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Understanding Student Mathematical Ways of Knowing: Relationships Among Mathematical Anxiety, Attitude Toward Learning Math, Gender, Ethnicity, and Separate and Connected Ways of Knowing

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Understanding Student Mathematical Ways of Knowing:
Relationships Among Mathematical Anxiety, Attitude Toward Learning Math, Gender,
Ethnicity, and Separate and Connected Ways of Knowing

by

Andrea L. Burnes

Dissertation Submitted in Partial Fulfillment

of the Requirements for the Degree of

Doctor of Education

In

Secondary Education

Mathematics

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December 2014

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

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Dedication

This dissertation is lovingly dedicated to my children, Dawson and Kyndall. I would like to thank them for their patience, support, and self-sacrifice. Without it, this dissertation would not have been possible or worth doing. I also dedicate this body of work to Dr. Alice Terry, for her continuous support from the moment we met as well as to Dr. Nita Paris, my mentor, my guardian angel, and my friend for her patience, prodding, and boundless encouragement. I am infinitely grateful.

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I wish to thank my Committee members for being so generous with their hard-earned expertise and precious time. A special thanks to my Committee Chair Dr. Nita Paris for her encouragement, patience, and unending support throughout this entire process. Thank you to Dr. Patricia Bullock and Dr. Mei-Linn Chang for agreeing to serve on my committee and contributing their knowledge to this body of work. Dr. Paris, Dr. Bullock, and Dr. Me-Linn made this arduous journey bearable.

I would also like to thank the students who graciously participated in my research. Their excitement and willingness to try and improve mathematics education for future students is admirable and is sincerely appreciated.

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Abstract

A quantitative, correlational, survey design with anecdotal qualitative data was used to investigate the relationships among mathematical anxiety, attitude toward learning math, gender, ethnicity, and separate and connected ways of knowing within the context of the mathematics classroom. Participants were 88 student volunteers enrolled in undergraduate mathematics classes at an open admissions technical college in the southeastern United States. Survey data consisted of demographic self-report items, Likert scale items, semantic differential scale items, and one qualitative free-response question. Quantitative data were analyzed by use of either a Spearman Rho, Pearson product moment correlation or an independent samples t-test of significance. These data were supplemented by participants' qualitative responses which were categorized. The results indicated that there was a significant positive relationship between attitudes toward math and separate knowing in that those who had more positive attitudes toward math were more likely to be separate knowers. Results also indicated that gender is related to one's way of knowing in that connected knowing correlated strongly with the female gender, and a significant difference existed between males and females with regard to connected knowing. However, results indicated that males' mean score on connected knowing was significantly higher than their mean score on separate knowing. Furthermore, results indicated a significant correlation between ethnicity and ways of knowing with historically underrepresented and marginalized individuals more likely to be separate knowers. Finally, results indicated that the mean scores for females differed significantly from those of males on two out of eight factors related to mathematical ways of knowing as measured by the Mathematical Dialectics Measure which was designed specifically for this study. The present findings indicate that relationships do exist among attitudes, anxiety, gender, ethnicity and ways of knowing in mathematics. Since

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this study was correlational, statements cannot be made about the causal effect of any of these variables on one another. Further research should use an experimental or quasi-experimental design to more thoroughly examine the impact of these variables on one another and on mathematics achievement in particular.

INDEX WORDS: Ways of knowing, Mathematical ways of knowing, Separate knowing, Connected knowing, Math anxiety, Attitude toward math, Gender, Ethnicity, Correlation, Survey, Semantic differential, Anecdotal data

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Chapter 1: Introduction and Rationale

The study of algebra in our common schools should aim chiefly to throw light on the principles and processes of arithmetic, and to train the pupils' power of abstraction and reasoning.

(Hewett, E. C., 1884. *A Treatise on Pedagogy for Young Teachers*, p.198.)

America is not educating students in mathematics adequately. The 2012 Programme for International Student Assessment (PISA) assessment revealed that the United States ranked 27th in mathematics of the 34 Organization for Economic Cooperation and Development (OECD) countries which gauge the knowledge and skills deemed essential for full participation in modern societies. In short, these results reflect not “what individuals know, but for what they can do with what they know” (OECD, 2014, p. 3).

Nations whose means scores on the mathematics portion that outperformed America included Korea, Japan, Switzerland, the Netherlands, Estonia, Finland, Canada, Poland, Belgium, Germany, Vietnam, Austria, Australia, Ireland, Slovenia, Denmark, New Zealand, the Czech Republic, France, the United Kingdom, Iceland, Luxembourg, Norway, Portugal, Italy, Spain, and Slovak (OECD, 2014). It is difficult to find a developed nation whose students do not outperform American students' in mathematics. The Program for International Student Assessment (PISA) that the OECD uses to assess mathematical proficiency is administered every three years, and every time it has been administered since its inception in 2000, the mathematical proficiency of American students has been significantly below average of other developed nations (OECD, 2014).

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PISA results are hardly the lone indicator of American mathematical performance. American employers continually note the lack of qualified American candidates for STEM jobs, and are often forced to look internationally in order to fill such positions (Salzman, Kuehn, & Lowell, 2013). Mathematics scores on standardized tests such as SAT and ACT have also steadily declined (ACT, 2013). ACT composite scores for the class of 2013 dropped to its lowest level in five years (ACT, 2013). The latest composite score was 20.9 out of 36, 20% lower than 2012 scores (ACT, 2013). To improve American mathematical education, structural changes in the way that mathematics is taught will almost certainly be necessary, but determining the precise changes in teaching strategies and methodologies that will increase student mathematics learning is challenging.

Many studies have been conducted to understand mathematics learning; however, many of these were limited in scope and lacked true generalizability. The studies of King, Wood, and Mines (1990) and Khine and Hayes (2010), for example, used samples consisting primarily of mathematics majors and education majors, respectively, do not necessarily possess the same attitudes or aptitudes held by the general populace. Studies by Schommer (1990), Buehl, Alexander, and Murphy (2002) and Hofer (2004, 2006) surveyed participants about their mathematical beliefs during non-mathematics classes. To more accurately assess student mathematical ways of knowing, students should be surveyed when and where they are directly utilizing and formulating mathematical ways of knowing in mathematical contexts—the mathematics classrooms. Therefore, this study focuses on a diverse sample of majors, not just mathematics or education majors, and administered all surveys in the mathematics domain.

Although one can speculate on ways to increase student learning in mathematics, more insight into student mathematical ways of knowing, how students come to know mathematics

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and view mathematical knowledge, is needed. Therefore, this study seeks to add to the current knowledge base on student mathematical ways of knowing in the hope of gaining a deeper understanding into the development and the functioning of students' beliefs. Moreover, this study will also explore the relationship between student mathematical ways of knowing and student ways of knowing, specifically separate knowing and connected knowing.

Past studies have shown that mathematics is a gender-sensitive domain despite similar results between males and females on math achievement tests. Although research shows there are gender tendencies in knowing, they are not gender specific. Both men and women are capable of using separate and connected ways of knowing and are established individually rather than determined solely by gender (Belenky, Clinchy, Goldberger, & Tarule, 1986, 1997; Love & Guthrie, 1999). However, women who tend to favor connected ways of knowing hold more negative mathematical dispositions, including attitudes and anxieties, than men who use more separate ways of knowing (Belenky et al., 1986, 1997; Galotti, 1998; Hofer & Pintrich, 1997; Muis, 2004). Just as mathematical dispositions have been shown to be significant in whether one pursues mathematics in course selection or career path, gender and ways of knowing are equally as important. Consequently, the relationships between student attitudes toward mathematics, mathematics anxiety, gender, and ways of knowing will be investigated in this study.

Research has also shown a significant need for more ethnically-balanced research. Just as inferring results to females based on an all-male sample is inappropriate, generalizing the dominant-culture paradigm to more ethnically diverse samples is equally unsuitable. As society becomes increasingly diverse, more ethnically-balanced research is needed since the experience, views, and problems of white students are not representative of the experience, views, and problems of those who identify with historically underrepresented and marginalized groups

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(Margolis, 2001). For that reason, this study will also investigate the relationship ethnicity (i.e. group identity) may have on students' ways of knowing and mathematical ways of knowing.

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Chapter 2: Review of Literature

Mathematics is notorious for evoking feelings of anxiety, fear, and utter disgust in students (Burns, 1998; Bursal & Paznokas, 2006; Dutton & Dutton, 1991; Gresham, 2004; Hembree, 1990). The prevalence of negative attitudes and anxiety about mathematics in students may be related to their previous experiences in learning mathematics and to their general beliefs about how one comes to learn or know. How one comes to know or learn any new content can be understood through epistemology.

Epistemology

Epistemology is the branch of philosophy dedicated to studying basic truths and principles associated with knowledge, including the origin of knowledge, the construction of knowledge, the limits of knowledge, and sources of knowledge (Ernest, 1991; Hofer & Pintrich, 1997; Muis, 2004). Initial research efforts in epistemology focused on the structure of knowledge, emphasizing models that viewed epistemological beliefs as developing linearly in a sequential and progressive manner (Ernest, 1991; Hofer & Pintrich, 1997; King & Kitchener, 1994; Muis, 2004). More recent research argues that epistemological beliefs develop associatively in a non-linear manner, rather than in a developmental or stage-like process (Muis, 2004). This recent epistemological research is more consistent with evolving theories of brain plasticity and neural networks, and the understanding that many brain processes are more associative and parallel than previously realized (Berlucchi & Buchtel, 2009; Hofer & Pintrich, 1997).

Epistemology involves questions about when and how beliefs are formed by individuals or qualify as knowledge. Many people speak as if there were a difference between believing something and knowing something. Some assert that a difference between belief systems and

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knowledge systems is that belief systems have an affective component that knowledge systems lack, or that belief systems have a relatively stronger affective component than knowledge systems (Abelson, 1979; Osterholm, 2010). Pehkonen and Pietilä (2003) see beliefs as a type of knowledge that are “subjective, experience-based, often implicit” (p. 2) and argue that beliefs have a closer connection to specific situations or experiences. As people use the word “know” for their strong beliefs, they may think that the difference between knowing something and believing something involves more than just the strength of a belief (Nilsson, 2014). Though sources of beliefs are not explicit and there is a lack of consensus on what constitutes beliefs and ultimately knowing, beliefs do have tangible sources (Op’t Eynde, De Corte, & Verschaffel, 2006).

Beliefs are formed in two ways. They arise from sensory inputs, especially seeing, hearing, touching, and reading (Nilsson, 2014). Beliefs are also formed by inventing explanations for phenomenon and drawing logical conclusions and consequences from what one already believes (Nilsson, 2014). A given belief or set of beliefs imposes intellectual constraints or logical or ethical corollaries that lead to other beliefs (Nilsson, 2014). In general, beliefs help people explain what they observe and determine what is true.

Personal epistemology research originated with William Perry (1970), who attempted to understand how college students interpreted their educational experiences (Ernest, 1991; Hofer & Pintrich, 1997; Muis, 2004). Perry’s seminal stage theory of epistemological development argued that individual beliefs about knowledge evolve from the simplistic to the complex through nine sequential stages (Ernest, 1991; Hofer & Pintrich, 1997; Muis, 2004; Valandies & Angeli, 2005).

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Acceptance of Perry's (1970) work was not universal. A primary criticism was that his participants were affluent white males (Denmark, Felipe Russo, Hanson Frieze, & Sechzer, 1988; Chickering, 1969; Erickson, 1950; Heath, 1964; Kohlberg, 1964, 1966, 1973; Perry, 1970). Gilligan (1982) expanded Perry's stage theory of epistemological development by including women as participants. Nevertheless, her participants were almost entirely white—raising questions about the generalizability of her findings to ethnically diverse populations.

Belenky, Clinchy, Goldberger, and Tarule (1986/1997) expanded Perry (1970) and Gilligan's (1982) research, investigating women's roles as knowers and learners (Hofer & Pintrich, 1997; Muis, 2004). Belenky et al. (1986/1997) found that Perry's stage theory did not explain how women view the nature of knowing and learning. Their seminal work, *Women's Ways of Knowing: The Development of Self, Voice, and Mind* (1986/1997), examined and described women's ways of knowing and the perspectives from which women view reality and their "assumptions about truth, knowledge, and authority" (Belenky et al., 1997, p. 14). Belenky et al. (1986/1997) identified five different perspectives or ways of knowing: silence, received knowledge, subjective knowledge, procedural knowledge, and constructed knowledge.

The procedural knowledge perspective of Belenky et al. (1986/1997), demarcated by what they refer to as *separate knowing* and *connected knowing*, has generated considerable research (Galotti, 1998; Knight, Elfenbein, & Messina, 1995; Love & Guthrie, 1999). Gilligan (1982) coined the terms separate knowing and connected knowing and used them to describe two different aspects of self when relating to others. Belenky et al. (1986/1997) redefined the terms and used them to describe relationships between the knower and the object or subject of knowing.

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Separate knowers and connected knowers are similar in that both emphasize and value the procedures by which one learns or comes to understand (Belenky et al., 1986, 1997). Both separate procedural knowers and connected procedural knowers need and want procedures when acquiring new knowledge (Belenky et al., 1986, 1997). Procedural knowers emphasize how to acquire knowledge rather than the actual acquiring of knowledge (Belenky et al., 1986, 1997). Procedural knowers also tend to prefer formal instruction where authorities tell them “what to think” rather than make them learn “how to think” (Love & Guthrie, 1999, p. 23).

Though separate knowers and connected knowers share the similarities described above, they diverge on just about all other aspects of learning. Separate knowers evaluate facts, maintain distance, and remain impartial to information as well as other individuals (Galotti, McVicker Clinchy, Ainsworth, Lavin, & Mansfield, 1999). The intent of separate knowers is rationalization, not connection, which is the emphasis of connected knowers (Belenky et al., 1986, 1997; Love & Guthrie, 1999). Connected knowers gauge situations relationally and contextually. The reality of connected knowers emerges through care and a spirit of reciprocity (Belenky et al., 1986, 1997; Love & Guthrie, 1999). Connected knowers consider the point of view of others, while separate knowers emphasize reasoned facts over personal views (Belenky et al., 1986, 1997). Connected knowers seek to understand the opinions of others because they believe this optimizes learning and the acquisition of truth (Belenky et al., 1986, 1997). Those who adopt a connected orientation often try to view the world through the lens of others to better understand it (Belenky et al., 1986, 1997; Love & Guthrie, 1999). Connected knowers do not connect with others merely to conform to external authorities (Belenky et al., 1986, 1997). Separate knowers try to please authority figures, constructing arguments that meet the standards of an impersonal authority, whether it is a person or subject, and often adopt the perspective

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which is congruent with that of the authority or subject matter (Belenky et al., 1986, 1997; Love & Guthrie, 1999).

Although separate ways of knowing and connected ways of knowing are not gender specific, women tend to favor connected ways of knowing, while men tend to favor separate ways of knowing (Belenky et al., 1986, 1997; Galotti, 1998; Hofer & Pintrich, 1997; Muis, 2004). While there are gender tendencies to ways of knowing, both males and females often use both ways of knowing (Belenky et al., 1986, 1997; Love & Guthrie, 1999).

