Uncertainty Shocks in a Model of Effective Demand*

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Abstract

Can increased uncertainty about the future cause a contraction in output and its components? An identified uncertainty shock in the data causes significant declines in output, consumption, investment, and hours worked. Standard general-equilibrium models with flexible prices cannot reproduce this comovement. However, uncertainty shocks can easily generate comovement with countercyclical markups through sticky prices. Monetary policy plays a key role in offsetting the negative impact of uncertainty shocks during normal times. Higher uncertainty has even more negative effects if monetary policy can no longer perform its usual stabilizing function because of the zero lower bound. We calibrate our uncertainty shock process using fluctuations in implied stock market volatility and show that the model with nominal price rigidity is consistent with empirical evidence from a structural vector autoregression. We argue that increased uncertainty about the future likely played a role in worsening the Great Recession.

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1 Introduction

Economists and the financial press often discuss uncertainty about the future as an important driver of economic fluctuations, and a contributor in the Great Recession and subsequent slow recovery. For example, Diamond (2010) says, "What's critical right now is not the functioning of the labor market, but the limits on the demand for labor coming from the great caution on the side of both consumers and firms because of the great uncertainty of what's going to happen next." Recent research by Bloom (2009), Bloom et al. (2014), Fernández-Villaverde et al. (2013), Born and Pfeifer (2014), and Gilchrist, Sim and Zakrajšek (2013) also suggests that uncertainty shocks can cause fluctuations in macroeconomic aggregates. However, most of these papers experience difficulty in generating business-cycle comovements among output, consumption, investment, and hours worked from changes in uncertainty.

We argue that this macroeconomic comovement is a key empirical feature of the economy's response to an uncertainty shock. Using a structural vector autoregression (VAR), we identify an uncertainty shock in the data as an exogenous increase in the implied volatility of future stock returns, an identification strategy that is consistent with our theoretical model. Empirically, an uncertainty shock causes statistically significant declines in output, consumption, investment, and hours, with a peak response occurring after about one year. A one standard deviation increase in uncertainty produces a peak decline in output of about 0.2 percent. Based on this empirical evidence, we view this macroeconomic comovement as a key minimum condition for business-cycle models driven by uncertainty fluctuations.

After presenting this stylized fact, we show why competitive, one-sector, closed-economy models generally cannot reproduce this comovement in response to changes in uncertainty. Under reasonable assumptions, an increase in uncertainty about the future induces precautionary saving and lower consumption. If households supply labor inelastically, then total output remains constant since the level of technology and capital stock remain unchanged in response to the uncertainty shock. Unchanged total output and reduced consumption together imply that investment must rise. If households can adjust their labor supply and consumption and leisure are both normal goods, an increase in uncertainty also induces "precautionary labor supply," or a desire for the household to supply more labor for any given level of the real wage. As current technology and the capital stock remain unchanged, the competitive demand for labor remains unchanged as well. Thus, higher uncertainty reduces consumption but raises output, investment, and hours worked. This lack of comovement is a robust prediction of simple neoclassical models subject to uncertainty fluctuations.

We also show that non-competitive, one-sector models with countercyclical markups through sticky prices can easily generate macroeconomic comovement after an uncertainty shock. An increase in uncertainty induces precautionary labor supply by the representative household, which reduces firm marginal costs of production. Falling marginal costs with slowly-adjusting prices imply an increase in firm markups over marginal cost. A higher markup reduces the demand for consumption, and especially, investment goods. Since output is demand-determined in these models, output and employment must fall when consumption and investment both decline. Thus, comovement is restored, and uncertainty shocks cause fluctuations that are consistent with our empirical evidence. Returning to Diamond's (2010) intuition, simple competitive business-cycle models do not exhibit movements in "the demand for labor" as a result of an uncertainty shock. However, uncertainty shocks easily cause fluctuations in the demand for labor in non-competitive, sticky-price models with endogenously-varying markups. Thus, the non-competitive model captures the intuition articulated by Diamond. Understanding the dynamics of the demand for labor explains why the two models behave so differently in response to a change in uncertainty.

Importantly, the non-competitive model is able to match the estimated effects of uncertainty shocks in the data. To analyze the quantitative impact of uncertainty shocks, we calibrate and solve a representative-agent, dynamic, stochastic general equilibrium (DSGE) model with capital accumulation and nominal price rigidity. We examine uncertainty shocks to household discount factors, which we interpret as demand uncertainty. We calibrate our uncertainty shock processes using the Chicago Board Options Exchange Volatility Index (VXO), which measures the expected volatility of the Standard and Poor's 100 stock index over the next thirty days. Using a third-order approximation to the model policy functions, we show that uncertainty shocks can produce contractions in output and all its components when prices adjust slowly. In particular, the declines in output, hours, consumption, and investment in the model are consistent with our empirical evidence. Importantly, we also show that our identifying assumptions in our empirical VAR are fully supported by our theoretical model.

Finally, we examine the role of monetary policy in determining the equilibrium effects of uncertainty shocks. Standard monetary policy rules imply that the central bank usually offsets increases in uncertainty by lowering its nominal policy rate. We show that increases in uncertainty have larger negative effects on the economy if the monetary authority is constrained by the zero lower bound on nominal interest rates. In these circumstances, our model predicts that an increase in uncertainty causes a much larger decline in output and its components. The

sharp increase in uncertainty during the financial crisis in late 2008 corresponds to a period when the Federal Reserve had a policy rate near zero. Thus, we believe that greater uncertainty may have plausibly contributed significantly to the large and persistent output decline starting at that time. Our results suggest that about one-fifth of the drop in output that occurred in late 2008 can plausibly be ascribed to increased uncertainty about the future.

Our emphasis on understanding the effects of uncertainty in a one-sector model does not deprecate alternative modeling strategies. For example, Bloom et al. (2014) examine changes in uncertainty in a heterogeneous-firm model with convex and non-convex adjustment costs. However, this complex model is unable to generate positive comovement of the four key macro aggregates following an uncertainty shock. Furthermore, heterogeneous-agent models are challenging technically to extend along other dimensions. For example, adding nominal price rigidity for each firm and a zero lower bound constraint on nominal interest rates would be difficult in the model of Bloom et al. (2014). We view our work as a complementary approach to modeling the business-cycle effects of uncertainty. The simplicity of our underlying framework allows us to tackle additional issues that we think are important for understanding the Great Recession.

2 Empirical Evidence

This section presents our key stylized fact: higher uncertainty about the future causes declines in output, consumption, investment, and hours worked. To document this feature of the data, we estimate a VAR with the following variables: a measure of uncertainty, gross domestic product (GDP), consumption, investment, hours worked, the GDP deflator, the M2 money stock, and a measure of the stance of monetary policy. We use the Chicago Board Options Exchange Volatility Index (VXO) as our observable measure of aggregate uncertainty for several reasons. The VXO is widely used in financial markets, it is easy to observe, and it maps exactly to a counterpart in our theoretical model. Most importantly for our purposes, the VXO is a *forward-looking* measure of the implied volatility of the Standard and Poor's 100 stock index. Uncertainty correctly defined is an ex ante concept, however, Bloom (2009) and others often use ex post measures of volatility when forward-looking measures are unavailable.

Since the data for the VXO begins in 1986, we estimate our baseline empirical model using quarterly data over the 1986-2014 sample period. With the exception of the monetary policy measure, all other variables enter the VAR in log levels. Figure 1 plots the time series of the VXO over time. Appendix A.1 provides further details on the data construction and additional responses for our baseline empirical model.

We identify an uncertainty shock using a Cholesky decomposition with the VXO ordered first. This ordering assumes that uncertainty shocks can have an immediate impact on output and its components. However, our identification scheme also assumes that the other non-uncertainty shocks do not affect implied stock market volatility on impact. In Section 7.5, we show that our theoretical model fully supports this identification strategy: First-moment or non-uncertainty shocks in the model have almost no effect on the expected volatility of future equity returns.[1]

Figure 2 plots the estimated responses to an identified uncertainty shock along with the 95% confidence intervals. A one-standard deviation uncertainty shock increases the level of the VXO to about 24.5%, from its unconditional average of about 21%. At impact, higher uncertainty causes statistically significant declines in output, consumption, and investment. After the initial shock, output, consumption, investment, and hours all decline together with their peak response occurring after about one year. The peak decline in investment is roughly twice as large as the decline in total output, while consumption moves by slightly less than output. About two years after the initial shock, the impulse responses are statistically indistinguishable from zero. The bottom panel of Figure 1 shows the time-series of the identified uncertainty shocks. The empirical model identifies large uncertainty shocks after the 1987 stock market crash, the failure of Lehman brothers, and the euro area sovereign debt crisis.[2]

Based on this empirical evidence, we argue that this macroeconomic comovement is a key litmus test for models of uncertainty fluctuations. In the following sections, we show that a standard model with nominal price rigidity is consistent with this empirical evidence, while the same model with flexible prices is not. Using our theoretical model, we show that monetary policy plays a key role in determining the equilibrium effects of higher uncertainty. At the end of 2008, the Federal Reserve became constrained by the zero lower bound on nominal interest rates. After that time, the central bank relied on less conventional policy tools to help stabilize the economy. In the later sections, we discuss this issue in detail using our theoretical model. From an econometric standpoint, however, it is less clear how to empirically model the stance of monetary policy over our 1986-2014 sample period. In our baseline VAR results, we used

[1] Appendix A.2 shows that our key stylized fact, macroeconomic comovment following an uncertainty shock, is robust to ordering the VXO last in our structural VAR. However, this identification scheme is not consistent with our theoretical model.

[2] Our results are quantitatively similar to the findings of Alexopoulos and Cohen (2009) and Jurado, Ludvigson and Ng (2015). These papers find that higher uncertainty decreases several monthly indicators of economic activity.

the Wu and Xia (2014) shadow rate as our indicator of monetary policy. Away from the zero lower bound, this series equals the federal funds rate. But at the zero lower bound, the shadow rate uses information from the entire yield curve to summarize the stance of monetary policy. However, this modeling choice is clearly not the only reasonable one. In Appendix A.2, we show that our stylized fact is robust to using different measures of monetary policy, different sample periods, alternative variable definitions, and using higher frequency estimation. In particular, we show that our key stylized fact of comovement survives even if we restrict our analysis to the pre-Great Recession sample period.

3 Intuition

The previous section argues that macroeconomic comovement is a robust empirical feature of the economy's response to an uncertainty shock. We now examine the ability of standard macroeconomic models to generate this comovement in response to uncertainty fluctuations. Using a few key equations that characterize a large class of one-sector business cycle models, we show that the causal ordering of these equations plays an important role in understanding the impact of uncertainty shocks. These equations link total output Y_t, household consumption C_t, investment I_t, hours worked N_t, and the real wage W_t/P_t. These equations comprise the national income accounts identity, an aggregate production function, a first-order labor supply condition for the representative household, and a first-order condition for labor demand by firms:

$$
\begin{aligned}
Y_t &= C_t + I_t, && (1)\\
Y_t &= F(K_t, Z_t N_t), && (2)\\
\frac{W_t}{P_t} U_1(C_t, 1 - N_t) &= U_2(C_t, 1 - N_t), && (3)\\
\frac{W_t}{P_t} &= Z_t F_2(K_t, Z_t N_t). && (4)
\end{aligned}
$$

Typical partial-equilibrium results suggest that an increase in uncertainty about the future should decrease both consumption and investment. When consumers face a stochastic income stream, higher uncertainty about the future induces precautionary saving by risk-averse households. Recent work by Bloom (2009) argues that an increase in uncertainty also depresses investment, particularly in the presence of non-convex costs of adjustment. If an increase in uncertainty lowers consumption and investment in partial equilibrium, Equation (1) suggests that it should lower total output in a general-equilibrium model. In a setting where output is demand-determined, economic intuition suggests that higher uncertainty should depress total

output and its components.

However, the previous intuition is incorrect in a general-equilibrium neoclassical model with a representative firm and a consumer with additively time-separable preferences. In this neoclassical setting, labor demand in Equation (4) is determined by the current level of capital K_t and technology Z_t, neither of which change when uncertainty increases. The first-order conditions for labor supply and labor demand can be combined to yield:

$$Z_t F_2(K_t, Z_t N_t) U_1(C_t, 1 - N_t) = U_2(C_t, 1 - N_t). \tag{5}$$

Equation (5) defines a positively-sloped "income expansion path" for consumption and leisure for given levels of capital and technology. If higher uncertainty reduces consumption, then Equation (5) shows that increased uncertainty must increase labor supply. However, Equation (2) implies that total output must rise. A reduction in consumption and an increase in total output in Equation (1) means that investment and consumption must move in opposite directions.[3]

In a non-neoclassical setting, Equations (1) and (3) continue to apply, but the first-order condition for labor demand now depends on the markup μ_t of price over marginal cost. Thus, Equations (4) and (5) are modified as follows:

$$\frac{W_t}{P_t} = \frac{1}{\mu_t} Z_t F_2(K_t, Z_t N_t), \tag{6}$$

$$\frac{1}{\mu_t} Z_t F_2(K_t, Z_t N_t) U_1(C_t, 1 - N_t) = U_2(C_t, 1 - N_t). \tag{7}$$

In such a setting, Equation (1) is causally prior to Equations (2) and (3). From Equation (1), output is determined by aggregate demand. Equation (2) then determines the necessary quantity of labor input for given values of K_t and Z_t. Finally, given C_t (determined by demand and other factors), the necessary supply of labor is made consistent with consumer optimization by having the markup taking on its required value. Alternatively, the wage moves to the level necessary for firms to hire the required quantity of labor, and the variable markup ensures that the wage can move independently of the marginal product of labor.

The previous intuition can also be represented graphically using simplified labor supply and labor demand curves with the real wage and hours worked on the axes. Figures 3 and 4 show the impact of an increase in uncertainty under both flexible prices with constant markups and sticky prices with endogenously-varying markups. An increase in uncertainty induces a

[3]This argument follows Barro and King (1984). Jaimovich (2008) shows that this prediction may not hold for certain classes of preferences that are not additively time-separable.

wealth effect on the representative household through the forward-looking marginal utility of wealth denoted by $\lambda_t = U_1(C_t, 1 - N_t)$. An increase in the marginal utility of wealth shifts the household labor supply curve outward. With flexible prices and constant markups, the labor demand curve remains fixed. In the flexible-price equilibrium, the desire of households to supply more labor translates into higher equilibrium hours worked and a lower real wage. When prices adjust slowly to changing marginal costs, however, firm markups over marginal cost rise when the household increases its labor supply. For a given level of the real wage, an increase in the markup decreases the demand for labor from firms. Figure 4 shows that equilibrium hours worked may fall as a result of the outward shift in the labor supply curve and the inward shift of the labor demand curve. The relative magnitudes of the changes in labor supply and labor demand depend on the specifics of the macroeconomic model and its parameter values. The following section shows that in a reasonably calibrated New-Keynesian sticky price model, firm markups increase enough to produce a decrease in equilibrium hours worked in response to a rise in uncertainty.

4 Model

This section outlines the baseline dynamic, stochastic general-equilibrium model that we use in our analysis of uncertainty shocks. Our model provides a specific quantitative example formalizing the general intuition of the previous section. The baseline model shares many features of the models of Ireland (2003), Ireland (2011), and Jermann (1998). The model features optimizing households and firms and a central bank that follows a Taylor rule to stabilize inflation and offset adverse shocks. We allow for sticky prices using the quadratic-adjustment costs specification of Rotemberg (1982). Our baseline model considers household discount rate shocks. These shocks have a time-varying second moment, which we interpret as the degree of uncertainty about future demand.

4.1 Households

In our model, the representative household maximizes lifetime utility given Epstein-Zin preferences over streams of consumption C_t and leisure $1 - N_t$. The key parameters governing household decisions are its risk aversion σ over the consumption-leisure basket and its intertemporal elasticity of substitution ψ. The parameter $\theta_V \triangleq (1 - \sigma)(1 - 1/\psi)^{-1}$ controls the household's preference for the resolution of uncertainty.[4] The household receives labor income

[4]Our main qualitative results are robust to using standard expected utility preferences. Epstein-Zin preferences allow us to calibrate our model using stock market data. Section 6 explains the details of our calibration method and discusses the role of risk aversion.

