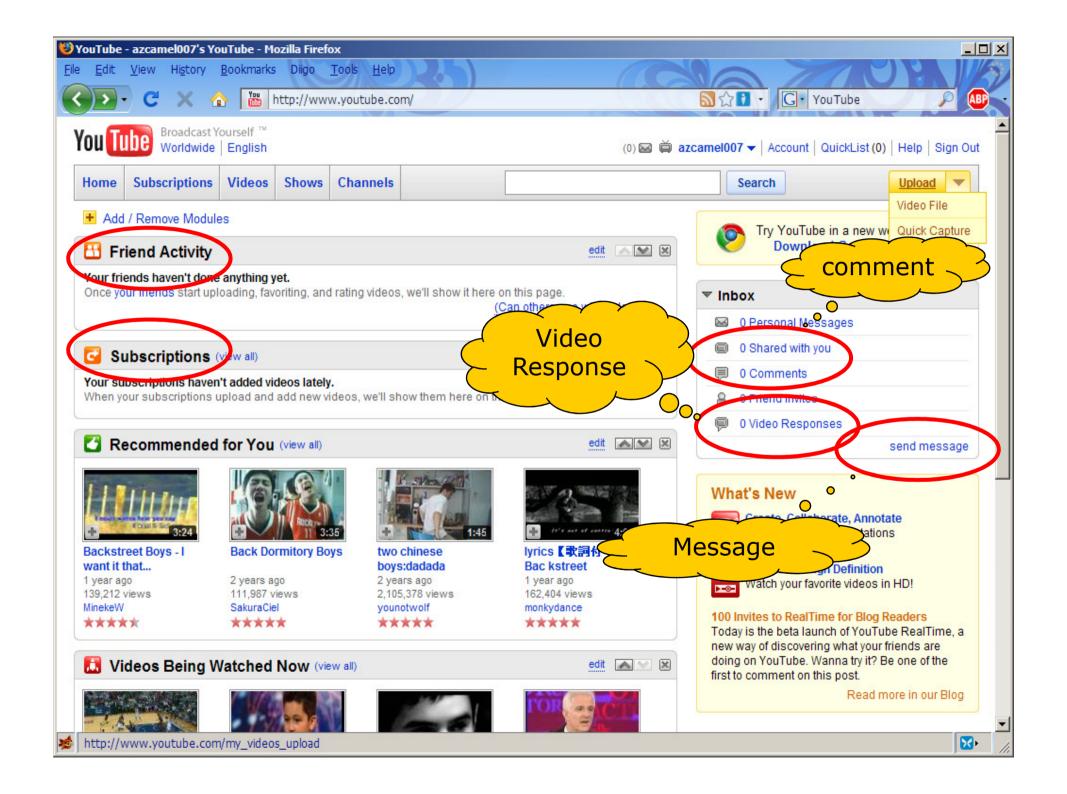
# Uncovering Groups via Heterogeneous Interaction Analysis

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### Multi-Dimensional Network

Different behaviors lead to heterogeneous Interactions



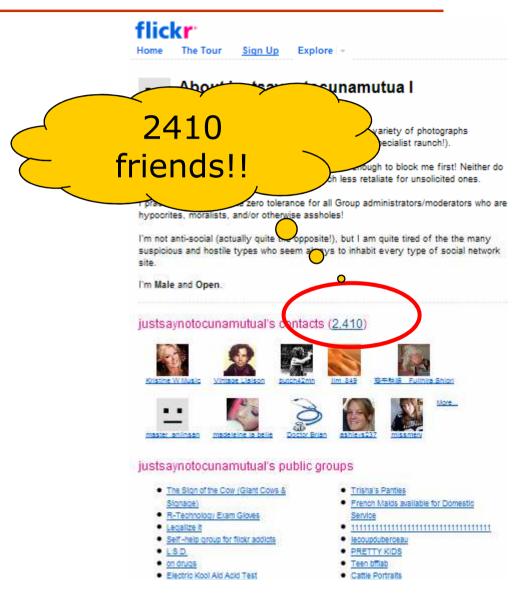
Network of Multiple Dimensions

### Heterogeneous Interaction Analysis

- A latent community structure is shared in a multi-dimensional network
  - E.g. a group sharing similar interest
- Goal: Find out the shared community structure by integrating the interactions at different dimensions

