBLOK) to calculate the impact of flows on one link on driving conditions on other links;
- the assignment process is based on a model of drivers' route choice which is a considerable simplification of reality – drivers consider more variables than are represented in the model, there is considerable variation between drivers, even when separate user classes are represented in the assignment and, as in the case of the matrix forecasting, the average values of the route choice parameters are estimated with error; one solution to these problems is to use a stochastic assignment procedure.

These errors mean that the route choice predictions for drivers are not correct, giving rise to errors in link flows.

The role of dynamic assignment methods in relation to these errors needs to be clarified. Dynamic assignment allows demand profiles to be modelled and can improve the representation of route choice in spatial and temporal dimensions. Relative to 'static' assignments, the nature of the errors is changed and the intention is to make a reduction in the error, but of course none of the errors is entirely eliminated.

All of these errors need to be quantified in order to assess their impact on the assignment results. Moreover, any correlations between them also need to be quantified, although in general the error sources discussed are somewhat independent and many correlations can be taken to be zero.

The next issue, assuming that the basic errors can be quantified, is to calculate their impact on the assignment.

Calculating error in assignment output

The essential outputs from assignment may be considered as the following:

- link flows and queues at local level, which are required to check the accuracy of the assignment and to indicate potential 'pinch-points' in the network;
- travel times and costs at matrix cell level, used as 'feed-back' to the demand model as part of the iterative process;
- evaluation criteria, such as total travel times, 'queue hours', etc., possibly disaggregated to regional level, which are used to assess the success of scenarios.

Each of these output criteria is subject to error as a function of the sources described in the previous section, but the calculation of these errors is not straightforward. Fortunately, many of the calculation procedures necessary have been developed by David Watling and his associates at Leeds and use can be made of this work.

The core of the Leeds work is a 'linearisation' procedure which relates errors in input matrices and in link parameters, i.e. capacities etc., to errors in the assignment outputs. Linearisation also exploits the derivatives of specific functions, in this case the functions formalising the fixed-point

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discussion is given by Hazelton, M. and Watling, D. (2004) Computation of equilibrium distributions of Markov traffic assignment models, *Trans. Science*, in press.

specification of equilibrium in the assignment. The idea is quite similar to that proposed for predicting error in matrices.

It is important to note that linearisation in a conventional user equilibrium assignment is a difficult procedure, but that if a stochastic assignment using MNL is undertaken then the procedure is quite different and substantially easier to undertake. Using other forms of stochastic assignment will however again give more difficult models, remaining distinct from the equilibrium assignment issues. For example, when a probit model is used the issue arises of the possible singularity of the covariance matrix of route utilities.

A further useful aspect of the Leeds work is to relate error in link flow to error in link cost, through analysis of the speed-flow curve. Given a level of error in the predicted flow on a link, the differential of the speed-flow curve can be used to indicate the approximate error in the link cost. There are considerable complications when link flows are inter-related, as in QBLOK, and because of correlation between flows – and hence costs – on successive links. By adding up the link cost errors, errors can be obtained in path costs, and hence in path choice, and in assignment outputs for scenario evaluation.

The final step in the Leeds work is the use of these formulae to indicate the critical parameters which determine the accuracy of the assignment process.

For use for LMS and OGM assignment error calculation, the Leeds procedures would require some amendment to adapt to the specific algorithms (QBLOK or dynamic procedures) used in The Netherlands. However, it appears that the theoretical development is largely in place.

Appendix 3. Derivation of analytical expressions for the model uncertainty

To find confidence intervals for the flow on individual links and groups of links on the Dutch transport network, it is necessary to know the variance of the estimates from the LMS model. The aim of this section to derive that variance for a mode-destination-time period model for a specific trip purpose (namely commuter trips), subsequent sections will consider the trip frequency models, and the interrelation between trip purposes.

Since the model is applied in an iterative manner, there is an element of feedback in the error. Rather than calculating variances directly, they will ultimately have to be found as the solution to a matrix equation; the solution will be the full covariance matrix for the flow on the links of the network[14]. Therefore, at each stage in the derivation, a full set of covariances must be considered.

Below is a discussion on the analytical derivatives of the mode-destination models. This discussion is more general than the impact of errors in the model coefficients on link flow error, it also deals with the impact of errors in the input variables. However the conclusions on model error are also valid for the –more restricted- situation in which we take the input values as given (but different in different simulations).

A3.1 Form of the mode-destination model

The LMS mode-destination-time choice model for commuter trips is a nested logit model. The non-choice variables it considers are origin zone *o* and demographic segment *n*. These form a single model segment *N*. Therefore, the choice set available in this model is three dimensional: time period *t*, mode *m* and destination *d*. These form a choice vector *C*. So we have:

$$N = (n, o)$$
$$C = (t, m, d)$$

[14] Alternatively, an intermediate matrix could be found

We can also define the following terms:

$$V_{NC} = \text{Observed utility of choice C from segment N}$$

$$P_{NC} = P(\text{choice} = \text{C} \mid \text{sector} = \text{N})$$

A nested logit model may be decomposed into sequential logits. In this case, mode choice is made after destination choice, so we have:

$$P_{NC|d} = P(\text{choice} = \text{C} \mid \text{destination} = \text{d and segment} = \text{N})$$

$$P_{N(t,m)|d} = P(\text{mode} = \text{m} \mid \text{destination} = \text{d and segment} = \text{N}) = P_{N(t,m,d)|d}$$

and

$$P_{Nd} = P(\text{destination} = \text{d} \mid \text{segment} = \text{N})$$

Although time period choice is theoretically made after mode choice, the structural coefficient in this case is 1. Therefore, time period and mode choice can be considered simultaneous.

The model is linear in parameters:

$$V_{NC} = \sum_i \beta_i x_{NCi}$$

and we must consider the full set of covariances:

$$\text{cov}(\beta_i, \beta_{i'})$$
$$\text{cov}(x_{NCi}, x_{N'C'i'})$$
$$\text{cov}(x_{NCi}, \beta_{i'})$$

It is assumed that the full covariances of the β terms, the coefficients, can be found by the standard estimation or bootstrap estimation of the model. These represent sampling and specification errors. The covariances of the x terms, the explanatory variables, are also to be found. Note that these covariances are *not* the correlation of the variables themselves across the sample, but describe errors in their measurement or forecasting. In the case of level-of-service variables, these may be functions of errors in the *input* to the network assignment process.

The nested logit model also contains a tree coefficient, λ, so we must consider:

$$\operatorname{cov}(x_{NCi}, \lambda)$$
$$\operatorname{cov}(\beta_i, \lambda)$$

Again, it is hoped that the second of these can found by jack-knife or bootstrap.

A3.2 Zero covariances

It would be helpful to assume that the remaining covariances are zero, namely:

$$\operatorname{cov}(x_{NCi}, \beta_{i'}) = 0$$
$$\operatorname{cov}(x_{NCi}, \lambda) = 0$$

The coefficients β and λ are derived from an estimation run of the model, based on observed level-of-service values. In a forecasting model run in equilibrium they are fixed, and level-of-service variables are derived from network assignment. The input and hence output of network assignment will depend on the values of the coefficients.

For example, suppose a coefficient of trip cost is overestimated, so that fewer trips are predicted to distant destinations, and more to closer destinations. Then in assignment there should be less traffic flow on large, long-distance roads, and more on smaller, local roads. The level-of service variables will change accordingly.

In other cases, it seems more reasonable to assume zero covariance: forecast public transport prices are unlikely to depend on the model outputs.

To properly address this issue requires detailed theoretical or practical analysis, for example the whole model could be run several times with different values of the coefficients. In the meantime we may assume these covariances are all zero.

A3.3 Covariance of the utility functions

Whether the full covariance matrix of the coefficients and explanatory variables (the inputs) is estimated, or whether several values are assumed to be zero, the covariance matrix of the utility functions can be derived as a result. The true covariances of the utility functions are not obvious, however a linear approximation is possible. It is easy to calculate:

$$\frac{\partial V_{NC}}{\partial \beta_i} = x_{NCi}$$

$$\frac{\partial V_{NC}}{\partial x_{N'C'i}} = \delta_{NN'}\delta_{CC'}\beta_i \text{ or more simply}$$

$$\frac{\partial V_{NC}}{\partial x_{NCi}} = \beta_i$$

This gives rise to the linear approximation:

$$\begin{aligned}\operatorname{cov}(V_{NC}, V_{N'C'}) \approx & \sum_i \sum_j \beta_i \operatorname{cov}(x_{NCi}, x_{N'C'j})\beta_j \\ & + \sum_i \sum_j \beta_i \operatorname{cov}(x_{NCi}, \beta_j) x_{N'C'j} \\ & + \sum_i \sum_j x_{NCi} \operatorname{cov}(\beta_i, x_{N'C'j})\beta_j \\ & + \sum_i \sum_j x_{NCi} \operatorname{cov}(\beta_i, \beta_j) x_{N'C'j}\end{aligned}$$

Assuming zero covariance between variables and coefficients, and adopting matrix notation, this becomes:

$$\operatorname{cov}(V_{NC}, V_{N'C'}) \approx \beta^T \Sigma_{x_{NC}, x_{N'C'}} \beta + x_{NC}^T \Sigma_\beta x_{N'C'}$$

where Σ is the covariance matrix.

We also have:

$$\operatorname{cov}\left(V_{NC},\lambda\right)\approx\beta^{T}\Sigma_{x_{NC},\lambda}+x_{NC}^{T}\Sigma_{\beta,\lambda}$$

which may be zero.

A3.4 Derivatives of the choice probabilities

The choice probabilities for the nested logit model are given by:

$$P_{NC} = \frac{\exp\left(V_{NC}\right)\left[\sum_{t',m'}\exp\left(V_{N(t',m',d)}\right)\right]^{\lambda-1}}{\sum_{d'}\left[\sum_{t',m'}\exp\left(V_{N(t',m',d')}\right)\right]^{\lambda}} \text{ where } C = \left(t, m, d\right)$$

This can be decomposed into two sequential logits:

$$P_{Nd} = \frac{\left[\sum_{t',m'}\exp\left(V_{N(t',m',d)}\right)\right]^{\lambda}}{\sum_{d'}\left[\sum_{t',m'}\exp\left(V_{N(t',m',d')}\right)\right]^{\lambda}}$$

$$P_{N(t,m)|d} = \frac{\exp\left(V_{N(t,m,d)}\right)}{\sum_{t',m'}\exp\left(V_{N(t',m',d)}\right)}$$

The derivatives with respect to the utility functions are:

$$\frac{\partial P_{NC}}{\partial V_{NC'}} = P_{NC}\left[\delta_{CC'} + (\lambda-1)\delta_{dd'}P_{N(t',m')|d'} - \lambda P_{NC'}\right]$$

$$N \neq N' \Rightarrow \frac{\partial P_{NC}}{\partial V_{N'C'}} = 0$$

We also need the derivative with respect to λ:

$$\frac{\partial P_{NC}}{\partial \lambda} = P_{NC}\left[\log\left(\sum_{t',m'}\exp\left(V_{N(t',m',d)}\right)\right) - \sum_{d'}\log\left(\sum_{t',m'}\exp\left(V_{N(t',m',d')}\right)\right)P_{Nd'}\right]$$

A3.5 Derivatives of the choice frequencies

The number of people from segment *N* predicted to make choice *C* is the product of the choice probability from the mode-destination choice model and the total number of trips output by the trip frequency model, and the derivatives of this product can be calculated:

$$T_{NC} = \omega_N P_{NC}$$

$$\frac{\partial T_{NC}}{\partial V_{N'C'}} = \omega_N \frac{\partial P_{NC}}{\partial V_{N'C'}} + P_{NC}\frac{\partial \omega_N}{\partial V_{N'C'}} = \omega_N \frac{\partial P_{NC}}{\partial V_{N'C'}}$$

The second term is zero: ω is not a function of *V*, although they both depend on the same explanatory variables *x* and therefore may be correlated. This could give rise to a covariance, to be examined when studying the trip frequency model, however this would certainly be very small and may be considered zero.

Similarly, we have:

$$\frac{\partial T_{NC}}{\partial \lambda} = \omega_N \frac{\partial P_{NC}}{\partial \lambda}$$

$$\frac{\partial T_{NC}}{\partial \omega_{N'}} = \delta_{NN'} P_{NC}$$

A3.6 Covariance of the choice frequencies

The covariance of two choice frequencies can be estimated using a first order approximation, which briefly is:

$$\operatorname{cov}\left(T_{NC}, T_{N'C'}\right) \approx T'^{\,T}_{NC}\left(V, \lambda, \omega\right) \Sigma_{V\lambda\omega}\, T'_{N'C'}\left(V, \lambda, \omega\right)$$

where *T'* is the vector of first derivatives with respect to the variables *V*, λ and ω, and Σ is the covariance matrix of those variables.