W_t for each unit of labor N_t supplied to the representative intermediate goods-producing firm. The representative household also owns the intermediate goods firm and holds equity shares S_t and one-period riskless bonds B_t issued by representative intermediate goods firm. Equity shares have a price of P_t^E and pay dividends D_t^E for each share S_t owned. The riskless bonds return the gross one-period risk-free interest rate R_t^R. The household divides its income from labor and its financial assets between consumption C_t and holdings of financial assets S_{t+1} and B_{t+1} to carry into next period. The discount rate of the household β is subject to shocks via the stochastic process a_t.

In principle, uncertainty can affect any exogenous variable in the model, such as technology. However, discussions of the Great Recession do not propose technological ferment as a significant source of uncertainty during that period. Instead, much of the commentary of the time discusses firms' uncertainty regarding the demand for their output. This discussion motivates us to ask whether such uncertainty contributed significantly to the depth of the recession and the slow pace of the recovery. Since our model is a standard dynamic general-equilibrium model without a government, any non-technological (demand) shocks must come from changes in preferences. We thus interpret changes in the household discount factor as demand shocks hitting the economy, and model uncertainty about demand as a change in the *ex ante* volatility of such shocks.

The representative household maximizes lifetime utility by choosing C_{t+s}, N_{t+s}, B_{t+s+1}, and S_{t+s+1} for all $s = 0, 1, 2, \ldots$ by solving the following problem:

$$V_t = \max \left[a_t \left(C_t^{\eta} (1 - N_t)^{1-\eta} \right)^{\frac{1-\sigma}{\theta_V}} + \beta \left(\mathbb{E}_t V_{t+1}^{1-\sigma} \right)^{\frac{1}{\theta_V}} \right]^{\frac{\theta_V}{1-\sigma}}$$

subject to its intertemporal household budget constraint each period,

$$C_t + \frac{P_t^E}{P_t} S_{t+1} + \frac{1}{R_t^R} B_{t+1} \leq \frac{W_t}{P_t} N_t + \left(\frac{D_t^E}{P_t} + \frac{P_t^E}{P_t} \right) S_t + B_t.$$

Using a Lagrangian approach, household optimization implies the following first-order conditions:

$$\frac{\partial V_t}{\partial C_t} = \lambda_t \tag{8}$$

$$\frac{\partial V_t}{\partial N_t} = \lambda_t \frac{W_t}{P_t} \tag{9}$$

$$\frac{P_t^E}{P_t} = \mathbb{E}_t \left\{ \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left(\frac{D_{t+1}^E}{P_{t+1}} + \frac{P_{t+1}^E}{P_{t+1}} \right) \right\} \tag{10}$$

$$1 = R_t^R \mathbb{E}_t \left\{ \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \right\} \tag{11}$$

where λ_t denotes the Lagrange multiplier on the household budget constraint. Epstein-Zin utility implies the following stochastic discount factor M between periods t and $t+s$:

$$M_{t+s} \triangleq \left(\frac{\partial V_t / \partial C_{t+s}}{\partial V_t / \partial C_t} \right) = \left(\beta^s \frac{a_{t+s}}{a_t} \right) \left(\frac{C_{t+s}^{\eta} \left(1 - N_{t+s}\right)^{1-\eta}}{C_t^{\eta} \left(1 - N_t\right)^{1-\eta}} \right)^{\frac{1-\sigma}{\theta_V}} \left(\frac{C_t}{C_{t+s}} \right) \left(\frac{V_{t+s}}{\mathbb{E}_t \left[V_{t+s}^{1-\sigma} \right]} \right)^{1-\frac{1}{\theta_V}}$$

Using the stochastic discount factor, we can eliminate λ and simplify Equations (9) - (11):

$$\frac{1-\eta}{\eta} \frac{C_t}{1 - N_t} = \frac{W_t}{P_t} \tag{12}$$

$$\frac{P_t^E}{P_t} = \mathbb{E}_t \left\{ M_{t+1} \left(\frac{D_{t+1}^E}{P_{t+1}} + \frac{P_{t+1}^E}{P_{t+1}} \right) \right\} \tag{13}$$

$$1 = R_t^R \mathbb{E}_t \Big\{ M_{t+1} \Big\} \tag{14}$$

Equation (12) represents the household intratemporal optimality condition with respect to consumption and leisure, and Equations (13) and (14) represent the Euler equations for equity shares and one-period riskless firm bonds.

4.2 Intermediate Goods Producers

Each intermediate goods-producing firm i rents labor $N_t(i)$ from the representative household to produce intermediate good $Y_t(i)$. Intermediate goods are produced in a monopolistically competitive market where producers face a quadratic cost of changing their nominal price $P_t(i)$ each period. The intermediate-goods firms own their capital stocks $K_t(i)$, and face convex costs of changing the quantity of installed capital. Firms also choose the rate of utilization of their installed physical capital $U_t(i)$, which affects its depreciation rate. Each firm issues equity shares $S_t(i)$ and one-period risk-less bonds $B_t(i)$. Firm i chooses $N_t(i)$, $I_t(i)$, $U_t(i)$, and $P_t(i)$ to maximize firm cash flows $D_t(i)/P_t(i)$ given aggregate demand Y_t and price P_t of the finished goods sector. The intermediate goods firms all have the same constant returns-to-scale Cobb-Douglas production function, subject to a fixed cost of production Φ.

Each firm producing intermediate goods maximizes discounted cash flows using the household's stochastic discount factor:

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} M_{t+s} \left[\frac{D_{t+s}(i)}{P_{t+s}} \right]$$

subject to the production function:

$$\left[\frac{P_t(i)}{P_t}\right]^{-\theta_\mu} Y_t \leq \left[K_t(i)U_t(i)\right]^{\alpha} \left[N_t(i)\right]^{1-\alpha} - \Phi,$$

and subject to the capital accumulation equation:

$$K_{t+1}(i) = \left(1 - \delta\Big(U_t(i)\Big) - \frac{\phi_K}{2}\left(\frac{I_t(i)}{K_t(i)} - \delta\right)^2\right) K_t(i) + I_t(i)$$

where

$$\frac{D_t(i)}{P_t} = \left[\frac{P_t(i)}{P_t}\right]^{1-\theta_\mu} Y_t - \frac{W_t}{P_t} N_t(i) - I_t(i) - \frac{\phi_P}{2}\left[\frac{P_t(i)}{\Pi P_{t-1}(i)} - 1\right]^2 Y_t$$

and depreciation depends on utilization via the following functional form:

$$\delta\Big(U_t(i)\Big) = \delta + \delta_1\Big(U_t(i) - U\Big) + \left(\frac{\delta_2}{2}\right)\Big(U_t(i) - U\Big)^2$$

The behavior of each firm i satisfies the following first-order conditions:

$$\frac{W_t}{P_t} N_t(i) = (1-\alpha)\Xi_t \left[K_t(i)U_t(i)\right]^{\alpha} \left[N_t(i)\right]^{1-\alpha}$$

$$\frac{R_t^K}{P_t} U_t(i) K_t(i) = \alpha \Xi_t \left[K_t(i)U_t(i)\right]^{\alpha} \left[N_t(i)\right]^{1-\alpha}$$

$$q_t \delta'\Big(U_t(i)\Big)\, U_t(i) K_t(i) = \alpha \Xi_t \left[K_t(i)U_t(i)\right]^{\alpha} \left[N_t(i)\right]^{1-\alpha}$$

$$\begin{aligned}\phi_P \left[\frac{P_t(i)}{\Pi P_{t-1}(i)} - 1\right]\left[\frac{P_t}{\Pi P_{t-1}(i)}\right] = (1-\theta_\mu)\left[\frac{P_t(i)}{P_t}\right]^{-\theta_\mu} + \theta_\mu \Xi_t \left[\frac{P_t(i)}{P_t}\right]^{-\theta_\mu - 1} \\ + \phi_P \mathbb{E}_t\left\{M_{t+1}\frac{Y_{t+1}}{Y_t}\left[\frac{P_{t+1}(i)}{\Pi P_t(i)} - 1\right]\left[\frac{P_{t+1}(i)}{\Pi P_t(i)}\frac{P_t}{P_t(i)}\right]\right\}\end{aligned}$$

$$\begin{aligned}q_t = \mathbb{E}_t\Bigg\{M_{t+1}\Bigg(U_{t+1}(i)\frac{R_{t+1}^K}{P_{t+1}} + q_{t+1}\Bigg(1 - \delta\Big(U_{t+1}(i)\Big) - \frac{\phi_K}{2}\left(\frac{I_{t+1}(i)}{K_{t+1}(i)} - \delta\right)^2 \\ + \phi_K\left(\frac{I_{t+1}(i)}{K_{t+1}(i)} - \delta\right)\left(\frac{I_{t+1}(i)}{K_{t+1}(i)}\right)\Bigg)\Bigg)\Bigg\}\end{aligned}$$

$$\frac{1}{q_t} = 1 - \phi_K\left(\frac{I_t(i)}{K_t(i)} - \delta\right)$$

where Ξ_t is the marginal cost of producing one additional unit of intermediate good i, and q_t is the price of a marginal unit of installed capital. R_t^K/P_t is the marginal revenue product per

unit of capital services $K_t U_t$, which is paid to the owners of the capital stock. Our adjustment cost specification is similar to the specification used by Jermann (1998) and allows Tobin's q to vary over time.

Each intermediate goods firm finances a percentage ν of its capital stock each period with one-period riskless bonds. The bonds pay the one-period real risk-free interest rate. Thus, the quantity of bonds $B_t(i) = \nu K_t(i)$. Total firm cash flows are divided between payments to bond holders and equity holders as follows:

$$\frac{D_t^E(i)}{P_t} = \frac{D_t(i)}{P_t} - \nu \left(K_t(i) - \frac{1}{R_t^R} K_{t+1}(i) \right). \tag{15}$$

Since the Modigliani and Miller (1958) theorem holds in our model, leverage does not affect firm value or optimal firm decisions. Leverage makes the payouts and price of equity more volatile and allows us to define a concept of equity returns in the model. We use the volatility of equity returns implied by the model to calibrate our uncertainty shock processes in Section 6.

4.3 Final Goods Producers

The representative final goods producer uses $Y_t(i)$ units of each intermediate good produced by the intermediate goods-producing firm $i \in [0, 1]$. The intermediate output is transformed into final output Y_t using the following constant returns to scale technology:

$$\left[\int_0^1 Y_t(i)^{\frac{\theta_\mu - 1}{\theta_\mu}} di \right]^{\frac{\theta_\mu}{\theta_\mu - 1}} \geq Y_t$$

Each intermediate good $Y_t(i)$ sells at nominal price $P_t(i)$ and each final good sells at nominal price P_t. The finished goods producer chooses Y_t and $Y_t(i)$ for all $i \in [0, 1]$ to maximize the following expression of firm profits:

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

subject to the constant returns to scale production function. Finished goods-producer optimization results in the following first-order condition:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta_\mu} Y_t$$

The market for final goods is perfectly competitive, and thus the final goods-producing firm earns zero profits in equilibrium. Using the zero-profit condition, the first-order condition for

profit maximization, and the firm objective function, the aggregate price index P_t can be written as follows:

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta_\mu} di \right]^{\frac{1}{1-\theta_\mu}}$$

4.4 Equilibrium

The assumption of Rotemberg (1982) (as opposed to Calvo (1983)) pricing implies that we can model our production sector as a single representative intermediate goods-producing firm. In the symmetric equilibrium, all intermediate goods firms choose the same price $P_t(i) = P_t$, employ the same amount of labor $N_t(i) = N_t$, and choose the same level of capital and utilization rate $K_t(i) = K_t$ and $U_t(i) = U_t$. Thus, all firms have the same cash flows and payout structure between bonds and equity. With a representative firm, we can define the unique markup of price over marginal cost as $\mu_t = 1/\Xi_t$, and gross inflation as $\Pi_t = P_t/P_{t-1}$.

4.5 Monetary Policy

We assume a cashless economy where the monetary authority sets the net nominal interest rate r_t to stabilize inflation and output growth. Monetary policy adjusts the nominal interest rate in accordance with the following rule:

$$r_t = \rho_r r_{t-1} + \big(1 - \rho_r\big)\big(r + \rho_\pi \left(\pi_t - \pi\right) + \rho_y \Delta y_t\big), \tag{16}$$

where $r_t = \ln(R_t)$, $\pi_t = \ln(\Pi_t)$, and $\Delta y_t = \ln(Y_t/Y_{t-1})$. Changes in the nominal interest rate affect expected inflation and the real interest rate. Thus, we include the following Euler equation for a zero net supply nominal bond in our equilibrium conditions:

$$1 = R_t \mathbb{E}_t \left\{ M_{t+1} \left(\frac{1}{\Pi_{t+1}} \right) \right\} \tag{17}$$

4.6 Shock Processes

In our baseline model, we are interested in capturing the effects of independent changes in the level and volatility of the preference shock process. The preference shock processes are parameterized as follows:

$$a_t = (1 - \rho_a)\, a + \rho_a a_{t-1} + \sigma_{t-1}^a \varepsilon_t^a$$

$$\sigma_t^a = (1 - \rho_{\sigma^a})\, \sigma^a + \rho_{\sigma^a} \sigma_{t-1}^a + \sigma^{\sigma^a} \varepsilon_t^{\sigma^a}$$

ε_t^a is a first-moment shock that captures innovations to the level of the stochastic process for household discount factors. We refer to $\varepsilon_t^{\sigma^z}$ as second-moment or "uncertainty" shock since it captures innovations to the volatility of the exogenous processes of the model. An increase in the

volatility of the shock process increases the uncertainty about the future time path of household demand. Both stochastic shocks are independent, standard normal random variables.[5]

4.7 Solution Method

Our primary focus is examining the effect of an increase in the second moment of the preference shock process. Using a standard first-order or log-linear approximation to the equilibrium conditions of our model would not allow us to examine second moment shocks, since the approximated policy functions are invariant to the volatility of the shock processes. Similarly, second moment shocks would only enter as cross-products with the other state variables in a second-order approximation, and thus we could not study the effects of shocks to the second moments alone. In a third-order approximation, however, second moment shocks enter independently in the approximated policy functions. Thus, a third-order approximation allows us to compute an impulse response to an increase in the volatility of the discount rate shocks, while holding constant the levels of those variables.

To solve the baseline model, we use the Dynare software package developed by Adjemian et al. (2011). Dynare computes the rational expectations solution to the model using third-order Taylor series approximation around the deterministic steady state of the model. Appendix B.1 contains all the equilibrium conditions for the baseline model.[6] As discussed in Fernández-Villaverde et al. (2011), approximations higher than first-order move the ergodic distributions of the model endogenous variables away from their deterministic steady-state values. In the main text, we compute the impulse responses in percent deviation from the stochastic steady state of the model. We define the stochastic steady state as the point where the third-order solution converges in absence of shocks. Koop, Pesaran and Potter (1996) advocates for an alternative generalized impulse response, which uses a simulation procedure around the ergodic mean of the endogenous variables. In Appendix B.2, we show that these two procedures produce nearly identical results. In the main text, we use the impulse response around the stochastic steady state since it allows us to analyze an increase in uncertainty about the future without any change in the realized volatility of the shock processes.[7]

[5]We specify the stochastic processes in levels, rather than in logs, to prevent the volatility σ_t^a from impacting average value of a_t through a Jensen's inequality effect. In principle, the normally-distributed processes in levels could allow for negative values of a_t or σ_t^a. However, a_t and σ_t^a always remain greater than zero in the model simulations.

[6]Previous versions of this paper used the Perturbation AIM algorithm and software developed by Swanson, Anderson and Levin (2006), which produced identical results but took significantly longer to compute the solution.