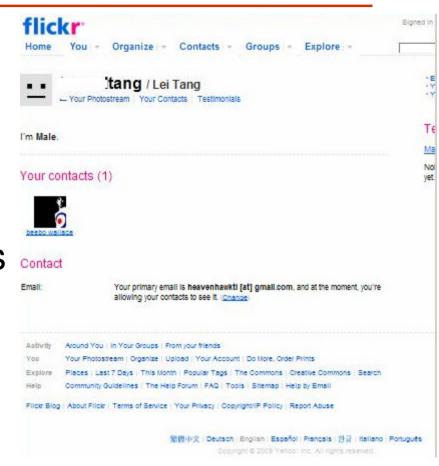
### Why not just friends network?

■ Too many friends?



### Why not just friends network?

- Too many friends?
- Too few friends?
- Friends network tells limited info for some users
- Interaction at other dimensions might help



### Recap of Modularity

- Modularity: A measure that compares the within group interaction with uniform random graph with the same node degree distribution
- □ In a network of m edges, for two nodes with degree d<sub>i</sub> and d<sub>j</sub>, respectively, the expected number of edges between them:

$$d_i d_j / 2m$$

- □ The connection strength in a group:  $\sum_{i \in C, j \in C} A_{ij} d_i d_j / 2m$
- □ To partition the network into multiple groups, we maximize

$$\frac{1}{2m} \sum_{C} \sum_{i \in C, j \in C} A_{ij} - d_i d_j / 2m$$

### Modularity Matrix

Modularity can be formulated in a matrix form

$$Q = \frac{1}{2m} Tr(S^T B S)$$

B is the modularity matrix

$$B_{ij} = A_{ij} - d_i d_j / 2m$$

With spectral relaxation, S corresponds to the top eigenvectors of the modularity matrix B

### Modularity in M-D Networks

- Average Modularity Maximization (AMM)
  - Average the network interaction of different dimensions

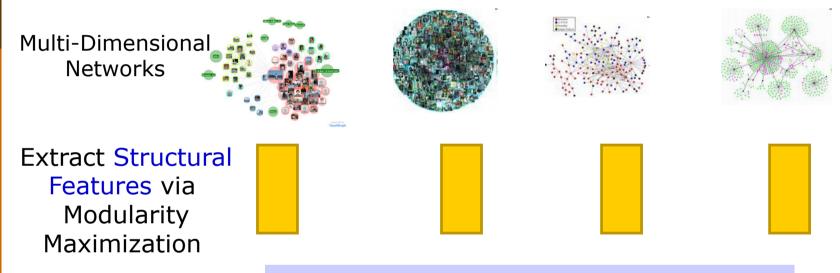
$$\bar{A} = \frac{1}{D}(A_1 + A_2 + \dots + A_D)$$

- Total Modularity Maximization (TMM)
  - the total sum of modularity at different dimensions

$$\max(Q_1 + Q_2 + \cdots + Q_D)$$

- Potential Cons
  - Not sure whether the interaction or modularity of different dimensions are comparable
  - Can be sensitive to networks of noisy dimensions

### Principal Modularity Maximization



Denoise the interaction at each dimension

- These structural features are not necessarily similar, but are highly correlated.
- Transform these features into a shared space such that their correlation is maximized.
- Solution: Generalized Canonical Correlation Analysis (CCA)