Mathematical Disposition

Damon (2007) defines dispositions as the beliefs and attitudes that direct the decisions people make, determining who they are and who they become. Mathematical dispositions are a specific type of disposition. Beyers (2008) describes them as a “tendency or inclination to have or experience particular attitudes, beliefs, feelings, emotions, moods, or temperaments with respect to mathematics” (p. 21). The National Council of Teachers of Mathematics (NCTM, 1989) asserts that mathematical dispositions are “not simply attitudes but a tendency to think and to act in positive ways” (p. 233) and considers positive dispositions as essential for successful mathematics education.

Attitude is a psychological tendency that is expressed by evaluating a particular entity with some degree of favor or disfavor (Eagly & Chaiken, 1998). Mathematics attitude is a psychological tendency that is expressed by evaluating mathematics with some degree of favor or disfavor. Mathematics attitude is defined by the following components: (a) *enjoyment of mathematics*, (b) *self-confidence about mathematics*, (c) *value of mathematics*, and (d) *motivation for mathematics* (Tapia & Marsh, 2004). Enjoyment of mathematics or *enjoyment* is a fondness for mathematical classes, mathematical problems, and mathematical tasks (Kalder &

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Lesik, 2011). Self-confidence about mathematics or *self-confidence* is confidence in one's ability to successfully deal with mathematical tasks and complete mathematical problems (Kalder & Lesik, 2011). Value of mathematics or *value* is the belief about the usefulness, relevance, and worth of mathematics to oneself, presently and in the future, and to society (Kalder & Lesik, 2011). Motivation for mathematics or *motivation* is an interest in mathematics and mathematical tasks and a desire to pursue studies in mathematics (Fennema & Sherman, 1976).

Related to student attitudes (Hauge, 1991), Terwilliger and Titus (1995) found that positive attitudes toward mathematics are inversely related to math anxiety. Anxiety is the feeling of tension and uneasiness related to the perceptions of one's ability as well as one's performance expectations (Meece, Wigfield, & Eccles, 1990; Wigfield & Meece, 1988). Mathematics anxiety is the feeling of tension and uneasiness related to the perceptions of one's mathematics ability as well as one's mathematics performance expectations. Mathematics anxiety affects millions of individuals on a daily basis (Burns, 1998) and is a result of mathematics teaching practices and teachers of mathematics (Furner & Gonzalez-DeHass, 2011; Williams, 1988).

As more than half of the variance in student performance and academic achievement in mathematics can be explained by student anxiety and attitudes toward mathematics (Sunn & Edwards, 1982), such affective variables should not be overlooked when trying to improve student mathematical proficiency—even though they often are. In recent years, affective roles have been repeatedly shown to play a critical role in teaching and learning mathematics, resulting in a resurgence of interest in them. Mathematics anxiety and negative attitudes about mathematics often translate into student disengagement, which inevitably leads to failure (Mayes, Chase, & Walker, 2008).

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Attitudes and Anxiety Toward Mathematics

A primary obstacle that college professors face when teaching mathematics is overcoming student beliefs about mathematics. Students often have strong negative preconceptions about mathematics and fears of mathematics. The NCTM asserts that student understanding of mathematics and beliefs about mathematics are shaped by the teaching students encounter in school and argues that teachers “exert a powerful influence on students’ evaluation of their own ability, on their willingness to engage in mathematical tasks, and on their ultimate mathematical disposition” (NCTM, 1989, p. 233). Nevertheless, teachers have a difficult time overcoming students’ negative perceptions about mathematics (Kloosterman, Raymond, & Emenaker, 1996).

Difficulties in mathematics may cause students to alter their educational paths and career ambitions. These alterations may delay academic progress or even cause students to abandon their pursuit of a college degree (Ashford, 2011; Attewell, Lavin, Domina, & Levey, 2006; Bahr, 2008; Bryk & Treisman, 2010; Noel-Levitz, 2006; Tobias, 1991). Mulvey (2008) argues that alterations in academic paths result from a students’ inability to complete developmental mathematics courses. Ashford (2011) and Tobias (1991) argue that students avoid mathematics courses because of fear of failure based on previous negative experiences with mathematics. Furthermore, scholars in mathematics education agree that the formal mathematics education students receive in school has a major influence on the development of their beliefs about mathematics (Buehl & Alexander, 2005; Kloosterman, et al., 1996; Schoenfeld, 1989; Szydlik, 2000).

Student understanding of mathematics and confidence in doing mathematics is shaped by the pedagogical practices students encounter in school (Bogdan & Biklen, 2007; Kena, Aud,

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Johnson, Wang, Zhang, Rathbun, A.,..., Kristapovich, 2014). Practices shown to negatively impact student beliefs about mathematics include teacher-centered classrooms (Muis, 2004; Szydlik, 2013), teaching mathematical concepts as isolated facts rather than as an integrated subject where concepts, procedures, and processes are interconnected (Muis, 2004; Stodolsky, Salk, & Glaessner, 1991, 1985; Szydlik, 2013), and having students work in isolation with little to no opportunity to interact with their classmates (Muis, 2004; Szydlik, 2013). These practices do little to energize or motivate students, and also reinforce student beliefs that the mathematics they learn in school has little to do with the mathematics used in real life (Muis, 2004, Stodolsky, et al., 1991; Szydlik, 2013).

As a result, many students have negative attitudes and untenable beliefs about mathematics. Many students believe that all mathematical problems have only one correct answer (Beghetto & Baxter, 2012; Chen & Pajares, 2010; Szydlik, 2013; Truatwein & Lüdtke, 2007) and that all mathematical problems can be solved quickly (Muis, 2004; Szydlik, 2013). Students also believe that those capable of doing mathematics, possess an innate gift, and those born without this gift cannot do mathematics (Asher, n.d.; Chen & Pajares, 2010; Rastegar, Jahromi, Haghghi, & Akbari, 2010). Students who cannot find the answer to a mathematics problem often feel that something is wrong with them or the math problem itself (Asher, n.d.; Chen & Pajares, 2010; Muis, 2004).

Student beliefs correlate strongly with academic achievement (Hailikaria, Nevgia, & Komulainena, 2008). Furthermore, student beliefs influence behaviors which strongly effect academic achievement. Behaviors shown to effect student academic achievement include the strategies students use for learning and assessing their own work (Muis, 2004), perseverance when faced with difficult problems (Muis, 2004), and persistence toward to a particular problem

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or to mathematics in general (Chen & Pajares, 2010; Kloosterman, 2002; Leder & Forgasz, 2002; Muis, 2004; Schoenfeld, 1992; Szydlik, 2013).

Most early research assumed that student beliefs about knowledge and learning generalized across subject domains (Belenky et al., 1986, 1997; Perry, 1970). Researchers eventually challenged this assumption and proved that student beliefs were not fixed, but rather varied across domains (Muis, 2004). Studies investigating domain specificity found that student beliefs about mathematics were generally more negative than other subject areas (Muis, et al., 2006). One positive implication that Muis (2004) did note, based on research conducted by Carter and Yackel (1989), Erickson (1993), Franke and Carey (1997), Higgins (1997), Lampert (1990), and Verschaffel, De Corte, Lasure, Van Vaerenbergh, Bogaerts, and Ratinckx (1999), was that student beliefs were not fixed, but could be altered if teachers utilized more student-centered approaches. For example, Pintrich, Marx, and Boyle (1993) as cited by Muis (2004), suggested that individuals must be (a) dissatisfied with current conceptions; (b) able to understand new concepts; (c) able to apply the new beliefs; and (d) successful (p. 355).

Research on the relationship between student anxiety about mathematics and student ways of knowing is limited, and is needed (Muis, 2004). Researchers believe that mathematics anxiety is rooted in student experiences with formal mathematics instruction from kindergarten through high school (Hauge, 1991; Jackson & Leffingwell, 1999). Anxiety about mathematics which arises during primary and secondary education accompanies matriculating students to the college mathematics classroom (Betz, 1978; Zakaria & Nordin, 2008). Anxiety about mathematics is so pronounced in some students that they avoid mathematics at all costs, some going as far as lowering achievement goals or quitting school altogether (Tobias, 1991).

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Separate and Connected Knowing

Separate knowing and connected knowing describe two contrasting epistemological orientations on how individuals understand and interact with their environment. Separate knowers rely on impersonal procedures for establishing truth whereas connected knowers gauge situations relationally as their primary means to understand (Belenky, et al., 1986, 1997; Love & Guthrie, 1999). This distinction is critical to educational outcomes because separate knowers and connected knowers learn in different ways, and may require different educational experiences for optimal learning (Schommer, 1990; Schoenfeld, 1985; Royce 1978).

Research has yet to adequately explore student ways of knowing, including separate knowing and connected knowing, specific to the domain of mathematics. For example, Ocean (1998) attempted to identify separate knowing and connected knowing in mathematics education, but her study was narrow in focus and was designed to identify individuals who had extreme separate knowing experiences in mathematics. She did not holistically investigate overall mathematical ways of knowing.

Models proposed by Schommer (1990), Schoenfeld (1985, 1989), and Royce (1978), suggest that one's predominant approach to knowing influences learning by determining how information is acquired and interpreted (Muis, 2004). Past research on connected and separate knowing has studied various cognitive aspects of learning mathematics but has overlooked the affective dimension of student mathematical dispositions. This study attempts to address this shortcoming, and will hopefully provide insight into approaching learning and teaching from different perspectives so as to improve student mathematical proficiency.

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Ethnicity and Ways of Knowing

Most foundational research on epistemological beliefs utilized a disconcertingly large proportion of white participants, especially affluent white participants (e.g., Baxter Magolda, 1992; King & Kitchener, 1994; Perry, 1970). The seminal studies of Perry (1970) used participants who were almost exclusively from the white, elite, economically-privileged males at Ivy-league universities. Baxter Magolda's (1992) participants consisted of 101 students of which 97% identified themselves as white and only 3% who identified themselves as ethnically diverse.

The U.S. Census Bureau estimates that by 2043, whites will no longer be the majority in America (Goedert, 2014). By 2043, historically underrepresented and marginalized groups will outnumber whites, who will be a minority (Goedert, 2014). However, according to the National Center for Education Statistics (NCES) as of 2014, white students are no longer the majority in P-12 schools (Kena, et al., 2014).

As America becomes more ethnically diverse, the dominant-culture paradigm which has dominated research must be replaced by cultural pluralism (Holmes, 2010; Reagan, 2009). In a dominant-culture research paradigm, the norms of the dominant group (e.g. White or Anglo) are assumed to be the norms of entire culture. Cultures besides the dominant culture differ from the dominant group where differences are treated as a dysfunction or deficiency rather than embracing diversity (Belenky et al., 1986, 1997; Paechter, 2000; Reagan, 2009). Educators then seek to eliminate the "deficiency" in students from non-dominant cultural backgrounds by assimilating them into the dominant group and forcing adoption of its norms (Margolis, 2001). It is this *one-size-fits-all* teaching methodology and obsession with standardization which is dysfunctional and deficient, and there is broad consensus that it must change (Holmes, 2010; Margolis, 2001; Paechter, 2000; Reagan, 2009).

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The experience, views, and problems of white students are not representative of the experience, views, and problems of those who identify with historically underrepresented and marginalized groups (Margolis, 2001). As the demographic makeup of America becomes increasingly diverse, participants comprised primarily of individuals who identify themselves as white cannot be used to reliably make inferences about or to generalize to ethnically diverse populations.

Collectivist societies are known for encouraging conformity, interdependence and success of the group, while individualist societies tend to stress autonomy, independence and the success of the individual (Maggioni, Risconscente, & Alexander, 2006; Triandis, 2001). Research that includes a broader cultural/ethnic cross-section will encompass greater diversity where collectivism and individualism are concerned. Students with more collectivist or individualist mindsets may have different ways of knowing and may have attitudes and emotions about math which differ. They may respond differently to same instruction, in previously uncorrelated ways, especially since the cultural practices of collectivism and individualism differ in their emphasis on the group versus the individual. In the case of collectivism, the central theme is the preservation and advancement of the group whereas the main premise of individualism is the preservation and advancement of the individual. Given that different cultures provide individuals with different ways of thinking--of seeing, hearing, and interpreting the world—it seems plausible to expect differences in beliefs about knowledge and knowing from different ethnicities (Neuling, 1999).

Gender and Ways of Knowing

Early educational research efforts have come under fire and have been widely criticized because of the universal approach by which females as well as the historically underrepresented

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and marginalized groups have been measured (Banks & McGee Banks, 2009). Past research, conducted by primarily white male researchers on primarily white male samples (Chickering, 1969; Erickson, 1950; Heath, 1964, Kohlberg, 1964, 1966, 1973; Perry, 1970) is not just biased but affects even new research. Considered biased because it was derived from and constrained by existing male-centric theories which did not take the experiences of *others* into account when being formulated (Belenky et al., 1986, 1997). Bias was also present in all stages of the research process from question formulation, study design, data analysis, and interpretation and conclusion formation (Easterly & Ricard, 2011). The male-centric bias, considered and accepted as the gold-standard in educational research, treat most any deviance from male-centric metrics as a deficit or dysfunction (Denmark, et al., 1988).

Most of the foundational research on epistemological beliefs took a male-centric view by using samples that were primarily male (Marrs & Benton, 2009). Perry's schema of adult development judges women as deficient against his almost entirely white, male sample (Perry, 1970).

An extremely recent review of gender bias in literature by Anderson (2012) concluded that, "efforts to incorporate sex and gender in research seem to be minimal. Women are included in trials if required, but the subsequent data analyses still are sex and gender insensitive" (p. 313) and the role of gender and sex is very weakly addressed in data analysis (NIH, 2001; Canadian Institutes of Health Research, 2003; European Commission, 2003, 2007). Other researchers, such as Marrocco and Stewart (2001) and Vidaver, Lafleur, Tong, Bradshaw, and Marts (2000), have drawn similar conclusions.

Research has shown that men and women often utilize different types of knowing, but male-centric research implicitly and erroneously assumes that the tendencies and preferences of

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males are the standard by which all others are measured. According to Erchick (1996), "If mathematics is (or is perceived to be) a formal system that threatens connectedness, a language that expresses power-over and control, a masculine space, a space that disallows subjectivism, perhaps most women really would choose not to participate in it" (p.120). This study will explore the influence that gender has on student ways of knowing.

Situated Cognition

The theory of situated cognition was formally introduced in the late 1980s and is credited with expanding the conception of cognition (Cobb, 1996). Situated cognition argued that cognition extends beyond the boundaries of the brain (Robbins & Aydede, 2009). Cognitive processes once thought to be strictly internal were re-conceptualized and viewed as beginning internally then expanding developing external components. The theory of situated cognition suggests that cognition is extended through the context of the external environment, beyond the brain, where it establishes its meaning (Robbins & Aydede, 2009; Hutchins, 1995).

The theoretical foundations of situated cognition arise from Vygotsky's work on sociocultural theory, which emphasizes the socially and culturally situated nature of activity (Cobb, 1996). The sociocultural perspective of cognition assumes that cognitive processes are subsumed by social and cultural processes (Cobb, 1996; Cobb & Yackel, 1996; Robbins & Aydede, 2009; Brown, Collins, & Duguid 1989; Hutchins, 1995).