[7]In our companion paper Basu and Bundick (2015), we provide a full analysis of both types of impulse

5 Calibration and Qualitative Results

5.1 Calibration

Table 1 lists the calibrated parameters of the model. We calibrate the model at a quarterly frequency, using standard parameters for one-sector models of fluctuations. Since our model shares many features with the estimated models of Ireland (2003) and Ireland (2011), we calibrate our model to match the estimated parameters reported in those papers. We use the estimates in these papers to calibrate the steady-state volatility for preference shocks σ^a (our value lies between their estimates). We calibrate the steady-state level of the discount factor process a to equal one. To assist in numerically calibrating and solving the model, we introduce constants into the period utility function and the production function to normalize the value function V and output Y to both equal one at the deterministic steady state.

We choose steady-state hours worked N and the model-implied value for η such that our model has a Frisch labor supply elasticity of 2. Our calibration of ϕ_K implies an elasticity of the investment-capital ratio with respect to marginal q of 4. The household intertemporal elasticity of substitution (IES) is calibrated to 0.8, which is consistent with the empirical estimates of Basu and Kimball (2002). The fixed cost of production for the intermediate-goods firm Φ is calibrated to eliminate pure profits in the deterministic steady state of the model. We calibrate δ_2 to 0.01, which is consistent with the estimated values of Fernández-Villaverde et al. (2013) and Christiano, Eichenbaum and Evans (2005). Risk aversion over the consumption and leisure basket σ is set to 30, which is slightly smaller that the estimated values of van Binsbergen et al. (2012) and Swanson and Rudebusch (2012), but larger than the values assumed by Gourio (2012).

We calibrate our price adjustment cost parameter ϕ_P to the estimate from Ireland (2003). In the following analysis, we compare the results from our baseline sticky-price calibration ($\phi_P = 160$), with a flexible-price calibration ($\phi_P = 0$).[8] We discuss our calibration of the uncertainty shock stochastic processes in depth in Section 6. In Section 6.4, we provide further insights into the calibration by perturbing several of the key asset-pricing parameters of the model.

responses both at and away from the zero lower bound.

[8]In a linearized New-Keynesian model, where Calvo and Rotemberg specifications generate identical Phillips curves, our calibration of ϕ_P implies that prices are reset about once every six quarters. This frequency of price adjustment is higher than the macro estimates of Smets and Wouters (2007), but is lower than micro estimates from Nakamura and Steinsson (2008). However, since we are using a nonlinear solution method, Calvo and Rotemberg pricing frictions are no equivalent.

5.2 Uncertainty Shocks & Business Cycle Comovements

Holding the calibrated parameters fixed, we analyze the effects of an exogenous increase in uncertainty associated with household demand. Figure 5 plots the impulse responses of the model to a demand uncertainty shock. The results are consistent with the intuition of Section 3 and the labor market diagrams in Figures 3 and 4. Uncertainty about household demand enters both Equation (5) and Equation (7) through the forward-looking marginal utility of wealth. An uncertainty shock induces wealth effects on the household which triggers precautionary labor supply.[9]

Households want to consume less and save more when uncertainty increases in the economy. In order to save more, households optimally wish to both reduce consumption and increase hours worked. Under flexible prices and constant markups, equilibrium labor supply and consumption follow the path that households desire when they face higher uncertainty. On impact of the uncertainty shock, the level of capital is predetermined, and thus labor demand is unchanged for a given real wage. Under flexible prices, the outward shift in labor supply combined with unchanged labor demand increases hours worked and output. After the impact period, households continue to save more, consume less, and work longer hours. Since firms owns the capital stock, higher household saving translates into higher capital accumulation for firms. Throughout the life of the uncertainty shock, consumption and investment move in opposite directions, which is inconsistent with our empirical evidence from Section 2.

Under sticky prices, households also want to consume less and save more in response to the uncertainty shock. On impact, households increase their labor supply and reduce consumption to accumulate more assets. With sticky prices, however, increased labor supply decreases the marginal costs of production of the intermediate goods firms. A reduction in marginal cost with slowly-adjusting prices increases firm markups. An increase in markups lowers the demand for household labor and lowers the real wage earned by the representative household. The decrease in labor demand also lowers investment in the capital stock by firms. In equilibrium, these effects combine to produce significant falls in output, consumption, investment, and hours worked, which are consistent with an identified uncertainty shock in the data. Thus, the desire by households to work more can actually lead to lower labor input and output in equilibrium. Equivalently, when output is determined by demand, the desire to save more depresses consumption demand, and thus lowers output and all its components.

[9]Previous versions of this paper show that an uncertainty shock about future technology can also produce comovement of the key macro variables.

6 Quantitative Results & Great Recession Application

6.1 Uncertainty Shock Calibration

The previous intuition and qualitative results suggest that uncertainty shocks can produce declines in output and its components when prices adjust slowly. This section shows that the previous sticky-price model closely matches our empirical evidence from Section 2. A related issue is determining the proper calibration of our uncertainty shock process. The transmission of uncertainty to the macroeconomy in our model crucially depends on the calibration of the size and persistence of the uncertainty shock processes. However, aggregate uncertainty shocks are an *ex ante* concept, which may be difficult to measure using *ex post* economic data. To ensure that our calibration of the stochastic process for uncertainty is reasonable, we discipline our model and uncertainty shock process to produce fluctuations in uncertainty that are consistent with the behavior of a well-known and readily-observable measure of aggregate uncertainty.

We choose the Chicago Board Options Exchange Volatility Index (VXO) as our observable measure of aggregate uncertainty due to its prevalence in financial markets, ease of observability, and the ability to generate a model counterpart. The VXO is a forward-looking indicator of the expected volatility of the Standard and Poor's 100 stock index. To link our model with the data, we want to create a model counterpart to our observable measure of aggregate uncertainty. Thus, we compute a model-implied VXO index as the expected conditional volatility of the return on the equity of the representative intermediate-goods producing firm. Using our third-order solution method, we define our model-implied VXO V_t^M as follows:

$$R_{t+1}^E = \frac{D_{t+1}^E + P_{t+1}^E}{P_t^E} \tag{18}$$

$$V_t^M = 100 * \sqrt{4 * \mathbb{VAR}_t\Big\{ R_{t+1}^E \Big\}} \tag{19}$$

$$\mathbb{VAR}_t\Big\{ R_{t+1}^E \Big\} = \mathbb{E}_t\Big\{ \left(R_{t+1}^E\right)^2 \Big\} - \Big(\mathbb{E}_t R_{t+1}^E\Big)^2 \tag{20}$$

where $\mathbb{VAR}_t(R_{t+1}^E)$ is the quarterly conditional variance of the return on equity R_{t+1}^E.[10] We annualize the quarterly conditional variance, and then transform the annual volatility units into percentage points.

[10] Technically, the VXO is the expected volatility of equity returns under the risk-neutral measure. In preliminary work, we found the results were quantitatively unchanged if we compute the model-implied VXO using the risk-neutral expectation.

Using our model-implied VXO, we calibrate the uncertainty shock parameters and firm leverage using a two-step process. Given the other parameters for the model and the unconditional shock variance σ^a, we set our uncertainty shock parameters such that a one standard deviation uncertainty shock generates an impulse response for the model-implied VXO that closely matches the actual VXO movements from our identified VAR. Specifically, we use the impulse-response matching methodology of Christiano, Eichenbaum and Evans (2005) to match the log of the model-implied VXO to the identified log VXO response in the structural vector autoregression. Conditional on the values of the endogenous state variables, our model-implied VXO has an AR(1) representation in the uncertainty shock process. Therefore, we are able to closely match the impulse response of the VXO in the data. We then choose the level of firm leverage such that the unconditional level of the model-implied VXO at the stochastic steady state matches the average level of the VXO in the data, 20.8 percent. Table 1 also shows the resulting calibration of our two-step procedure.

6.2 Quantitative Impact of Uncertainty Shocks

Our calibration strategy produces general-equilibrium results which are qualitatively consistent with the empirical evidence from the structural vector autoregression. Our previous Figure 2 also plots our baseline model results for a demand uncertainty shock versus the estimated impulse responses from the vector autoregression. Our baseline model replicates both the qualitative comovement among the four key macroeconomics aggregates and reasonably matches the quantitative implications. Similar to identified shock in the data, the peak decline in output in the model is around 0.2 percent and the model generates a decline in investment that is significantly larger than the response of consumption. With the exception of the impact effect for some variables, the model impulse responses fall completely within the 95% confidence intervals of the empirical model.[11] Our results suggest that nominal price rigidity likely plays a key role in understanding the transmission of uncertainty to the macroeconomy.

6.3 The Role of Uncertainty Shocks in the Great Recession

The previous section shows that uncertainty shocks associated with household demand have quantitatively significant effects on output and its components. Many economists and the fi-

[11] The model generally predicts that the impact effect is large for most variables, while the data show somewhat more hump-shaped responses. While we could address these small discrepancies by adding adjustment costs for flows, such as habit formation in consumption, this strategy would add several state variables. Our third-order solution could easily accommodate these additional states, but the additional computational burden would be significant for our global solution method in Section 8. Therefore, we choose a more parsimonious model which we can solve both at and away from the zero lower bound.

nancial press believe the large increase in uncertainty in the Fall of 2008 may have played a role in the Great Recession and subsequent slow recovery. For example, Kocherlakota (2010) states, "I've been emphasizing uncertainties in the labor market. More generally, I believe that overall uncertainty is a large drag on the economic recovery." The bottom plot of Figure 1 shows a 2.75 standard deviation VXO-implied uncertainty shock around the collapse of Lehman Brothers in September of 2008. Feeding this size shock into our theoretical model predicts that this increase in uncertainty in the Fall of 2008 should have lowered output by about 0.7 percent.[12]

This decline in output may seem a small number relative to the size of the output drop in 2008-2009. For example, the CBO estimates that the output gap was -5.0 percent in 2008Q4.[13] However, as we will emphasize rigorously in Section 7.1, the assumptions regarding monetary policy are crucial in determining the effects of changes in uncertainty on the macroeconomy. The fed funds target rate hit the zero lower bound on December 16, 2008. From then on, the Federal Reserve could no longer fully offset the contractionary effects of higher uncertainty on the economy. Under these circumstances, the predicted macroeconomic effects of uncertainty are substantially larger. In Section 8, we explore this idea by rigorously modeling the impact of an uncertainty shock at the zero lower bound.

One potential criticism of using our model to determine the role of uncertainty shocks in the Great Recession is that our model lacks a realistic financial sector and abstracts from financial frictions. Thus, one might argue that what we term an exogenous uncertainty shock is actually due to a financial crisis. We are quite sympathetic to the idea that a financial crisis can raise uncertainty, but we believe that it is important to investigate the full set of channels through which financial market disruptions can affect the macroeconomy. A financial market disruption, such as the failure of Lehman Brothers in the Fall of 2008, is a single event which can have multiple effects, just as a war might increase government expenditure, raise distortionary taxes, and lead to rationing, each of which has different macroeconomic effects. Recent work by Iacoviello (2015), Gertler and Karadi (2011), and many others focuses on the first-moment effects of the financial market disruption, such as a higher cost of capital and tighter borrowing constraints for households and firms. In this paper, we analyze the likely effects of the concurrent rise in uncertainty and its effect on the economy during the Great Recession, which are second-moment effects.

[12]Given the AR(1) law of motion for volatility shocks in our third-order approximation to the policy functions, the impulse responses for the model scale approximately linearly in the size of the uncertainty shock.

[13]Since flexible-price output only increases slightly after an uncertainty shock, the output gap is very close to output in our baseline model.

Financial frictions can easily cause second-moment effects as well; for example, even firms that experience no decline in current demand might know that the purchasers of their goods may become credit constrained at some point in the future, leading the firm's future path of demand to become more uncertain.[14] To analyze this independent mechanism and the effects of the increase in uncertainty, we choose to model uncertainty in a simple but reasonable macroeconomic model that abstracts from financial frictions. Our paper complements other work on the Great Recession, since one could easily combine the first-moment and second-moment analyses to obtain a complete picture of the effects of the financial crisis. Adding a detailed financial sector to our model would obscure the transmission mechanism of uncertainty to the macroeconomy, and we eschew this course of action for the sake of clarity.

6.4 Exploring Asset-Pricing Features

Our model is consistent with both the qualitative comovement and quantitative predictions of an identified uncertainty shock in the data. While our model remains relatively simple and tractable, it embeds some features from the asset-pricing literature into a macroeconomic model with nominal rigidities. In this section, we illustrate the role of leverage and risk aversion in helping the model match the identified VAR results. While these features help the model match the data quantitatively, the model can still generate the qualitative comovement of output and its components, which is our key stylized fact, without these additional features.

Figure 6 shows the impulse responses after an uncertainty shock for several different calibrations of leverage ν, risk aversion σ, and the size of the uncertainty shock. In the model, the amount of leverage helps the model match the unconditional volatility of equity returns and affects the equity premium. Table 2 reports some unconditional asset pricing features of our model. Using our calibrated value of $\nu = 0.87$, the model is able to exactly match the average VXO in the data of 20.8%, implies an average risk-free rate of around 1.5%, and generates an equity premium over the risk-free rate of about 8.5%. All of these values are well within the standard errors for the data as computed by Bansal and Yaron (2004). While this calibrated value for leverage is quite high, two important caveats are important to keep in mind. First, the model only contains household demand shocks. Adding additional shocks (such as technology, government spending, and monetary policy) would allow the model to match the volatility of the equity return with a much smaller amount of leverage. Second, since the Modigliani & Miller (1963) theorem holds in our model, the amount of leverage does not affect firm decisions or firm value. If we remove leverage $\nu = 0$, Figure 6 shows that the responses of the key macro

[14] Fulford (2015) documents that many consumer credit lines (credit card borrowing limits) were cut sharply during the Great Recession.

variables are unchanged. Without leverage, however, the average model-implied VXO drops to around 3%. Thus, our main results would be unchanged if we eliminated the second step in our calibration process, chose not to match the average stock market volatility, and examined the log VXO response in deviations from its steady state value.

In loose terms, the precautionary labor supply by households after an uncertainty shock depends on the 'price' of risk multiplied by the 'quantity' of risk. The price of risk is mainly influenced by risk aversion σ and the quantity of risk is determined by the size of the uncertainty shock. We find the model can match the VAR evidence with a calibration of $\sigma = 30$ and an uncertainty shock that increases the volatility of shocks by around 20% relative to its steady state value.[15] If we divide the risk aversion parameter by three ($\sigma = 10$), then Figure 6 shows that the resulting impulse responses are roughly one-third as large as the baseline calibration. However, if set $\sigma = 10$ but triple the size of the uncertainty shock ($\sigma^{\sigma^a} = 0.016$), the model can generate responses that look like the baseline model even with substantially less risk-averse households. Thus, the inclusion of Epstein-Zin preferences allow us match the VAR evidence with smaller movements in the expected volatility of the exogenous shocks.

6.5 Model-Based Support for Empirical Identification

In our empirical evidence from Section 2, we identified an uncertainty shock in the data using a Cholesky decomposition with the VXO ordered first. This ordering assumes that uncertainty shocks can have an immediate impact on output and its components. However, our identification scheme also assumes that the other non-uncertainty shocks do not affect the implied stock market volatility at impact. In this section, we show that this identification strategy is supported by our theoretical model.

Figure 7 plots the impulse responses to both a first- and second-moment demand shock in our model. Consistent with our identifying assumptions in our VAR, a first-moment demand shock in the model has little effect on the expected volatility of future equity returns.[16] Despite causing a decline in investment and equity prices, a first-moment demand shock does not change the expected volatility of future equity returns. This result is also reflected in the evolution of the equity premium. An uncertainty shock increases the likelihood of bad outcomes, so households require much higher compensation for holding firm equity. However, first-moment

[15] Since households can adjust their labor margin, Swanson (2013) shows that σ in our model is not comparable to fixed-labor risk aversion estimates.