### Canonical Correlation Analysis

$$R(i,j) = (S_i w_i)^T (S_j w_j) = w_i^T (S_i^T S_j) w_j = w_i^T C_{ij} w_j$$

$$\max \sum_{i=1}^{d} \sum_{j=1}^{d} w_i^T C_{ij} w_j$$

$$s.t. \sum_{i=1}^{d} w_i^T C_{ii} w_i = 1$$



$$\max \sum_{i=1}^{d} \sum_{j=1}^{d} w_{i}^{T} C_{ij} w_{j} \\
s.t. \sum_{i=1}^{d} w_{i}^{T} C_{ii} w_{i} = 1$$

$$\Rightarrow \begin{bmatrix}
C_{11} & C_{12} & \cdots & C_{1d} \\
C_{21} & C_{22} & \cdots & C_{2d} \\
\vdots & \vdots & \ddots & \vdots \\
C_{d1} & C_{d2} & \cdots & C_{dd}
\end{bmatrix}
\begin{bmatrix}
w_{1} \\
w_{2} \\
\vdots \\
w_{d}
\end{bmatrix}$$

$$= \lambda \begin{bmatrix}
C_{11} & 0 & \cdots & 0 \\
0 & C_{22} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & C_{dd}
\end{bmatrix}
\begin{bmatrix}
w_{1} \\
w_{2} \\
\vdots \\
w_{d}
\end{bmatrix}$$



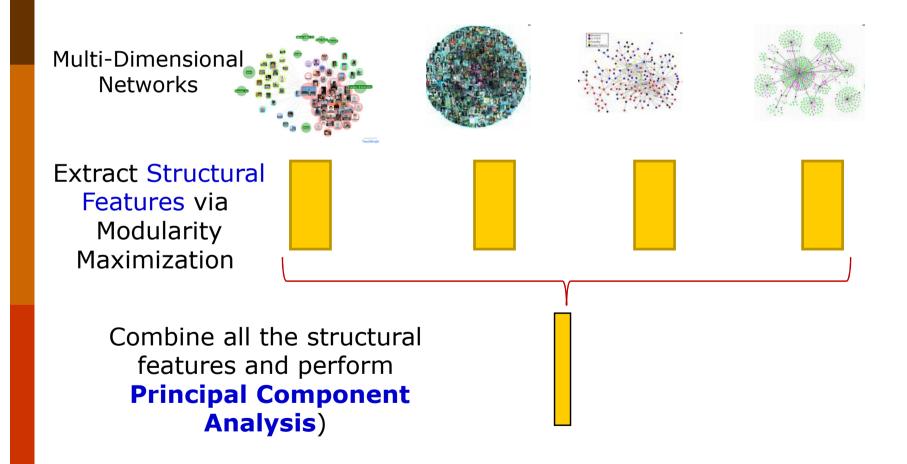
$$C_{11} = S_1^T S_1 = I$$

$$C_{22} = S_2^T S_2 = I$$
....

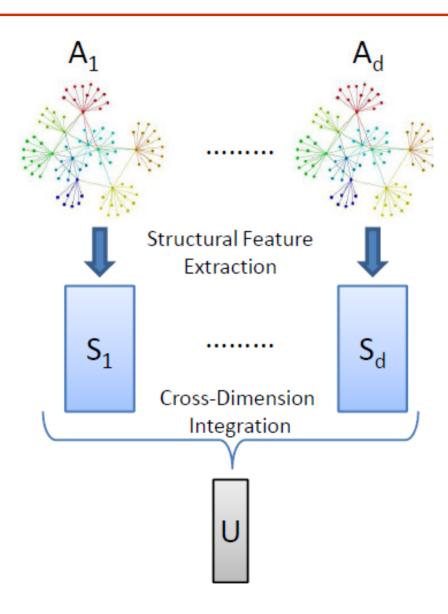
**Principal Component Analysis** (PCA)