Lave and Wenger (1991) introduced situational learning, which views human beings as actively engaged in creating their world. In situational learning, humans are viewed as interpreters and definers of the world around them and not simply as responders to stimuli (Bogdan & Biklen, 2007). Knowledge is seen as being distributed among people. Aggregates of

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people are viewed as a dynamic system that changes as knowledge is contributed and utilized collaboratively (Boaler, 2002; Cobb 2000; Sawyer & Greeno, 2009).

The guiding principle of situated learning is that all learning is situated in a specific social and physical context, which is co-constructed by all participants in that context (Lave, 1988, 1996; Sawyer & Greeno, 2009). The relational interdependency between individuals and the context as well as the multifarious field of relations that exist between them in turn creates a symbiotic and interactive learning process (Lave, 1988, 1996; Sawyer & Greeno, 2009).

Contextual epistemological theories, which include situated learning, view students as wanting to become a part of a community of practice where they are socialized to the values and beliefs of the academic enterprise (Hofer & Pintrich, 1997). Situated knowing occurs as learners participate in the learning community (Sawyer & Greeno, 2009).

Situated learning in a mathematics classroom involves individual students as participants and contributors to the development of the mathematic practices (Cobb & Bowers, 1999; Hofer & Pintrich, 1997; Sawyer & Greeno, 2009). Cobb and Bowers (1990) use the term *communal practices* to refer to situated learning classroom practices. They argue that these communal practices are influential as students are socialized to the values and beliefs of the academic discipline (Cobb & Bowers, 1999).

Student mathematical dispositions are shaped by participating in classroom practices (Cobb & Bowers, 1999; Muis, 2004). One such practice is mathematical discourse, also known as *mathematizing*. The term mathematizing was coined in 1968 by Hans Freudenthal, the Dutch mathematician and mathematics educator who saw such a practice as a form of organizing mathematical matter--the activity of interpreting one's "lived world" mathematically (Sfard, 2008, p. 297). The process of mathematizing is demarcated by its distinct vocabulary,

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imaginings, and narratives which occur as students participate in the collaborative, rational, and logical processes common to mathematics (Boaler, 2002; Cobb & Bowers, 1999; Muis, 2004).

As individuals develop an understanding of mathematics and formulate beliefs about mathematics, their classmates also actively reorganize their beliefs and values. More than just mathematical socializing, mathematizing is also form of enculturation because it is embedded within a more comprehensive process which socialize students to the cultural norms of the dominant culture deemed for necessary for learning (Perrenet & Taconis, 2009).

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Chapter 3: Methods and Design

Purpose

The purpose of this study was to investigate: (1) the relationship between student attitudes toward learning mathematics and student ways of knowing; (2) the relationship between student mathematics anxiety and student ways of knowing; (3) the relationship between gender and student ways of knowing; (4) the relationship between ethnicity and student ways of knowing; (5) the relationship between students' *dominant way of knowing* and student *mathematical ways of knowing*; (6) the relationship between gender and student mathematical ways of knowing; (7) the relationship between ethnicity and student mathematical ways of knowing; (7) the usefulness of the Mathematics Dialectics Measure in measuring and assessing student mathematical ways of knowing.

Research Questions

This study investigated the following research questions:

1. Is there a relationship between student attitudes toward learning mathematics and student ways of knowing?
2. Is there a relationship between student mathematics anxiety and student ways of knowing?
3. Is there a relationship between gender and student ways of knowing?
4. Is there a relationship between ethnicity and student ways of knowing?
5. Is there relationship a relationship between students' dominant way of knowing on student mathematical ways of knowing?
6. Is there a relationship between gender and ethnicity and student mathematical ways of knowing?

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7. Does the Mathematics Dialectics Measure (MDM) provide any new insight into student mathematical ways of knowing?

Operational Definitions of Key Terms

Many of the variables in this study are latent variables, and cannot be observed directly (e.g. attitude, anxiety). Furthermore, multiple interpretations and definitions of such variables exist and have been documented in prior research. For the purpose of this study, definitions of key terms were established.

Key Terms. The following definitions were utilized within the context of this study.

Ways of Knowing. How students make meaning of their educational experiences (Hofer & Pintrich, 1997).

Separate Knowing. A way of knowing that ignores personal feelings and beliefs, favoring detached, analytical evaluation of the object to be known (Belenky et al., 1986, 1997).

Connected Knowing. A way of knowing that favors connection with others and empathy to understand the object of knowing (Belenky et al., 1986, 1997).

Dominant Way of Knowing. The higher of the two scores for separate knowing and connected knowing on the ATTLS. Dominant knowing is one's dominant way of learning and preferential way of knowing.

Mathematical Ways of Knowing. Beliefs about mathematical knowing and mathematical knowledge based on educational experiences.

Mathematical Knowing. Beliefs about the nature of how one comes to know mathematics (e.g. source and structure of knowing).

Mathematical Knowledge. Beliefs about the nature of mathematical knowledge (e.g. simplicity and certainty of mathematical knowledge).

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Mathematics Dialectics Measure (MDM). A customized instrument to help determine student mathematical ways of knowing (i.e. beliefs about mathematics, including beliefs about mathematical knowledge and beliefs about how one comes to know mathematics).

Attitude. A psychological tendency that is expressed by evaluating a particular entity or phenomenon with some degree of favor or disfavor (Eagly & Chaiken, 1998).

Self-Confidence about Mathematics. Confidence in one's ability to successfully deal with mathematical tasks and complete mathematical problems (Kalder & Lesik, 2011).

Enjoyment of Mathematics. A fondness for mathematical classes, mathematical problems, and mathematical tasks (Kalder & Lesik, 2011).

Motivation for Mathematics. Interest in mathematics and mathematical tasks and a desire to pursue studies in mathematics (Fennema & Sherman, 1976).

Value of Mathematics. The belief about the usefulness, relevance, and worth of mathematics to oneself, presently and in the future, and to society (Kalder & Lesik, 2011).

Mathematics Anxiety. The feeling of tension and anxiety about mathematics, which is related to perception of mathematics ability and performance expectations (Meece, et al., 1990).

Research Design

This quantitative, correlational study utilized a non-experimental design with an anecdotal free response question. Relationships between seven variables were investigated. The seven variables were; way of knowing, mathematical way of knowing, dominant way of knowing, attitude toward mathematics, mathematics anxiety, gender, and ethnicity. Participants were drawn from a sample of convenience. Participants were given a survey consisting of four different instruments which include

1. Attitude Toward Thinking and Learning Scale (ATTLS), (Appendix A);

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2. Attitudes Toward Mathematics Inventory (ATMI), (Appendix B);
3. Mathematics Anxiety Scale (MAS), (Appendix C); and
4. Mathematics Dialectics Measure (MDM), (Appendix D).

These instruments provided the quantitative measures used to investigate variable pairings in this study. Variable pairings are presented in Table 1 along with the instrument used to measure each variable.

Table 1

Variable Pairings and Related Instruments

Variable 1	Instrument	Variable 2	Instrument
Attitudes Toward Mathematics	ATTLS	Way of Knowing	ATTLS
Mathematics Anxiety	MAS	Way of Knowing	ATTLS
Gender	Self-report	Way of Knowing	ATTLS
Ethnicity	Self-report	Way of Knowing	ATTLS
Dominant Way of Knowing	ATTLS	Mathematical Way of Knowing	MDM
Gender	Self-report	Mathematical Way of Knowing	MDM
Ethnicity	Self-report	Mathematical Way of Knowing	MDM

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Instruments and Measures

The Attitudes Toward Thinking and Learning Scale (ATTLS), (Appendix A) created by Galotti et al. (1999) was used to measure student separate and connected knowing. The ATTLS is based on the seminal work of Belenky et al. (1986/1997), *Women's Ways of Knowing*. Tapia and Marsh's (2004) Attitude Toward Mathematics Inventory (ATMI; Appendix B) and Mahmood and Khatoon's (2011) Mathematics Anxiety Scale (MAS; see Appendix C) were used to measure of student anxiety and student attitudes towards learning mathematics.

The ATTLS, MAS, and ATMI instruments were selected because of their reliability and validity. Equally important, all three instruments have been shown to accurately assess targeted research variables efficiently and effectively (Fennema, 1989; Galotti et al., 1999; Richardson & Suinn, 1972).

As there is very little research on the affective dimensions of student beliefs specific to the mathematics domain, a validated instrument to measure student mathematical ways of knowing was not available. Therefore, the Mathematics Dialectic Measure (MDM; see Appendix D) was created specifically to assess mathematical ways of knowing. The MDM is based on the 10 educational dialectics used by Belenky et al. (1997, p. 237) as coding categories during their contextual analysis "to capture the ways in which women ... experience their learning environments" (p. 16). Since males have been previously adequately represented in instruments and metrics, as has been noted, Belenky et al. utilized the dialectics of in an attempt to create a more gender balanced instrument.

The 83 item survey utilized 20 items from the ATTLS (see Appendix A), 40 questions from the ATMI (see Appendix B), 14 items from the MAS (see Appendix C), and 9 items from

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the MDM 14 items from the MAS (see Appendix D). The items drawn from each of these instruments will be reviewed, as will the construct, reliability, and validity of each instrument.

Attitude Toward Thinking and Learning Scale (ATTLS). The 20-item Attitude Toward Thinking and Learning Scale (see Appendix A) measures connected procedural knowing and separate procedural knowing. Developed by Galotti et al., (1999), the ATTLS utilizes quotations and descriptions taken directly from the work of Belenky et al. (1986/1997). The ATTLS (see Appendix A) includes 10 statements exemplifying separate knowing (SK) and 10 statements exemplifying connected knowing (CK). A seven point Likert-type scale ranging from 1 (strongly disagree) to 7 (strongly agree) generates one score for separate knowing and one for connected knowing. High scores represent a stronger agreement with a particular way of knowing and range from 10 to 70.

Galotti et al. (1999) tested and validated the ATTLS on four different samples. Similar tests were conducted to ensure the validity and reliability of the ATTLS using this particular sample. A principal component factor analysis with varimax rotation was also conducted for this sample. Chronbach alpha coefficients for this sample for SK (.684) and CK (.814) were similar to those obtained by Galotti et al., (1999) which indicated SK (.83) and CK (.77) measured their intended constructs.

A closer inspection of the data analysis for separate knowing items indicated that scale reliability could be marginally improved by eliminating items 3, 6, and 18 because of low loadings. However, removal of these items would have increased the Cronbach alpha coefficient to .692, a marginal increase of .008. In an effort to preserve the integrity of the original scale, it was decided that none of the items would be removed. Although the Chronbach alpha coefficient

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for SK was not ideal, the ATTLS (see Appendix A), approached acceptable levels of internal reliability.

Unlike the results obtained in the original study, the correlation coefficient between SK and CK for this sample was not statistically significant. However, the correlation coefficient was low ($r = -0.07$) comparable to the coefficient ($r = .017$) obtained by Galotti and her colleagues; reinforcing previous claims that the two ways of knowing are indeed independent and not at opposite ends of a unidimensional scale (Galotti et al., 1999).

Attitude Toward Mathematics Inventory (ATMI). The 40-item Attitude Toward Mathematics Inventory (ATMI, see Appendix B) measures student attitudes toward mathematics. Tapia and Marsh (2004) designed the ATMI to assess four underlying dimensions: enjoyment, self-confidence, motivation, and value. The ATMI includes 7 items which measure enjoyment, 15 items which measure self-confidence, 8 items which measure motivation, and 10 items which measure value (See Appendix B). A 7 point Likert-type scale (1= strongly disagree to 7 = strongly agree) having a total score ranging from 40 - 280. Subscales for the four underlying dimensions are generated by summing only the items specific to each dimension. Ten of the 40 items are reversed. Reversed items are scored by taking the value of the item and subtracting it from 8 to ensure the appropriate value for data analysis is used. High scores represent a more positive attitude about mathematics, and low scores represent negative attitudes about mathematics.

The ATMI (see Appendix B) had a high degree of internal consistency for this sample and produced the same Cronbach alpha coefficient (.97) as Tapia and Marsh (2004) when validating the ATMI using the original sample. A factor analysis with a varimax, orthogonal, rotation which produced high Cronbach alpha scores for the subscales (a) *enjoyment* (0.89), (b)

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self-confidence (0.95), (c) *motivation* (0.88), and (d) *value* (0.89) established the items measured their intended construct and had coherent internal structure. Test-retest reliability results, produced by Tapia and Marsh (2004,) showed moderate to almost perfect alignment of the scales as indicated by the large Pearson correlation coefficients.

Mathematics Anxiety Scale (MAS). The Mathematics Anxiety Scale (MAS, see Appendix C) measures student anxiety about mathematics. It is a 14-item bi-dimensional scale created by Mahmood & Khatoon (2011) which assesses both positive and negative dimensions of mathematics anxiety. The MAS consists of seven positively worded items and seven negatively worded items. A 7 point Likert-type scale (1= strongly disagree to 7 = strongly agree) results in a total score ranging from 14 - 98. Negative effect items are assigned values from 1 to 7, whereas positive effect values are reversed. High scores represent high levels of mathematics anxiety and low scores represent low levels of mathematics anxiety.

The MAS returned a high split-half reliability coefficient (0.89) when Mahmood and Khatoon (2011) compared participants' total odd-item scores and total even-item scores. The MAS generated a much lower split-half reliability coefficient (0.653), below the acceptable level for this sample used. To get a better estimate of the reliability of the full test (Oluwatayo, 2012), the Spearman-Brown correction was applied improving the reliability of the MAS to .790.

A high Cronbach alpha coefficient (0.82) similar the coefficient (.87) found by Mahmood and Khatoon (2011) indicated a high degree of internal consistency and corroborates the improved coefficient generated using the Spearman-Brown correction.

Mathematics Dialectics Measure (MDM). The Mathematics Dialectics Measure (MDM) (Appendix D) measures student *mathematical ways of knowing*, including student beliefs about what it means to know mathematics and student beliefs about mathematical

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knowledge. The MDM was specifically created for this study and was developed using the *educational dialectics* scale of Belenky et al. (1986/1997). The MDM consists of 9 items, 8 semantic-differential-scaled statements and 1 free response item. Each semantic differential-scaled statement consists of adjective word-pairs which are scored on a bimodal scale (-3 = *relates more with the word on the left* to 3 = *relates more with the word on the right*) where negative values represent adjectives on the left side of the scale (*Procedure, Discovery, Rational, Relational, Listening, Student-Directed, Discrete, and Extensive*) and positive values represent adjectives listed on the right side of the bimodal scale (*Purpose, Instruction, Intuitive, Impersonal, Speaking, Teacher-Directed, Related, Concentrated*).

Belenky et al. (1997) developed 10 *educational dialectics* (p. 237) to measure “the ways in which women ... experience their learning environments” (p. 16). When Belenky et al. (1986/1997) surveyed women about ways of knowing, their responses failed to fit neatly into Perry’s (1970) stages of development. They suggested it was likely due to the lack of homogeneity of their sample relative to Perry’s homogenous male sample from Harvard.

The 10 modes of thought referred to as *educational dialectics* emerged during their contextual analysis of the data and were used as coding categories to sort the interview data from the 135 women they sampled. The 10 different modes consisted of adjective word-pairs located on opposite ends of a non-numerical bipolar scale. There was no Likert-type scaling; rather the measure was nominal and dichotomous, with one of two words from the word pair being selected. Belenky et al. (1986/1997) suspected women would tend to favor one mode and conventional educational practice would favor the opposite mode.