[16] For comparision, we choose the size of the first-moment shock such that it generates the same decline in investment as the second-moment shock.

shocks primarily only affect the mean of future outcomes, which doesn't translate into a higher equity premium in our model. Figure 7 also shows that an uncertainty shock in our model can generate a decline in equity prices with an empirically-plausible IES less than one. This result contrasts with the long-run risk literature of Bansal and Yaron (2004), which typically requires an IES significantly greater than one for equity prices to fall after an increase in the expected volatility of future consumption.

7 Discussion and Connections

7.1 Specific Example of General Principle

The differences in economic responses to uncertainty fluctuations under flexible and sticky prices is a specific instance of the general proposition established by Basu and Kimball (2005). They show that "good" shocks that cause output to rise in a flexible-price model generally tend to have contractionary effects in a model with nominal price rigidity. Basu and Kimball (2005) also show that the response of monetary policy is critical for determining the equilibrium response of output and other variables. If monetary policy follows a sensible rule, for example the celebrated Taylor (1993) rule, then the monetary authority typically lowers its nominal policy rate to offset the negative short-run effects of the shock. Our results show, however, that under standard parameter values this effect is not strong enough to offset the contractionary effects of higher uncertainty.

If the interest-rate rule allowed the monetary authority to conduct policy optimally and replicate the flexible-price equilibrium allocations, then monetary policy could undo the negative effects of the uncertainty shock. If we replace our policy rule in Equation (16) with the following policy rule, Figure 6 shows that the monetary authority can replicate the flexible price allocation even when prices adjust slowly:

$$r_t = r_t^n + \pi + \rho_\pi \left(\pi_t - \pi\right) + \rho_x x_t, \tag{21}$$

where r_t^n is the "natural" real rate of interest from the equivalent flexible-price economy and x_t is the output gap between equilibrium and flexible-price output.[17] In keeping with the bulk of the literature, we do not model why the monetary policy rule does not react more aggressively to uncertainty in normal times. However, we do investigate in depth one particular barrier to expansionary monetary policy that is critical for understanding the Great Recession: the zero lower bound constraint on nominal interest rates. If uncertainty increases when the monetary

[17] For this example, we calibrate $\rho_\pi = 1.5$ and $\rho_x = 0.125$.

authority is unable to further lower its nominal policy rate, as was the case in late 2008, then the central bank cannot replicate the flexible-price allocations. Thus, the short-run contractionary effect of the "good" shock dominates, and the equilibrium response of output becomes robustly negative. We explore this issue in Section 8.

7.2 Extension to Sticky Nominal Wages

Our exposition so far suggests that the mechanism we have identified works only in the special case where nominal prices are sticky but wages are flexible. Indeed, our intuition for the channel through which an increase in uncertainty raises the markup has emphasized these two elements. We argued that higher uncertainty induces households to work at lower wages, the reduction in the wage reduces firms' marginal costs, but since their output prices are fixed, lower marginal costs translate to contractionary higher markups. However, various types of evidence suggests that nominal wages are sticky, not flexible, especially at high frequencies. At the macro level, Christiano, Eichenbaum and Evans (2005) find that nominal wage stickiness is actually more important than nominal price stickiness for explaining the observed impact of monetary policy shocks. At the micro level, Barattieri, Basu and Gottschalk (2014) find that the wages of individual workers change less than once a year on average.

In this subsection, we show that our results extend readily to the case where nominal wages are sticky. Rather than writing down an extended model with two nominal frictions, we make our point heuristically using the graphical labor supply-labor demand apparatus of Section 3. As we argued above, if households act competitively in the labor market:

$$U_2(C_t, 1 - N_t) = \lambda_t W_t, \tag{22}$$

where W is the nominal wage and λ_t is now the utility value of a marginal dollar. Assuming firms have market power, we can reorganize Equations (6) and (7) as follows:

$$W_t = \frac{P_t}{\mu_t^P} Z_t F_2(K_t, Z_t N_t). \tag{23}$$

$$\frac{U_2(C_t, 1 - N_t)}{\lambda_t P_t} = \frac{1}{\mu_t^P} Z_t F_2(K_t, Z_t N_t), \tag{24}$$

where μ_t^P is the price-markup over marginal cost.

Now assume a new model, where households also have market power, and set wages with a markup over their marginal disutility of work. Equation (3) and the resulting equilibrium are modified as follows:

$$W_t = \mu_t^W \frac{U_2(C_t, 1 - N_t)}{\lambda_t}. \tag{25}$$

$$\frac{U_2(C_t, 1-N_t)}{\lambda_t P_t} = \frac{1}{\mu_t^W}\frac{1}{\mu_t^P} Z_t F_2(K_t, Z_t N_t) \tag{26}$$

Compared with the competitive labor market model, we can replace the labor supply curve in Figures 3 and 4 with $U_2(C_t, 1-N_t)/\lambda_t P_t$. This quantity has the interpretation of the disutility faced by the household of supplying one more unit of labor, expressed in units of real goods (the real marginal cost of supplying labor). On the vertical axis, we now plot the equilibrium level of the real marginal disutility of work. This alternative 'supply curve' is shifted in exactly the same way by uncertainty as the standard labor supply curve – higher uncertainty raises λ, which shifts the supply curve out. But now the 'demand curve' in the right-hand side of Equation (26) is shifted by both price and wage markups – only the product of the two matters.

Take the polar opposite of the case we have analyzed so far: Assume perfect competition in product markets, but Rotemberg wage setting by monopolistically competitive households in the labor market. Then the price markup is always fixed at 1, but the wage markup would jump up in response to an increase in uncertainty (since the marginal cost of supplying labor falls but the wage is sticky). This alternative assumption would make the qualitative outcome exactly the same as in our previous results. Thus, while introducing nominal wage stickiness would certainly affect quantitative magnitudes, it would not change our qualitative results.

7.3 Connections with Existing Literature

Our framework can be used to understand the economic mechanisms at work in some recent papers in the literature. Recent work by Bloom et al. (2014), Chugh (2014), and Gilchrist, Sim and Zakrajšek (2013) uses flexible-price models to show that shocks to uncertainty can lead to fluctuations that resemble business cycles. Their modeling approach is to drop Equation (2) and use multi-sector models of production. Follow the insight of Bloom (2009), the normal industry equilibrium in these models features resource reallocation from low- to high-productivity firms. Higher uncertainty impedes this reallocation process through a real options effect. These models use multi-sector production and costly factor adjustment to transform a change in the expected future dispersion of total factor productivity (TFP) into a change in the current mean of the TFP distribution.[18] This approach may allow equilibrium real wages, consumption and labor supply to move in the same direction. However, all three papers experience difficulties in

[18]This intuition also helps understand the recent work of Bidder and Smith (2012), which embeds stochastic volatility and preferences for robustness in a business-cycle model. In their setting, an increase in volatility of technology shocks affects the expected mean of the technology distribution by changing the conditional worst case distribution of the robustness-seeking agent. In a related paper, Ilut and Schneider (2014) embed ambiguity-averse agents in the model of Smets and Wouters (2007). They show that exogenous changes in the agents' beliefs about the worst-case scenario can produce business-cycle comovements.

getting the desired comovements, at least for calibrations that are consistent with steady-state growth. We view these approaches are complementary to ours since both mechanisms (cyclical markups and cyclical reallocation) could be at work simultaneously. However, we view our approach as a realistic and tractable alternative, since non-linear heterogeneous-agent models are computationally difficult to analyze. Our model of time-varying markups allows us to analyze uncertainty in the same representative-agent DSGE framework used to study other real and monetary shocks.

A recent paper by Fernández-Villaverde et al. (2011) studies the effects of uncertainty in a small open economy setting, where they directly shock the exogenous process for the real interest rate. Since a small open economy analysis is effectively done in a partial-equilibrium framework, they experience no difficulties in getting business-cycle comovements from an uncertainty shock. As we show, the difficulties come when the real interest rate is endogenous in a general equilibrium framework. In this setting, our mechanism changes the qualitative predictions of baseline DSGE models, and makes the model predictions consistent with the empirical evidence.

Another recent paper by Gourio (2012) follows Rietz (1988) and Barro (2006) and introduces a time-varying "disaster risk" into an otherwise-standard real business cycle model. This shock can be viewed as bad news about the future first-moment of technology combined with an increase in the future dispersion of technology. Thus, a higher risk of disaster is a combination of a negative news shock and a shock that increases uncertainty about the future. However, a key difference between Gourio (2012) and our work is that a realized disaster affects the level of both technology and the capital stock. In our model, a realized innovation does not affect the level of capital at the impact of the shock. The additional assumption in Gourio (2012) implies that an increase in the probability of disaster directly lowers the risk-adjusted rate of return on capital. In order for investment to fall when the probability of disaster increases, Gourio must assume an intertemporal elasticity of substitution (IES) greater than one. With an IES greater than one, the substitution effect dominates the wealth effect when the probability of disaster increases. The lower risk-adjusted rate of return on investment induces the household to decrease investment. Since the return on investment is low, households supply less labor which lowers total output. Since leisure and consumption are normal goods, an increase in risk results in lower equilibrium output, investment, and hours, but higher equilibrium consumption. For the reasons we discuss in Section 3, his competitive one-sector model is unable to match the comovement implied by the empirical evidence.

In independent and simultaneous work, papers by Fernández-Villaverde et al. (2013) and

Born and Pfeifer (2014) examine the role of fiscal uncertainty shocks in a model with nominal wage and price rigidities. Fernández-Villaverde et al. (2013) shows that uncertainty regarding future fiscal policy is transmitted to the macroeconomy primarily through uncertainty about future taxes on income from capital. As we discuss in the Introduction, an increase in uncertainty with nominal rigidities changes markups and creates macroeconomic comovement. We view these works as highly complementary to our paper. Our work emphasizes the basic mechanism in a stripped-down model and shows why fluctuations in uncertainty can create business cycle comovement. These two papers show that the mechanism we identify can have important economic effects in the benchmark medium-scale model of Smets and Wouters (2007). Other than sharing a mechanism for generating comovement, these two papers differ greatly from our work. We focus on demand uncertainty, rather than policy uncertainty. In addition, we follow a very different calibration strategy, which allows us to closely link the model with the data using an observable *ex ante* measure of stock market volatility. The object of our paper is to understand the role of increased uncertainty in generating the Great Recession and the subsequent slow recovery. We also analyze the interaction between the zero lower bound on nominal interest rates and uncertainty shocks, which we view as important for understanding the economics of this period.[19]

8 Uncertainty Shocks and the Zero Lower Bound

Finally, we examine the role of monetary policy in determining the general-equilibrium effects of uncertainty shocks. In our model, the monetary authority follows a standard interest-rate rule that responds to inflation and output growth. The impulse responses in Figure 5 show that the monetary authority aggressively lowers the nominal interest rate in response to a demand uncertainty shock. However, the calibrated interest-rate rule does not decrease the policy rate enough to offset the negative impact on output and the other model variables. In Section 6.1, we showed that monetary policy could undo the negative effects of the uncertainty shock if their policy rule allowed the monetary authority to target the natural rate of interest and replicate the flexible-price equilibrium allocations. However, monetary policy cannot replicate the flexible-price allocations when they are constrained by the zero lower bound. The sharp increase in uncertainty during the financial crisis in late 2008 corresponds to a period when the Federal Reserve had a policy rate near zero. Thus, we believe that the zero lower bound may have plausibly contributed significantly to the large and persistent output decline starting at that time. In this section, we show that increases in uncertainty have larger effects on output

[19]Since circulating the original draft of our paper, Fernández-Villaverde et al. (2013) now also examine the impact of an uncertainty shock at the zero lower bound.

when monetary policy is constrained by the zero lower bound. Our results suggest that the second-moment effects of the financial crisis may be important for understanding the large declines in output and employment in late 2008.

8.1 Solution Method and Calibration

To analyze the impact of the zero lower bound, we solve a modified version of our baseline model using the policy function iteration method of Coleman (1990). This global approximation method allows us to model the occasionally-binding zero lower bound constraint. This method discretizes the state variables and solves for the policy functions which satisfy all the equilibrium conditions of the model. Appendix C contains the details of the policy function iteration algorithm. To make the model computationally feasible using policy function iteration, we simplify our baseline model by reducing the number of state variables and Euler equations. We eliminate two Euler equations by removing leverage and assuming that households receive firm dividends as a lump-sum payment. To keep the number of grid points reasonable, we also slightly lower the volatility of the exogenous shocks as well.[20]

8.2 Uncertainty, Monetary Policy, & the Zero Lower Bound

In addition to the difficulty of modeling changes in uncertainty at the zero lower bound, increases in uncertainty can produce an additional source of fluctuations beyond the precautionary working and saving channel. This additional amplification mechanism, which we define as the contractionary bias in the nominal interest rate distribution, can dramatically affect the economy when uncertainty increases at the zero lower bound. The contractionary bias emerges from the interaction of uncertainty and the zero lower bound when monetary policy follows a standard Taylor (1993)-type policy rule.

For this discussion, assume monetary policy implements policy using the following rule:

$$r_t^d = r + \rho_\pi \left(\pi_t - \pi\right) \tag{27}$$

$$r_t = \max\left(0, r_t^d\right) \tag{28}$$

where r_t^d is the desired policy rate of the central bank and r_t is the actual policy rate subject to the zero lower bound. A higher volatility of exogenous shocks in the economy leads to more volatile inflation. Through the monetary policy rule, the volatility of inflation dictates the volatility of the desired nominal policy rate. However, since the zero lower bound left-truncates

[20] For Section 8 only, we set $\sigma^a = 0.01$ and $\sigma^{\sigma^a} = 0.005$.

the *actual* policy rate distribution, more volatile desired policy rates lead to higher average actual policy rates.[21] Figure 8 illustrates this effect through the distributions of the desired and actual policy rates under low and high levels of exogenous shock volatility. This contractionary bias in the actual policy rate distribution can have very large general-equilibrium effects. The left panel of Figure 9 plots the average Fisher relation $r = \pi + r^r$ and the average policy rule under both high and low levels of volatility. The upper-right intersection of the monetary policy rule and the Fisher relation dictates the normal general-equilibrium average levels of inflation and the nominal interest rate. Under the simple policy rule in Equation (27), an increase in volatility shifts the policy rule inward and increases the average nominal interest rate for a given level of inflation. Higher volatility thus raises average expected real interest rates, since it implies a higher level of the nominal interest rate for a given level of inflation. All else equal, higher real interest rates discourage consumption and investment and depress output in the economy.

In our companion paper, Basu and Bundick (2015), we fully examine the effects of the contractionary bias using a simple model of nominal price rigidities. We show that changes in the contractionary bias caused by higher uncertainty at the zero lower bound can cause very large declines in output. In addition, we show that the contractionary bias can become so large that a rational expectations equilibrium may fail to exist if policy follows a standard Taylor (1993)-type rule. The right panel of Figure 9 illustrates this potential disequilibrium that can be caused by the contractionary bias. If the volatility of the shocks becomes too large and policy follows a Taylor (1993)-type rule, the policy rule may far enough to the left such that it no longer intersects the Fisher relation. In our companion paper, we show that the form of the monetary policy rule is crucial for avoiding this bad outcome. We argue that global solutions methods are crucial for uncovering the full set of policy implications resulting from uncertainty at the zero lower bound. We also discuss how the existing literature, such as Fernández-Villaverde et al. (2013), Nakata (2013), and Johannsen (2013), either fails to uncover the contractionary bias or conflates the two distinct contractionary bias and precautionary working mechanisms.

In the current paper, our primary interest in modeling an uncertainty shock at the zero lower bound is to quantify the role of uncertainty shocks during the Great Recession. Therefore, we choose a highly conservative assumption and eliminate this contractionary bias mechanism from our following results. However, since we are removing an amplification mechanism, our quanti-

[21] Mendes (2011) proves analytically that the average nominal interest rate is increasing in the volatility of the exogenous shocks when monetary policy follows a simple Taylor (1993)-type rule but is subject to the ZLB constraint.

tative implications represent a conservative lower bound on the effects of changes in uncertainty at the zero lower bound. If we assumed that central bank follows the same simple Taylor rule at the zero lower bound that it does during normal times, then we could explain the entire output drop in the Great Recession as being due to increased uncertainty!