eigvenvector of the covariance matrix

### Principal Modularity Maximization



### Overview of PMM



### PMM Algorithm

- □ Given: a multi-dimensional network
- Output: shared group structure

#### **□** Algorithm:

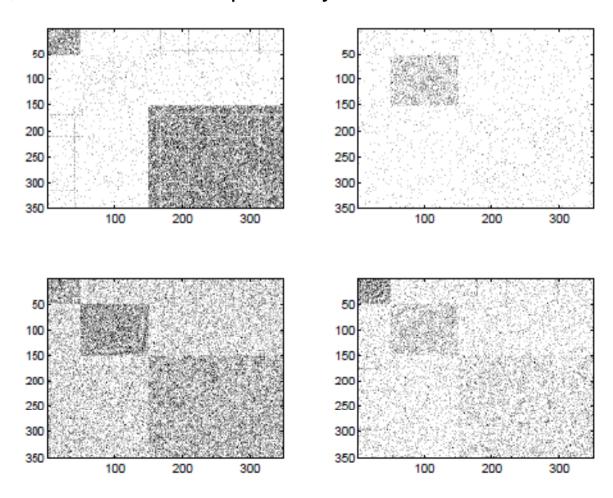
- Phase I: Extract structural features from each dimension of the network
- Phase II: Combine all the extracted features of each dimension and perform PCA
- Apply K-means to obtain the discrete partition

### Experiments

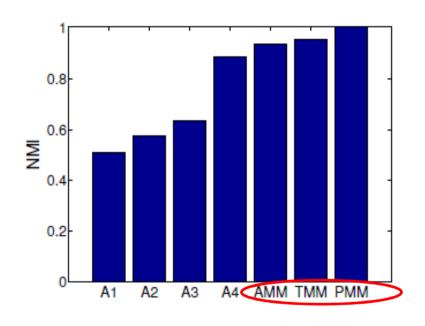
- Compare different community detection strategies
  - AMM, TMM, PMM
  - Modularity maximization on a single dimension
- Verify the sensitivity to noise for different methods
- Data Sets
  - Synthetic Data
    - controlled noise and ground truth information
  - Real-World Data
    - collected from YouTube

# Synthetic Data

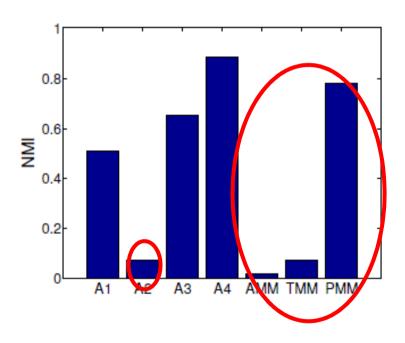
- 4 dimensions, 3 communities
- □ 50, 100, 150 members respectively



### Performance on Synthetic Data



Small Noise



Substantial Noise in one dimension

# Average Performance

Table 1: Average Performance Over 100 Runs

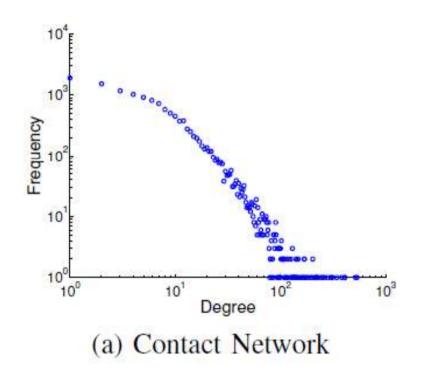
	Strategy	Performance	
	$A_1$	$0.7237 \pm 0.1924$	
Single-Dimensional	$A_2$	$0.6798 \pm 0.1888$	
	$A_3$	$0.6672 \pm 0.1848$	
	$A_4$	$0.6906 \pm 0.1976$	
	AMM	$0.7946 \pm 0.1623$	
Multi-Dimensional	TMM	$0.9157 \pm 0.1137$	
	PMM	$0.9351 \pm 0.1059$	

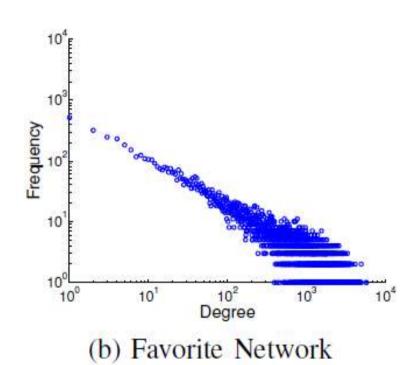
Single < AMM < TMM < PMM PMM: Low Variance

### YouTube Data

- Collect contact network, subscription network, and users' favorite videos
- □ Crawl 30,522 user profiles reaching in total 848,003 users and 1,299, 642 favorite videos
- □ 15,088 active users
- Construct a 5-dimensional network
  - Contact
  - Share Contacts
  - Share subscription
  - Followed by the same set of people
  - Share favorite videos

