

Understanding and supporting mathematics teachers' knowledge for teaching

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Published online: 26 February 2015
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Mathematics teachers' knowledge for teaching mathematics has received significant attention in mathematics education research. It has been considered from different perspectives, with different constructs to describe it, resulting in a complex landscape of what it is about and what it entails. While category-based perspectives based on content knowledge and pedagogical content knowledge have received the most visibility in studies of the mathematics teacher, they provide a limited or biased representation of this knowledge. For example, teachers' ways of knowing, thinking, and holding knowledge are important aspects of what is needed to teach mathematics and what we need to understand in order to help teachers to enhance their practice and students' learning. Thus, ongoing research of this knowledge is needed to reveal details of it and issues associated with it from different perspectives and contexts.

The articles in this issue of the *Journal of Mathematics Teacher Education* contribute to our understanding of this knowledge in a variety of ways. Collectively, they deal with topics that include: unpacking issues of mathematics knowledge for teaching (MKT) as a construct based on specific categories; supporting the development of teachers' knowledge in using challenging mathematics tasks in their teaching; understanding prospective teachers' knowledge in terms of how their conceptions of number develop during their engagement in well-designed tasks; and understanding teachers' knowledge as beliefs.

Natasha Speer, Karen King, and Heather Howell draw attention to issues that arise when the MKT construct derived from research into elementary teachers' practices is used in research on secondary and post-secondary teachers who typically differ from elementary teachers in their content preparation. In particular, they examined the definitions of common content knowledge (CCK) and specialized content knowledge (SCK) and their relationships to typical characteristics of elementary teachers implicit in those definitions and compared those characteristics with characteristics typical of secondary and post-

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secondary teachers and the nature of the teaching-related tasks they face. They discussed two major issues emerging from this analysis of the definitions that deal with the relationship of CCK to SCK for teachers holding a bachelors or higher degree in mathematics and the relationship between the type of work mathematicians do in their research and while teaching mathematics. For example, they criticized the distinction made between CCK and SCK (i.e., there is mathematical work that is required while teaching that is not required in the other contexts) by pointing out that in both the teaching and research contexts, the mathematician needs to make sense of the mathematical ideas and reasoning presented by someone else and determine whether the reasoning is correct. They suggested that the potential generalizing of this elementary-teacher-based theoretical framing of MKT is likely to lead to unproductive interventions for improvement in teachers' learning.

Peter Sullivan, Mike Askew, Jill Cheeseman, Doug Clarke, Angela Mornane, Anne Roche, and Nadia Walker explored ways to support primary and secondary school teachers in converting challenging mathematics tasks into classroom lessons and in encouraging their students to engage with those tasks. They examined the characteristics of appropriately challenging tasks, what happened when teachers posed such tasks to students, and the actions teachers could take to encourage students to persist on those tasks. In framing the work, they recognized the perspective that teachers' actions are informed by their intentions, which, in turn, are influenced by teachers' mathematics knowledge, pedagogical beliefs, and expectation of opportunities and constraints they might experience when teaching. Their intervention involved suggesting particular tasks, lessons, and pedagogies, which were different from those commonly used by the teachers, for them to use with their students. This included suggestions of lessons based on challenging tasks to match content they intended to teach and suggestions about ways of encouraging students to persist. They found that the lesson structure they proposed was helpful, and the elements of the lessons suggested to teachers were both necessary and sufficient for supporting students in engaging with the challenging tasks.

Eva Thanheiser investigated how prospective elementary school teachers' conceptions of number developed during their engagement in well-designed tasks and how these tasks should be designed. The tasks focused on multi-digit whole numbers and addition and subtraction and were designed to enable the participants to experience reform-based mathematics learning focused on sense making and conceptual understanding. This included one task that focused on connecting the digits to their values and the other that focused on connecting the values of the digits to each other. The tasks and the task implementation were developed to prevent participants from relying upon their prior algorithmic understanding. The intent of the tasks was to assist participants in developing more sophisticated conceptions and to encourage sense making and ownership. The results include information on what teachers can learn from working with the two tasks, conceptual difficulties in their learning based on the two tasks, and how artifacts of children's mathematical thinking can be used to help them address their conceptions. The author concluded that the tasks and artifacts can be used to develop mathematical content knowledge but conceptual difficulties may persist even with well-designed tasks.

Dionne Cross-Francis investigated the mathematics-related beliefs and experiences of two elementary school (early childhood) teachers to determine their mathematics beliefs profiles, the ways in which their beliefs and practices appeared to be misaligned and cohered, and possible reinterpretations of the perceived inconsistencies. The teachers participated in a 3-year (summer–spring) professional development project for elementary teachers with goals to improve the teachers' mathematics content knowledge and expose them to reform-based teaching strategies. This included a focus on extending the teachers'

understandings of number and geometry concepts in year two of the project. Teachers worked on inquiry-based tasks and watched, critiqued, and reflected on practices observed in video exemplars of student-centered pedagogy. Results showed that non-mathematics beliefs and contextual factors took precedence in certain classroom situations and that contextual factors had an intervening influence on the actualization of beliefs. For example, in addition to psychological factors, there were factors external to the teachers that impacted how they engaged students. This included time, testing concerns, and the influence of parent expectations as constraints that impacted the teachers' lesson planning and implementation in ways that were inconsistent with their descriptions of practices they valued.

These four studies offer insights about the nature of mathematics teachers' knowledge, ways in which we can support the development of specific aspects of this knowledge, the importance of beliefs and contextual factors on this knowledge, and issues regarding the definition of MKT that need ongoing attention in research to understand MKT more generally.