The educational dialectics of Belenky et al. (1986/1997) are a general educational instrument which is not domain-specific. The MDM, therefore, is intended to adapt the

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educational dialectics of Belenky et al., developed for this study into a domain-specific tool that can measure the mathematics domain. The *educational dialectics* of Belenky et al. measured *ways of knowing*, and the mathematics dialectics of this study measures *mathematical ways of knowing*. Sample items from the mathematical dialectic measure are shown in Table 2 along with the educational dialectics created by Belenky et al. (1986/1997).

The eight mathematical dialectic items on the MDM (Appendix D), used the sentence stem, *Mathematics is...*, and contained adjective word-pairs contrasting educational practices. Located on opposite ends of a non-numerical bimodal scale, adjective word-pairs were designed specifically, to avoid using value-laden terms, which are neither positive nor negative. Special care was taken when choosing word pairs since research has shown that individuals tend to have a psychological propensity to select responses that use positive words over those that use negative words (Lubian, 2010). A non-numerical bipolar scale was used so that participants could assign their own personal meaning to various positions on the scale based on their personal assumptions, perceptions, aims, and values (Kilpatrick & Cantril, 1960).

Table 2

Sample Educational Dialectics vs. Mathematics Dialectics

Educational Dialectics	Mathematics Dialectics
<i>Rational vs. Intuitive</i> What methods are used for analysis? What methods are valued?	<i>Rational vs. Intuitive</i> What method is used for interpretation/understanding mathematics?
<i>Discrete vs. Related</i> What is the relationship between learning and life?	<i>Discrete vs. Related</i> What is the relationship between mathematical concepts?
<i>Inner vs. Outer</i> What factors control goal setting, pacing, decision making, and evaluation?	<i>Student Directed vs. Teacher Directed</i> Who controls goal setting, pacing, decision making, and evaluation in mathematics?

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Given that the MDM is a new instrument created specifically for this study, no pre-existing reliability and validity metrics were available. This study did not attempt to validate the MDM as it was a pilot instrument designed to explore student ways of knowing in mathematics specific to this study.

Anecdotal Qualitative Item. Participants were asked to respond to the prompt, Please describe what would be your optimal condition for learning mathematics. The free response question was located on the demographic portion of the survey (Appendix E) and was analyzed qualitatively as well as quantitatively using coding categories (Appendix F) to group similar responses. Responses to the qualitative item were coded in order to quantify qualitative data which describe what participants would consider an optimal environment for learning mathematics.

Participants

Participants in the study consisted of 88 college students enrolled in undergraduate mathematics classes at an open-admission technical college located in the southeast United States. Participants were 18 years or older, currently enrolled in a mathematics class, and seeking certificates, diplomas, or degrees. Participation was not compulsory; students were invited to participate in this research. Participants represented diverse majors and ethnicities, and included 51 women.

The open-admissions technical college offers a wide variety of programs leading to certificates, diplomas and associates degrees with campuses located in five different counties. In 2013, the annual report of the college stated that 8,565 students were enrolled in credit courses. The student population was predominately female (66.1%) with males representing (33.1%) of the population. Students identified with a several ethnic/racial groups. Specifically, students

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identified themselves as American Indian (0.2%), Asian (0.6%), Black (11.7%), Hispanic (6%), Multiracial (1.5%) and White (80%).

Participants were from six different general education mathematics courses required of students seeking certificates, diplomas, or degrees. A total of 125 students were enrolled in the six mathematics courses. Only students who were 18 years or older and returned signed consent forms were allowed to participate in this study. A total of 92 students returned signed consent forms, a response rate of approximately 73.6%. However, 4 students were below the age of 18 and were therefore excluded, leaving 88 students in the sample.

Procedure

The researcher and two additional mathematics professors administered surveys to students in the six different classes over the course of a two-week period. Surveys were administered during a single class period near the end of the 2014 spring semester.

The purpose of the study was explained to the participants followed by a brief explanation of the contents of each packet. After consent forms were signed, students were given the remainder of class time to complete the survey. All participants completed the survey packet within 30 minutes. Packets without signatures were removed.

The survey contained 83 items, 82 Likert-scaled or semantic-differential-scaled statements and 1 free-response open-ended question to which participants described their optimal environment for learning mathematics. Participants also provided demographic information. Each survey contained a consent form, and no survey was used if its consent form was not complete and signed.

As noted, the survey combined questions from four different instruments, the ATTLS (Appendix A), the ATMI (Appendix B), the MAS (Appendix C), and the MDM (Appendix D)

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designed specifically for this study. To avoid an ordering effect, four different survey booklets were created and the order of the instruments was changed in each booklet. Booklets utilized the following ordering:

Booklet 1: ATTLS, MDM, MAS, ATMI, Free Response, Demographics

Booklet 2: MDM, ATTLS, ATMI, MAS, Free Response, Demographics

Booklet 3: ATMI, MDM, MAS, ATTLS, Free Response, Demographics

Booklet 4: MAS, ATMI, ATTLS, MDM, Free Response, Demographics

Data Analysis

Descriptive statistics were calculated for the demographic data provided by the participants. In addition, group means and standard deviations for four of the test variables (way of knowing, mathematical way of knowing, attitude toward mathematics, and mathematical anxiety) were calculated. Examination of the data for each of these four variables indicated that the data followed closely to a normal distribution, with only mild skew. Therefore, non-parametric corrections for further statistical analyses were not necessary. Three of the test variables (dominant way of knowing, gender, and ethnicity) were dichotomous variables not appropriate for mean and standard deviation calculations.

To determine if significant relationships existed between the seven paired variables, further analyses were conducted using the Pearson product-moment correlations between each pairing of variables. The significance level was set at $\alpha = .05$. In calculating the correlations between the variables, three of the variables were coded as dichotomous variables. For analysis purposes, the variable, *ethnicity*, was coded with the two categories set as *White* and *Ethnically Diverse* with those students who identified themselves as American Indian, Asian, Black, Hispanic, and Multiracial included in the *Ethnically Diverse* category. The variable, *dominant*

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way of knowing, was determined by an individual's highest score on the ATTLS resulting in categories of *connected knower* and *separate knower*. Finally, *gender* was a dichotomous variable with categories of *male* and *female*. Participants who did not indicate a gender were not included in the study.

Data obtained from free response items were analyzed quantitatively and qualitatively. The researcher read through all of the participants' responses. All responses were then transcribed into word format. Finally, similar responses were tabulated, coded, and categorized.

Chapter 4: Results and Discussion

The purpose of this study was to investigate if relationships exist between student anxiety and attitudes toward mathematics, gender, ethnicity, mathematical ways of knowing, and separate and connected ways of knowing. Several significant findings emerged from this study.

Participant Demographics

The participant ages ranged from 18 to 55 years old ($M = 26.49$, $SD = 9.036$). The mean of 26.49 is higher than the age traditionally associated with college undergraduate students. Non-traditional age students are more prevalent on college campuses which are technical colleges as individuals tend to select these colleges when they return to pursue continuing education.

The gender and ethnic identification of the participants in the study is provided in Table 3. The sample size was modest but adequate ($N = 88$) and it generally reflected the gender and ethnic makeup of the campus as a whole with the sample being slightly more ethnically diverse than that of the campus.

Table 3

Participant Demographic Variables

Ethnicity	Males		Females		Combined	
	n ^a	%	n ^a	%	N ^a	%
Asian	1	2.8			1	2.8
Black	5	13.9	11	21.2	16	18.0
Hispanic	6	16.7	3	3.00	9	10.1
Multiracial	1	2.80	3	5.80	4	4.50
Unspecified	1	2.80	--	0.00	--	2.20
White	22	61.1	35	67.3	57	64.0
Total	35	39.7	52	59.1	88	100

^aParticipants who failed to identify gender were omitted.

It is noteworthy that the sample was more ethnically diverse than the samples utilized in previous research on this topic. This is beneficial because an ethnically diverse sample is a primary construct of this study.

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Separate knowing scores for all participants ranged from 26 to 63 ($M = 42.1$, $SD = 8.28$).

Connected knowing scores for all participants ranged from 27 to 70 ($M = 52.1$, $SD = 8.70$).

Table 4 shows mean scores and standard deviations for connected knowing and separate knowing, sorted by gender and ethnicity. Differences between groups for connected knowing (CK) and separate knowing (SK) are also displayed in Table 4.

Correlation Analysis

Using the Pearson product-moment correlation, zero-order correlation coefficients were calculated for all variable pairings. Correlation coefficients were tested for significance at a level of $\alpha = 0.05$ and are presented in Table 4. Before proceeding with a discussion of the presence or absence of significant relationships between the seven test variables, it is worth noting that a weak positive correlation, $r(79) = 0.264$, $p = 0.017$, was found between separate knowing and connected knowing. This weak correlation between separate knowing and connected knowing is consistent with prior research and gives credence to the idea that separate and connected ways of knowing are “two interconnected, but distinct, tracks” (Love & Guthrie, 1999, p. 23; Knight et al., 1995) as identified by Belenky et al. (1986/1997) and that “separate knowing is not the opposite of connected knowing but, rather, a style of thinking that is independent of connected knowing” (Galotti, 1998, p. 282).

A strong negative correlation, $r(79) = -0.925$, $p = 0.001$, was found between attitude toward mathematics and mathematics anxiety. This negative correlation implies that students with more negative attitudes toward mathematics are likely to exhibit higher levels of anxiety in mathematics than students with positive attitudes who are likely to experience less anxiety in mathematics. This negative correlation between mathematics attitude and mathematics anxiety

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is consistent with past research conducted by Betz (1978), Meece, Wigfield, and Eccles (1990) and Richardson and Suinn (1972) and further support the results reported in this study.

Table 4

Zero-Order Correlation Coefficients

Variable		Gender	Ethnicity	CK	SK	Attitude	Anxiety
Gender	Pearson	—					
	Sig. (2-tailed)						
	N	88					
Ethnicity	Pearson	-.046	—				
	Sig. (2-tailed)	.673					
	N	87	87				
Connected Knower	Pearson	.266*	.093	—			
	Sig. (2-tailed)	.017	.414				
	N	80	80	81			
Separate Knower	Pearson	-.054	.246*	.264*	—		
	Sig. (2-tailed)	.633	.027	.017			
	N	81	81	81	82		
Attitude	Pearson	-.159	.414**	.187	.311**	—	
	Sig. (2-tailed)	.141	.000	.097	.005		
	N	87	86	80	81	88	
Anxiety	Pearson	.139	-.294**	-.146	-.226*	-.925**	—
	Sig. (2-tailed)	.219	.008	.198	.043	.000	
	N	80	80	79	80	79	80

*Correlation is significant at the 0.05 level (2-tailed).

** Correlation is significant at the 0.01 level (2-tailed).

Attitude Toward Learning Mathematics and Ways Of Knowing. Attitude scores for all participants ranged from 73 to 269 ($M = 175.3$, $SD = 54.7$). As noted in Table 5, a significant positive correlation, $r(79) = 0.311$, $p = 0.005$, was found between attitude towards mathematics and separate knowing, whereas no significant correlation was found between attitudes toward mathematics and connected knowing ($r(78) = 0.187$, $p = .097$). In other words, separate knowing was positively related to students' attitudes toward math and indicates that students with a more positive attitude about mathematics tend to have higher separate knowing scores than students with negative attitudes toward mathematics.

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Separate knowers, who show a penchant for critical thinking and are often well-versed in evaluating arguments (Belenky et al., 1986, 1997), are likely to display more positive attitudes toward mathematics since this way of knowing tends to be predominant in mathematics classrooms (Brown, et al., 1989; Clancey, 1997) and is consistent with the situated view of learning where the context influences students' attitudes (Op't Eynde, et al., 2002).

The positive correlation between attitude toward mathematics and separate knowing makes sense because separate knowers are adept at constructing arguments and may be more likely to actively participate in mathematics classrooms as a result. Findings parallel to this line of thinking were reported by Boaler (2002) and Samuelsson and Granstrom (2007) who found that students who contribute during classroom discussions tend to have more positive attitudes about mathematics than students who do not take part.

Mathematics Anxiety and Ways of Knowing. As noted in Table 5, anxiety scores ranged from 26 to 86 ($M = 57.2$, $SD = 15.5$). A small but significant negative correlation, $r(78) = -0.226$, $p = 0.043$, was found between mathematics anxiety and separate knowing, whereas no significant relationship was found between connected knowing and mathematics anxiety ($r(77) = -0.146$, $p = 0.198$). Put succinctly, separate knowing was negatively related to students' math anxiety. This indicates that students with a lower mathematics anxiety tend to have higher separate knowing scores and those with higher mathematics anxiety are apt to have lower separate knowing scores.

Separate knowers, who are often skilled at analyzing and evaluating arguments, are likely to be "less vulnerable" (Belenky et al., 1997, p. 105) to anxiety which has been shown to influence the cognitive processes of thinking and reasoning (Hofer & Pintrich, 1997). Because

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they report lower anxiety in mathematics, separate knowers may feel more secure about the subject of mathematics. Thereby, making separate knowers ultimately less anxious than connected knowers in the mathematics classroom. Qualitative responses from the participants of this study support this notion. “I get nervous when I have to ask for help” is a response from a student who was categorized as a connected knower.

Results for student mathematics anxiety paralleled those of student attitudes toward mathematics. Students with a lower mathematics anxiety tend to have higher separate knowing scores. Higher separate knowing scores correspond to more positive attitudes towards mathematics who would be more inclined usually have lower mathematical anxiety. This relationship can be viewed as a recursive, self-feedback loop. Higher anxiety causes a more negative attitude, which then causes higher anxiety. In the same way, a negative attitude about mathematics would likely cause higher anxiety. Viewed in aggregate, the two self-reinforcing factors of negative attitudes about mathematics and high mathematical anxiety may contribute to lower achievement in mathematics (Ma, 1999) and to a lack of self-confidence in regard to mathematics which has been shown to cause students to be reluctant to participate in the classroom (Miller & Mitchell, 1994).

Ways of Knowing and Gender and Ethnicity. Table 5 provides a summary and breakdown of ways of knowing (connected = CK; separate = SK) by gender and ethnicity. Also, a “Difference” score was calculated by subtracting the mean separate knowing (SK) from the mean connected knowing (CK) score. To understand more completely the relationships between these three variables, the data were not only examined in total but were also disaggregated into groups by gender and ethnicity. The overall means as well as group means and standard deviations are presented in Table 5.