# Degree Distribution





### Evaluation on Real-World Data

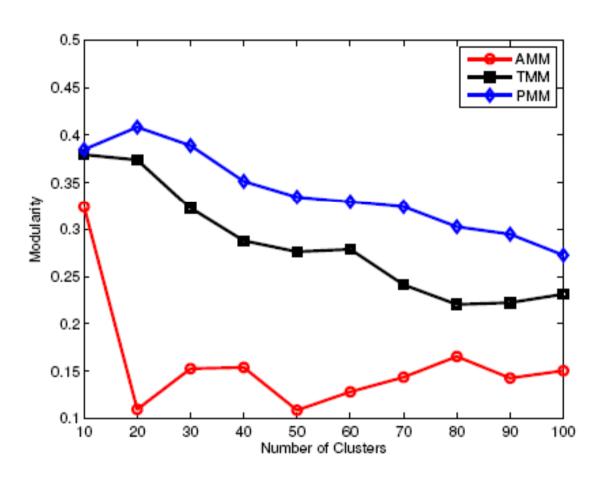
- Challenges
  - No ground truth
  - Need a smart way to do the comparative study
- Evaluation -- Cross Dimension Validation
  - Follow the idea of cross validation
  - For a network of D dimensions
    - □ learn the community structure from (D-1) dimensions
    - evaluate it on the remaining dimension in terms of modularity

### Performance on YouTube Data

Methods	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$
$A_1$		.0007	.0008	.0008	.0002
$A_2$	.1548		.0133	.0361	.0076
$A_3$	.0712	.0275		.0446	.0140
$A_4$	.0584	.0569	.0186		.0108
$A_5$	.0314	.0135	.0095	.0180	
AMM	.1096	.0001	.0018	.0053	.0070
TMM	.3740	.1856	.1246	.1800	.0706
PMM	.4085	.2063	.1307	.1844	.0947

> PMM tends to be the winner

### PMM compared with AMM & TMM



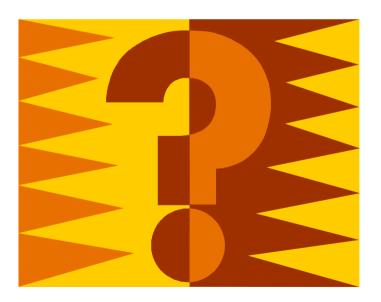
AMM < TMM < PMM

### Conclusions & Future Work

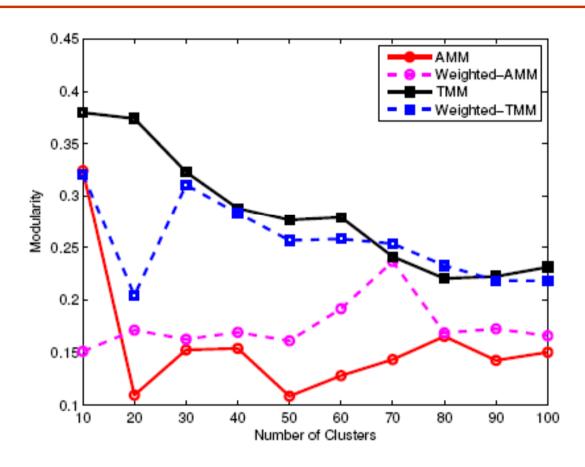
- Networks in social media are multi-dimensional and noisy
- Propose an effective Principal Modularity Maximization to extract the shared group structure
  - Extract Structural Features via Modularity Maximization
  - Perform Cross-Dimensional Integration via PCA
- Can be applied similarly to other spectral clustering methods
- Future Work:
  - Determine whether two network dimensions share the same community structure?
  - Need to remove noisy interaction dimensions?
  - One actor assigned to multiple different groups?
  - Scalability?

# Acknowledgments

■ Thanks to the sponsorship of AFOSR and ONR.



### Weighted AMM & TMM



Assigning a proper weight to each dimension is not easy!