Ignoring zero derivatives (but including covariances that may be zero) this is:

$$T'^{T}_{NC}\, \Sigma\, T'_{N'C'}$$

where:

$$T'^{T}_{NC} = \left(\frac{\partial T_{NC}}{\partial V_{NC_1}}, \frac{\partial T_{NC}}{\partial V_{NC_2}}, \ldots, \frac{\partial T_{NC}}{\partial V_{NC_i}}, \ldots, \frac{\partial T_{NC}}{\partial \lambda}, \frac{\partial T_{NC}}{\partial \omega_N} \right)$$

and:

$$\Sigma = \left(\begin{array}{ccc|c|c} \ddots & & \iddots & \vdots & \vdots \\ & \Sigma_{V_{NC_i}, V_{N'C_j}} & & \Sigma_{V_{NC_i}, \lambda} & \Sigma_{V_{NC_i}, \omega_{N'}} \\ \iddots & & \ddots & \vdots & \vdots \\ \hline \cdots & \Sigma_{\lambda, V_{N'C_j}} & \cdots & \operatorname{var}(\lambda) & \operatorname{cov}(\lambda, \omega_{N'}) \\ \hline \cdots & \Sigma_{\omega_N, V_{N'C_j}} & \cdots & \operatorname{cov}(\omega_N, \lambda) & \operatorname{cov}(\omega_N, \omega_{N'}) \end{array} \right)$$

C_i and C_j range over all time-mode-destination triples, of which there are 3×5×1308=19620, so the covariance is the sum of $(19620+2)^2 > 385{,}000{,}000$ terms; and that for only one pair of output values. Taking one term as an example (and this assuming zero covariance between explanatory variables and coefficients):

$$\frac{\partial T_{NC}}{\partial V_{NC_i}} \operatorname{cov}\left(V_{NC_i}, V_{N'C_j}\right) \frac{\partial T_{N'C}}{\partial V_{N'C_j}}$$
$$\approx \omega_N P_{NC}\left[\delta_{CC_i} + (\lambda - 1)\delta_{dd_i} P_{N(t_i,m_i)|d_i} - \lambda P_{NC_i}\right]$$
$$\times \left[\beta^T \Sigma_{x_{NC_i}, x_{N'C_j}} \beta + x_{NC_i}^T \Sigma_\beta x_{N'C_j}\right]$$
$$\times \omega_{N'} P_{N'C'}\left[\delta_{CC_j} + (\lambda - 1)\delta_{dd_j} P_{N(t_j,m_j)|d_j} - \lambda P_{NC_j}\right]$$

The *P* terms here are output by the model, and λ and ω are known, so the first and third components are easy to calculate; there are 19620 for each segment *N*. Still, there are at least 1308 segments, since the segment incorporates origin zone. This results in over 25 million calculations. An added complication is wrought by the demographic segmentation, but since this is largely implemented through on-off variables, huge simplifications may be possible.

The second component of the product is a cell of the covariance matrix, and will differ for each choice pair C_i and C_j, and each pair of segments *N* and *N'*. Since the segment *N* incorporates origin zone *o*, the explanatory variables for different segments will differ significantly. Even if we disregard demographic segmentation, there are $1308^2 \times 5 \times 3 > 25$m origin-destination-mode-time sets, and a separate calculation is necessary for each pair of these: over 6.5×10^{14}.

Since modern processors perform at rates of around 1Gflops (10^9 floating point operations per second), and even considering no division is necessary, a full set of covariances would take a matter of weeks to calculate. Incorporating all six trip purposes would be expected to increase this time by a factor of 36. It would take, as the saying goes, “all year”.

Our goal though is not only to calculate the covariance matrix of model output for a given covariance of the inputs, but also to solve an equation for the covariances of an iterated supply-demand model that is in equilibrium. Since we will know the equilibrium values of ω, *P*, λ, β and *x*, and claim to know the covariance of β, perhaps having theoretically derived any cross covariance cov(β, *x*), the product above is in fact a first order function of the covariance of the explanatory variables:

$$\operatorname{cov}\left(T_{NC}, T_{N'C'}\right) =$$
$$\sum_{C_1,C_2} A_{NCN'C'C_1C_2} \sum_{i,j} \beta_i \beta_j \operatorname{cov}\left(x_{NC_1i}, x_{N'C_2j}\right) + C_{NCN'C'C_1C_2}$$

or simply:

$$\Sigma_T = AB\Sigma_x + C$$

The same is true of the other terms in the equation: those coming from the variances of λ and ω. This is no accident; it is caused by our using linear approximations for the covariance calculations.

A3.7 Covariance of the flow on a link

The flow assigned to a link L of the transport network at a given time period t depends on the number of trips predicted between each origin-destination pair, regardless of demographic segment, and is a linear combination of these values:

$$F^{Lt} = \sum_{o,d} \alpha_{od}^{Lt} \sum_{n} T_{(n,o)(t,M,d)}$$

This formula needs comment. Firstly, the choice frequencies T are summed over demographic population segments n. In certain forecasting scenarios, for example if a monetary toll were enforced on parts of the network, this would not be possible. For the sake of this analysis though, we shall assume route choice is independent of demographics.

Then α is the fraction of travellers from origin o to destination d choosing routes that include link L. Mode m here has been replaced by M, which is taken to represent car travel. We are only interested in car journeys in this project.

α is not a constant, it depends on all (or at least some) of the values of ΣT. It is hoped that we can estimate the derivatives of α with respect to the values of T, or rather the sums ΣT, at least for a given assignment when the model is in equilibrium. This is an area of debate, and is considered by Watling to be possible for a probabilistic assignment process. If it is possible, then given:

$$\left.\frac{\partial \alpha_{o'd'M}^{Lt}}{\partial S_{o(t,M,d)}}\right| = \left.\frac{\partial \alpha_{o'd'M}^{Lt}}{\partial T_{(n,o)(t,M,d)}}\right| \quad \text{where } S_{o(t,M,d)} = \sum_{n} T_{(n,o)(t,M,d)}$$

(where the bar indicates that this is a numerical value; we will not know an analytical formula) we can derive:

$$\frac{\partial F^{Lt}}{\partial T_{(n,o)(t,M,d)}} = \sum_{o',d'} \left[\frac{\partial \alpha^{Lt}_{o'd'}}{\partial S_{o(t,M,d)}} S_{o'(t,M,d')} + \delta_{oo'} \delta_{dd'} \alpha^{Lt}_{o'd'} \right]$$

$$= \sum_{o',d'} \frac{\partial \alpha^{Lt}_{o'd'}}{\partial S_{o(t,M,d)}} S_{o'(t,M,d')} + \alpha^{Lt}_{od}$$

This is independent of *n*, as would be expected. We can use another first order approximation to find the covariance of the flow on two links, possibly in two different time periods. There are two approaches at this stage: we can either use the formula above for the derivative of *F* with respect to each *T*; or treat *F* as a function of the $S = \Sigma T$ terms, and find the covariance of these terms. Computationally these approaches are equivalent, the second is presented here:

$$\operatorname{cov}\left(S_{oC}, S_{o'C'}\right) = \sum_{n,n'} \operatorname{cov}\left(T_{(n,o)C}, T_{(n',o')C'}\right)$$

$$\operatorname{cov}\left(F^{Lt}, F^{L't'}\right) = \sum_{\substack{o,d \\ o',d'}} \frac{\partial F^{Lt}}{\partial S_{o(t,M,d)}} \operatorname{cov}\left(S_{o(t,M,d)}, S_{o'(t',M,d')}\right) \frac{\partial F^{L't'}}{\partial S_{o'(t',M,d')}}$$

where as above for *T*:

$$\frac{\partial F^{Lt}}{\partial S_{o(t,M,d)}} = \sum_{o',m',d'} \frac{\partial \alpha^{Lt}_{o'd'}}{\partial S_{o(t,M,d)}} S_{o'(t,M,d')} + \alpha^{Lt}_{od}$$

Again, this is a linear function of the covariance of the *T* terms by design. It is also computationally intensive; we appear to need the derivative of $|L|\times 5\text{m}\ \alpha$ terms (where $|L|$ is the number of links on the network, around 40,000) with respect to 1m *S* terms in each case.

However, many of these will be zero. For any origin-destination pair, only a small number of links will be used by the routes considered in assignment. The assignment program can identify these links for us, all other α values will be zero. Conversely, for any non-zero α, we need only find derivatives with respect the traffic demand between origin-destination pairs that that use the corresponding link.

If a is the maximum number of links that a given o-d pair can feasibly use, and b the number of o-d pairs that can feasibly use a given link, then we calculate at most $a\times b\times 5$m derivatives, which is much fewer than the $|L|\times 5\times 10^{12}$ suggested above, since $a<<|L|$ and $b<<1$m.

A3.8 Completing the circle

We now have a formula for the covariance matrix of the flow on the links of the network, as a first order function of the covariance of the explanatory variables:

$$\Sigma_F = A\Sigma_x + B$$

It is now hoped that we can express the covariance of the explanatory variables in terms of the covariance of the network flow. This really requires understanding of the assignment process. However, let us consider one explanatory variable. Suppose the time of a journey on link L in time period t is a known function of the flow on that link:

$$\tau^{Lt} = \tau^{Lt}(F^{Lt})$$

The predicted time of travel from origin zone o to destination zone d by mode m will be:

$$\tau_{(n,o)C} = \sum_L \alpha_{od}^{Lt} \tau^{Lt} \text{ where } C = (t, M, d)$$

A linear approximation for the covariance of the explanatory variables must be possible if the derivatives of the cost functions are known, and we will have an equation:

$$\Sigma_F = A\Sigma_F + B$$
$$\Rightarrow (A - I)\Sigma_F + B = 0$$

If $(A - I)$ is invertible, then the equation has a unique solution. Rather than invert such a large matrix though, it may be easier to find fixed points of the transformation **AX+B** by other means, especially since we know **A** as the product of several matrices that we have calculated separately. An exploration of efficient numerical algorithms might be useful.

A3.9 Summary on the NSES models

It is theoretically possible, using first order approximations, to estimate the covariance matrix of the flow assigned to the links of the transport network, given the covariance of the coefficients of the mode-destination-time period model which arises out of specification and sampling error. From this, confidence intervals for the flow on individual links and groups of links could be derived.

However, this involves finding and possibly inverting a matrix, which is at least as large as the square of the number of links on the network. Considering time period increases this size by a factor of nine. This is around 10^{10} entries. Even assuming a lot of intermediate values to be zero, the number of calculations involved is absolutely prohibitive. If other methods for generating confidence intervals are available, they should be preferred. We recommend to use simulation methods for the NSES model error instead of analytical derivatives.

A3.10 Form of the Trip Frequency Models

The following text describes calculations that would produce estimates of variance and covariance for the output of the LMS trip frequency models, along with projections of the amount of computer processing time required. These projections assume around one billion operations per second.

There are eleven trip frequency models for the LMS, corresponding to eleven trip purposes[15]; each consists of two sub-models which are estimated and run separately. The first determines the probability that an individual makes any trips at all for the relevant purpose, while the second is a stop-go model which determines, for n>1, the probability of making an nth trip given that (n-1) have already been made.

The explanatory variables for the models are determined by demographic population segment; there is no segmentation over origin zone and nor are any level-of-service variables used. We can assume moreover that they are the same for each model; if not they may be treated as separate variables, this is merely a matter of notation.

All of the sub-models are binary logit models, and linear in parameters. The expected trip frequency per person output for purpose *p*, segment *n* is therefore:

[15] Commuter trips for workers and non-workers, for example, are treated as separate purposes.

(1) $$\omega_{np} = \frac{P_{np}}{1 - Q_{np}} = P_{np} R_{np} \quad , \quad R_{np} = \frac{1}{1 - Q_{np}}$$

where:

(2) $$P_{np} = \frac{\exp(V_{np})}{1 + \exp(V_{np})} \ , \ R_{np} = 1 + \exp(W_{np})$$

$$V_{np} = \sum_i \beta_{pi} x_{ni} \ , \ W_{np} = \sum_i \gamma_{pi} x_{ni}$$

The explanatory variables *x* are all Boolean on-off indicators, so we shall assume that they are known exactly. Any error will be dealt with by simulation, when considering the number of people in each population segment. The constant term in the utility functions is treated as a coefficient, with corresponding variable fixed to one.