To remove the contractionary bias, we assume that the monetary authority implements policy using the following history-dependent monetary policy rule:

$$r_t^d = r_{t-1}^d + \rho_\pi \left(\pi_t - \pi\right) \tag{29}$$

$$r_t = \max \left(0, r_t^d\right) \tag{30}$$

When the monetary authority is unconstrained by the zero lower bound, this policy rule responds exactly as a Taylor (1993)-type policy rule with interest-rate smoothing. However, when the monetary authority encounters the zero lower bound, the history-dependent monetary policy rule lowers future desired policy rates to offset the previous higher-than-desired nominal rates caused by the zero lower bound. Since deviations from the desired path of the policy rate are offset exactly one-for-one, the average expected nominal policy rate does not rise when volatility increases. Thus, the history-dependent monetary policy rule removes the contractionary bias and allow us to isolate the effects of precautionary saving and working due to uncertainty at the zero lower bound.[22]

8.3 Impulse Response Analysis

Figure 10 plots the impulse responses of an uncertainty shock for our simplified model at the stochastic steady state.[23] These impulse responses replicate our previous experiments using this alternative model and calibration. Holding the level of the discount factor shock constant, an increase in uncertainty about the future decreases output by 0.21 percent. In our following analysis of the zero lower bound, we focus on the relative amount that the zero lower bound

[22]In Basu and Bundick (2015), we also solve for optimal monetary and fiscal policy under commitment in response to an uncertainty shock at the zero lower bound. In addition to removing the contractionary bias, the simple history-dependent rule in Equation (29) implements many of the features of optimal monetary policy at the zero lower bound. Thus, our quantitative results about the effects of an uncertainty shock at the zero lower bound are a conservative lower bound because (1) we removed the contractionary bias and (2) chosen a rule which is a reasonable approximation for optimal policy at the zero lower bound.

[23]In Section 8 only, we plot two-standard deviation uncertainty shocks for the simplified model. Under the policy rule assumed in Equation (29), this slightly larger shock generates the same size output response as our baseline model from Sections 4-6.

amplifies the effects of an uncertainty shock compared to this steady state impulse response.

To compute the impulse response of an uncertainty shock at the zero lower bound, we generate two time paths for the economy. In the first time path, we simulate an economy hit by a series of negative first-moment demand shocks, of a size chosen to make the zero lower bound bind for about two years. In the second time path, we simulate the same series of first-moment demand shocks, but also simulate an uncertainty shock. After the uncertainty shock, neither economy is hit with any further shock. We present the (percent) difference between the time paths of variables in the two simulations as the impulse response to the uncertainty shock at the zero lower bound.

Figure 10 also shows the impulse response to the uncertainty shock when the central bank unable to change its current nominal policy rate. At the zero lower bound, the uncertainty shock produces a 0.27 percent drop in output on impact, and causes larger declines in consumption, investment, and hours worked. When compared with the impulse response at the stochastic steady state, these results suggest that the zero lower bound amplifies uncertainty shocks by about a factor of about 1.25.[24] As we emphasize in Basu and Bundick (2015), this amplification emerges from the endogenous volatility generated by the zero lower bound. Since the monetary authority can not longer play its usual stabilizing role, households understand that the economy faces higher expected volatility at the zero lower bound. The exogenous uncertainty shock amplified by the endogenous volatility generated by the zero lower bound further increases precautionary saving and labor supply by households. This higher desire by households to work and save more at the zero lower bound translates into a larger drop in equilibrium hours worked and investment.

In addition to removing the contractionary bias, simple history-dependent rules like Equation (29) act as a form of commitment by the monetary authority to keep interest rates lower after encountering the zero lower bound. This promise of future lower nominal rates stimulates the economy throughout the zero lower bound episode, but the effect is not strong enough to prevent significant contractions in output and its components. As the monetary authority maintains zero policy rates during the beginning of the recovery, output and its components rise slightly above the unconstrained impulse responses. As the first-moment demand shock subsides and the economy exits the zero lower bound, the time-paths for output and its components

[24]This finding is quantitatively congruent with the work of Ireland (2011) and Gust, López-Salido and Smith (2013). Using likelihood-based estimation methods, these papers show that output would have been about 20% higher if monetary policy had not been constrained during the Great Recession.

closely follow the impulse responses in the neighborhood of the steady state.

8.4 Revisiting Uncertainty Shocks in the Great Recession

The previous impulse responses suggest that adverse effects of uncertainty shocks are amplified by the zero lower bound. The bottom plot of Figure 1 shows a 2.75 standard deviation VXO-implied uncertainty shock during the end of 2008. Our larger baseline model, without accounting for the zero lower bound, suggests that this large uncertainty shock may explain up to a 0.7 percent drop in output during that period. The results of our zero lower bound experiments, however, suggest that the zero lower bound amplifies uncertainty shocks by about a factor of about 1.25. Thus, our results suggest that the increase in uncertainty when the zero lower bound constraint was binding may have accounted for about a 0.9 percent drop in output during the Great Recession. The Congressional Budget Office currently estimates that the gap between actual and potential output for the fourth quarter of 2008 was negative 5.0 percent. Our results suggest that a non-trivial fraction of the decline in output during the Great Recession - roughly one-fifth - can be explained by increased uncertainty about the future. Note again that our results are a lower bound on the effects of uncertainty during the recent crisis, since we assumed that monetary policy succeeds in fully offsetting the contractionary bias. We view our findings as highly complementary to other work on the financial crisis, since our results can be combined with investigations of other channels through which financial crises affect the macroeconomy to obtain a complete picture of the Great Recession.

9 Conclusion

This paper examines the transmission mechanism of uncertainty to the macroeconomy. We argue that macroeconomic comovement between output, consumption, investment, and hours worked is a key empirical feature of the economy's response to an identified uncertainty shock in the data. We show that a standard representative-agent general equilibrium model can replicate this stylized fact if prices adjust slowly to changing economic conditions. We calibrate our model to be consistent with a well-known and observable index of *ex ante* stock market volatility. We find that the dramatic increase in uncertainty during the Fall of 2008, combined with the zero lower bound on nominal interest rates, may be an important factor in explaining the large and persistent decline in output starting at that time.

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Table 1: Baseline Calibration

Parameter	Description	Calibrated Value
α	Capital's Share in Production	0.333
β	Household Discount Factor	0.994
δ	Steady State Depreciation Rate	0.025
δ_1	First-Order Utilization Parameter	$\beta^{-1} - 1 + \delta$
δ_2	Second-Order Utilization Parameter	0.01
ϕ_K	Adjustment Cost to Changing Investment	10
ϕ_P	Adjustment Cost to Changing Prices	160
Π	Steady State Inflation Rate	1.005
ρ_r	Central Bank Interest Rate Smoothing Coefficient	0.50
ρ_π	Central Bank Reaction Coefficient on Inflation	1.50
ρ_y	Central Bank Reaction Coefficient on Output Growth	0.50
σ	Parameter Affecting Household Risk Aversion	30
ψ	Intertemporal Elasticity of Substitution	0.80
η	Consumption Weight in Period Utility Function	0.32
θ_μ	Elasticity of Substitution Intermediate Goods	6.0
ν	Firm Leverage	0.87
ρ_a	First Moment Preference Shock Persistence	0.90
σ^a	Steady-State Volatility of Preference Shock	0.03
ρ_{σ^a}	Second Moment Preference Shock Persistence	0.70
σ^{σ^a}	Volatility of Second Moment Preference Shocks	0.005

Table 2: Asset-Pricing Implications of Baseline Model

Moment	Formula	Data	Model
Real Interest Rate	$400 * \log\left(R^R\right)$	0.86 (0.04, 1.68)	1.37
Equity Premium	$400 * \log\left(R^E\right) - 400 * \log\left(R^R\right)$	6.33 (2.11, 10.54)	8.63
Equity Return Volatility	$100 * \sqrt{4 * \mathbb{VAR}\left(R^E\right)}$	19.42 (13.40, 25.44)	20.78

Note: All values are reported in annualized percentage points. The data moments come from Bansal and Yaron (2004). Confidence intervals appear in parenthesis and are computed using +/- 1.96 of the standard error estimates. The average VXO in the data is 20.78 over the 1986-2014 sample period. Therefore, we target that moment in our calibration rather than the value reported by Bansal and Yaron (2004).