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Table 5

Ways of Knowing by Gender and Ethnicity

Group	Ways of Knowing			
	n	CK ^a M (SD)	SK ^a M (SD)	Difference CK - SK
Overall Sample	82	5.21 (.87)	4.21 (.83)	1.00
Overall Gender				
Male	32	4.91 (.89)	4.27(.82)	0.64
Female	49	5.39 (.82)	4.18 (.85)	1.21
Overall Ethnicity				
Black	15	5.57 (.73)	4.59 (.84)	0.98
Hispanic	9	4.83 (.73)	4.39 (.60)	0.44
White	51	5.14 (.89)	4.06 (.80)	1.08
Asian	1	5.10 (0.0)	5.10 (0.0)	---
Multiracial	4	5.45 (1.3)	4.15 (1.5)	1.30
Black				
Male	5	5.20 (.93)	4.03 (.34)	1.17
Female	10	5.76 (.58)	4.38 (.74)	1.38
Hispanic				
Male	6	4.47 (.58)	3.83 (.61)	0.64
Female	3	5.57 (.25)	5.10 (.00)	0.47
White				
Male	18	4.94 (.99)	4.15 (.81)	0.79
Female	33	5.25 (.82)	4.01 (.80)	1.29
Asian				
Male	1	5.10 (0.0)	5.10 (0.0)	--
Female	--	--	--	--
Multiracial				
Male	1	5.40 (0.0)	3.40 (0.0)	2.00
Female	3	5.50 (1.6)	4.40 (1.6)	1.10
Ethnically Diverse				
Male	13	4.87 (.76)	4.45 (.84)	0.42
Female	16	5.67(.76)	4.52 (.86)	1.15

^aScores range from 1 to 7 where high scores represent a stronger agreement with a particular way of knowing.

Gender and Ways of Knowing. Data analysis using Pearson product moment correlation indicated that a significant relationship exists between gender and connected knowing $r(78) = 0.266$, $p = 0.017$ indicating that females tend to score higher on the connected knowing

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scale than males. Further examination of the data using an independent sample t-test indicated that there exists a significant difference between the group mean score for connected knowing between males ($M=4.91$, $SD=0.89$) and females ($M = 5.39$, $SD = 0.82$), $t(78) = -2.44$, $p = .017$. These quantitative findings are consistent with qualitative findings of Belenky, et al., (1986/1997) who found that women tend to be connected knowers more so than males. However, differences in mean scores between males ($M=4.27$, $SD=0.82$) and females ($M = 4.18$, $SD = 0.85$) for separate knowing were not significant, $t(79) = 0.48$, $p = 0.633$.

As one might expect, males had higher separate knowing scores ($M = 5.02$, $SD = 0.95$) than females ($M = 4.15$, $SD = 0.83$) indicating that men may tend to favor separate ways of knowing more so than females which has been well established (Belenky et al., 1986, 1997). The data indicate that both male and female group mean scores for connected knowing were higher than their group mean scores for separate knowing. That is, both males and females favored connected knowing to separate knowing. This finding contradicts previous research by Belenky, et al (1986). Yet, the degree to which each gender favored connected knowing over separate knowing in mathematics was different. Females preference for connected knowing to separate knowing (*Difference* score of 1.21) was nearly twice as large relative to males (*Difference* score of 0.64). Some examples of response received from males follow.

Since learning is socially constructed and is “articulated within a social context” (Clancy, 2013, p. 17) in regard to social roles and norms, it follows that females are more likely to be connected knowers than males. Females, who have been enculturated to the societal roles, values and practices endorsed and organized around relationships (Gilligan 1982; Miller 1976) often, prefer settings that emphasize the affective domain (Philbin, Meier, Huffman, & Boverie, 1995).

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As expected, females had significantly larger connected knowing scores than males. Given this much larger margin, learning environments which tend to encourage and value separate knowing could be problematic for females. Particularly, since female learners prefer settings that emphasize the affective, group interaction, and doing (Philbin, et al., 1995). Fox, Tobin, and Brody (1979) posit the negative view of mathematics among females may stem from the lack of support for their preferred learning style and from social stereotyping present in so many mathematics classrooms. Possible explanations include those of Belenky and her colleagues who suggest that females who favor connected learning are “unable to visualize themselves into the educational context, leaving them feeling disconnected from the learning environment” (Belenky et al., 1997, p. 105).

The finding that may be the most significant is that connected knowing scores for all demographics are appreciably higher than separate knowing scores. The mean connected knowing score for all participants was appreciably higher than the mean separate knowing score, a difference of 1.0 (see Table 5), which may indicate a fundamental student preference for connected knowing. The strong preference for connected knowing these data reveal seems to indicate that the mathematics classroom status quo which emphasizes separate knowing in mathematics may not be helpful for all students—especially those who favor connected ways of knowing.

Ethnicity and Ways of Knowing. As presented in Table 4, data analysis using Pearson product moment correlation indicated that a significant relationship exists between ethnicity and separate knowing, $r(79) = 0.246$, $p = 0.027$. More specifically, participants who identify themselves as ethnically diverse tended to score higher in separate ways of knowing ($M = 4.49$)

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than White participants ($M = 4.06$). Independent samples t-tests between group means for SK score results, $t(79) = -2.256$, $p = 0.027$, further support this result.

Table 6

Separate and Connected Means By Ethnicity

Ethnicity	N	CK ^a		SK ^a		CK - SK
		M	SD	M	SD	
White	51	5.14	0.89	4.06	0.80	1.08
Ethnically Diverse	29	5.31	0.85	4.49	0.83	0.82

^aScores range from 1 to 7 where high scores represent a stronger agreement with a particular way of knowing.

Although both groups' means for connected knowing are higher than their scores for separate knowing, the finding that ethnically diverse groups score significantly higher on separate knowing than their white peers seems to contradict the ideals consistent with collectivism which is common in many ethnic groups. Collectivist societies value interdependence over individuality and success of the group over that of the individual (Falicov, 1998; Parham, 2002; Yeh, 2000). Encouraging interdependence, cooperation, and success of the group is contrary to the ideals ascribed to by separate knowers (Belenky et al., 1986, 1997; Galotti, et al., 1999). Separate knowers also often prefer individual work to group work (Belenky et al., 1986, 1997; Love & Guthrie, 1999) which is more consistent with individualist societies.

It seems plausible that individuals who come from more collectivist cultures might act counter to values advocated at home. Particularly, if there has been a continual reinforcement of one's inequality of status and a history of unresolved conflict that result from cultural/ethnic differences (Gaertner, Dovidio, Nier, Ward, & Banker, 1999). Research conducted by Ryan and David (2003) on delineating group differences seemed to confirm this premise, reporting

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significantly higher levels of separate knowing in students when they responded as a member of an *out-group*. The balance of *in-group/out-group* (i.e. group composition) may play a larger role for individuals who are ethnically diverse since the dominant group often controls the environment.

Situational factors can help or hinder whether or not individuals select *in-group* or *out-group* status. Environments that encourage group members to form alliances and work together toward common goals are more likely to counteract out-group status than those that endorse competitive tasks, supporting individualism, and often predominate. Distinctions such as these affect how an individual view one's place relative to the larger group and are significant when trying to develop a worldview that is more multicultural in nature rather than continue to accept the worldview of the dominant culture.

Dominant Way of Knowing and Mathematical Ways of Knowing. A growing number of researchers posit that human cognition is domain-specific (Hirschfeld & Gelman, 1994); therefore, a new domain-specific customized instrument, was used to investigate two aspects of mathematical ways of knowing: beliefs about how one comes to know mathematics and beliefs about the nature of mathematics.

Trying to segregate connected knowing from separate knowing is not realistic, given that numerous researchers support the idea that individuals think and learn using both separate and connected ways of knowing. However, it is useful to attempt to determine the degree of relationship and degree of predominance of each way of knowing, and the segregation aids in this purpose. Students were classified as either connected or separate knowers are presented in Table 7. Clearly, there were an overwhelming number of participants whose dominant knowing style was *connected knowing*.

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Table 7

<i>Dominant Knowing Style</i>		
	n	%
Separate Knowers	10	11.2
Connected Knowers	69	77.5
<hr/>		
Total	89	100.0

Independent sample t-tests were run for each MDM item to test for significant differences between the two groups. Group mean scores on the Mathematics Dialectics Measure (MDM) are shown below in Table 8. Independent t-test found a significant difference between connected and separate knowers' mean scores on the item Relational or Impersonal ($t(76) = 2.393$ $p = .019$). This result indicates that connected knowers, the majority in this sample, do favor contexts that endorse group work and reciprocity which aligns with past research (Belenky, et al., 1986, 1997; Love & Guthrie, 1999). It follows then that students who prefer relational aspects of would believe working with other students would improve their learning outcomes (Belenky et al., 1986, 1997).

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Table 8

MDM Item Means for Dominate Knowing Style

	Separate n = 9	Connected n = 67	Total N = 76
Procedure Purpose	.11	-.36	-.23
Discovery Instruction	1.30	.68	.67
Rational Intuitive	-1.40	-1.22	-1.30
Discrete Related	1.80	1.25	1.39
Extensive Concentrated	.10	-.18	-.14
Listening Speaking	-.80	-.49	-.57
Student Teacher	1.30	.99	1.05

^aScores range from -3 to 3 where negative scores relate more with the first word listed and positive scores relate more with the second word listed.

Independent t-test revealed a significant difference between group means between connected and separate knowers on the *Relational or Impersonal* item ($t(76) = 2.393$ $p = .019$). This result indicates that connected knowers, the majority in this sample, do favor contexts that support and encourage group work and reciprocity which aligns with past research (Belenky, et al., 1986, 1997; Love & Guthrie, 1999). It follows then that students who prefer relational aspects of learning would believe working with other students would improve their learning outcomes (Belenky et al., 1986, 1997).

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Gender and Ethnicity and Mathematical Ways of Knowing. Independent sample t-test did not show a significant difference between group mean scores between males and females on any of the MDM items (Table 9).

Table 9

MDM Item Means by Gender

	Gender	N	M	.SD
Procedure/Purpose	Male	34	-.06	1.71
	Female	51	-.35	1.56
Discovery/Instruction	Male	35	.26	1.67
	Female	52	.92	1.60
Rational/Intuitive	Male	34	-1.62	1.54
	Female	52	-1.08	1.71
Discrete/Related	Male	35	1.49	1.17
	Female	52	1.35	1.36
Relational/Impersonal	Male	35	-.37	1.70
	Female	52	-.58	1.61
Extensive/Concentrated	Male	35	-.11	1.92
	Female	52	-.17	1.74
Listening/Speaking	Male	35	-.80	1.64
	Female	52	-.42	2.30
Student/Teacher	Male	35	.83	1.72
	Female	52	1.17	1.56

^aScores range from -3 to 3 where negative scores relate more with the word on the left and positive scores relate more with the word on the right

Furthermore, independent t-test did not find a significant difference between the group mean scores between participants who identify themselves as White and those who identify themselves as Ethnically Diverse on any of the MDM items (Table 10). That is, when examining students' mathematical ways of knowing based on gender or ethnicity, no significant differences exist. Hence, students are more similar than different in their mathematical ways of knowing. This is also substantiated by the only significant difference seen between connected and separate knowers' mean scores on the MDM was the *Relational/Impersonal* item which showed students

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overwhelmingly preferred connected knowing in mathematics, and indicates that students are more alike than different when groups are based on students' dominant way of knowing.

Table 10

MDM Item Means By Ethnicity

	Ethnicity	N	M	SD
Procedure/Purpose	White	56	-.38	1.590
	Ethnically Diverse	28	.00	1.678
Discovery/Instruction	White	56	.77	1.716
	Ethnically Diverse	30	.50	1.526
Rational/Intuitive	White	56	-1.32	1.674
	Ethnically Diverse	29	-1.17	1.649
Discrete/Related	White	56	1.45	1.387
	Ethnically Diverse	30	1.30	1.088
Relational/Impersonal	White	56	-.41	1.735
	Ethnically Diverse	30	-.60	1.476
Extensive/Concentrated	White	56	-.14	1.813
	Ethnically Diverse	30	-.10	1.807
Listening/Speaking	White	56	-.54	2.080
	Ethnically Diverse	30	-.67	2.090
Student/Teacher	White	56	.98	1.668
	Ethnically Diverse	30	1.10	1.583

^aScores range from -3 to 3 where negative scores relate more with the word on the left and positive scores relate more with the word on the right

Insight from Mathematical Dialectics Measure. One of the research questions investigated in this study was, *Does the Mathematics Dialectics Measure (MDM) provide any new insight into student mathematical ways of knowing?* This study is an initial attempt to understand student mathematical ways of knowing, which is framed using the situated view of cognition where students are socialized to the values and beliefs of the academic enterprise (Cobb & Bowers, 1999) In this case, that enterprise is mathematics. Previous mathematics education experiences interact with student beliefs and attitudes towards mathematics teaching and learning as well as learning outcomes (Furinghetti & Pehkonen, 2002; Leder, Pehkonen, & Törner, 2002; Pehkonen & Pietilä, 2003; Schoenfeld, 1989; Thompson, 1992) and are the

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primary means by which students come to know and understand mathematics (Cobb & Yackel, 1996). Data from the MDM show that students believe:

1. Mathematics is about procedures not the purpose.
2. Mathematics is rational and logical not intuitive.
3. Instruction is the primary means used to dispense knowledge.
4. Mathematical knowledge consists of interrelated facts not discrete facts.
5. Mathematics is more about listening than speaking.
6. Mathematics is more teacher-centered than student-centered.

The findings above are well-documented by previous research. One finding contradicts previous research. Contrary to past research which posited that a large majority of students hold simplistic views of mathematics and view mathematics as the study of disparate, isolated facts (Brown et al., 1989; Garofalo, 1989; Schommer, 1990), these results showed that students believe mathematics consists of highly interrelated concepts. This finding could be attributed to a number of factors such as (a) the sample consisted of college students who were largely non-traditional and may have experienced more real-world situations requiring them to use math as part of their work or daily lives, and (b) the sample consisted of some students from the researcher's classes where math is consistently presented as interrelated concepts.

Participants in this study indicate that mathematics is rational and logical rather than a subject that is intuitively known. While this may be true, other researchers found that students often ignore more rational and logical approaches when solving mathematics problems preferring to try numerous approaches before picking the answer that made the most sense (Lester, Garofalo, & Kroll 1989; Schoenfeld, 1985, 1989). Though this is a seeming dichotomy,

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the characterization of trying numerous approaches and choosing the most sensible answer could be construed as logical by some.

A majority of participants associate school mathematics with rules and memorization driven by procedures, as opposed to concepts. This finding confirms previous research (Brown et al., 1989; Diaz-Obando, Plasencia-Cruz, & Solano-Alvarado, 2003; Frank, 1988; Garofalo, 1989; Spangler, 1992). Males who preferred separate ways of knowing viewed mathematics as a means to an end while the rest tended to view mathematics as set of procedures to follow to produce an answer, emphasizing the process of obtaining the answer rather than the usefulness of the answer.

Participants indicated that mathematics is more teacher-directed than student-directed. This finding is consistent with prior research indicating that mathematics teachers are often viewed as the source of knowledge because they dictate student tasks and procedures as well as determine whether student answers are right or wrong (Brown et al., 1989; Diaz-Obando, et al., 2003; Frank, 1988; Garofalo, 1989; Schoenfeld, 1985, 1989; Stodolsky, et al., 1991).

As expected, participants indicated their role in mathematics is to listen while teachers speak. Much prior research has confirmed the passive nature of learning in mathematics (Diaz-Obando, et al., 2003; Frank, 1988; Garofalo, 1989). Conversely, prior researchers have found that students report the best lessons in mathematics are those in which teachers talk less and students talk more (Lee & Johnston-Wilder, 2013; Alexander, 2008; Mercer & Littleton, 2007) and that mathematics teachers talk too much (Lee and Johnston-Wilder, 2013).