It is worth commenting that, for a well specified model, the coefficients β take exact values. Therefore it makes no sense to calculate their variance, or the variance of other values derived from them. However, we *are* interested in the variance of their estimators, which in this case are maximum likelihood estimators. We should really write:

$$\hat{V}_{np} = \sum_i \hat{\beta}_{pi} \hat{x}_{ni}$$ (although we are treating *x* as fixed)

We could use this notation throughout the document, but since all the variables studied are estimators it is unnecessary to make any distinctions. Similarly, it is confusing to express variances and covariances as functions of random variables; they should take precise values. However, we are actually expressing estimators of variance as functions of estimators.

A3.11 Derivation of Variance

The variance and covariance of the *estimators* of coefficients β or γ will be found for each sub-model by a bootstrap resampling technique. Cross-model covariances such as cov(β,γ), and cov(β,β) for different purposes, might also be found. We can calculate directly:

$$\text{var}(V_{np}) = x_n^T \Sigma_{\beta_p} x_n \ , \ \text{var}(W_{np}) = x_n^T \Sigma_{\gamma_p} x_n$$

$$\mathrm{cov}\left(V_{np}, V_{n'p}\right) = x_n^T \Sigma_{\beta_p} x_{n'} \ , \ \mathrm{cov}\left(W_{np}, W_{n'p}\right) = x_n^T \Sigma_{\gamma_p} x_{n'}$$

and possibly the cross-model terms including most generally:

$$\mathrm{cov}\left(V_{np}, W_{n'p'}\right) = x_n^T \Sigma_{\beta_p, \gamma_{p'}} x_{n'}$$

In many cases we will not be able to calculate cross-purpose covariances, for example where the models apply to disjoint subsets of the population. It will be both necessary and possibly correct to assume that these are zero.

Suppose that we calculate covariances within each sub-model, and between the two separate sub-models for each purpose. This would require at most $|n|^2 \times 3|p|$ covariance calculations, each derived as a sum of $|i|^2$ terms, where |n| is the number of population segments, |p|=11 the number of purposes, and |i|=38 the maximum number of explanatory variables that enter any model (including the constant). The number |n| of population segments is not immediately obvious, as it is determined by the specification of the explanatory variables.

In reality, the Boolean nature of the explanatory variables means that the vast majority of the terms will be zero, and selection of those that are not will be simple. The largest model[16] has 38 explanatory variables, but they occur in 10 categorical groups within each of which at most one variable will be non-zero. This means we need add only 10 terms per pair of segments, not 38^2=1444. This model implicitly defines almost 1 million "segments" through these variables though – but many of those may be empty. In the worst case scenario that there *were* a million segments, we would need at most 3.3×10^{14} additions to calculate the full set of covariances (for all purposes and across the two sub-models). This should be possible within a week, or at very most two.

Given the variance and covariance of the utility functions, we wish to perform similar calculations for the functions *P* and *R*, as in equation (2). Without knowing the sampling distributions of *V* and *W*, only approximations can be used. We use a standard first order approximation, first calculating the derivatives exactly:

$$\frac{dP_{np}}{dV_{np}} = P_{np}\left[1 - P_{np}\right] \ , \ \frac{dR_{np}}{dW_{np}} = R_{np} - 1$$

[16] Or at least, the model with the most variables.

$$\mathrm{cov}(P_{np}, P_{n'p}) \approx P_{np}\left[1 - P_{np}\right]\mathrm{cov}(V_{np}, V_{n'p})P_{n'p}\left[1 - P_{n'p}\right]$$

$$\mathrm{cov}(R_{np}, R_{n'p}) \approx \left[R_{np} - 1\right]\mathrm{cov}(W_{np}, W_{n'p})\left[R_{n'p} - 1\right]$$

$$\mathrm{cov}(P_{np}, R_{n'p}) \approx P_{np}\left[1 - P_{np}\right]\mathrm{cov}(V_{np}, W_{n'p})\left[R_{n'p} - 1\right]$$

The variance of an individual term is just a special case where *n*=*n'*. Allowing again for a very worst case scenario of a million segments, there are 33×10[12] covariance calculations here. To perform them we must know in total 22 million *P* and *R* values. This will take a day or two at most.

It remains to calculate the covariance of the ω values, as in equation (1). It would be consistent with the previous calculations to use another first order approximation, which is very simple:

$$\frac{\partial \omega_{np}}{\partial P_{np}} = R_{np} \;,\; \frac{\partial \omega_{np}}{\partial R_{np}} = P_{np}$$

$$\begin{aligned}\mathrm{cov}(\omega_{np}, \omega_{n'p}) \approx\; & R_{np}\,\mathrm{cov}(P_{np}, P_{n'p})R_{n'p} \\ & + R_{np}\,\mathrm{cov}(P_{np}, R_{n'p})P_{n'p} \\ & + P_{np}\,\mathrm{cov}(R_{np}, P_{n'p})R_{n'p} \\ & + P_{np}\,\mathrm{cov}(R_{np}, R_{n'p})P_{n'p}\end{aligned}$$

The number of calculations here is similar to the previous stage, and again a couple of days may be required.

It is worth stating explicitly the special case for variance:

$$\mathrm{var}(\omega_{np}) \approx R_{np}^2\,\mathrm{var}(P_{np}) + P_{np}^2\,\mathrm{var}(R_{np}) + 2R_{np}P_{np}\,\mathrm{cov}(P_{np}, R_{np})$$

This can be compared with an exact formula if *P* and *R* are independent:

$$\begin{aligned}\operatorname{var}(\omega) &= E(R)^2 \operatorname{var}(P) + E(P)^2 \operatorname{var}(R) + \operatorname{var}(P)\operatorname{var}(R) \\ &\approx R^2 \operatorname{var}(P) + P^2 \operatorname{var}(R) + \operatorname{var}(P)\operatorname{var}(R)\end{aligned}$$

The second line replaces the expectations of the estimators of *R* and *P* with their values. It is certainly true that any statistic is an unbiased estimator of it's own expectation.

We can see the magnitude of the error brought about by using an approximation; it is approximately var(*P*)×var(*R*). The smaller the variances themselves, the less significant this error will be.

A3.12 Inputs to the mode-destination model

The mode destination choice models use fewer demographic segments than the trip frequency models. They take as input total trip counts, rather than trip counts per person. Therefore the inputs are:

$$\Omega_{Np} = \sum_{n \subseteq N} \eta_n \omega_{np}$$

Here η is the number of people in trip frequency segment *n*, and the sum is over trip frequency segments which are a subset of the given mode-destination segment *N*.

Errors in the values of η will be handled by simulation, so in this analysis we can consider them fixed. The expression for the covariance of Ω is therefore exact:

$$\operatorname{cov}(\Omega_{Np}, \Omega_{N'p}) = \sum_{n \subseteq N} \sum_{n' \subseteq N'} \eta_n \operatorname{cov}(\omega_{np}, \omega_{n'p}) \eta_{n'}$$

Each pair of trip frequency segments *n* and *n'* will be included in exactly one pair of mode-destination segments *N* and *N'*, so the number of calculations is no more than 1.1×10^{13}, which should take less than a day to perform.

A3.13 Overall processing time for the tour frequency models

The predicted processing times in the text above are very pessimistic, and yet predict a total computer processing time of only around two weeks. Calculations for each purpose could be performed independently, and perhaps 24 hours would be needed for each. This is feasible, but still rather long. For this reason, and to be consistent with the procedure used for the mode-destination models, it was decided not to use the analytic methods for model uncertainty, but to use simulation methods.

Appendix 4. Detailed simulation outcomes

Table A4.1 LMS NSES simulation results at a national level for the reference situation (with congestion). (1) Input uncertainty, (2) Model uncertainty, (3) Both.

Total	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers
Mean	11704962	343362408	3899493	106871986	700622	71352153	709868	20761188	15392973	80110208	32407918	622457941
Median	11739404	344608185	3960201	106446048	696777	70234959	692583	20363086	15343995	79884268	32474231	623133774
Minimum	9361704	272488228	3343520	88154416	555926	57687749	615142	18389135	14476592	74835762	31362100	573369065
Maximum	13731458	385846069	4308965	128423936	885335	90814915	854440	24506992	16915291	89678397	33291106	676961110
St. deviation	1025047	21204826	204475	8956772	82900	7946380	66688	1656062	473259	2875377	458827	21558267
Percentile 5	9870036	299886757	3473376	91785036	568546	58369185	618934	18584399	14633749	76087248	31594922	583909863
Percentile 20	10644098	338761769	3752997	100993953	646850	65957299	668882	19714659	15135798	78297424	31998336	610589123
Percentile 80	12332377	359242395	4018936	112488193	795959	80030907	791648	22883731	15520655	80542112	32727877	634273707
Percentile 95	13298717	374541424	4153534	123939509	830452	83325049	840220	23961031	16464636	85904288	33086322	659928364

(1)	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers
Mean	11743430	342817888	3789352	105193837	700645	71153691	718028	20912244	15405249	80253005	32356703	620330665
Median	12107122	348043725	3835753	106113232	673047	68225641	683491	20019271	15238549	79519377	32477110	624132193
Minimum	9433600	272488228	3343520	88154416	555926	57753818	615142	18389135	14476592	74835762	31362100	573369065
Maximum	13731458	385846069	4161919	125939629	864797	87393387	854440	24506992	16840643	89410463	33114260	671489756
St. deviation	1371745	28428148	240878	10808622	107503	10256395	88155	2182409	630490	3748600	575570	27591406
Percentile 5	9666051	287260481	3354641	88856206	560922	58007013	616047	18399436	14484657	75291398	31384360	578512920
Percentile 20	10330603	325323254	3627077	96462272	579055	59555650	622953	18644880	15065763	77696081	31883008	591824190
Percentile 80	13073349	360742260	3968482	112833419	817126	82498321	810827	23152719	15758287	83325218	32940726	638521641
Percentile 95	13510353	376749415	4148677	121717154	830235	83474123	844940	24172488	16801392	87200542	33078828	664833632

(2)	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers
Mean	11704730	344384801	3991079	107584407	696903	70933522	698117	20492879	15374718	79819162	32465547	623214769
Median	11723400	344253447	3995040	106446048	704165	71308540	696728	20455834	15377525	79865158	32472659	623005750
Minimum	11545587	339794722	3932082	101679125	653676	67288643	681905	19719053	15223594	78956579	32125519	614291122
Maximum	11841625	348291673	4046001	114680354	718996	73477516	721200	21632004	15520973	80526137	32762906	633929125
St. deviation	91017	2256386	35087	4215577	16794	1797091	9507	428529	80012	428658	180407	5607201
Percentile 5	11563461	340339260	3936361	102061137	671035	67700583	684698	19883505	15255001	79154711	32240833	614925703
Percentile 20	11623000	343103666	3957805	103757421	683630	69463520	691443	20237890	15302385	79435344	32279047	618519069
Percentile 80	11791493	346357355	4018936	112228911	707978	72591549	704171	20683613	15430157	80101338	32663870	628936598
Percentile 95	11832040	347615203	4041638	113286476	717905	73317917	711933	21147580	15507309	80491297	32698344	629926869

(3)	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers
Mean	11628492	342406660	3936602	108803442	708015	72586338	717051	20995695	15404930	80406705	32395091	625198839
Median	11912344	347885214	3938544	106288936	668734	69063775	685908	19973697	15266155	80119538	32482066	623901573
Minimum	9361704	273755101	3531537	93619676	569644	57687749	618021	18759541	14486198	75513543	31586843	578963607
Maximum	13355913	372483576	4308965	128423936	885335	90814915	844190	24371992	16915291	89678397	33291106	676961110
St. deviation	1403185	28284780	239746	11954983	114907	11007019	86637	2112806	660140	4039436	601499	28067822
Percentile 5	9765486	294994771	3609476	94818326	571115	59195788	619838	18815839	14714519	76284054	31641053	589630920
Percentile 20	10329855	335131240	3761905	99531229	618054	64696147	637450	19017972	15042056	77319127	31783322	604229117
Percentile 80	13106525	360841426	4148273	116195672	827052	83321086	804386	23016140	15648406	82577296	32952629	640910584
Percentile 95	13286065	368083745	4241066	128085635	861387	87446769	831636	23840662	16492727	86759208	33202399	666604641

TableA4.2. LMS NSES simulation results at a national level for the project situation (with congestion). (1) Input uncertainty, (2) Model uncertainty, (3) Both.