Figure 1: VXO and Estimated Uncertainty Shocks

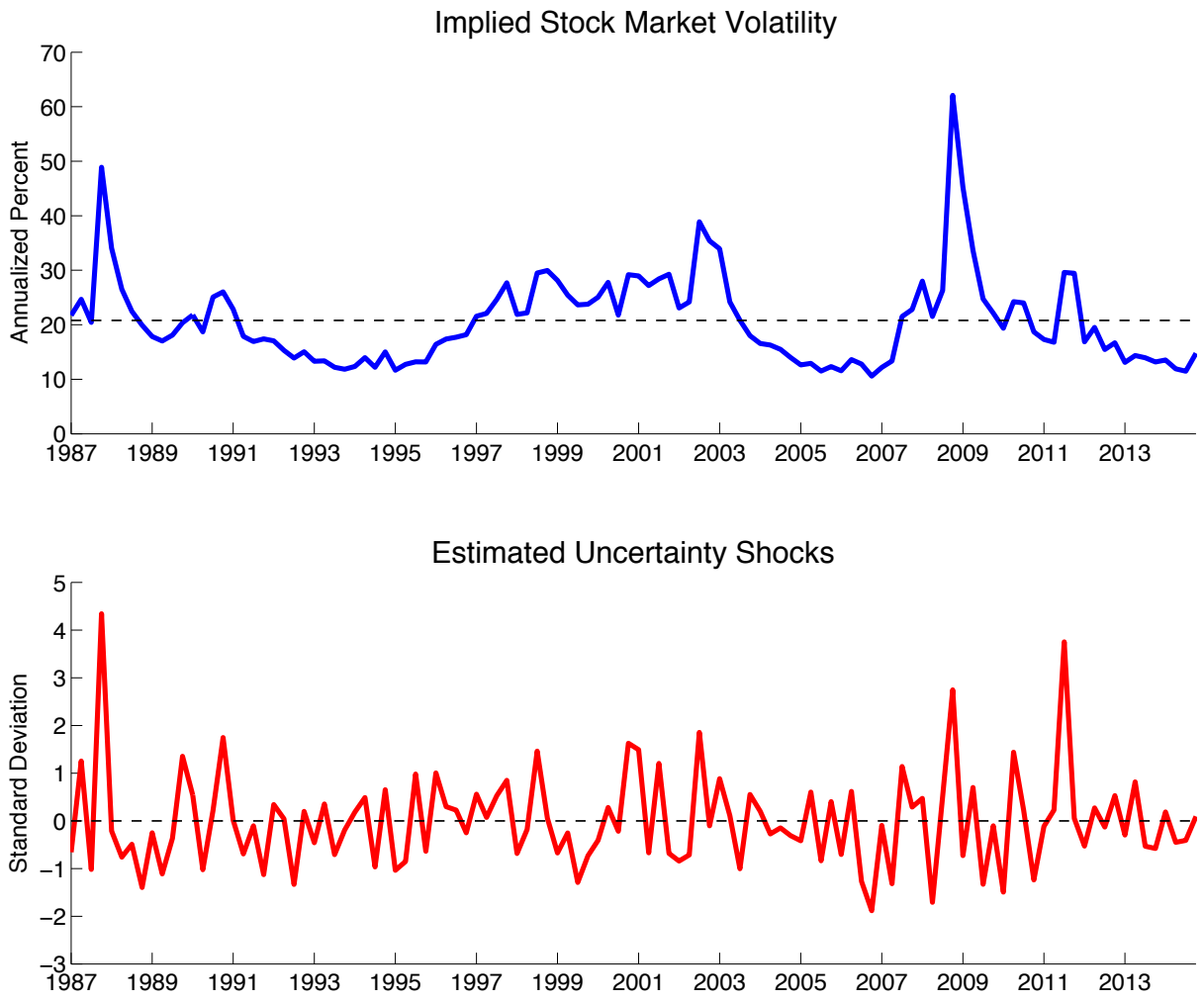


Figure 2: Empirical & Model-Implied Impulse Responses to Uncertainty Shock

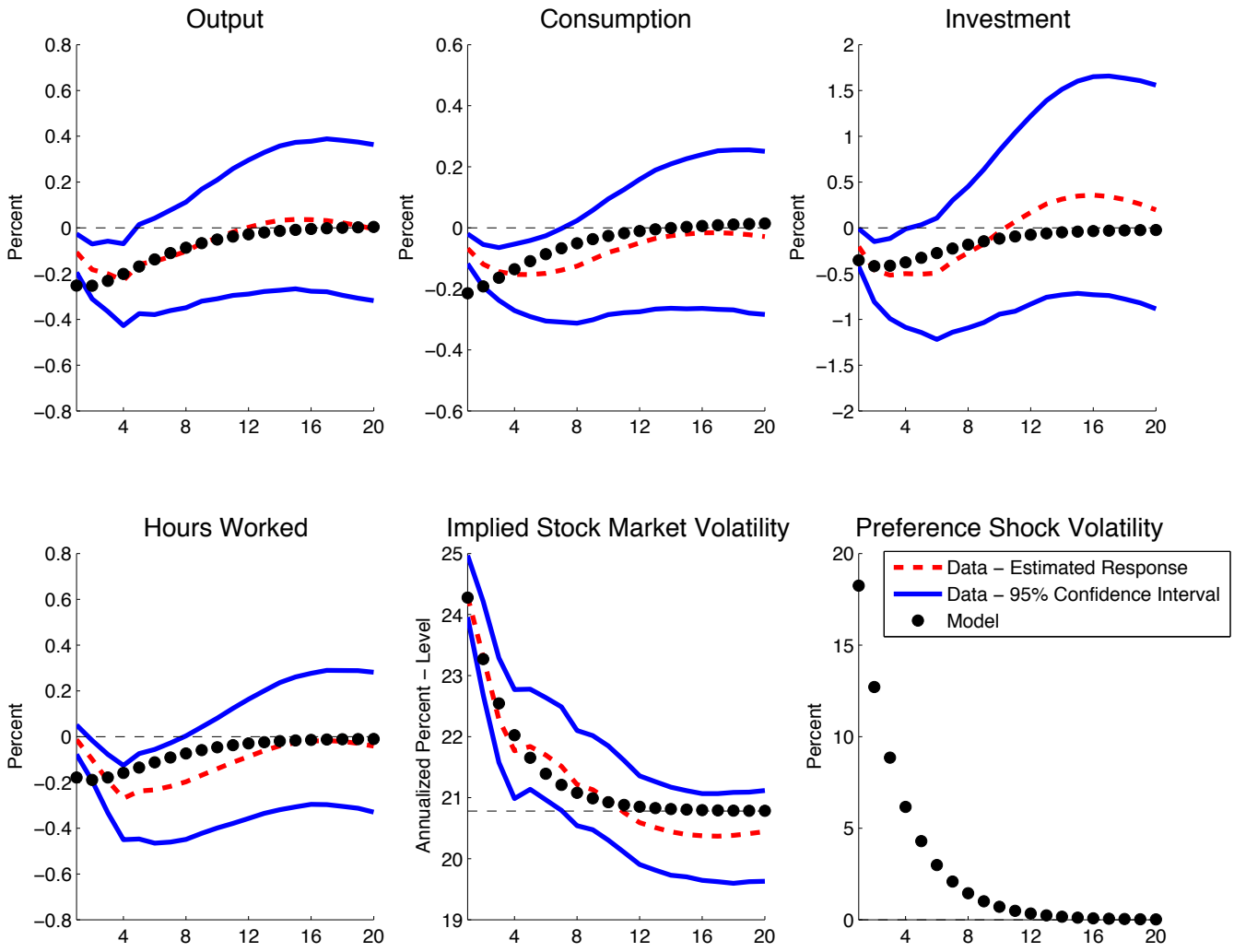


Figure 3: Flexible Price Model Intuition

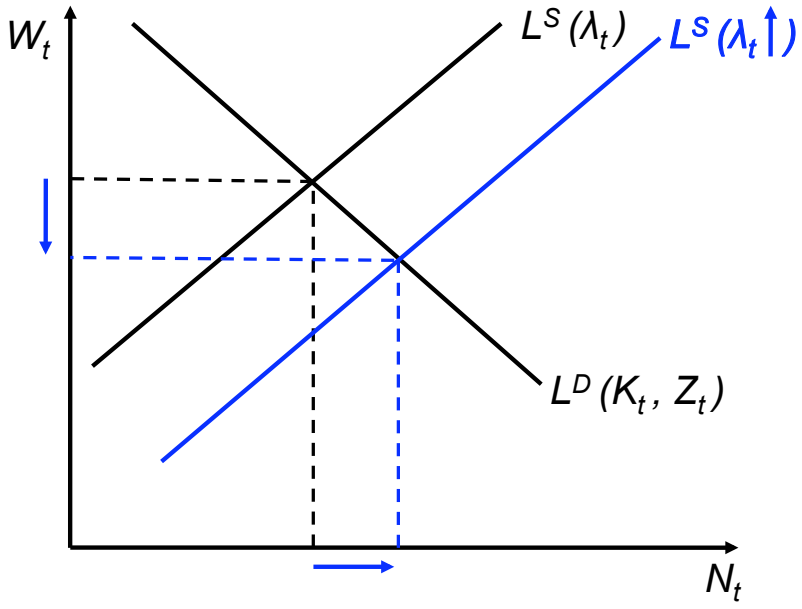


Figure 4: Sticky Price Model Intuition

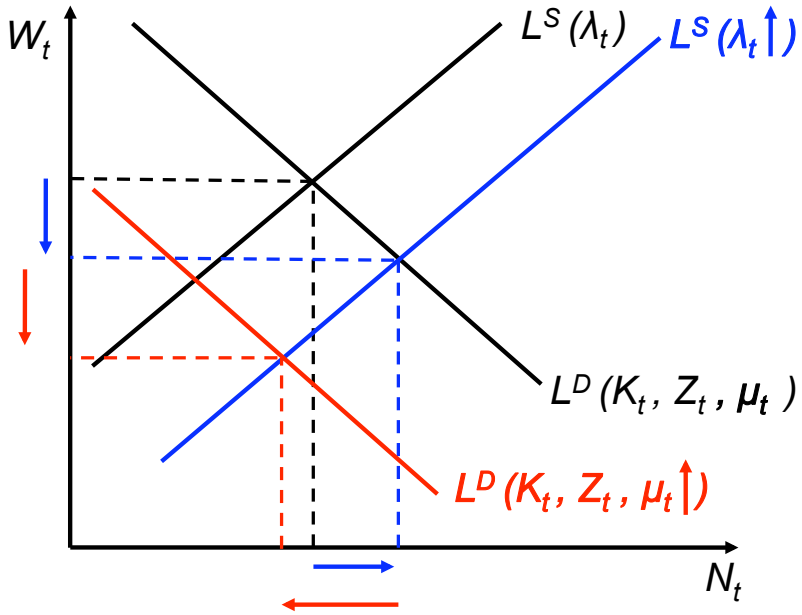
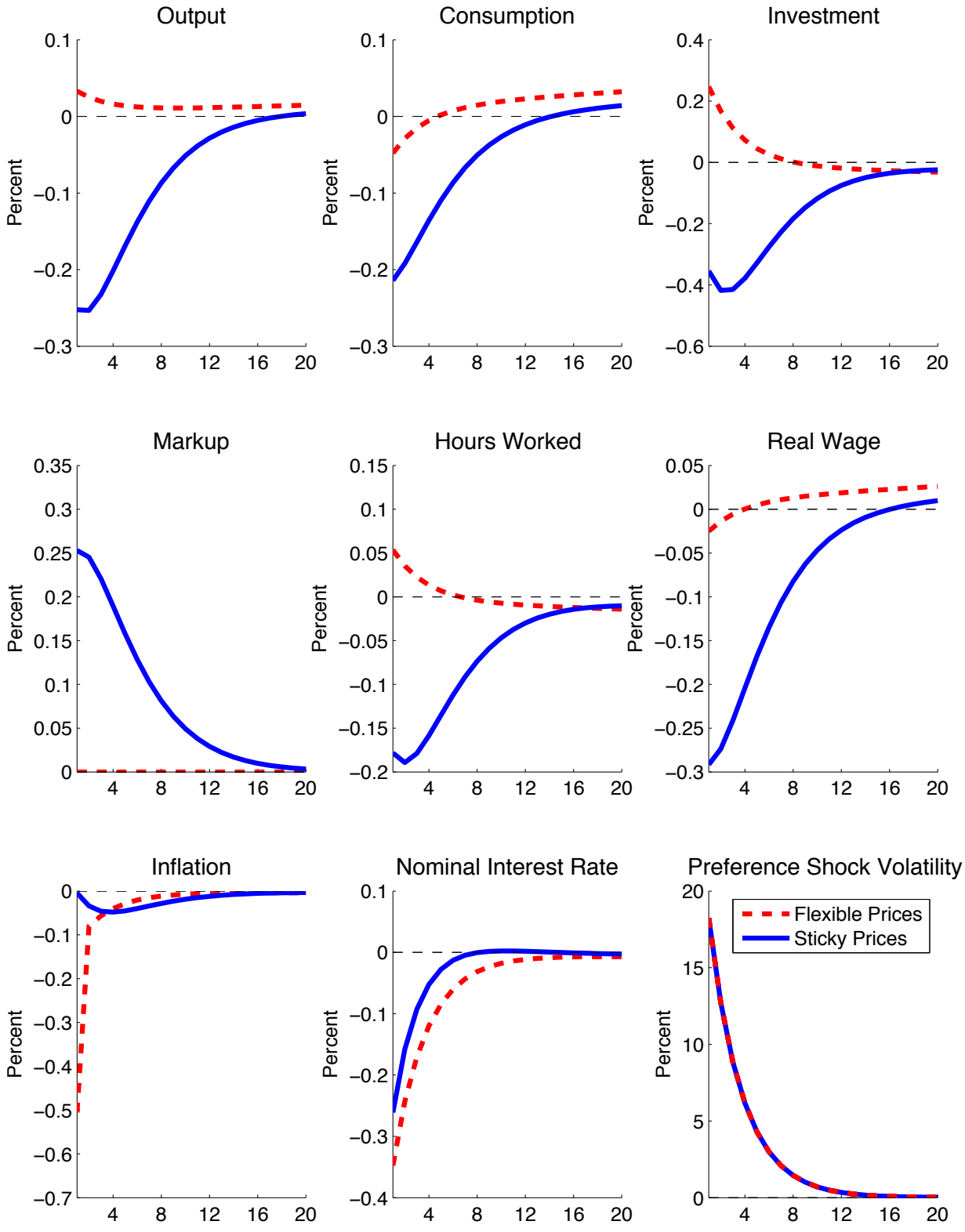


Figure 5: Impulse Responses to Second-Moment Preference Shock



Note: The impulse responses for inflation and the nominal interest rate are plotted in annualized percent deviations.

Figure 6: Perturbing Various Model Features

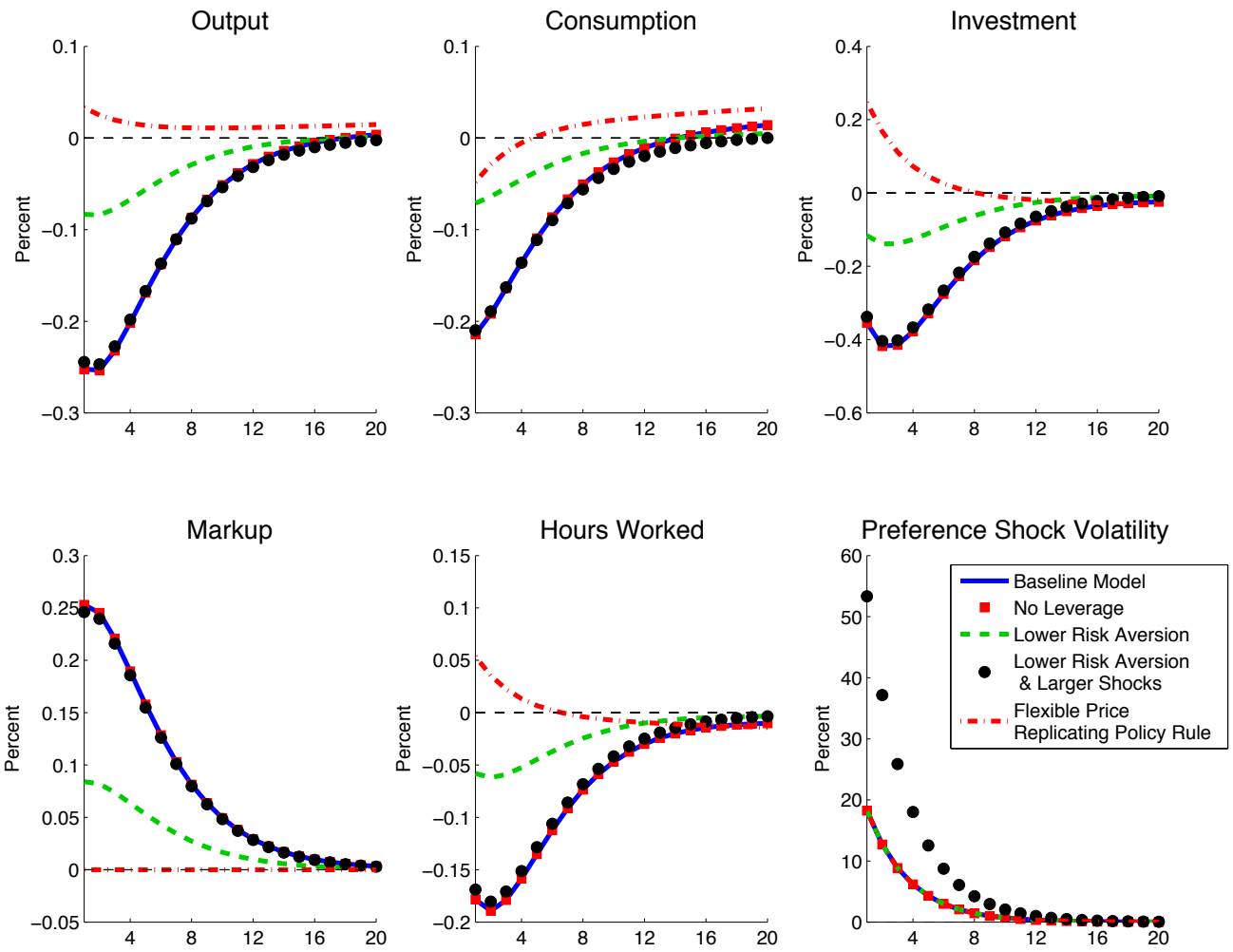
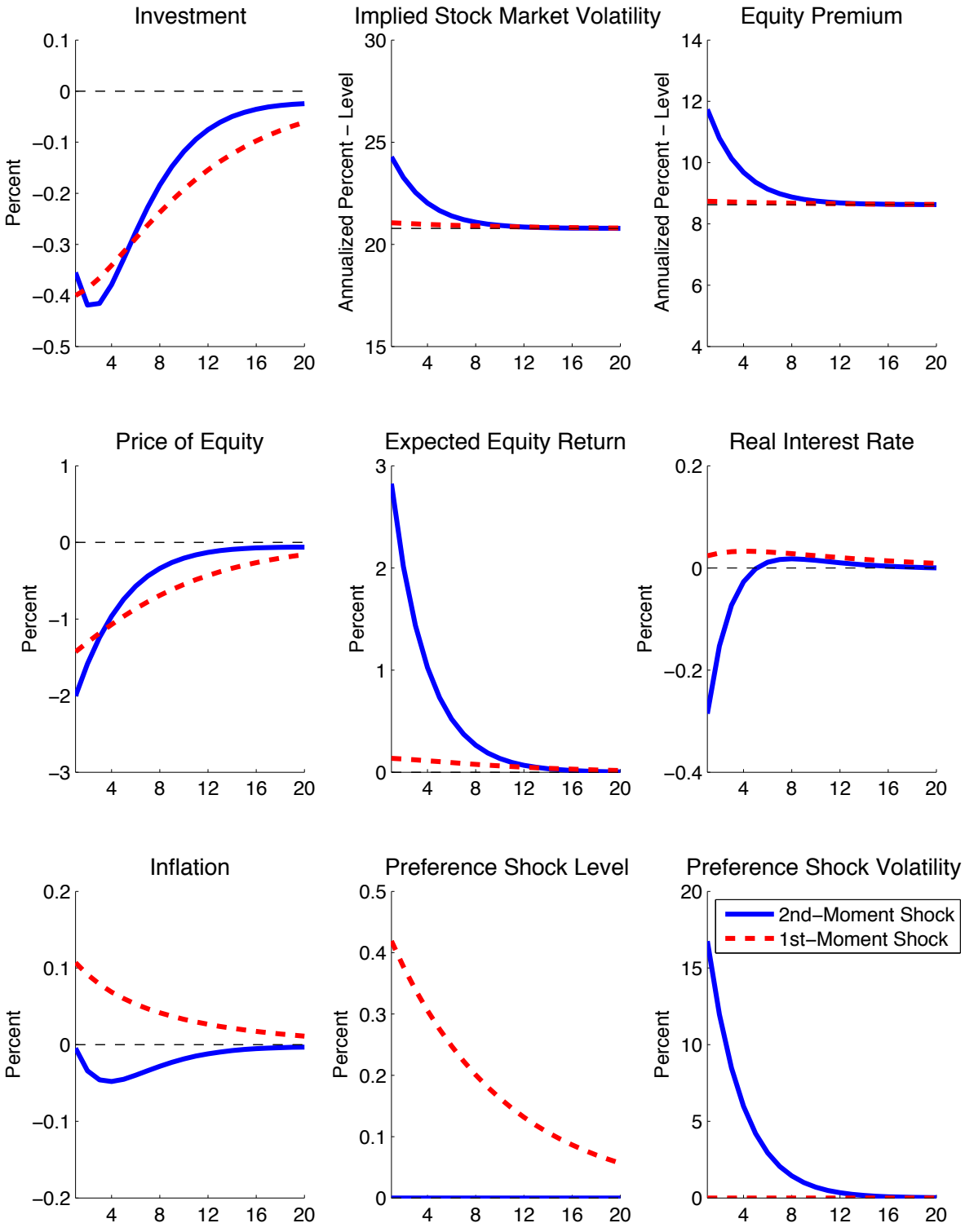


Figure 7: Model-Based Support for Empirical Identification Scheme



Note: The impulse responses for inflation, expected equity return, and the real interest rate are plotted in annualized percent deviations.