None of the participants associated mathematics with discovery-type learning, but rather indicated mathematics is learned through teacher instruction. This finding is not surprising since

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teacher instruction has been shown to be the dominant way students come to know mathematics (Doyle, 1988; Garofalo, 1989; Schoenfeld, 1989; Stodolsky, et al., 1991).

A large majority of participants in this study do not believe they are capable of using logic or reasoning to construct their own mathematical knowledge, and believe they are merely copiers of mathematical knowledge when learning mathematics. This may explain why most students in this study had lower scores on separate knowing than connected knowing. Students often experience mathematics teaching which discourages discovery-type learning and the independent creation of mathematical knowledge, thereby discouraging development of a key aspect of separate knowing and a critical habit of mind for mathematical thinking.

In summary, the results indicate students believe:

1. Mathematics is about procedures not the purpose.
2. Mathematics is rational and logical not intuitive.
3. Instruction is the primary means used to dispense knowledge.
4. Mathematical knowledge consists of interrelated facts not discrete facts.
5. Mathematics is more about listening than speaking.
6. Mathematics is more teacher-centered than student-centered.

Optimal Environment for Learning Mathematics. Of the 88 students who took the survey, 59 students completed the free response portion yielding a response rate of approximately 67%. Participants who completed the free response item consisted of 34 females (57.6%) and 25 males (42.4%). There were 44 Whites (74.6%), 11 Blacks (18.6%), 3 Hispanics (5.1%), and 1 participant who did not specify ethnicity (1.7%). All of the free responses and the codes assigned to them are provided in Table 11. Free responses are reproduced exactly as students wrote them, including spelling and punctuation.

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Table 11

Free Responses, Gender, Ethnicity, and Coding

Response Number	Gender*	Ethnicity**	Free Response	Coding												
				Guided Practice	Ask Questions	Thorough Explanation	Examples	Steps	Slower Paced	Smaller Classes	Individual Help	Interaction	Related To Real Life	Repetition	Working With Others	Teacher Knows Material
1	M	W	I'm more of a visual learner. Show more pictures and examples work better for me. And doing group work also helps me because I can get different ideas and different ways to solve a problem				✓							✓		
2	M	W	Nothing higher than College Algebra; Quiet classroom. Too much noise and I lose concentration when in a learning environment. Very structured layout. i.e. This is what we are going to do 1...2...3...etc					✓								✓
3	M	W	For me, learning math is easiest and best when seeing several examples done in a class setting and then trying several examples on my own both in class and for homework. I think having a structured environment helps me put everything together and see how all math is related and builds on itself. Learning math is best when making mistakes on problems and learning from these mistakes to apply that knowledge to future problems.	✓		✓										✓
4	M	W	On the computer													
5	M	W	By making it fun, sing songs to remember equations. If test are given without knowing it is a test. Most people freeze when it comes to a test.													
6	M	W	A small to medium sized class with a teacher than makes learning interesting													✓
7	M	W	In a small class with a teacher who is willing to talk you throughing you don't understand	✓												
8	M	W	In a classroom and no longer than an hour and a half long that way you can retain everything you learn													
9	M	W	One on one. Someone showing me how to do it and telling me how to work problems	✓	✓									✓		
10	M	W	Hybrid class with teachers available to help.		✓											
11	M	W	A course where I learn at my own pace and if I ever needed help on something a teacher would be there to help		✓											
12	M	B	Teacher teaching math													
13	M	B	I like to be taught in steps broken down. Then bring it all together so I can form my own ways to complete the process			✓	✓									
14	M	B	Math should be taught by the teacher. She or he should show us how to do the work first then let us try it and if the student doesn't know how to do the work, he or she should help us personally	✓		✓							✓			
15	M	B	My optimal condition for learning math would be smaller classes with a more personalized approach													✓
16	M	W	I think math should be put more into context when it's being taught. By that I mean being shown what you are trying to solve instead of being taught only the formulas.			✓							✓			
17	M	B	I would love to walk into an environment with everyone on the same page. Math is a new journey when you're learning something new. Only in math do I like a know-it-all teacher or professor. Seeing all the work written out step by step That way what work was done can be looked over for evaluation			✓	✓									✓
18	M	W	To focus on an objective at the time with no rush													✓
19	M	N	I think learning math while building something would be optimal. For instance finding an equation then graphing and using zero method to size a machine part											✓		
20	M	W	Going over math problems on a board and being able to visually see them being worked on a board			✓										

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Free Responses, Gender, Ethnicity, and Coding

Response Number	Gender*	Ethnicity**	Free Response	Coding														
				Guided Practice	Ask Questions	Thorough Explanation	Examples	Steps	Slower Paced	Smaller Classes	Individual Help	Interaction	Related To Real Life	Repetition	Working With Others	Teacher Knows Material	Structured Environment	
21	M	W	Learning a way to look at problems simply to allow the mind to follow fairly simply as well as deduce answers with understanding and logic, one way may not be the best way so I choose to try my own way of grasping each subject the best could. Showing all ways of doing math is one way however it confused me because it varied from how I've learned in the past this is something I'd choose to allow the students to choose their own way as long as they could show how they logically came to their conclusion															
22	M	W	I feel like just finishing would be good for me But obviously practice would help.															
23	M	B	You're doing great. I just need to better manage my time.															
24	M	W	Learning more, studying more															
25	M	W	A pill that makes me smarter															
26	F	W	Classroom setting with patient and thorough instructor			✓												
27	F	W	Quiet room with good, lighting, large tables, and all the tools necessary to be able to work problems; math book and an instructor that is approachable and thoroughly explains what they are teaching	✓	✓						✓							
28	F	W	A teacher that isn't rushed and steps written for each new things we learn as well as a teacher that relates the math we are learning to life and allows us to work WITH each other in class, keeps it interesting, and does more than read examples out of a textbook					✓	✓			✓	✓			✓		
29	F	W	Quiet environment, few people. I really teach myself once I have the basics. Doing a lot of extra homework helps me for tests.								✓					✓		
30	F	B	Working on computers			✓		✓										
31	F	W	Step by step detailed process of how to work problems			✓		✓										
32	F	B	A quiet focused environment with great teacher help, lots of student participation and happy environment	✓								✓						
33	F	B	Coming to class and listening to the instructor; ask questions if one does not understand; try to find what other problem solving works best for you	✓														
34	F	B	I learn better in the classroom															
35	F	W	I thrive in a very explained, written out slow moving classroom. I have found that a hybrid course were 90% of my work is done in a Pearson or similar program that I learn the most. The option to repeat similar problems in the program while guided with instruction at my own pace is best for me	✓		✓				✓						✓		
36	F	W	It is easiest for me to learn when it is shown to me doing a problem on the board and explaining everything as we go helps me understand and remember			✓	✓											
37	F	W	Learning one way to solve a problem that's easiest for me. Having take-home tests seems very helpful to me on learning the subject. I also prefer having a lecture math class over having the class online so I can ask questions if needed and also so I can actually see the problem being done on the board.			✓		✓										
38	F	W	Small class numbers with an interactive teacher. An environment where I feel comfortable asking questions and answering questions. A teacher doesn't always teacher by the book, but by the easiest method of solving a problem. An environment focused on the students and make sure that no one is being left behind			✓					✓		✓					
39	F	W	Your Class! LOL Seriously I would have to say my optimal conditions would have to be in a classroom where I feel would	✓														✓

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Free Responses, Gender, Ethnicity, and Coding

Response Number	Gender	Ethnicity	Free Response	Coding													
				Guided Practice	Ask Questions	Thorough Explanation	Examples	Steps	Slower Paced	Smaller Classes	Individual Help	Interaction	Related To Real Life	Repetition	Working With Others	Teacher Knows Material	Structured Environment
			have to be in a classroom where I feel comfortable enough to ask questions and not feel stupid. I need a teacher who actually cares whether or not I understand the material and knows what is actually going on, as well as someone who is more than willing to take things back to "the basics" if needed														
40	F	W	If the teacher could explain the math problems in more depth. Sometimes it is hard to understand how a math problem is solved when the teacher explains it too fast. I get nervous when I have to ask for help, but I do it anyway cause math is a hard subject for me and it takes me longer now to comprehend how to solve it.		✓	✓				✓							
41	F	W	Described in detail how to work out the difficult version of a problem; working some on computers instead of always from a text book.			✓		✓									
42	F	W	I would have lecture, computer, hands on, and groups as well as individual	✓												✓	
43	F	W	I see an understanding in learning things that we use everyday. I do not understand why we are graded on things we will never use again.										✓				
44	F	W	My choice would be to make math just an option. But, if I have to take a math course, I'd prefer a teacher that can really teach the subject and is able to help math strugglers like me.														✓
45	F	W	My optimal condition for learning math would be hands on. I like to look at things in order to learn them. Also, I think repeating things make it easier to learn.	✓										✓			
46	F	W	What I am currently doing with Math Matters; a combination of online work with teacher instruction														
47	F	W	I believe hands on during class helps and breaking down the problems with the simplest solution	✓				✓									
48	F	W	I believe I would learn better one on one and hands on when it comes to math. Being in a large environment makes me feel as though I may not be as advanced as some of the others especially ones that are younger than me.	✓							✓						
49	F	W	To do it over and over again until its right													✓	
50	F	W	I like to learn hands on; do stuff to learn better	✓													
51	F	W	Finding a hands on way to learn it; small class and a teacher that goes slowly	✓						✓	✓						
52	F	H	Hands on, working problems instead of just listening	✓													
53	F	H	I learn by listening and interacting with the teacher. I tend to ask questions when I'm not sure of something and I learn visually. If a teacher explains it well enough I tend to understand it better		✓	✓							✓				
54	F	H	Teachers using examples and explaining step by step what to do in a quiet room where I can hear and no tall person in front of me so that I can see					✓	✓								
55	F	B	Good classmates and good environment that way I would be comfortable discussing the things I don't understand in front of others; and we all are able to help one another		✓											✓	
56	F	W	I only take the cases required for my major. I don't want to take them because I want to but because I have to.														
57	F	W	I love math and love working with math problems. It's very comforting for me. I am planning to teach high school math as a professor.														
58	F	W	I really enjoy doing math. I tend to learn the techniques of problem solving fairly easily. Math happens to be my favorite subject.														

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Free Responses, Gender, Ethnicity, and Coding

Response Number	Gender*	Ethnicity**	Free Response	Coding																
				Guided Practice	Ask Questions	Thorough Explanation	Examples	Steps	Slower Paced	Smaller Classes	Individual Help	Interaction	Related To Real Life	Repetition	Working With Others	Teacher Knows Material	Structured Environment			
59	F	W	Math is what you make it out to be. If you absolutely hate math then you're not going to want to learn but if you enjoy it then you'll do great. You should strive to do your best in each subject though																	

*Gender codings: M = Male, F = Female

** Ethnicity codings: W = White, B = Black, H = Hispanic, N = None

In some cases, a participant's free response met the requirements for multiple coding categories. Coding categories are subjective because a researcher must determine whether a given free response qualifies for a given coding, and different researchers might have different interpretations of which free responses qualify for different coding categories. However, coding categories are the only way to provide a quantitative, descriptive statistical analysis of free responses. The names used for the 14 free response coding categories, shown in Table 12, reflect the language used by students in free responses, rather than more formal academic terminology, so as to retain as much of the qualitative aspect of the data as possible.

The mode for the coding categories was Guided Practice (12, 20.3%). Other predominant coding category include Ask Questions (11, 18.6%), Thorough Explanation (11, 18.6%), Examples (9, 15.3%), and Steps (9, 15.3%). Appreciable numbers of students felt that the optimal condition for learning mathematics involves a thorough explanation of mathematical concepts by instructors, step-by-step instructions, the use of examples, and guided practice where students are able to ask questions freely and feel comfortable doing so.

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Table 12

Free Response Items Coding Categories

Coding	Number of Free Response Participants Who Gave Answer	Percentage of Free Response Participants Who Gave Answer*
Guided Practice	12	20.3%
Ask Questions	11	18.6%
Thorough Explanation	11	18.6%
Examples	9	15.3%
Steps	9	15.3%
Slower Paced	5	8.5%
Smaller Classes	5	8.5%
Individual Help	4	6.8%
Interaction	4	6.8%
Related To Real Life	4	6.8%
Repetition	4	6.8%
Working With Others	4	6.8%
Teacher Who Knows Material	3	5.1%
Structured Environment	2	3.4%

*Percentage of 59 free response participants, not all 88 students who took survey completed free response item

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Chapter 5: Conclusion

For years, researchers in psychology, education, and mathematics have been trying to identify factors which contribute to student reluctance to participate in mathematics. Study after study indicates that past experiences of students with formal mathematics instruction greatly influences their beliefs about mathematics, as well as their emotional feelings about and emotional responses to mathematics, including anxiety and other negative feelings (Bursal & Paznokas, 2006; Harper & Daane, 1998; Jackson & Leffingwell, 1999; NCTM, 1989, 2000; Sloan, Daane & Giesen, 2002). The fear and loathing of and anxiety toward mathematics that many students develop early in their educational career usually stays with them over the long term, in many cases for the rest of their lives (Burns, 1998; Bursal & Paznokas, 2006; Dutton & Dutton, 1991; Gresham, 2004; Hembree, 1990). Furthermore, student conceptions of mathematical knowledge and mathematical knowing play a significant role in the formation of students' mathematical dispositions and beliefs about mathematics. This fear and loathing of mathematics causes students to shun mathematics, avoid STEM degrees, and in many cases abandon of the pursuit of a degree entirely (Tobias, 1991).

A quantitative, correlational, survey design with anecdotal qualitative data was used to investigate the relationships among mathematical anxiety, attitude toward learning math, gender, ethnicity, and separate and connected ways of knowing within the context of the mathematics classroom. Participants were 88 student volunteers enrolled in undergraduate mathematics classes at an open admissions technical college in the southeastern United States. Survey data consisted of demographic self-report items, Likert scale items, semantic differential scale items, and one qualitative free-response question. This study utilized an established, though generalized, instrument for *measuring ways of knowing* (ATTLS) which has been traditionally

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been applied on a cross-domain basis. However, this is the first such research to apply the ATTLS rigorously on a domain-specific basis in conjunction with domain-specific instruments to investigate the relationship between students' beliefs about mathematics and ways of knowing. Research examining student ways of knowing specifically studying the domain of mathematics is limited, and is a primary reason this study was undertaken.

In this study, a strong negative correlation was found between attitudes toward mathematics and math anxiety. This finding is consistent with prior research on mathematics attitude and anxiety (e.g. Terwilliger & Titus, 1990; Furner & Gonzalez, 2011; Ramirez, Gunderson, Levine, Beilock, 2013; Williams, 1988). Specifically, those with positive attitudes toward mathematics have lower anxiety toward the subject while those with more negative attitudes tend to have higher anxiety toward mathematics. The importance of emotional factors in learning cannot be underestimated. Researchers in neurobiology and neuroscience continue to expand our understanding of the role that emotions play in learning and the effects of stress on the brain (e.g. McEwen, Gray & Nasca, 2015). Stress has been shown to have “detrimental effects on learning and neurocognitive functioning that can challenge student learners as they navigate through their college years” (Palmer, 2013, p. 322). Given the results of this and other studies, P-12 teachers and college professors should strive to reduce the stressful nature of the mathematics classroom.