Total	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers
Mean	11708471	343603601	3900473	106925765	700118	71316613	709572	20752721	15389285	80087925	32407918	622686624
Median	11743362	344921357	3961296	106526261	696238	70196033	692301	20354627	15340151	79863682	32474230	623400929
Minimum	9365827	272667457	3343754	88165525	555300	57655106	614876	18376273	14472326	74819683	31362100	573764890
Maximum	13733954	386037312	4310303	128496106	884635	90762789	854199	24500247	16911387	89652444	33291106	677233494
St. deviation	1024958	21187221	204618	8965476	82856	7943640	66676	1655621	473283	2874675	458827	21551281
Percentile 5	9873557	300165485	3473912	91819379	568022	58345166	618681	18575336	14629828	76063675	31594922	584197727
Percentile 20	10648179	339043972	3754122	101048387	646454	65923324	668615	19706538	15131292	78274974	31998336	610967190
Percentile 80	12335006	359386651	4019741	112543998	795452	80001483	791426	22874194	15516787	80517153	32727877	634488575
Percentile 95	13301973	374809395	4154782	124028303	829907	83293788	839880	23951309	16461017	85880880	33086322	660132692

(1)	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers
Mean	11746854	343056970	3790258	105238344	700144	71119374	717740	20904015	15401708	80231240	32356703	620549944
Median	12110093	348320699	3836703	106160491	672574	68193863	683244	20013508	15234897	79495972	32477110	624367803
Minimum	9436254	272667457	3343754	88165525	555300	57707256	614876	18376273	14472326	74819683	31362100	573764890
Maximum	13733954	386037312	4163171	126023659	864335	87363257	854199	24500247	16837712	89391740	33114259	671846732
St. deviation	1371780	28421940	241123	10824682	107467	10253795	88143	2181792	630529	3748065	575570	27574824
Percentile 5	9669258	287515818	3355252	88895602	560742	57994584	615663	18390917	14481856	75264887	31384360	578689984
Percentile 20	10334069	325550134	3627658	96497780	578550	59519494	622657	18641353	15062805	77677145	31883008	592126171
Percentile 80	13076558	360815494	3969464	112868046	816574	82450472	810485	23144263	15754346	83300580	32940726	638637866
Percentile 95	13514910	377112359	4150109	121811241	829720	83430149	844632	24162383	16797994	87178893	33078828	664944655

(2)	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers
Mean	11708212	344623727	3992133	107644513	696396	70897503	697821	20484379	15370985	79796922	32465547	623447043
Median	11726590	344629618	3996076	106526261	703571	71279179	696444	20446666	15373918	79842685	32472659	623328950
Minimum	11549018	340010757	3933153	101728315	653193	67253220	681602	19710545	15219973	78933975	32125519	614428584
Maximum	11844304	348552236	4046917	114736984	718409	73449642	720956	21625203	15516695	80502458	32762906	634156769
St. deviation	90830	2248632	35066	4209041	16785	1797207	9513	428752	80031	428938	180407	5629276
Percentile 5	11566830	340604984	3937457	102122582	670549	67668298	684379	19875182	15251194	79133053	32240835	615128009
Percentile 20	11626624	343443839	3958935	103830983	683183	69427824	691128	20229948	15298781	79412200	32279047	618611130
Percentile 80	11794539	346433299	4019741	112291409	707456	72555323	703888	20676038	15426570	80077211	32663870	629144062
Percentile 95	11834963	347898832	4042290	113347385	717469	73280127	711642	21139245	15503514	80465633	32698344	630163687

(3)	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers	Tours	Kilometers
Mean	11632222	342656611	3937582	108863110	707510	72549310	716739	20986817	15401038	80383300	32395091	625439148
Median	11916501	348146784	3939150	106318855	668203	69024240	685575	19963720	15261981	80097201	32482066	624010808
Minimum	9365827	274130900	3532523	93687513	569223	57655106	617703	18753927	14482431	75490329	31586843	579290751
Maximum	13360005	372629118	4310303	128496106	884635	90762789	843831	24360567	16911387	89652444	33291106	677233494
St. deviation	1402978	28246572	239797	11960919	114816	11002287	86621	2112459	660236	4038455	601499	28063065
Percentile 5	9770033	295256110	3610678	94879031	570796	59169165	619544	18808143	14710775	76263535	31641053	589897186
Percentile 20	10332599	335369695	3762857	99574308	617564	64659494	637175	19008850	15038283	77297488	31783322	604461017
Percentile 80	13109537	361099984	4149229	116260076	826608	83292610	804037	23006127	15643787	82547929	32952629	641180462
Percentile 95	13289535	368283502	4242292	128161517	860747	87402420	831260	23832447	16489374	86738431	33202399	666915700

Table A4.3. LMS NSES simulation results at a national level for the reference situation (without congestion feedback). (1) Input uncertainty, (2) Model uncertainty, (3) Both.

Total Scenario	Tours	Kilometers
Mean	11794237	359837735.7
Median	11834011	361645260.5
Minimum	9394905	275860878
Maximum	13856394	411430726
St. Deviation	1056449	25950401.99
Percentile 5	9907141	306382722.5
Percentile 20	10797649	355402508
Percentile 80	12413220	378215700.4
Percentile 95	13392356	397475705.2

(1) Scenario	Tours	Kilometers
Mean	11828979	358976705.2
Median	12196028	363309857
Minimum	9462508	275860878
Maximum	13856394	411430726
St. Deviation	1385011	34394148.68
Percentile 5	9663120	290132914.2
Percentile 20	10406062	338235651.6
Percentile 80	13169227	382155785.6
Percentile 95	13628865	400703369.7

(2) Scenario	Tours	Kilometers
Mean	11796223	361027201.7
Median	11815039	360978186.5
Minimum	11636467	355509893
Maximum	11933067	365835606
St. Deviation	91730.23	2719292.685
Percentile 5	11651570	356062534.6
Percentile 20	11715883	359594100
Percentile 80	11885719	363229067.4
Percentile 95	11924422	365106793.6

(3) Scenario	Tours	Kilometers
Mean	11720780	359180864.9
Median	12007062	365419021.5
Minimum	9394905	277778198
Maximum	13469660	395653691
St. Deviation	1414342	33884801.02
Percentile 5	9825038	302164926.1
Percentile 20	10401533	350798610.2
Percentile 80	13209729	381095752.8
Percentile 95	13379958	391559605.9

Table A4.4. Selected link results for Reference 2020: hours and flows

TOTAL	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht	
Scenario	Hours	Flow (Total)	Hours	Flow (Total)	Hours	Flow (Total)
Mean	2466.66	83338.7	4550.5	86660.68	5428.28	114703.78
Median	2464.5	83474	4579	86746	5451.5	115138
Minimum	2227	75428	3195	77465	4295	92406
Maximum	2688	90199	5728	94711	6332	133304
St. deviation	84.505	2663.013	467.038	3053.015	366.358	7289.782
Percentile 5	2315.4	78342.1	3692.7	81021.1	4729.25	100911.8
Percentile 20	2441	82828.4	4394.2	85625.6	5294	111960.4
Percentile 80	2522	84558.2	4858.4	88339	5649	118935.4
Percentile 95	2600.15	87274.25	5206	91103.05	5937.75	124879

(1)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht	
Scenario	Hours	Flow (Total)	Hours	Flow (Total)	Hours	Flow (Total)
Mean	2469.5	83305.25	4549.65	86697.25	5428.5	114780.6
Median	2484	84147	4704.5	87728.5	5558	117122
Minimum	2227	75428	3195	77465	4303	92567
Maximum	2688	90199	5728	94711	6332	133304
St. deviation	110.146	3481.372	599.491	3999.851	479.134	9577.602
Percentile 5	2280.2	77162.7	3468.6	79612.95	4543.35	97192.55
Percentile 20	2377.6	80476.6	4082.8	83642.6	5047.8	107287.8
Percentile 80	2531.4	85602.4	4882	88948.4	5657.4	119704.4
Percentile 95	2635.75	88444.35	5343.25	92260	6095.45	128326.95

(2)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht	
Scenario	Hours	Flow (Total)	Hours	Flow (Total)	Hours	Flow (Total)
Mean	2462.75	83369.55	4550.9	86636.45	5421.4	114506.9
Median	2464	83388.5	4534	86522.5	5408.5	114356.5
Minimum	2441	82818	4402	85910	5256	111078
Maximum	2487	83904	4826	87876	5600	117745
St. deviation	13.094	281.247	124.382	549.449	78.678	1532.135
Percentile 5	2441	82830	4407	86027	5296	112040
Percentile 20	2452	83216	4435	86176	5375	113472
Percentile 80	2472	83589	4647	87033	5472	115627
Percentile 95	2484	83716	4783	87537	5537	116577

(3)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht	
Scenario	Hours	Flow (Total)	Hours	Flow (Total)	Hours	Flow (Total)
Mean	2468.8	83343.9	4551.4	86636	5441.6	114943.9
Median	2487	83870.5	4675.5	87740.5	5620.5	118606
Minimum	2230	75520	3221	77576	4295	92406
Maximum	2633	88486	5382	91864	5994	126175
St. deviation	113.351	3584.945	629.439	4040.985	482.418	9515.805
Percentile 5	2293.45	77747.5	3548.6	80069.45	4645.1	99371.55
Percentile 20	2400.6	81372.4	4172.2	84758.6	5237	110727.4
Percentile 80	2547.6	85841	5028.4	89382.6	5721.6	120372
Percentile 95	2595.65	87347.95	5230.8	90759.25	5885.1	123723.85

Table A4.5. Selected link results for Reference 2020: Q-hours and flows

Total	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht	
Scenario	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	37.26	83338.7	1481.08	86660.68	15.02	114703.78
Median	35	83474	1490.5	86746	16	115138
Minimum	25	75428	818	77465	0	92406
Maximum	57	90199	2083	94711	29	133304
St. deviation	7.323	2663.013	251.098	3053.015	6.912	7289.782
Percentile 5	28.45	78342.1	1017.55	81021.1	2.25	100911.8
Percentile 20	31	82828.4	1361.8	85625.6	11	111960.4
Percentile 80	44	84558.2	1657.2	88339	22	118935.4
Percentile 95	51.55	87274.25	1856.15	91103.05	23.55	124879

(1)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht	
Scenario	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	40	83305.25	1488.15	86697.25	13.3	114780.6
Median	36.5	84147	1570.5	87728.5	14	117122
Minimum	25	75428	818	77465	0	92567
Maximum	57	90199	2083	94711	24	133304
St. deviation	8.903	3481.372	320.894	3999.851	7.241	9577.602
Percentile 5	28.8	77162.7	913.95	79612.95	0	97192.55
Percentile 20	33.8	80476.6	1207.4	83642.6	5.8	107287.8
Percentile 80	47.8	85602.4	1677.6	88948.4	18	119704.4
Percentile 95	56.05	88444.35	1941.45	92260	23.05	128326.95

(2)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht	
Scenario	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	34.75	83369.55	1470.6	86636.45	16.8	114506.9
Median	31	83388.5	1460	86522.5	16	114356.5
Minimum	28	82818	1368	85910	11	111078
Maximum	47	83904	1624	87876	28	117745
St. deviation	5.857	281.247	70.700	549.449	5.012	1532.135
Percentile 5	28	82830.35	1380.35	86026.85	11	112040.35
Percentile 20	31	83215.8	1413.4	86176.2	11	113472.4
Percentile 80	38.8	83589.2	1513.8	87033	22	115627
Percentile 95	45.1	83715.9	1589.8	87536.85	23.25	116577.45

(3)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht	
Scenario	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	36.8	83343.9	1487.9	86636	14.9	114943.9
Median	36.5	83870.5	1548	87740.5	16	118606
Minimum	30	75520	826	77576	0	92406
Maximum	44	88486	1970	91864	29	126175
St. deviation	4.614	3584.945	339.009	4040.985	9.098	9515.805
Percentile 5	30.9	77747.5	966.4	80069.45	2.25	99371.55
Percentile 20	33.6	81372.4	1239.6	84758.6	5.8	110727.4
Percentile 80	40.6	85841	1756.8	89382.6	23	120372
Percentile 95	43.55	87347.95	1875.5	90759.25	26.3	123723.85

Table A4.6. Selected link results for project situation 2020: hours and flows

total	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht		A20-A13		A13-A20	
Scenario	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)
Mean	2230.74	77227.98	2399.54	74000.46	5409.9	114316.72	967.58	37553.8	764.26	30263.92
Median	2223.5	77038.5	2393.5	73522.5	5429	114742	972	37806.5	769.5	30645.5
Minimum	2054	71155	2167	67168	4295	92374	685	26923	471	19277
Maximum	2426	83361	2704	82787	6303	132554	1145	43890	966	37764
St. deviation	70.557	2332.071	89.545	2707.880	366.716	7259.484	93.926	3428.976	93.240	3667.518
Percentile 5	2115.35	73295.65	2251.15	69865.4	4687.35	100224.7	782.15	30724.6	589.4	21963
Percentile 20	2202.2	76298	2351.4	72737	5279	111499.6	929.8	36184.6	755.8	29555
Percentile 80	2268	78486.8	2441.2	75669	5618.4	118901.8	1024.8	39624.8	818.2	32329
Percentile 95	2355.3	81205.3	2553.2	78700.7	5923.15	124631.25	1114.85	42752	900.35	35362.45