Figure 8: Nominal Interest Rate Distributions

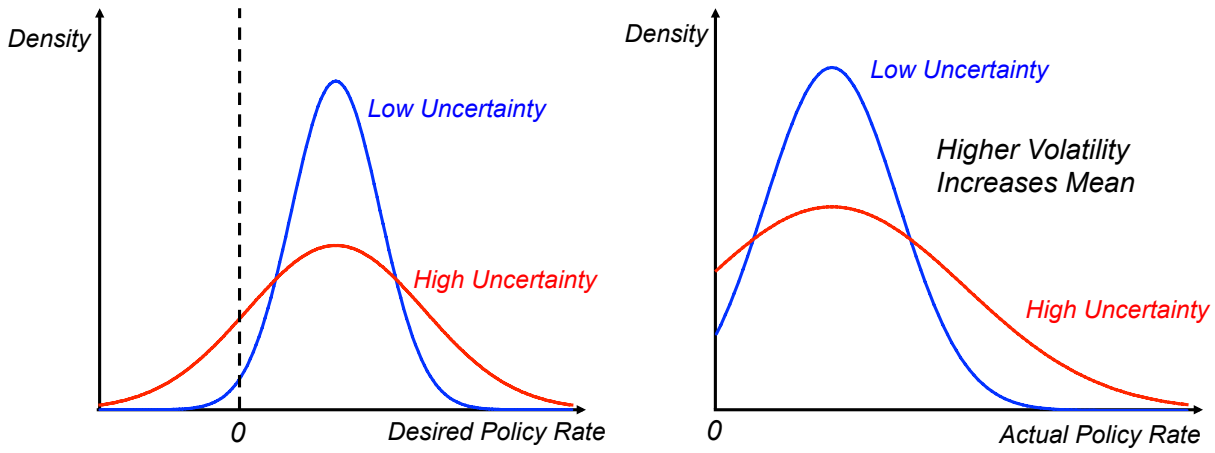


Figure 9: General-Equilibrium Effects of the Contractionary Bias

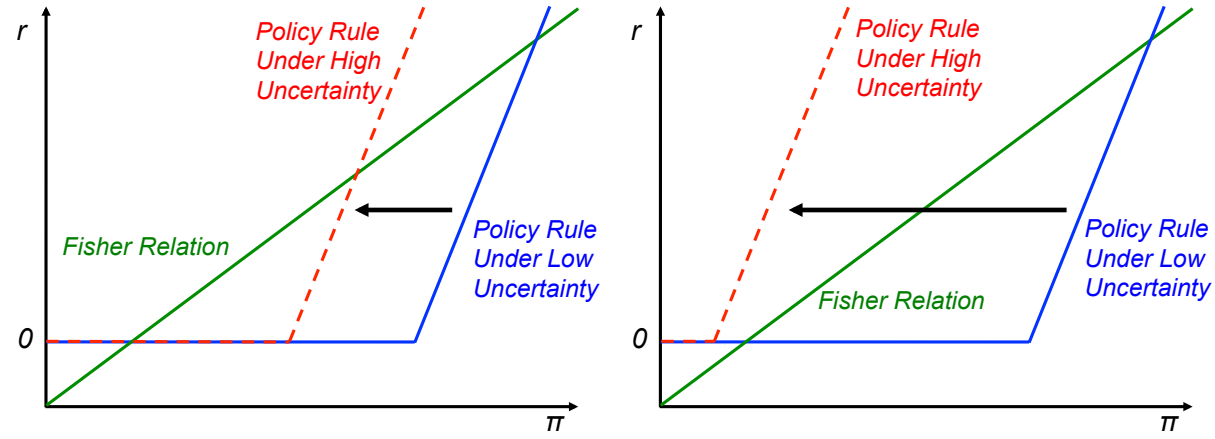
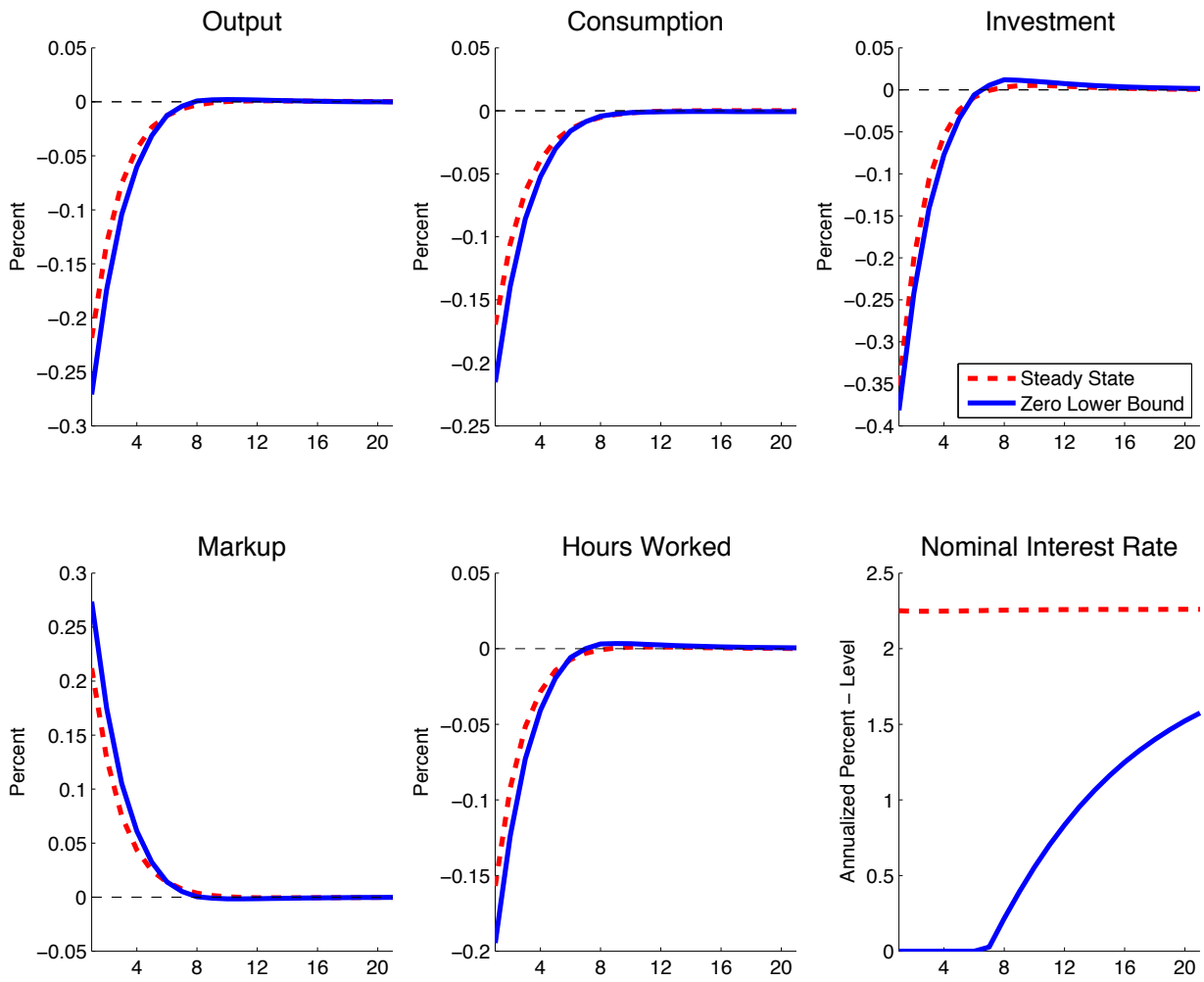


Figure 10: Demand Uncertainty Shock at Zero Lower Bound Under History-Dependent Rule



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Technical Appendix

A Additional Details Concerning Empirical Evidence

A.1 Data Construction and Estimation

This section provides additional details on the data construction and estimation procedure for the empirical evidence from Section 2 of the main text. We estimate our baseline VAR using data on the VXO, GDP, consumption, investment, hours worked, the GDP deflator, the M2 money stock, and the Wu and Xia (2014) shadow rate. To match the concept in the model, we measure consumption in the data as the sum of non-durable and services consumption. Then, we use the sum of consumer durables and private fixed investment as a measure of investment in our baseline empirical model. To match the quarterly frequency of the macroeconomic data, we average a weekly VXO series for each quarter. Thus, our measure of uncertainty captures the average implied stock market volatility within a quarter. We convert output, consumption, investment, and hours work to per-capita terms by dividing by population. Except for the shadow rate, all other variables enter the VAR in log levels. We include four lags in the estimation of the VAR and generate our confidence intervals using the Bayesian method outlined in Sims and Zha (1999).[1]

A.2 Robustness of Macroeconomic Comovement

We argue that macroeconomic comovement between output, consumption, investment, and hours worked is a key stylized after an identified uncertainty shock. In this section, we show that our key empirical result is robust along several dimensions. In our baseline specification, we treated consumer durables as a form of investment. If we instead use the standard National Income and Product Accounts definitions of consumption and investment, Figure A.2 shows a larger impact effect on consumption with a slight over-shoot after three years. The response of investment, however, remains similar to our baseline results.

Our baseline results are also robust to using higher frequency estimation. In our baseline model, we aggregate a weekly VXO series to quarterly frequency. However, the VXO reflects the expected S&P 100 volatility over the next 30 days, not over the next quarter. To

[1] We are grateful to Andrew Lee Smith for many helpful discussions and for sharing his code for computing the Sims and Zha (1999) confidence intervals.

ensure our results are robust to this aggregation strategy, we estimate a version of our empirical model using monthly frequency data on the VXO, output, non-durable plus services consumption, durable consumption, hours worked, the personal consumption expenditure price index, the M2 money stock, and the shadow rate. To construct a monthly GDP series, we splice together monthly GDP from Macroeconomic Advisers beginning in 1992 with Stock and Watson's (2010) monthly GDP estimates from the NBER Business Cycle Dating committee website.[2] Figure A.2 shows that our results are nearly unchanged if we use this higher frequency data. However, data on investment are not available at a monthly frequency. Therefore, we rely on the aggregated quarterly data for our baseline empirical results.

In addition, we compute the impulse response to an uncertainty shock with the VXO ordered last in our structural VAR. Figure A.2 also shows our main stylized fact regarding macroeconomic comovement remains under this alternative identification scheme, which allows contemporaneous macroeconomic events to affect the level of uncertainty. While this ordering is not consistent with our theoretical model, it shows that our baseline identification scheme alone is not crucial for our main result. We compute this robustness check using monthly data, to match the interpretation of the VXO as closely as possible. However, the results with quarterly data produce similar findings.

Figure A.3 contains three additional specifications, which examine alternative assumptions about monetary policy. As we discuss in the main text, the Federal Reserve hit the zero lower bound on nominal interest rates at the end of 2008. While we model this outcome rigorously using our theoretical model, it is less clear how to model the stance of monetary policy during our 1986-2014 sample period econometrically. As an alternative to the shadow rate, we can use the 5-year Treasury rate as a control for monetary policy. Since hitting the zero lower bound, the Federal Reserve used a variety of large-scale asset purchases and forward guidance to help stabilize the economy. Longer-term Treasury rates reflect the effects of these unconventional policies. An alternative modeling assumption is to use the federal funds rate but end the sample period before the zero lower bound binds for too long. Figure A.2 shows that either of these alternative assumptions actually produces responses that are larger than our baseline model. An different modeling assumption is to remove the post-2008 period altogether. If we use the 1962Q3-2008Q2 sample of Bloom (2009) with the federal

[2]Since 2010, the Bureau of Economic Analysis now includes intellectual property as a form of investment. Thus, we splice the two series together using the growth rates from the Stock and Watson estimates to fix issues with the actual level of the data series.

funds rate as the measure of monetary policy, our stylized fact remains: Higher uncertainty generates declines in output, consumption, investment and hours worked.[3]

B Additional Model Details

B.1 Complete Model

In the symmetric equilibrium, the baseline model in Dynare notation is as follows:

```
y + fixedcost  = productionconstant*n^(1 - alpha)*(u*k(-1))^(alpha);

c + leverageratio*k/rr  = w*n  + de + leverageratio*k(-1);

w  = ((1 - eta)/eta)*c/(1 - l);

vf  = (utilityconstant*a*(c^(eta)*(1 - n)^(1 - eta))^((1 - sigma)/thetavf)  +
       beta*expvfsigma^(1/thetavf))^(thetavf/(1 - sigma));

expvfsigma  = vf(+1)^(1 - sigma);

w*n  = (1 - alpha)*(y + fixedcost)/mu;

rrk*u*k(-1)  = alpha*(y + fixedcost)/mu;

q*deltauprime*u*k(-1)  = alpha*(y + fixedcost)/mu;

k  = ((1 - deltau) - (phik/2)*(inv/k(-1) - delta0)^(2))*k(-1) + inv;

deltau = delta0 + delta1*(u-1) + (delta2/2)*(u-1)^(2);

deltauprime = delta1 + delta2*(u-1);

sdf = beta*(a/a(-1))
      *((c^(eta)*(1 - n)^(1 - eta))
```

[3]Since the VXO is first measured in 1986, we use the spliced volatility series constructed by Bloom (2009). This series splices together predicted volatility from a time-series model in the pre-1986 period with the ex ante VXO measure of implied volatility after 1986.

```
      /(c(-1)^(eta)*(1 - n(-1))^(1 - eta)))^((1 - sigma)/thetavf) ...
      *(c(-1)/c)*(vf^(1 - sigma)/expvfsigma(-1))^(1 - 1/thetavf);

1 = rr*sdf(+1);

1 = r*sdf(+1)*(pie(+1))^(-1);

1 = sdf(+1)*(de(+1) + pe(+1))/pe;

log(r) = rhor*log(r(-1))
         + (1 - rhor)*(log(rss) + rhopie*log(pie/piess) + rhoy*log(y/y(-1)));

de = y - w*n - inv - (phip/2)*(pie/piess - 1)^(2)*y - leverageratio*(k(-1) - k/rr);

1 = sdf(+1)*(u(+1)*rrk(+1) +
     q(+1)*((1 - deltau(+1)) - (phik/2)*(inv(+1)/k - delta0)^(2)
     + phik*(inv(+1)/k - delta0)*(inv(+1)/k)))/q;

1/q = 1 - phik*(inv/k(-1) - delta0);

phip*(pie/piess - 1)*(pie/piess) = (1 - thetamu) + thetamu/mu +
                    sdf(+1)*phip*(pie(+1)/piess - 1)*(y(+1)/y)*(pie(+1)/piess);

expre = (de(+1) + pe(+1))/pe;

expre2 = (de(+1) + pe(+1))^(2)/pe^(2);

varexpre = expre2 - (expre)^(2);

a = (1 - rhoa)*ass + rhoa*a(-1) + vola(-1)*ea;

vola = rhovola*vola(-1) + (1 - rhovola)*volass + volvola*evola;
```

Since the capital stock is predetermined, we lag the capital stock K variables by one period relative to the timing in the main text. The replication code is available from the Federal Reserve Bank of Kansas City website.

B.2 Impulse Response Construction

In our main text, we present impulse responses to an uncertainty shock at the stochastic steady state of the model. These impulse responses allow us to characterize the impact of an increase in uncertainty about the future without any change in actual realized shock volatility. To construct these responses, we set the exogenous shocks in the model to zero and iterate our third-order solution forward. After a sufficient number of periods, the endogenous variables of the model converge to a fixed point, which we denote the stochastic steady state. We then hit the economy with a one standard deviation uncertainty shock but assume the economy is hit by no further shocks. We compute the impulse response as the percent deviation between the equilibrium responses and the pre-shock stochastic steady state.

By default, Dynare uses an alternative simulation-based procedure to construct impulse responses for 2nd-order and higher model solutions. This method is based on the generalized impulse response of Koop, Pesaran and Potter (1996). As opposed to being centered around the stochastic steady state, these alternative responses are computed in deviations from the ergodic mean of the endogenous variables. In addition, these responses combine both the effects of higher uncertainty about future shocks with higher realized volatility of the actual shocks hitting the economy. Figure B.1 shows that these two alternative impulse responses produce almost identical results for the baseline model. In the main text, we show the impulse responses at the stochastic steady state for two reasons. First, we want to highlight the effects of higher uncertainty in isolation without any change in actual shock volatility. Second, the generalized impulse responses require many simulations to produce adequate results. This additional computational time becomes burdensome when we calibrate the model-implied VXO to the VAR evidence using impulse response matching.[4]

For consistency with the impulse responses, we report the unconditional asset-pricing implications of the model from Section 7.4 at the stochastic steady state. If we instead use a simulation procedure and report them from the ergodic mean, Table B.1 shows that the quantitative predictions are nearly unchanged. This result occurs because the stochastic steady state for each endogenous variable is close to its ergodic mean.

[4]See Section 7.1 of the main text for the details of our calibration strategy.