Another important finding from this study is that participants indicated that mathematics is procedural in nature and is a subject that is learned passively through expository teaching methods and direct instruction. Prior research indicates that one's experiences in mathematics shapes one's perceptions and beliefs about how one learns and comes to know math (NCTM, 1989). Traditional teaching practices, such as those commonly encountered by students in P-12

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and college classrooms, where teachers talk and students listen, position students as receivers of knowledge, socializing them to believe mathematics is learned passively rather than actively. These findings suggest mathematics classrooms should resemble more constructive, problem-based environments. Students should be given ample time to work cooperatively with their classmates on personally relevant, authentic problems which offer multiple paths for solutions. In keeping with tenets of situated cognition learning and prior research (Paris & Glynn, 2004), learning is enhanced when learners find personal relevance in the learning activities. The processes of problem-solving rather than the procedures to be followed become more valuable in these learning situations. In problem-based classrooms, the teacher becomes a facilitator or coach rather than the sole source of knowledge.

A very important finding from this study related to gender, ethnicity and ways of knowing. Specifically, regardless of gender or ethnicity, students are more alike than they are different in their ways of knowing. Overwhelmingly, the majority of participants in this study indicated they were connected knowers. This finding contradicts previous qualitative research by Belenky et.al (1986) who suggested that females tended to be connected knowers while males tended to be separate knowers. Nevertheless, the mean scores for females on connected knowing were significantly higher than that of males. This study showed that connected knowing is preferred by all genders and ethnicities in the mathematics domain. These findings suggest that mathematics pedagogies should incorporate approaches that facilitate learning for connected knowers. Such practices may include more group interactions where discussion and team building are fostered. Cooperative rather than competitive approaches to learning should predominate in the mathematics classroom. Coupled with the findings related to attitude and anxiety, pedagogical practices which facilitate learning with connected knowers in mind would

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likely reduce anxiety toward mathematics learning as well. The solution to the mathematical woes seen in mathematics education may not be static changes to the material or message delivered to individual students, or the way students are taught on an individual basis, but rather *reformation of the mathematical learning context and the socialization which accompanies it*. As Brown et al. (1989) suggested nearly 25 years ago,

the activity and context in which learning currently takes place is often regarded as merely ancillary to learning—pedagogically useful, of course, but fundamentally distinct and even neutral with respect to what is learned (p.32).

Limitations

A primary limitation to this study is that causal inferences cannot be drawn as this study was a descriptive, correlational study. Nevertheless, the benefits of correlational research such as this should not be overlooked since they are instrumental exposing variables or conditions which other researchers may then pursue. Many foundational studies began as a result of correlational research providing the information to build upon. Descriptive statistics, for instance, help identify variables, relationships between variables leading to other types of research designs, and play a role in developing theoretical models to help explain relationships between variables.

Another limitation of this study was the development and use of a new instrument. Although the Mathematics Dialectic Measure was tested, it should be further developed to establish reliability and validity ensuring that the items measure the intended construct reliably.

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Recommendations for Future Research

This was an initial, descriptive exploratory study. Future research should utilize experimental designs as well as longitudinal studies as these approaches would expand our understanding of the effects of interventions as well as how experiences may shape long term outcomes and attitudes. Other possible research studies might explore the degree to which a dominant knowing style is favored. Moreover, longitudinal studies that distinctly investigate preferences of *separate* and *connected* students would likely provide new insight into both learning and teaching.

Certainly, understanding the impact of one's "way of knowing" and how or when one actually learns should be explored in future research. Factorial randomized research designs, such as those advanced by Pashler, McDaniel, Rohrer, and Bjork (2008), should be developed to explore interactions between participants' *ways of knowing* and different instructional treatments. Matching one's expressed way of knowing to one's learning environment or instructional strategy and the effect of this matching (or mismatch) on learning outcomes should be undertaken.

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Appendix A

Attitude Toward Thinking and Learning (ATTLS)

Directions: Please rate the degree to which you agree with each statement using the following scale. Answer the questions quickly without dwelling too long on any question (i.e., go with your first reaction). Please do not change your responses to items once you have marked them.

Strongly Disagree	Somewhat Disagree	Slightly Disagree	Neither Agree Nor Disagree	Slightly Agree	Somewhat Agree	Strongly Agree
1	2	3	4	5	6	7

1. _____ When I encounter people whose opinions seem alien to me, I make a deliberate effort to “extend” myself into that person, to try to see how they could have those opinions.
2. _____ I like playing devil’s advocate—arguing the opposite of what someone is saying.
3. _____ It’s important for me to remove myself from analysis of something and remain as objective as possible.
4. _____ I can obtain insight into opinions that differ from mine through empathy.
5. _____ I tend to put myself in other people’s shoes when discussing controversial issues, to see why they think the way they do.
6. _____ In evaluating what someone says, I focus on the quality of their argument, not on the person who’s presenting it.
7. _____ I find that I can strengthen my own position through arguing with someone who disagrees with me.
8. _____ I’m more likely to try to understand someone else’s opinion than to try to evaluate it.
9. _____ One could call my way of analyzing things “putting them on trial”, because of how careful I am to consider all of the evidence.
10. _____ I try to think with people instead of against them.
11. _____ I often find myself arguing with the authors of books I read, trying to logically figure out why they’re wrong.
12. _____ I have certain criteria I use in evaluating arguments.
13. _____ I try to “shoot holes” in what other people are saying to help them clarify their arguments.
14. _____ I feel that the best way for me to achieve my own identity is to interact with a variety of other people.
15. _____ I am always interested in knowing why people say and believe the things they do.
16. _____ I spend time figuring out what’s “wrong” with things; for example, I’ll look for something in a literary interpretation that isn’t argued well enough.
17. _____ I enjoy hearing the opinions of people who come from backgrounds different from mine—it helps me understand how the same things can be seen in such different ways.
18. _____ I value the use of logic and reason over the incorporation of my own concerns when solving problems.
19. _____ The most important part of my education has been learning to understand people who are very different from me.
20. _____ I like to understand where other people are “coming from”, what experiences have led them to feel the way they do.

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Appendix B

Attitude Toward Mathematics Instrument (ATMI)

Directions: This inventory consists of statements about your attitude toward mathematics. Read each item carefully. Please think about how you feel about each item. Choose how the statement best describes your feelings. There are no correct or incorrect responses. Use the following response scale to respond to each item.

Strongly Disagree	Somewhat Disagree	Slightly Disagree	Neither Agree Nor Disagree	Slightly Agree	Somewhat Agree	Strongly Agree
1	2	3	4	5	6	7

1. _____ Mathematics is a very worthwhile and necessary subject.
2. _____ I want to develop my mathematical skills.
3. _____ I get a great deal of satisfaction out of solving a mathematics problem.
4. _____ Mathematics helps develop the mind and teaches a person to think.
5. _____ Mathematics is important in everyday life.
6. _____ Mathematics is one of the most important subjects for people to study.
7. _____ High school math courses would be very helpful no matter what I decide to study.
8. _____ I can think of many ways that I use math outside of school.
9. _____ Mathematics is one of my most dreaded subjects.
10. _____ My mind goes blank and I am unable to think clearly when working with mathematics.
11. _____ Studying mathematics makes me feel nervous.
12. _____ Mathematics makes me feel uncomfortable.
13. _____ I am always under a terrible strain in a math class.
14. _____ When I hear the word mathematics, I have a feeling of dislike.
15. _____ It makes me nervous to even think about having to do a mathematics problem.
16. _____ Mathematics does not scare me at all.
17. _____ I have a lot of self-confidence when it comes to mathematics
18. _____ I am able to solve mathematics problems without too much difficulty.
19. _____ I expect to do fairly well in any math class I take.
20. _____ I am always confused in my mathematics class.
21. _____ I feel a sense of insecurity when attempting mathematics.
22. _____ I learn mathematics easily.
23. _____ I am confident that I could learn advanced mathematics.
24. _____ I have usually enjoyed studying mathematics in school.
25. _____ Mathematics is dull and boring.
26. _____ I like to solve new problems in mathematics.
27. _____ I would prefer to do an assignment in math than to write an essay.
28. _____ I would like to avoid using mathematics in college.
29. _____ I really like mathematics.
30. _____ I am happier in a math class than in any other class.
31. _____ Mathematics is a very interesting subject.
32. _____ I am willing to take more than the required amount of mathematics.
33. _____ I plan to take as much mathematics as I can during my education.
34. _____ The challenge of math appeals to me.
35. _____ I think studying advanced mathematics is useful.

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36. _____ I believe studying math helps me with problem solving in other areas.
37. _____ I am comfortable expressing my own ideas on how to look for solutions to a difficult problem in math.
38. _____ I am comfortable answering questions in math class.
39. _____ A strong math background could help me in my professional life.
40. _____ I believe I am good at solving math problems.

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Appendix C

Mathematics Attitude Survey (MAS)

Directions: Please rate the degree to which you agree with each statement using the following scale. Answer the questions quickly without dwelling too long on any question (i.e., go with your first reaction). There are no correct or incorrect responses. Please do not change your responses to items once you have marked them.

Strongly Disagree	Somewhat Disagree	Slightly Disagree	Neither Agree Nor Disagree	Slightly Agree	Somewhat Agree	Strongly Agree
1	2	3	4	5	6	7

1. _____ I feel comfortable and relaxed with math.
2. _____ Math is my most dreaded subject.
3. _____ I feel apprehensive before entering math class.
4. _____ I find math interesting.
5. _____ Math is one of my favorite subjects.
6. _____ I am afraid of math exams.
7. _____ Solving math problems is enjoyable for me.
8. _____ I feel nervous when I am about to do math homework.
9. _____ I feel happy and excited in a math class compared to any other class.
10. _____ I would prefer taking math classes over other subjects.
11. _____ Math is a headache for me.
12. _____ I am afraid to ask questions in math class.
13. _____ Math does not scare me at all.
14. _____ My mind goes blank when the teacher asks me math questions.

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Appendix D

Mathematics Dialectic Measure (MDM)

Directions: Mark a space between the word pairs which best represents your beliefs about mathematics. The space closet to a word indicates a very strong feeling. The middle space indicates your feelings are neutral or undecided about the item. Do not dwell too long on any question (i.e., go with your first reaction). Please do not omit or change your response to an item once you have marked it. There is no right or wrong answer.

Word 1 Strongly Agree : Agree : Slightly Agree : Neutral : Slightly Agree : Agree : Strongly Agree Word 2

Mathematics is...

Procedures _____ : _____ : _____ : _____ : _____ : _____ Purpose
(Means/Process Oriented) (Ends/ Goal Oriented)

Discovery _____ : _____ : _____ : _____ : _____ : _____ Instruction
(Constructing Knowledge) (Receiving knowledge)

Rational _____ : _____ : _____ : _____ : _____ : _____ Intuitive
(Logical, analytical, objective) (Gut feeling, subjective)

Discrete _____ : _____ : _____ : _____ : _____ : _____ Related
(Isolated /Compartmentalized) (Connected/Synthesis)

Relational _____ : _____ : _____ : _____ : _____ : _____ Impersonal
(Cooperative/ Collaborative) (Competitive/ Solitary)

Extensive _____ : _____ : _____ : _____ : _____ : _____ Concentrated
(Generalizable /Broad) (Specialized /Narrow)

Listening _____ : _____ : _____ : _____ : _____ : _____ Speaking
(Observing) (Participating)

Student Directed _____ : _____ : _____ : _____ : _____ : _____ Teacher Directed

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Appendix E

Age: _____

Sex: Male Female

Ethnicity (Circle one): Asian Black Hispanic

Native American/Hawaiian /Other Pacific Islander White

Academic major or program of study: _____

Current mathematics course: _____

Please describe what would be your optimal condition for learning mathematics.

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Appendix F

Participant	Age	Gender	Ethnicity	ATMI_1V	ATMI_2V	ATMI_3E	ATMI_4V	ATMI_5V	ATMI_6V	ATMI_7V
Student 1	19	1	1	4	6	6	7	5	5	6
Student 2	35	1	3	3	6	2	6	3	3	6
Student 3	28	0	3	7	6	7	7	6	6	7
Student 4	26	1	3	1	1	1	5	2	2	1
Student 5	21	1	1	7	7	7	7	7	7	5
Student 6	23	1	3	1	7	1	7	7	7	7
Student 7	21	0	3	4	7	1	4	4	3	6
Student 8	34	1	1	5	7	4	7	6	7	7
Student 9	38	1	3	5	7	2	5	5	4	5
Student 10	28	0	3	7	7	7	7	7	8	8
Student 11	19	1	1	7	6	6	7	7	7	7
Student 12	19	1	1	6	7	7	7	7	6	5
Student 13	36	0	1	7	7	5	7	7	7	7
Student 14	46	1	3	6	6	5	7	7	7	3
Student 15	24	1	3	3	6	5	5	1	5	1
Student 16	29	0	3	7	6	6	6	7	5	6
Student 17	24	1	3	1	6	3	6	6	1	7
Student 18	19	1	3	3	5	5	5	4	3	2
Student 19	34	1	3	7	7	7	7	7	7	7
Student 20	31	0	5	7	6	7	7	6	7	6
Student 21	20	1	3	5	3	1	6	7	4	5
Student 22	21	1	3	5	5	4	5	5	4	4
Student 23	19	1	5	5	5	1	5	5	5	4
Student 24	22	1	3	6	7	7	5	4	5	7
Student 25	27	1	3	3	6	6	4	2	2	3
Student 26	20	0	3	6	4	4	5	7	5	5
Student 27	51	0	2	6	6	6	6	5	5	6
Student 28	22	1	2	6	4	7	6	7	7	5
Student 29	19	0	2	5	4	4	5	5	6	4
Student 30	18	1	3	7	7	6	7	5	6	7
Student 31	18	1	1	6	6	6	7	7	6	6
Student 32	28	0	2	7	6	7	7	6	7	7
Student 33	24	1	3	7	7	5	7	6	5	6
Student 34	26	0	3	6	5	4	7	6	6	5
Student 35	32	0	1	7	7	7	7	7	7	7
Student 36	19	1	3	6	6	5	6	7	6	6
Student 37	39	1	3	7	7	5	7	7	7	7
Student 38	43	1	3	7	7	7	7	7	5	4
Student 39	19	0	3	4	4	1	1	1	1	6

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Student 40	22	0	3	5	5	3	6	6	4	6
Student 41	19	0	3	6	6	3	7	7	6	6
Student 42	21	0	1	7	7	7	7	7	6	6
Student 43	21	1	1	6	6	5	6	6	6	6
Student 45	30	1	3	4	5	4	4	4	3	4
Student 46	24	1	3	3	7	6	4	5	6	6
Student 47	21	0	1	5	5	3	6	4	4	5