(1)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht		A20-A13		A13-A20	
Scenario	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)
Mean	2238.2	77448.8	2408.45	74514.3	5408.55	114380.75	968.2	37534.75	763.85	30398.15
Median	2245.5	77753.5	2403.5	74811	5520.5	116510.5	999	38611	783.5	31239
Minimum	2060	71367	2167	67471	4317	92863	685	26923	471	19294
Maximum	2426	83361	2704	82787	6303	132554	1145	43890	966	37764
St. deviation	91.651	3024.347	120.761	3543.811	482.363	9577.032	122.676	4475.479	121.109	4492.017
Percentile 5	2085.65	72268.55	2218.3	69030.9	4488	96468.25	731.55	28932.25	535.6	21829.55
Percentile 20	2162.2	74904.4	2332.6	71803.4	5030.8	106907.4	871.4	34158.6	675	27157.2
Percentile 80	2286.2	79132.2	2477.6	76191.6	5659.6	119676.6	1057.8	40901.4	843.8	33289.2
Percentile 95	2420.3	83176.7	2598.55	80374.95	6058.85	127528.5	1127.9	43219.3	942.25	36866.25

(2)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht		A20-A13		A13-A20	
Scenario	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)
Mean	2220.75	76929.75	2380.35	73275.5	5407.6	114181.8	966.8	37576.85	766.25	30118.15
Median	2219.5	76884	2380	73348	5406.5	114170.5	967.5	37628.5	765	30485
Minimum	2195	76058	2341	72689	5243	110906	944	36705	757	21963
Maximum	2248	77798	2418	73949	5573	117187	998	38472	778	31033
St. deviation	12.793	438.345	24.487	357.347	80.509	1533.685	12.800	440.699	6.034	1932.164
Percentile 5	2204	76343	2345	72712	5286	111611	951.6	37015.65	757	29778.65
Percentile 20	2211	76600	2360	72939	5351	113046	955.6	37152.8	761.8	30353.2
Percentile 80	2230	77268	2403	73558	5467	115466	974.4	37914.6	772.6	30684.8
Percentile 95	2240	77645	2417	73710	5556	116261	985.65	38083.45	775.15	30913.3

(3)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht		A20-A13		A13-A20	
Scenario	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)
Mean	2235.8	77382.8	2420.1	74422.7	5417.2	114458.5	967.9	37545.8	761.1	30287
Median	2231	77304	2437	75563.5	5586	117947	978.5	37959	795	31677
Minimum	2054	71155	2205	67168	4295	92374	693	27392	471	19277
Maximum	2394	82288	2573	79130	5956	125484	1130	43487	935	36622
St. deviation	93.012	3092.243	100.690	3344.709	476.668	9394.414	126.142	4617.108	127.559	4760.661
Percentile 5	2106.2	72968.95	2250.9	68970.25	4632.5	99084.4	773.55	30425	557.4	22641.65
Percentile 20	2180.4	75545.2	2394.2	72858.6	5195.4	110040.4	912	35550.4	685.4	27572.4
Percentile 80	2299.2	79583.6	2466.2	76114	5716	120105.2	1052	40625	825.4	32600
Percentile 95	2355.3	81205.3	2539.7	78181.4	5867.8	123344.7	1112.9	42780.5	897.2	35242.3

Table A4.7. Selected link results for project situation 2020: Q-hours and flows

Total	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht		A20-A13		A13-A20	
Scenario	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	0.06	77227.98	276.62	74000.46	15.02	113906.14	0.16	37553.8	0	30263.92
Median	0	77038.5	278	73522.5	16.5	114701	0	37806.5	0	30645.5
Minimum	0	71155	234	67168	0	92374	0	26923	0	19277
Maximum	3	83361	330	82787	51	132554	2	43890	0	37764
St. deviation	0.424	2332.071	21.722	2707.880	8.679	7663.148	0.548	3428.976	0.000	3667.518
Percentile 5	0	73295.65	246.45	69865.4	0	96658	0	30724.6	0	21963
Percentile 20	0	76298	257.8	72737	10	110870.6	0	36184.6	0	29555
Percentile 80	0	78486.8	292	75669	18	118901.8	0	39624.8	0	32329
Percentile 95	0	81205.3	315	78700.7	23.55	124631.25	2	42752	0	35362.45

(1)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht		A20-A13		A13-A20	
Scenario	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	0.15	77448.8	269.05	74514.3	12.55	114380.75	0.2	37534.75	0	30398.15
Median	0	77753.5	272	74811	12	116510.5	0	38611	0	31239
Minimum	0	71367	234	67471	0	92863	0	26923	0	19294
Maximum	3	83361	299	82787	24	132554	2	43890	0	37764
St. deviation	0.671	3024.347	18.069	3543.811	6.809	9577.032	0.616	4475.479	0.000	4492.017
Percentile 5	0	72268.55	241.6	69030.9	0	96468.25	0	28932.25	0	21829.55
Percentile 20	0	74904.4	248.8	71803.4	6	106907.4	0	34158.6	0	27157.2
Percentile 80	0	79132.2	286	76191.6	17.2	119676.6	0	40901.4	0	33289.2
Percentile 95	0.15	83176.7	289.5	80374.95	23.05	127528.5	2	43219.3	0	36866.25

(2)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht		A20-A13		A13-A20	
Scenario	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	0	76929.75	279.1	73275.5	18.1	113155.35	0.2	37576.85	0	30118.15
Median	0	76884	281	73348	17	114102	0	37628.5	0	30485
Minimum	0	76058	246	72689	0	96658	0	36705	0	21963
Maximum	0	77798	317	73949	51	116212	2	38472	0	31033
St. deviation	0.000	438.345	20.016	357.347	9.199	4114.621	0.616	440.699	0.000	1932.164
Percentile 5	0	76343	246.95	72711.8	10.45	110193.6	0	37015.65	0	29778.65
Percentile 20	0	76600.4	258.8	72939.2	16	112875.2	0	37152.8	0	30353.2
Percentile 80	0	77268	297	73558.2	22	115147.4	0	37914.6	0	30684.8
Percentile 95	0	77645.05	309.4	73709.6	24.4	115803.5	2	38083.45	0	30913.3

(3)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht		A20-A13		A13-A20	
Scenario	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	0	77382.8	286.8	74422.7	13.8	114458.5	0	37545.8	0	30287
Median	0	77304	285.5	75563.5	14	117947	0	37959	0	31677
Minimum	0	71155	250	67168	0	92374	0	27392	0	19277
Maximum	0	82288	330	79130	36	125484	0	43487	0	36622
St. deviation	0.000	3092.243	27.888	3344.709	9.864	9394.414	0.000	4617.108	0.000	4760.661
Percentile 5	0	72968.95	253.15	68970.25	2.25	99084.4	0	30425	0	22641.65
Percentile 20	0	75545.2	259.4	72858.6	5.8	110040.4	0	35550.4	0	27572.4
Percentile 80	0	79583.6	315	76114	17.2	120105.2	0	40625	0	32600
Percentile 95	0	81205.3	323.25	78181.4	27.9	123344.7	0	42780.5	0	35242.3

Table A4.8. Difference between project and reference situation in hours and flows at selected links.

TOTAL	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht	
Scenario	Hours	Flow (Total)	Hours	Flow (Total)	Hours	Flow (Total)
Mean	-235.92	-6110.72	-2150.96	-12660.22	-18.38	-387.06
Median	-240.5	-6126	-2180.5	-12818	-26	-499.5
Minimum	-269	-7209	-3024	-14755	-89	-1550
Maximum	-167	-4061	-1016	-9994	81	1095
St. deviation	21.868	644.482	389.203	1004.027	27.471	472.999
Percentile 5	-265.1	-6872.1	-2677.35	-14083.85	-51.55	-972.5
Percentile 20	-251.4	-6657.4	-2427	-13537.8	-37.2	-718.6
Percentile 80	-222	-5817.6	-1993	-11917	2.8	-20.4
Percentile 95	-198.25	-5046.45	-1433	-10776.8	21	316.9

(1)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht	
Scenario	Hours	Flow (Total)	Hours	Flow (Total)	Hours	Flow (Total)
Mean	-231.3	-5856.45	-2141.2	-12182.95	-19.95	-399.85
Median	-230	-5953.5	-2295.5	-11986.5	-29	-524
Minimum	-269	-7139	-3024	-13823	-89	-1550
Maximum	-167	-4061	-1028	-9994	81	1095
St. deviation	25.992	683.863	486.682	970.243	35.098	587.533
Percentile 5	-264.25	-6853.05	-2744.7	-13542.75	-60.5	-1108.25
Percentile 20	-254.6	-6167.8	-2473.4	-13185.2	-37.6	-755.6
Percentile 80	-212.2	-5380.2	-1753.2	-11829.4	3.2	127
Percentile 95	-194.55	-4894.15	-1250.3	-10582.05	17.35	335.95

(2)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht	
Scenario	Hours	Flow (Total)	Hours	Flow (Total)	Hours	Flow (Total)
Mean	-242	-6439.8	-2170.55	-13360.95	-13.8	-325.1
Median	-243.5	-6496	-2133.5	-13168	-19	-417
Minimum	-263	-7209	-2447	-14755	-52	-1222
Maximum	-219	-5852	-1995	-12128	29	604
St. deviation	11.309	380.998	127.750	655.693	22.369	424.345
Percentile 5	-260.15	-6906.9	-2389.05	-14215.4	-42.5	-786.9
Percentile 20	-251	-6723.2	-2276.2	-14027.2	-29.4	-615.8
Percentile 80	-233.6	-6025.6	-2062.2	-12923.2	6.8	-7.2
Percentile 95	-221.85	-5884.3	-2013.05	-12422.5	21.4	347.5

(3)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht	
Scenario	Hours	Flow (Total)	Hours	Flow (Total)	Hours	Flow (Total)
Mean	-233	-5961.1	-2131.3	-12213.3	-24.4	-485.4
Median	-239	-6136.5	-2205.5	-12099.5	-28.5	-541
Minimum	-267	-6671	-2809	-13577	-45	-835
Maximum	-176	-4365	-1016	-10408	8	2
St. deviation	27.717	747.288	537.339	879.839	18.957	299.242
Percentile 5	-266.55	-6663.35	-2720.8	-13434.35	-45	-819.7
Percentile 20	-249.2	-6572.4	-2575.4	-12839.2	-41	-727.4
Percentile 80	-220.2	-5352.8	-1778	-11858.2	-7.2	-273.6
Percentile 95	-187.25	-4778.55	-1297.7	-11005.15	4.4	-13.3

Table A4.9. Difference between project and reference in Q-hours and flows at selected links.

Total	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht	
Scenario	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	-37.2	-6110.72	-1204.46	-12660.22	0	-797.64
Median	-35	-6126	-1203.5	-12818	0	-587.5
Minimum	-57	-7209	-1794	-14755	-16	-14420
Maximum	-25	-4061	-535	-9994	28	2575
St. deviation	7.354	644.482	249.268	1004.027	6.931	2320.772
Percentile 5	-51.55	-6872.1	-1591.25	-14083.85	-11	-2191.5
Percentile 20	-44	-6657.4	-1379.6	-13537.8	-6	-1460.4
Percentile 80	-31	-5817.6	-1076.4	-11917	6	105.4
Percentile 95	-28.45	-5046.45	-763.4	-10776.8	7	1775.7

(1)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht	
Scenario	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	-39.85	-5856.45	-1219.1	-12182.95	-0.75	-399.85
Median	-36.5	-5953.5	-1302.5	-11986.5	0	-524
Minimum	-57	-7139	-1794	-13823	-11	-1550
Maximum	-25	-4061	-570	-9994	6	1095
St. deviation	9.016	683.863	313.285	970.243	5.250	587.533
Percentile 5	-56.05	-6853.05	-1667.65	-13542.75	-10.05	-1108.25
Percentile 20	-47.8	-6167.8	-1424.4	-13185.2	-6	-755.6
Percentile 80	-33	-5380.2	-937.4	-11829.4	5.2	127
Percentile 95	-28.8	-4894.15	-671.65	-10582.05	6	335.95

(2)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht	
Scenario	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	-34.75	-6439.8	-1191.5	-13360.95	1.3	-1351.55
Median	-31	-6496	-1186	-13168	0	-1369.5
Minimum	-47	-7209	-1343	-14755	-16	-14420
Maximum	-28	-5852	-1080	-12128	28	2575
St. deviation	5.857	380.998	74.128	655.693	9.286	3600.239
Percentile 5	-45.1	-6906.9	-1331.6	-14215.4	-11.25	-5724.65
Percentile 20	-38.8	-6723.2	-1228.4	-14027.2	-5.2	-1750.4
Percentile 80	-31	-6025.6	-1132.6	-12923.2	6	1248.4
Percentile 95	-28	-5884.3	-1097.1	-12422.5	11.85	2324.2

(3)	A20 Rotterdam-Gouda		A20 Gouda-Rotterdam		A2 Amsterdam-Utrecht	
Scenario	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	-36.8	-5961.1	-1201.1	-12213.3	-1.1	-485.4
Median	-36.5	-6136.5	-1253	-12099.5	0	-541
Minimum	-44	-6671	-1669	-13577	-6	-835
Maximum	-30	-4365	-535	-10408	7	2
St. deviation	4.614	747.288	344.423	879.839	3.872	299.242
Percentile 5	-43.55	-6663.35	-1595.65	-13434.35	-6	-819.7
Percentile 20	-40.6	-6572.4	-1503.6	-12839.2	-5.2	-727.4
Percentile 80	-33.6	-5352.8	-933.8	-11858.2	0	-273.6
Percentile 95	-30.9	-4778.55	-685.3	-11005.15	3.85	-13.3

Table A4.10 NRM RSES simulation results at the study area level for the reference situation (with congestion). (1) Input uncertainty, (2) Model uncertainty, (3) Both.