C Solving Model with a Zero Lower Bound Constraint

To analyze the impact of uncertainty shocks at the zero lower bound, we solve our model using the policy function iteration method of Coleman (1990) and Davig (2004). This global approximation method allows us to model the occasionally-binding zero lower bound constraint. This section provides the details of the numerical solution algorithm. The algorithm is implemented using the following steps:

1. Discretize the state variables of the model: $\left\{K_t \times R^d_{t-1} \times a_t \times \sigma^a_t\right\}$

2. Conjecture initial guesses for the policy functions of the model $N_t = N(K_t, R^d_{t-1}, a_t, \sigma^a_t)$, $U_t = U(K_t, R^d_{t-1}, a_t, \sigma^a_t)$, $I_t = I(K_t, R^d_{t-1}, a_t, \sigma^a_t)$, $\Pi_t = \Pi(K_t, R^d_{t-1}, a_t, \sigma^a_t)$, and $\mathbb{E}_t V^{1-\sigma}_{t+1} = EV(K_t, R^d_{t-1}, a_t, \sigma^a_t)$.

3. For each point in the discretized state space, substitute the current policy functions into the equilibrium conditions of the model. Use interpolation and numerical integration over the exogenous state variables a_t and σ^a_t to compute expectations for each Euler equation. This operation generates a nonlinear system of equations. The solution to this system of equations provides an updated value for the policy functions at that point in the state space.

4. Repeat Step (3) for each point in the state space until the policy functions converge and cease to be updated.

We implement the policy function iteration method in `FORTRAN` using the nonlinear equation solver `DNEQNF` from the `IMSL` numerical library.

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Figure A.1: Empirical Impulse Responses to Identified Uncertainty Shock

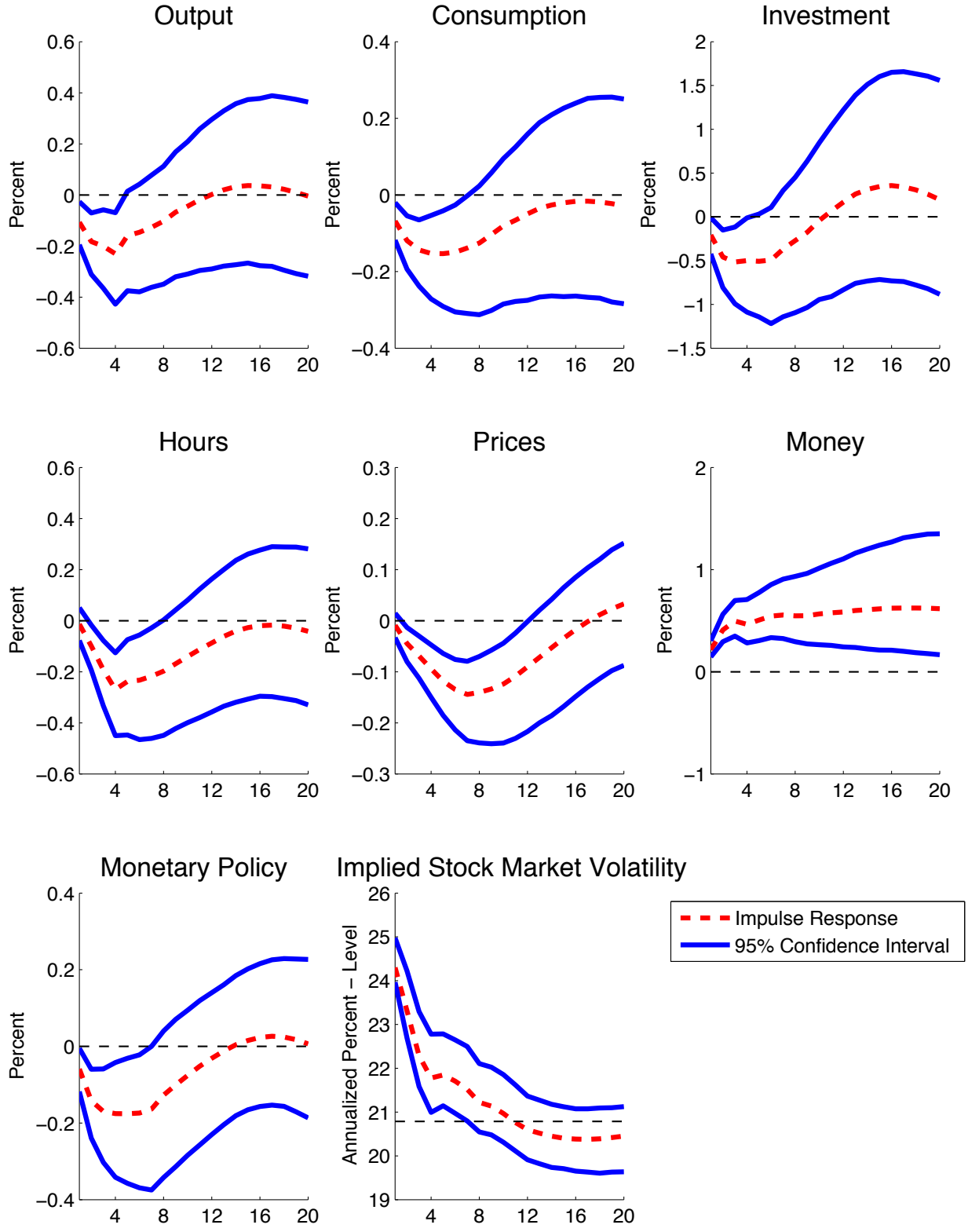
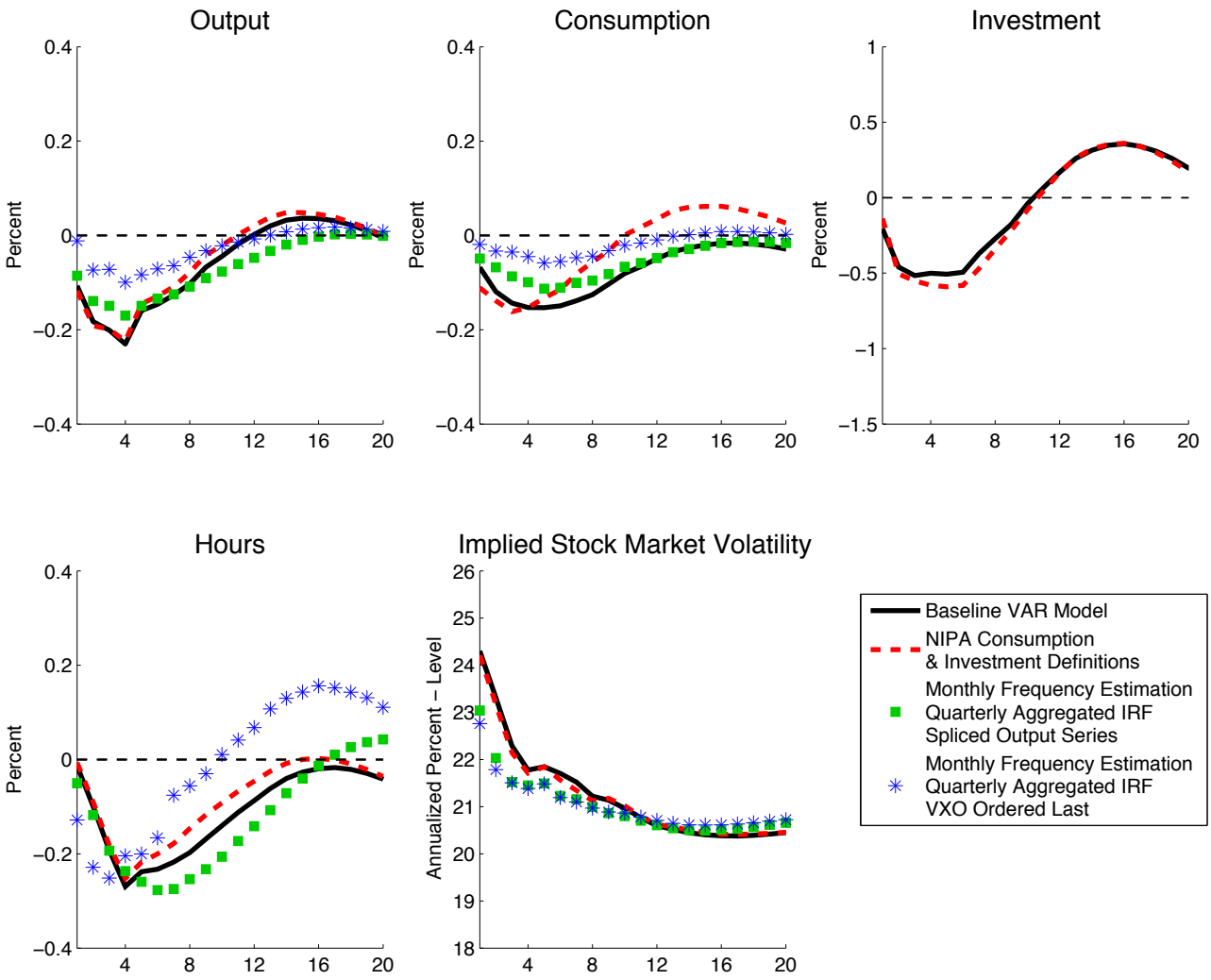
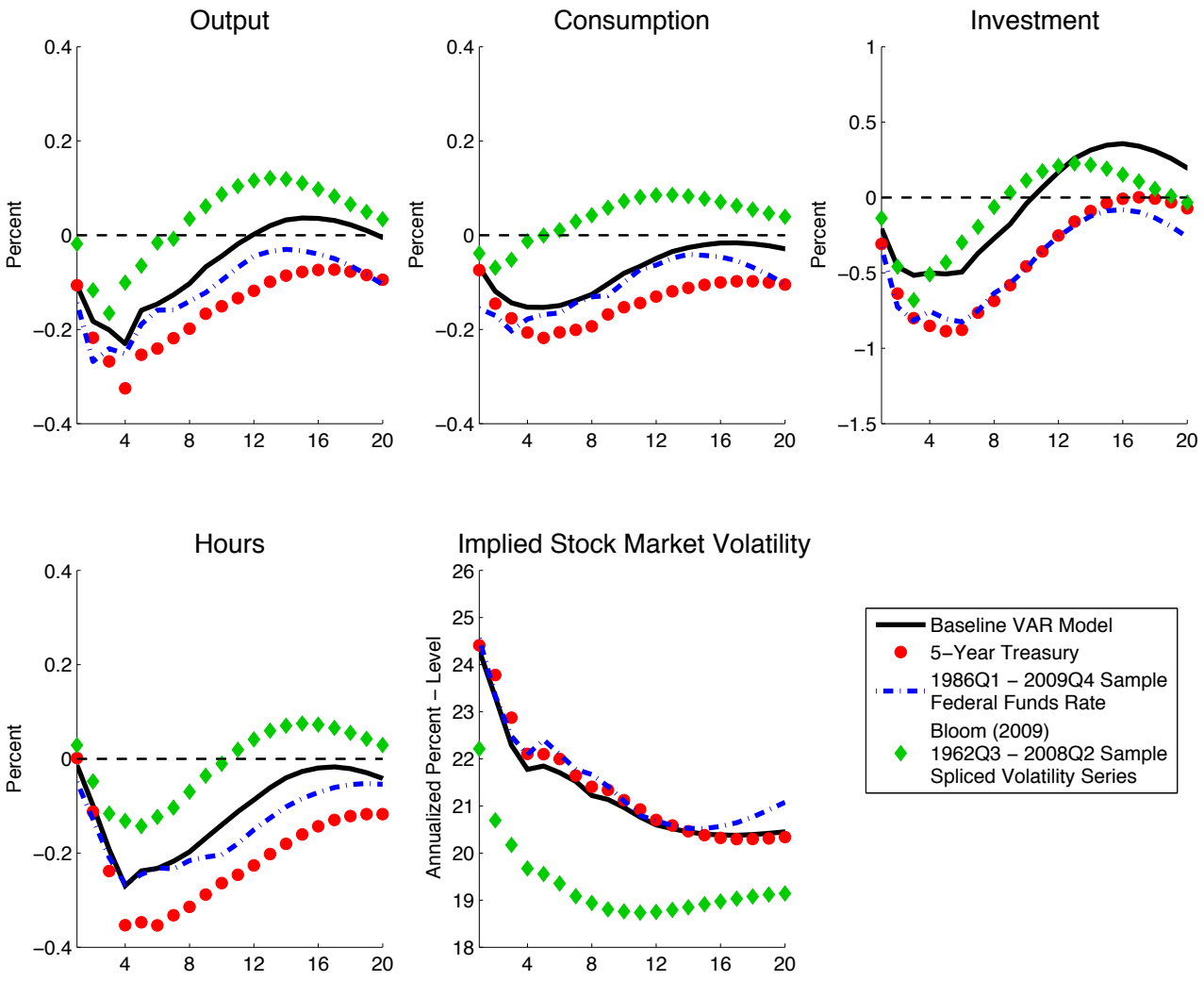


Figure A.2: Alternative Empirical Specifications



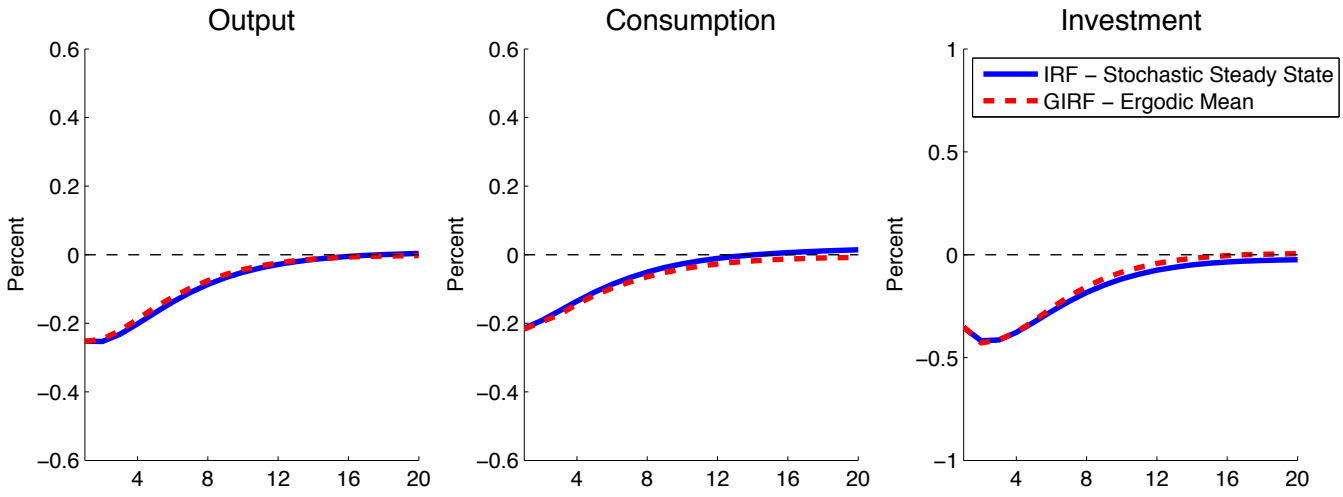
Note: Each series represents point estimates from a different empirical specification.

Figure A.3: Alternative Specifications for Monetary Policy



Note: Each series represents point estimates from a different structural vector autoregression.

Figure B.1: Alternative Impulse Response Construction



Note: Impulse responses are plotted in percent deviations from either the stochastic steady state or their ergodic mean.

Table B.1: Asset-Pricing Implications of Baseline Model

Moment	Formula	Stochastic Steady State	Ergodic Mean
Real Interest Rate	$400 * \log\left(R^R\right)$	1.37	1.35
Equity Premium	$400 * \log\left(R^E\right) - 400 * \log\left(R^R\right)$	8.63	8.64
Equity Return Volatility	$100 * \sqrt{4 * \mathbb{VAR}\left(R^E\right)}$	20.78	19.44

Note: All values are reported in annualized percentage points.