Total	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometer	Tours	Kilometer	Tours	Kilometers	Tours	Kilometers
Mean	1799497	54681137	587876	16803267	80252	8087678	62202	1996264	1964821	9108326	4494649	90676674
Median	1817301	55214599	588995	16850905	79839	8026735	61017	1960084	1952535	9034147	4497878	90810402
Minimum	1488162	44736738	507125	13919029	64595	6466979	51766	1684171	1854317	8579217	4367793	82723765
Maximum	2015207	59559751	646711	20099874	96938	9821562	73908	2369099	2195829	10437212	4608588	98586911
St. deviation	123411	2928948	32969	1569530	8653	904493	5563	179429	65419	378503	61838	3595740
Percentiles												
5	1562704	48937884	520807	14235794	67047	6733182	53523	1703347	1892528	8669481	4393399	84393567
20	1668829	53396782	562590	15525159	72760	7350515	58160	1866812	1922989	8856517	4438781	88575014
80	1889870	56826587	614723	18105972	88366	8940363	67466	2155234	1998891	9298958	4540498	93026063
95	1969681	58181408	635823	19608840	94954	9610684	71658	2303800	2104136	9835244	4597498	97037990
(1)	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometer	Tours	Kilometer	Tours	Kilometers	Tours	Kilometers
Mean	1794520	54312690	574072	16621888	78687	7859795	63082	2023838	1979909	9186814	4490270	90005026
Median	1853732	54893632	583277	16842015	74698	7459719	60048	1932398	1959851	9108946	4497407	90254098
Minimum	1495421	44736738	507125	13919029	64595	6466979	56860	1824880	1884944	8654630	4367793	82723765
Maximum	2015207	59559751	637782	20099874	96938	9647350	73908	2369099	2156474	10306091	4608588	98586911
St. deviation	163834	3851488	39194	1897066	10344	1033545	5560	171416	77563	442569	77188	4392439
Percentiles												
5	1527567	47156029	510796	14128957	66103	6632721	57974	1862652	1889908	8685982	4372808	84237115
20	1630514	51720646	540355	14828734	68518	6777401	58160	1875103	1923204	8826414	4421748	85657662
80	1935206	57626245	608540	18217858	90254	8941395	68705	2198985	2042623	9517476	4572971	93335597
95	1990336	58767890	630133	19335373	91952	9220791	71242	2271421	2150529	9972988	4594601	97142696
	9.1%	7.1%	6.8%	11.4%	13.1%	13.1%	8.8%	8.5%	3.9%	4.8%	1.7%	4.9%
(2)	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometer	Tours	Kilometer	Tours	Kilometers	Tours	Kilometers
Mean	1815040	55193363	597649	16749260	79951	8087068	60640	1945213	1946703	8999949	4499984	90974852
Median	1812935	55214599	589264	16625313	80704	8108343	60860	1951570	1950395	9007098	4499846	90832361
Minimum	1783956	54457956	564317	15412666	67213	6794797	51766	1684280	1903378	8837335	4453978	88914043
Maximum	1856117	55911351	633429	18993369	95664	9821562	68204	2237142	1986644	9208017	4548520	94706600
St. deviation	17789	379768	20573	1071119	6662	740346	5172	170264	25087	109234	27043	1536078
Percentiles												
5	1788691	54571552	565318	15538215	70583	7108249	51766	1684280	1912308	8869863	4467697	88993370
20	1797348	54947901	582271	15786524	73833	7410257	55110	1753796	1921629	8889832	4473465	89468666
80	1830008	55513738	615627	17576327	83190	8388278	64998	2114379	1972007	9088841	4529536	92217019
95	1838297	55651838	629683	18525755	89727	9416969	67695	2160360	1978860	9164079	4538993	93139669
	1.0%	0.7%	3.4%	6.4%	8.3%	9.2%	8.5%	8.8%	1.3%	1.2%	0.6%	1.7%
(3)	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometer	Tours	Kilometer	Tours	Kilometers	Tours	Kilometers
Mean	1778364	54393580	595939	17274042	83983	8544666	63567	2043219	1970882	9168104	4492737	91423612
Median	1836195	55168939	590966	17039804	82433	8604644	62997	2016035	1954423	9157317	4498014	92086253
Minimum	1488162	45675729	557105	14803516	74123	7447593	53023	1684171	1854317	8579217	4391247	83462142
Maximum	1969317	58240281	646711	20066721	96412	9637164	73701	2352701	2195829	10437212	4608370	98310453
St. deviation	163949	3757448	34081	1782187	8608	892758	5990	200159	90283	547696	80757	4635550
Percentiles												
5	1547215	48526326	557820	15092860	74192	7456855	55931	1789706	1879334	8612640	4397778	85021813
20	1636613	52313989	561888	15615528	74840	7546299	60275	1929307	1928676	8692066	4409533	86937599
80	1912103	57171689	629078	18858261	93621	9570225	66556	2194182	2002946	9381798	4570899	94512290
95	1961864	57882872	646013	19976175	95366	9610684	72986	2344510	2109344	9971246	4604815	97722063
	9.2%	6.9%	5.7%	10.3%	10.2%	10.4%	9.4%	9.8%	4.6%	6.0%	1.8%	5.1%

TableA4.11. NRM RSES simulation results at the study area level for the project situation (with congestion). (1) Input uncertainty, (2) Model uncertainty, (3) Both.

Total	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometer	Tours	Kilometer	Tours	Kilometers	Tours	Kilometers
Mean	1802838	55268062	589012	16960870	79689	8034253	62326	1998924	1961602	9084201	4495467	91346311
Median	1820990	55872393	589968	17059594	78956	7928672	61011	1963546	1948738	9006517	4498305	91599628
Minimum	1490300	45035822	507678	14031092	64208	6428478	51535	1676469	1851384	8558355	4367791	83238830
Maximum	2018121	60177525	647255	20228804	96581	9754757	73736	2363138	2193888	10421190	4608641	99209448
St. deviation	123508	3012786	33517	1588656	8633	900805	5229	169499	65823	380067	61912	3648111
Percentiles												
5	1565185	49308182	521525	14361204	66612	6692985	54475	1734887	1889396	8646481	4393385	84831356
20	1672704	53915055	563056	15542021	72670	7336959	58037	1867927	1919862	8828320	4438938	89244346
80	1893241	57542803	616216	18247927	87789	8887877	67210	2146973	1995895	9280095	4540466	93622706
95	1972508	58790921	636472	19736770	94431	9559761	71465	2297941	2101883	9817856	4597565	97662974

(1)	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometer	Tours	Kilometer	Tours	Kilometers	Tours	Kilometers
Mean	1797517	54849001	574762	16747443	78244	7815135	62863	2016486	1976909	9164987	4490296	90593052
Median	1857067	55454079	583971	16981276	74197	7408547	59842	1925467	1957154	9091044	4497370	90953599
Minimum	1497289	45035822	507678	14031092	64208	6428478	56661	1818323	1881262	8633512	4367791	83238830
Maximum	2018121	60177525	638542	20228804	96581	9612108	73736	2363138	2154575	10290359	4608641	99209448
St. deviation	163940	3946763	39228	1903109	10340	1033345	5568	171656	78051	444518	77198	4449259
Percentiles												
5	1529526	47461638	511223	14224967	65763	6598889	57772	1853668	1886671	8660892	4372793	84618146
20	1633977	52197923	541159	14950325	68069	6731938	57963	1867927	1919997	8805246	4421756	86239020
80	1937494	58205023	609182	18357230	89763	8889624	68460	2190756	2039800	9495204	4572946	93926265
95	1993484	59328420	630817	19459665	91524	9187582	71075	2265870	2148565	9957643	4594682	97747302

(2)	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometer	Tours	Kilometer	Tours	Kilometers	Tours	Kilometers
Mean	1818772	55839885	599505	16955529	79233	8023874	61280	1962968	1943214	8972826	4502003	91755082
Median	1815555	55872393	595482	16900155	79305	7928672	60909	1965673	1944817	8979484	4502905	91753501
Minimum	1786835	55037201	564782	15389678	66705	6741933	51535	1676469	1900521	8816903	4453976	89611669
Maximum	1859352	56442283	633943	19141272	95023	9754757	67937	2228859	1983207	9182946	4548503	95404037
St. deviation	17405	389503	22082	1128417	6597	731675	4473	150090	25137	105277	26884	1510559
Percentiles												
5	1791552	55119804	565783	15514707	70076	7055754	53795	1716962	1908855	8847142	4467623	89684841
20	1808701	55565606	582780	15905015	73512	7410177	58408	1847387	1918345	8864523	4475439	90171163
80	1833519	56158883	618253	18149410	82695	8337091	64756	2106172	1968596	9063596	4529532	92872415
95	1841821	56316333	630218	18668809	89130	9352224	67453	2151252	1977420	9139176	4538959	93810923

(3)	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometer	Tours	Kilometer	Tours	Kilometers	Tours	Kilometers
Mean	1781614	54962540	596528	17398405	83490	8493246	63343	2035715	1967764	9145380	4492739	92035285
Median	1839735	55769453	591613	17168612	81845	8547212	62771	2007892	1950908	9137684	4497968	92672359
Minimum	1490300	45997525	557625	14907055	73638	7397630	52795	1676594	1851384	8558355	4391226	83822174
Maximum	1972296	58871353	647255	20207903	95843	9578869	73520	2345489	2193888	10421190	4608373	98975321
St. deviation	164056	3865739	34155	1791896	8597	890814	6005	200524	90722	549386	80767	4720465
Percentiles												
5	1549673	48921948	558302	15195123	73762	7410552	55692	1781991	1876070	8589633	4397760	85466699
20	1640102	52791441	562331	15733239	74436	7508007	60038	1920905	1925032	8669628	4409586	87516348
80	1914673	57834491	629818	18982112	93218	9529590	66324	2186409	2000543	9359247	4570962	95250099
95	1964989	58543762	646614	20112823	94882	9559761	72790	2337785	2107028	9949076	4604841	98380709

Table A4.12. NRM RSES simulation results at the study area level for the reference situation (without congestion feedback). (1) Input uncertainty, (2) Model uncertainty, (3) Both.

Total	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometer	Tours	Kilometer	Tours	Kilometers	Tours	Kilometers
Mean	1826625	60984452	593900	18321583	76238	7672728	60281	1926877	1932669	8897612	4489714	97803251
Median	1846281	61726338	595558	18344532	75731	7564791	58988	1882304	1919573	8822284	4492667	98103502
Minimum	1501617	47474188	511296	15067098	61298	6128280	49907	1617461	1819115	8352550	4363904	86913470
Maximum	2047469	68010929	652403	22049244	94934	9443618	72954	2333306	2180304	10328809	4604009	107401789
St. deviation	124956	4079631	33634	1731924	8653	902089	5653	181852	70412	401593	61439	4627766
Percentiles												
5	1582625	52968691	526198	15576901	63564	6352509	51752	1639707	1854899	8420974	4388881	89007557
20	1708648	58998062	567525	16918631	69041	6925555	56178	1790884	1886788	8644402	4435316	94859604
80	1923087	64060828	620443	19712635	83955	8510849	65421	2077223	1968356	9100089	4535320	100624787
95	1996490	66070697	642616	21471479	91129	9273792	70224	2260989	2082697	9690150	4591994	105315430

(1)	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometer	Tours	Kilometer	Tours	Kilometers	Tours	Kilometers
Mean	1821100	60458559	580189	18130294	74897	7472261	61181	1954981	1948170	8978597	4485537	96994692
Median	1881418	61153027	588217	18250734	70645	7020998	58096	1861712	1930730	8881905	4492335	97690292
Minimum	1506740	47474188	511296	15067098	61298	6128280	55158	1764114	1841371	8407203	4363904	87604108
Maximum	2047469	68010929	645498	22049244	94934	9443618	72954	2333306	2142214	10205963	4604009	107401789
St. deviation	166137	5384052	40087	2063202	10505	1052185	5724	176934	84984	478108	76623	5738836
Percentiles												
5	1541360	50611231	514077	15230306	63329	6293369	55668	1788963	1853026	8409237	4369140	88108942
20	1659483	56697035	546956	16360705	64190	6373552	56337	1807026	1886611	8603569	4417177	91266998
80	1954329	65139784	615004	19747593	85439	8519980	66422	2111377	2015128	9323041	4567886	101248504
95	2021048	66823196	638077	21255813	88714	8979168	70130	2230085	2132382	9851483	4589558	105777735

(2)	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometer	Tours	Kilometer	Tours	Kilometers	Tours	Kilometers
Mean	1842750	61684891	603727	18284930	75734	7648948	58673	1874305	1913932	8785685	4494816	98278760
Median	1840050	61726338	595666	18142464	76726	7654569	58782	1867963	1916571	8786695	4494445	98142451
Minimum	1808756	60592194	570523	16876586	63464	6402660	49907	1617461	1873375	8641285	4448849	96210980
Maximum	1884721	62726130	639094	20681452	90718	9304675	66135	2159018	1952675	8984692	4543227	101973166
St. deviation	18844	548137	20477	1158562	6451	716555	5106	167304	24212	103631	27067	1562094
Percentiles												
5	1815124	60682053	571313	16950945	66556	6692002	49907	1617461	1880166	8666628	4462562	96210980
20	1824412	61361784	588252	17233002	69747	6987824	53241	1687139	1890047	8678765	4468242	96719207
80	1858993	62150335	621194	19157333	78876	7945661	63069	2044609	1938788	8872237	4524700	99589302
95	1866850	62343623	635536	20191282	84960	8902851	65592	2084902	1945558	8945178	4533837	100428179

(3)	Car driver		Car passenger		Train		BTM		Slow		Total	
Scenario	Tours	Kilometers	Tours	Kilometers	Tours	Kilometer	Tours	Kilometer	Tours	Kilometers	Tours	Kilometers
Mean	1805428	60635358	601669	18777466	79927	8121221	61698	1975813	1939140	8959495	4487862	98469354
Median	1864064	61624712	597368	18768631	77916	8112438	61135	1949956	1917100	8973988	4492876	99246123
Minimum	1501617	48782260	560464	16006438	70303	7051164	51186	1618498	1819115	8352550	4386829	86913470
Maximum	1998849	66219499	652403	21940165	91794	9327692	72600	2294371	2180304	10328809	4603627	107157124
St. deviation	165333	5196620	35470	2055766	8661	896710	6211	206009	96462	573453	80252	6092118
Percentiles												
5	1564974	52572873	560595	16084692	70584	7054763	53798	1719017	1844466	8389737	4393214	89682045
20	1667100	57682128	565917	16779381	71069	7202290	58252	1848882	1891813	8501913	4405860	93231097
80	1933923	64494671	636887	20370933	90471	9151345	64316	2115568	1976199	9128969	4565082	102610810
95	1989185	65605980	652230	21824036	91646	9286451	71620	2292728	2088906	9817145	4599569	106121107

Table A4.13. Selected link results for Reference 2020: hours and flows

Total	A2 West Eindhoven N-Z Hours	Flow (total)	A2 West Eindhoven Z-N Hours	Flow (total)	A58 Bergen op Zoom-Roosendaal Hours	Flow (total)
Mean	969.2	46,497	282.7	43,480	433.6	39,863
Median	975.5	46,606	283.0	43,479	435.5	39,980
Minimum	824.0	44,264	276.0	42,628	355.0	36,173
Maximum	1,058.0	47,749	289.0	44,107	483.0	41,656
St. deviation	47.1	649	3.0	326	26.4	1,137
Percentile 5	886.9	45,244	277.0	42,951	386.2	37,787
Percentile 20	935.4	46,136	281.0	43,191	418.6	39,349
Percentile 80	1,006.4	46,911	285.0	43,786	447.6	40,725
Percentile 95	1,028.5	47,180	287.0	43,892	471.8	41,348

(1)	A2 West Eindhoven N-Z Hours	Flow (total)	A2 West Eindhoven Z-N Hours	Flow (total)	A58 Bergen op Zoom-Roosendaal Hours	Flow (total)
Mean	960.4	46,427	283.3	43,453	433.0	39,794
Median	970.5	46,477	282.5	43,428	438.0	39,957
Minimum	824.0	44,264	276.0	42,628	355.0	36,173
Maximum	1,058.0	47,749	289.0	44,107	483.0	41,656
St. deviation	56.3	848	3.5	407	34.7	1,466
Percentile 5	874.4	44,797	277.0	42,791	373.1	37,335
Percentile 20	917.0	46,043	281.0	43,146	402.8	38,634
Percentile 80	1,003.6	47,080	286.2	43,786	463.2	40,994
Percentile 95	1,034.3	47,552	288.1	44,086	483.0	41,415

(2)	A2 West Eindhoven N-Z Hours	Flow (total)	A2 West Eindhoven Z-N Hours	Flow (total)	A58 Bergen op Zoom-Roosendaal Hours	Flow (total)
Mean	981.2	46,649	282.4	43,557	434.4	40,003
Median	977.5	46,675	283.0	43,515	435.0	39,999
Minimum	936.0	46,348	278.0	43,098	423.0	39,810
Maximum	1,053.0	47,030	285.0	43,892	440.0	40,229
St. deviation	32.8	147	2.0	231	5.2	120
Percentile 5	945.5	46,457	279.0	43,227	423.0	39,837
Percentile 20	950.0	46,543	281.0	43,398	431.8	39,899
Percentile 80	1,014.4	46,752	284.0	43,811	439.0	40,114
Percentile 95	1,018.8	46,831	285.0	43,892	440.0	40,152

(3)	A2 West Eindhoven N-Z Hours	Flow (total)	A2 West Eindhoven Z-N Hours	Flow (total)	A58 Bergen op Zoom-Roosendaal Hours	Flow (total)
Mean	962.7	46,333	282.3	43,380	433.2	39,718
Median	977.0	46,465	283.0	43,418	441.5	40,260
Minimum	859.0	44,415	276.0	42,897	362.0	36,387
Maximum	1,023.0	47,112	287.0	43,838	469.0	41,286
St. deviation	51.1	798	3.6	300	34.7	1,544
Percentile 5	884.7	45,106	276.5	42,969	379.6	37,262
Percentile 20	928.0	46,034	279.4	43,135	410.6	38,815
Percentile 80	1,004.2	46,944	284.4	43,650	467.2	40,932
Percentile 95	1,016.7	47,040	286.6	43,770	468.6	41,169

Table A4.14. Selected link results for Reference 2020: Q-hours and flows

Total	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal	
	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	470.7	46,497	5.7	43,480	41.5	39,863
Median	472.5	46,606	5.0	43,479	41.0	39,980
Minimum	398.0	44,264	2.0	42,628	0.0	36,173
Maximum	514.0	47,749	11.0	44,107	77.0	41,656
St. deviation	24.1	649	2.1	326	15.9	1,137
Percentile 5	427.8	45,244	3.0	42,951	13.1	37,787
Percentile 20	452.8	46,136	5.0	43,191	30.0	39,349
Percentile 80	489.6	46,911	7.0	43,786	50.0	40,725
Percentile 95	502.2	47,180	9.0	43,892	68.6	41,348

(1)	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal	
	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	466.3	46,427	5.8	43,453	41.6	39,794
Median	472.0	46,477	5.0	43,428	41.0	39,957
Minimum	398.0	44,264	3.0	42,628	0.0	36,173
Maximum	514.0	47,749	9.0	44,107	77.0	41,656
St. deviation	28.3	848	1.6	407	20.8	1,466
Percentile 5	419.9	44,797	3.0	42,791	4.8	37,335
Percentile 20	448.6	46,043	5.0	43,146	25.8	38,634
Percentile 80	488.2	47,080	7.0	43,786	58.8	40,994
Percentile 95	504.5	47,552	9.0	44,086	71.3	41,415

(2)	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal	
	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	477.1	46,649	5.4	43,557	41.1	40,003
Median	474.5	46,675	5.0	43,515	41.0	39,999
Minimum	452.0	46,348	2.0	43,098	30.0	39,810
Maximum	513.0	47,030	11.0	43,892	47.0	40,229
St. deviation	17.9	147	2.6	231	4.5	120
Percentile 5	455.8	46,457	2.0	43,227	30.0	39,837
Percentile 20	460.6	46,543	3.0	43,398	40.6	39,899
Percentile 80	493.0	46,752	7.0	43,811	45.0	40,114
Percentile 95	500.7	46,831	11.0	43,892	45.1	40,152

(3)	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal	
	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	466.9	46,333	6.4	43,380	42.5	39,718
Median	476.0	46,465	7.0	43,418	43.0	40,260
Minimum	416.0	44,415	3.0	42,897	3.0	36,387
Maximum	498.0	47,112	9.0	43,838	69.0	41,286
St. deviation	25.4	798	1.8	300	20.5	1,544
Percentile 5	428.6	45,106	3.9	42,969	12.0	37,262
Percentile 20	448.8	46,034	5.0	43,135	28.6	38,815
Percentile 80	485.0	46,944	8.0	43,650	62.6	40,932
Percentile 95	494.0	47,040	8.6	43,770	67.2	41,169

Table A4.15. Selected link results for project situation 2020: hours and flows

Total	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal		Oostelijke Randweg Eindhoven N-Z		Oostelijke Randweg Eindhoven Z-N	
	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)
Mean	487.2	44,976	273.1	42,755	431.5	39,797	370.1	43,699	312.5	44,987
Median	485.0	45,085	274.0	42,883	434.0	39,978	373.5	43,874	315.0	45,123
Minimum	440.0	43,053	265.0	41,475	352.0	35,840	288.0	40,355	256.0	42,653
Maximum	524.0	45,990	277.0	43,418	477.0	41,731	401.0	45,046	349.0	46,146
St. deviation	17.7	550	2.7	425	26.6	1,234	21.6	877	21.4	706
Percentile 5	460.8	43,870	267.9	41,965	384.9	37,648	328.4	42,201	274.6	43,806
Percentile 20	475.8	44,665	271.0	42,441	413.0	39,157	361.8	43,350	294.6	44,643
Percentile 80	504.2	45,357	275.0	43,090	453.0	40,820	384.0	44,257	329.2	45,558
Percentile 95	511.6	45,601	277.0	43,296	469.1	41,289	393.1	44,562	338.0	45,819

(1)	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal		Oostelijke Randweg Eindhoven N-Z		Oostelijke Randweg Eindhoven Z-N	
	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)
Mean	483.4	44,898	272.9	42,726	429.7	39,706	366.3	43,616	311.4	44,952
Median	480.5	45,039	273.5	42,841	433.0	39,931	370.0	43,772	315.5	45,111
Minimum	440.0	43,053	265.0	41,475	352.0	35,840	288.0	40,355	256.0	42,653
Maximum	509.0	45,990	277.0	43,418	477.0	41,731	401.0	45,046	349.0	46,146
St. deviation	18.2	705	3.4	535	34.5	1,578	26.8	1,127	27.2	896
Percentile 5	461.9	43,646	266.9	41,840	373.9	37,294	312.7	41,689	270.3	43,565
Percentile 20	468.8	44,524	270.8	42,338	398.8	38,287	352.6	43,063	287.8	44,233
Percentile 80	501.8	45,406	276.0	43,231	463.6	41,219	384.0	44,470	336.2	45,709
Percentile 95	508.1	45,666	277.0	43,322	470.4	41,387	394.4	44,990	349.0	45,875

(2)	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal		Oostelijke Randweg Eindhoven N-Z		Oostelijke Randweg Eindhoven Z-N	
	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)
Mean	490.0	45,089	273.6	42,826	433.8	39,939	374.4	43,860	315.4	45,049
Median	487.5	45,085	274.0	42,886	434.0	39,957	376.0	43,881	315.0	45,123
Minimum	469.0	44,672	272.0	42,555	425.0	39,541	356.0	43,655	298.0	44,692
Maximum	512.0	45,356	275.0	42,978	444.0	40,307	386.0	44,056	328.0	45,307
St. deviation	12.2	149	0.9	131	5.6	196	8.8	103	10.7	195
Percentile 5	470.9	44,922	272.0	42,602	426.0	39,602	359.8	43,708	301.8	44,747
Percentile 20	482.8	45,002	272.8	42,719	428.8	39,807	366.4	43,755	303.8	44,878
Percentile 80	500.6	45,205	274.0	42,926	439.0	40,081	381.2	43,945	328.0	45,242
Percentile 95	511.1	45,252	275.0	42,971	441.2	40,173	385.1	43,991	328.0	45,268

(3)	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal		Oostelijke Randweg Eindhoven N-Z		Oostelijke Randweg Eindhoven Z-N	
	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)
Mean	489.4	44,906	272.7	42,673	430.4	39,696	369.3	43,544	309.0	44,934
Median	489.0	45,093	273.5	42,785	438.5	40,455	375.0	43,759	310.0	45,249
Minimum	445.0	43,215	266.0	41,566	355.0	36,021	300.0	40,513	257.0	42,778
Maximum	524.0	45,600	277.0	43,295	471.0	41,280	394.0	44,499	338.0	45,789
St. deviation	25.1	708	3.6	566	35.4	1,696	27.9	1,178	25.8	959
Percentile 5	451.3	43,791	267.4	41,804	377.5	36,930	324.3	41,681	272.3	43,416
Percentile 20	474.2	44,550	269.8	42,279	405.8	38,707	361.2	43,174	292.6	44,333
Percentile 80	511.0	45,406	276.0	43,162	459.4	40,945	391.2	44,406	334.4	45,720
Percentile 95	520.0	45,551	276.6	43,266	466.5	41,166	393.1	44,472	337.1	45,776

Table A4.16. Selected link results for project situation 2020: Q-hours and flows

Total	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal		Oostelijke Randweg Eindhoven N-Z		Oostelijke Randweg Eindhoven Z-N	
	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	141.3	44,976	0.0	42,755	40.1	39,797	125.7	43,699	55.8	44,987
Median	141.5	45,085	0.0	42,883	41.0	39,978	128.0	43,874	56.0	45,123
Minimum	119.0	43,053	0.0	41,475	0.0	35,840	62.0	40,355	31.0	42,653
Maximum	160.0	45,990	0.0	43,418	71.0	41,731	150.0	45,046	72.0	46,146
St. deviation	10.0	550	0.0	425	14.9	1,234	17.2	877	9.7	706
Percentile 5	125.9	43,870	0.0	41,965	14.4	37,648	92.6	42,201	41.0	43,806
Percentile 20	133.8	44,665	0.0	42,441	30.0	39,157	117.6	43,350	48.6	44,643
Percentile 80	149.4	45,357	0.0	43,090	49.4	40,820	138.0	44,257	65.0	45,558
Percentile 95	157.7	45,601	0.0	43,296	63.1	41,289	143.6	44,562	68.6	45,819

(1)	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal		Oostelijke Randweg Eindhoven N-Z		Oostelijke Randweg Eindhoven Z-N	
	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	139.3	44,898	0.0	42,726	39.0	39,706	122.7	43,616	54.6	44,952
Median	137.5	45,039	0.0	42,841	39.5	39,931	127.0	43,772	54.5	45,111
Minimum	119.0	43,053	0.0	41,475	0.0	35,840	62.0	40,355	31.0	42,653
Maximum	159.0	45,990	0.0	43,418	71.0	41,731	150.0	45,046	72.0	46,146
St. deviation	11.3	705	0.0	535	19.4	1,578	21.2	1,127	11.5	896
Percentile 5	126.6	43,646	0.0	41,840	8.6	37,294	79.1	41,689	40.5	43,565
Percentile 20	127.8	44,524	0.0	42,338	22.6	38,287	112.4	43,063	44.2	44,233
Percentile 80	149.4	45,406	0.0	43,231	55.2	41,219	136.0	44,470	65.6	45,709
Percentile 95	159.0	45,666	0.0	43,322	67.2	41,387	148.1	44,990	69.2	45,875

(2)	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal		Oostelijke Randweg Eindhoven N-Z		Oostelijke Randweg Eindhoven Z-N	
	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	142.6	45,089	0.0	42,826	41.1	39,939	128.9	43,860	58.1	45,049
Median	143.0	45,085	0.0	42,886	41.0	39,957	129.5	43,881	58.0	45,123
Minimum	130.0	44,672	0.0	42,555	34.0	39,541	111.0	43,655	49.0	44,692
Maximum	155.0	45,356	0.0	42,978	49.0	40,307	141.0	44,056	67.0	45,307
St. deviation	6.5	149	0.0	131	4.3	196	8.7	103	5.5	195
Percentile 5	130.0	44,922	0.0	42,602	35.9	39,602	114.8	43,708	50.9	44,747
Percentile 20	138.8	45,002	0.0	42,719	37.6	39,807	121.0	43,755	52.0	44,878
Percentile 80	147.0	45,205	0.0	42,926	45.0	40,081	135.6	43,945	63.2	45,242
Percentile 95	151.2	45,252	0.0	42,971	49.0	40,173	140.1	43,991	66.1	45,268

(3)	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal		Oostelijke Randweg Eindhoven N-Z		Oostelijke Randweg Eindhoven Z-N	
	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	142.9	44,906	0.0	42,673	40.2	39,696	125.5	43,544	53.5	44,934
Median	145.5	45,093	0.0	42,785	40.0	40,455	130.5	43,759	51.0	45,249
Minimum	119.0	43,215	0.0	41,566	2.0	36,021	73.0	40,513	32.0	42,778
Maximum	160.0	45,600	0.0	43,295	64.0	41,280	144.0	44,499	69.0	45,789
St. deviation	13.1	708	0.0	566	19.2	1,696	21.3	1,178	12.1	959
Percentile 5	121.7	43,791	0.0	41,804	11.5	36,930	90.6	41,681	37.4	43,416
Percentile 20	134.6	44,550	0.0	42,279	28.6	38,707	119.2	43,174	45.6	44,333
Percentile 80	153.6	45,406	0.0	43,162	58.4	40,945	142.2	44,406	65.2	45,720
Percentile 95	158.2	45,551	0.0	43,266	62.2	41,166	143.6	44,472	67.7	45,776

Table A4.17. Difference between project and reference situation in hours and flows at selected links.

Total	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal	
	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)
Mean	-482.0	-1,521	-9.6	-724	-2.1	-65
Median	-484.0	-1,560	-10.0	-705	-2.5	-47
Minimum	-590.0	-1,893	-14.0	-1,339	-17.0	-385
Maximum	-384.0	-1,148	-4.0	-219	18.0	329
St. deviation	42.6	173	2.0	250	5.8	172
Percentile 5	-544.6	-1,761	-12.6	-1,119	-9.1	-332
Percentile 20	-517.2	-1,647	-11.0	-918	-6.0	-224
Percentile 80	-447.8	-1,350	-8.0	-522	2.2	73
Percentile 95	-414.0	-1,205	-6.5	-330	6.0	200

(1)	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal	
	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)
Mean	-477.1	-1,529	-10.4	-727	-3.3	-88
Median	-484.5	-1,584	-10.5	-786	-3.5	-59
Minimum	-590.0	-1,893	-13.0	-1,153	-17.0	-385
Maximum	-384.0	-1,148	-7.0	-219	7.0	329
St. deviation	52.1	204	1.6	246	5.8	193
Percentile 5	-555.8	-1,770	-12.1	-970	-16.1	-336
Percentile 20	-509.2	-1,680	-12.0	-912	-6.0	-279
Percentile 80	-432.0	-1,337	-9.0	-516	1.0	22
Percentile 95	-406.8	-1,208	-8.0	-323	6.1	206

(2)	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal	
	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)
Mean	-491.2	-1,560	-8.9	-730	-0.6	-64
Median	-486.5	-1,580	-9.0	-705	-1.0	-59
Minimum	-569.0	-1,784	-11.0	-1,021	-10.0	-330
Maximum	-439.0	-1,250	-6.0	-490	18.0	160
St. deviation	34.4	132	1.6	165	6.2	137
Percentile 5	-532.0	-1,753	-11.0	-1,000	-8.1	-325
Percentile 20	-523.4	-1,651	-10.0	-865	-6.0	-195
Percentile 80	-461.0	-1,429	-7.0	-583	3.2	56
Percentile 95	-449.5	-1,408	-6.0	-507	6.6	103

(3)	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal	
	Hours	Flow (total)	Hours	Flow (total)	Hours	Flow (total)
Mean	-473.3	-1,427	-9.6	-707	-2.8	-22
Median	-483.0	-1,478	-9.5	-571	-3.5	-25
Minimum	-528.0	-1,620	-14.0	-1,339	-8.0	-366
Maximum	-414.0	-1,190	-4.0	-302	4.0	303
St. deviation	36.4	160	3.0	395	4.6	201
Percentile 5	-515.9	-1,607	-14.0	-1,335	-7.6	-332
Percentile 20	-499.4	-1,565	-10.8	-1,129	-7.0	-108
Percentile 80	-446.4	-1,270	-8.6	-399	2.2	118
Percentile 95	-416.7	-1,195	-5.4	-324	3.6	256

Table A4.18. Difference between project and reference in Q-hours and flows at selected links.

Total	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal	
	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	-329.4	-1,521	-5.7	-724	-1.5	-65
Median	-329.5	-1,560	-5.0	-705	-2.0	-47
Minimum	-395.0	-1,893	-11.0	-1,339	-15.0	-385
Maximum	-271.0	-1,148	-2.0	-219	19.0	329
St. deviation	23.8	173	2.1	250	5.3	172
Percentile 5	-363.4	-1,761	-9.0	-1,119	-8.1	-332
Percentile 20	-348.0	-1,647	-7.0	-918	-5.0	-224
Percentile 80	-312.8	-1,350	-5.0	-522	2.0	73
Percentile 95	-290.5	-1,205	-3.0	-330	6.1	200

(1)	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal	
	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	-327.0	-1,529	-5.8	-727	-2.6	-88
Median	-328.5	-1,584	-5.0	-786	-2.5	-59
Minimum	-395.0	-1,893	-9.0	-1,153	-15.0	-385
Maximum	-271.0	-1,148	-3.0	-219	8.0	329
St. deviation	30.5	204	1.6	246	5.4	193
Percentile 5	-372.2	-1,770	-9.0	-970	-13.1	-336
Percentile 20	-345.6	-1,680	-7.0	-912	-5.2	-279
Percentile 80	-300.2	-1,337	-5.0	-516	1.2	22
Percentile 95	-281.5	-1,208	-3.0	-323	5.2	206

(2)	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal	
	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	-334.5	-1,560	-5.4	-730	0.1	-64
Median	-333.0	-1,580	-5.0	-705	0.0	-59
Minimum	-373.0	-1,784	-11.0	-1,021	-6.0	-330
Maximum	-310.0	-1,250	-2.0	-490	19.0	160
St. deviation	17.6	132	2.6	165	5.4	137
Percentile 5	-355.0	-1,753	-11.0	-1,000	-5.1	-325
Percentile 20	-349.0	-1,651	-7.0	-865	-4.0	-195
Percentile 80	-316.8	-1,429	-3.0	-583	2.4	56
Percentile 95	-312.9	-1,408	-2.0	-507	4.8	103

(3)	A2 West Eindhoven N-Z		A2 West Eindhoven Z-N		A58 Bergen op Zoom-Roosendaal	
	Q-hours	Flow (total)	Q-hours	Flow (total)	Q-hours	Flow (total)
Mean	-324.0	-1,427	-6.4	-707	-2.3	-22
Median	-327.0	-1,478	-7.0	-571	-2.5	-25
Minimum	-352.0	-1,620	-9.0	-1,339	-9.0	-366
Maximum	-291.0	-1,190	-3.0	-302	7.0	303
St. deviation	19.3	160	1.8	395	4.8	201
Percentile 5	-347.5	-1,607	-8.6	-1,335	-8.1	-332
Percentile 20	-339.6	-1,565	-8.0	-1,129	-7.0	-108
Percentile 80	-310.0	-1,270	-5.0	-399	1.2	118
Percentile 95	-294.2	-1,195	-3.9	-324	4.7	256

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