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Participant	ATMI_8V	ATMI_9CR	ATMI_10CR	ATMI_11CR	ATMI_12CR	ATMI_13CR	ATMI_14CR
Student 1	4	4	3	4	5	3	4
Student 2	3	1	1	1	1	1	1
Student 3	6	7	7	7	7	7	7
Student 4	1	1	3	3	3	3	2
Student 5	7	1	2	1	1	1	3
Student 6	7	1	7	1	1	1	1
Student 7	2	4	4	4	4	3	3
Student 8	5	7	5	5	6	4	3
Student 9	5	2	2	1	4	4	3
Student 10	7	4	7	5	7	7	7
Student 11	7	3	3	5	6	5	5
Student 12	5	6	6	6	7	7	6
Student 13	7	6	6	4	7	6	7
Student 14	6	2	1	2	2	2	3
Student 15	5	1	3	5	5	4	1
Student 16	7	7	6	6	6	7	7
Student 17	1	1	4	1	1	2	1
Student 18	1	1	5	1	1	1	1
Student 19	7	5	6	6	6	6	6
Student 20	7	7	7	7	7	4	7
Student 21	3	1	1	1	1	1	1
Student 22	3	5	5	4	6	5	5
Student 23	1	1	5	6	7	7	3
Student 24	3	2	2	1	3	3	5
Student 25	1	1	1	1	1	2	1
Student 26	5	4	4	5	3	5	4
Student 27	6	1	1	1	1	1	1
Student 28	7	7	7	7	7	7	7
Student 29	3	6	4	6	4	5	3
Student 30	5	3	5	5	7	6	7
Student 31	6	3	5	3	5	5	5
Student 32	6	1	3	7	7	7	7
Student 33	7	1	3	3	3	3	3
Student 34	7	4	4	4	6	7	6
Student 35	7	7	7	7	7	7	7
Student 36	5	4	6	6	6	6	6
Student 37	7	1	2	1	1	2	2
Student 38	5	1	2	1	1	3	2
Student 39	5	7	1	1	1	5	1
Student 40	5	2	4	5	5	5	5
Student 41	7	3	3	4	4	4	5
Student 42	7	2	7	7	7	7	7

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Student 43	6	6	3	5	5	3	5
Student 45	4	2	3	3	3	3	3
Student 46	3	5	5	6	6	6	6
Student 47	3	6	2	5	4	3	4

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Participant	ATMI_15CR	ATMI_16C	ATMI_17C	ATMI_18C	ATMI_19C	ATMI_20CR	ATMI_21CR	ATMI_22C
Student 1	4	4	6	5	6	1	4	6
Student 2	1	1	1	1	1	1	1	1
Student 3	7	6	6	6	6	7	7	7
Student 4	4	2	1	3	3	3	4	3
Student 5	2	1	5	1	7	1	1	5
Student 6	1	1	1	1	1	1	1	1
Student 7	3	4	2	2	5	4	4	3
Student 8	7	4	1	2	4	5	6	4
Student 9	3	2	2	3	4	5	5	3
Student 10	7	7	6	7	6	4	6	7
Student 11	6	4	4	5	5	5	3	5
Student 12	7	5	5	5	7	6	6	5
Student 13	7	5	5	5	5	7	7	5
Student 14	3	2	1	2	3	2	2	2
Student 15	1	1	7	1	5	6	6	1
Student 16	6	6	6	6	7	5	6	4
Student 17	1	1	1	1	4	3	1	1
Student 18	1	1	1	1	1	1	1	2
Student 19	7	5	4	4	7	7	6	4
Student 20	7	7	7	7	7	2	7	7
Student 21	3	3	1	1	1	1	1	1
Student 22	5	5	5	4	5	5	5	5
Student 23	7	5	5	5	6	7	7	5
Student 24	3	7	6	5	6	6	4	6
Student 25	2	1	1	3	1	3	1	1
Student 26	6	5	4	4	4	6	7	4
Student 27	1	1	1	1	1	7	7	1
Student 28	7	7	7	6	6	7	7	7
Student 29	6	5	5	5	5	6	6	4
Student 30	7	6	5	4	5	6	7	4
Student 31	5	3	3	3	5	5	5	3
Student 32	7	7	7	7	7	7	7	7
Student 33	3	3	2	3	3	3	3	3
Student 34	6	4	2	6	6	4	5	4
Student 35	7	7	6	5	7	7	7	4
Student 36	6	6	5	4	5	4	6	5
Student 37	1	1	1	1	1	1	1	1
Student 38	3	1	1	2	5	5	3	2
Student 39	1	1	1	1	1	2	2	1
Student 40	5	6	5	6	6	5	5	4
Student 41	5	4	3	5	4	5	4	3
Student 42	7	7	6	5	7	3	7	6

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 43	5	5	5	5	5	4	4	5
Student 45	4	2	2	2	2	3	3	2
Student 46	6	5	5	5	5	5	5	6
Student 47	6	1	4	3	4	5	6	4

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	ATMI_23M	ATMI_24E	ATMI_25ER	ATMI_26E	ATMI_27E	ATMI_28MR	ATMI_29E	ATMI_30E
Student 1	6	5	3	6	4	5	5	5
Student 2	1	1	3	2	1	1	1	1
Student 3	7	6	6	7	7	7	7	4
Student 4	2	4	7	3	7	4	3	2
Student 5	5	7	5	5	7	7	5	1
Student 6	1	7	1	1	1	1	1	7
Student 7	5	4	4	2	1	1	1	1
Student 8	7	4	3	5	3	4	2	3
Student 9	2	2	6	3	7	3	3	4
Student 10	6	7	7	7	7	7	7	7
Student 11	5	3	4	3	6	5	4	3
Student 12	5	5	5	5	6	5	5	5
Student 13	4	7	7	4	7	7	5	5
Student 14	2	1	5	4	3	4	3	2
Student 15	1	1	1	1	1	1	1	1
Student 16	6	6	6	6	7	7	7	6
Student 17	1	1	7	1	2	1	1	1
Student 18	1	1	1	1	7	1	1	1
Student 19	7	4	7	6	7	7	6	4
Student 20	7	7	7	7	7	7	7	7
Student 21	1	1	2	7	1	1	1	1
Student 22	5	5	6	4	6	5	5	4
Student 23	5	3	1	1	5	1	3	1
Student 24	5	4	5	6	7	6	5	4
Student 25	1	1	1	4	7	1	1	1
Student 26	1	1	2	4	2	1	1	4
Student 27	1	1	7	2	2	6	2	2
Student 28	6	7	7	6	7	7	7	7
Student 29	3	3	4	4	4	4	4	3
Student 30	5	7	7	7	7	7	6	6
Student 31	5	4	5	4	1	4	3	3
Student 32	5	6	7	7	7	7	7	7
Student 33	3	1	3	2	1	2	2	1
Student 34	4	4	4	5	7	4	6	4
Student 35	7	7	7	7	1	6	6	6
Student 36	4	5	6	5	7	7	5	5
Student 37	1	1	4	1	1	1	1	1
Student 38	1	1	4	3	5	4	3	4
Student 39	1	1	1	1	1	7	1	1
Student 40	5	3	2	4	4	4	3	3
Student 41	5	3	3	3	5	4	4	3
Student 42	5	7	2	5	7	1	5	4

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 43	6	6	5	6	6	5	5	5
Student 45	2	3	4	4	2	3	2	2
Student 46	6	6	2	6	7	5	6	4
Student 47	3	2	4	3	1	4	3	3

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	ATMI_31E	ATMI_32M	ATMI_33M	ATMI_34M	ATMI_35V	ATMI_36V	ATMI_37E	ATMI_38E
Student 1	4	5	4	6	5	6	5	6
Student 2	4	1	1	1	1	5	1	1
Student 3	7	7	5	7	6	6	7	6
Student 4	3	1	1	1	1	5	4	3
Student 5	5	1	1	1	5	7	4	5
Student 6	1	1	1	1	1	7	1	1
Student 7	2	2	2	1	4	2	1	1
Student 8	5	4	6	5	7	7	6	6
Student 9	4	1	1	2	4	4	4	4
Student 10	7	7	7	7	7	7	6	6
Student 11	4	2	3	3	5	5	3	3
Student 12	5	1	1	5	5	6	5	5
Student 13	6	1	4	5	7	7	5	6
Student 14	5	5	4	3	6	7	3	3
Student 15	2	1	1	1	2	2	4	5
Student 16	6	4	4	6	6	6	6	6
Student 17	1	1	1	1	1	1	1	5
Student 18	1	1	1	1	1	1	1	1
Student 19	6	7	7	7	7	7	7	7
Student 20	7	7	7	7	7	7	7	7
Student 21	1	1	1	1	5	2	1	1
Student 22	5	4	1	3	5	4	3	5
Student 23	1	1	1	1	4	5	5	6
Student 24	5	1	2	3	2	5	6	5
Student 25	2	1	1	1	1	2	1	1
Student 26	1	1	1	2	4	4	3	4
Student 27	2	2	2	2	2	6	2	1
Student 28	7	7	4	6	6	6	5	6
Student 29	3	4	3	4	4	4	4	5
Student 30	6	6	6	5	5	6	4	4
Student 31	6	3	6	7	5	6	5	5
Student 32	7	7	7	7	5	7	7	7
Student 33	2	4	2	1	5	5	3	3
Student 34	4	5	4	4	7	6	6	4
Student 35	7	2	1	7	7	7	7	7
Student 36	5	5	5	6	6	7	7	6
Student 37	4	1	1	1	7	5	5	4
Student 38	4	4	4	4	4	4	5	6
Student 39	1	1	1	1	1	1	1	1
Student 40	4	3	4	3	4	5	4	5
Student 41	3	3	7	3	6	5	4	5
Student 42	7	1	4	5	5	7	7	7

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 43	5	3	5	5	5	7	7	5
Student 45	4	1	1	1	2	4	3	5
Student 46	3	2	2	4	2	3	4	6
Student 47	3	1	1	2	4	4	5	5

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	ATMI_39V	ATMI_40C	ATTLS_1C	ATTLS_2S	ATTLS_3S	ATTLS_4C	ATTLS_5C	ATTLS_6S
Student 1	7	7	7	3	3	6	5	6
Student 2	5	1	6	5	4	6	6	7
Student 3	7	7	6	1	6	7	7	3
Student 4	3	3	4	1	5	5	5	6
Student 5	3	4	6	1	7	5	6	7
Student 6	7	7	5	1	1	1	7	1
Student 7	4	4	4	1	4	4	7	7
Student 8	7	4	7	5	7	4	7	4
Student 9	5	3	4	1	4	4	5	6
Student 10	7	7	7	5	4	6	6	7
Student 11	7	5	5	5	4	5	3	5
Student 12	5	5	5	2	3	5	5	6
Student 13	7	5	1	6	5	5	6	7
Student 14	6	3	6	2	4	5	5	6
Student 15	1	1	4	4	7	5	7	4
Student 16	7	7	5	4	4	5	6	6
Student 17	6	1	3	1	4	4	4	3
Student 18	7	1	5	6	6	1	3	5
Student 19	7	6	6	4	6	7	5	5
Student 20	7	7	4	5	1	1	6	6
Student 21	3	1	6	3	3	5	7	4
Student 22	1	5	4	5	3	2	1	3
Student 23	6	6	3	1	3	5	4	5
Student 24	4	5	3	1	2	6	5	4
Student 25	6	2	5	1	5	5	5	3
Student 26	7	4	4	1	4	4	6	7
Student 27	6	2	6	6	6	6	6	7
Student 28	4	7	5	5	4	5	5	5
Student 29	5	5	4	4	4	4	5	5
Student 30	6	5	4	6	4	5	6	6
Student 31	6	6	4	6	5	6	7	6
Student 32	7	7	4	3	5	6	5	1
Student 33	4	3	5	2	2	6	7	6
Student 34	6	4	4	4	2	5	5	6
Student 35	7	7	1	7	7	3	1	7
Student 36	5	5	4	4	4	5	5	5
Student 37	7	1	4	1	4	1	7	7
Student 38	7	5	5	4	4	5	5	5
Student 39	1	1	4	6	6	4	6	6
Student 40	6	5	5	2	3	3	6	6
Student 41	6	6	1	1	1	4	4	7
Student 42	7	6	4	4	4	4	6	6

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 43	5	5	3	3	3	5	5	5
Student 45	4	2	5	3	3	5	5	5
Student 46	4	6	6	6	7	7	6	7
Student 47	4	3	5	2	4	6	6	5

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	ATTL5_7S	ATTL5_8C	ATTL5_9S	ATTL5_10C	ATTL5_11S	ATTL5_12S	ATTL5_13S	ATTL5_14C
Student 1	5	6	4	6	5	4	7	7
Student 2	4	5	3	7	1	5	1	7
Student 3	1	7	4	6	1	1	1	7
Student 4	1	6	2	6	1	5	2	6
Student 5	7	7	5	5	1	4	2	7
Student 6	1	7	1	7	7	1	7	7
Student 7	3	6	5	7	1	4	4	2
Student 8	6	5	7	7	1	4	7	7
Student 9	4	6	2	6	1	1	1	5
Student 10	4	6	5	6	4	6	6	7
Student 11	3	6	4	7	2	3	3	6
Student 12	3	6	6	6	1	5	3	6
Student 13	7	5	5	4	1	5	5	1
Student 14	2	5	6	4	3	4	3	5
Student 15	4	5	2	6	4	5	4	6
Student 16	6	4	7	5	4	6	4	3
Student 17	2	3	6	5	1	1	1	1
Student 18	1	2	1	5	1	1	1	7
Student 19	4	4	6	5	3	4	4	6
Student 20	1	5	5	6	6	1	1	7
Student 21	5	6	6	5	5	3	5	6
Student 22	4	5	5	4	3	2	4	5
Student 23	4	2	5	4	1	1	1	5
Student 24	4	6	7	5	6	4	3	7
Student 25	1	5	7	5	1	4	4	4
Student 26	7	3	4	4	4	6	6	7
Student 27	3	7	6	3	3	2	6	2
Student 28	7	4	6	4	4	5	4	6
Student 29	5	4	4	4	3	4	4	4
Student 30	5	5	5	4	4	5	6	6
Student 31	7	6	5	6	5	6	7	7
Student 32	3	4	5	3	3	2	5	4
Student 33	2	5	2	5	1	5	2	4
Student 34	2	5	6	4	2	4	4	3
Student 35	7	1	7	5	1	7	7	7
Student 36	4	5	4	5	3	4	4	6
Student 37	5	6	6	6	6	6	6	6
Student 38	4	4	5	6	3	4	5	6
Student 39	6	4	4	5	3	5	4	5
Student 40	3	4	5	5	1	4	3	6
Student 41	3	6	4	6	4	4	6	6
Student 42	5	6	4	7	1	1	4	6

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 43	5	4	5	6	5	5	4	5
Student 45	5	5	5	5	3	5	4	4
Student 46	7	7	3	6	3	3	5	6
Student 47	3	5	6	5	2	7	6	7

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	ATTLS_15C	ATTLS_16S	ATTLS_17C	ATTLS_18S	ATTLS_19C	ATTLS_20C	MAS_1PR	MAS_2N
Student 1	7	4	6	7	5	6	2	3
Student 2	7	2	7	5	1	5	7	7
Student 3	7	4	7	5	7	7	2	1
Student 4	6	4	6	5	5	6	6	5
Student 5	7	4	7	3	7	7	3	5
Student 6	7	7	7	7	7	7	7	7
Student 7	6	5	7	4	4	7	5	5
Student 8	6	1	7	7	6	7	6	1
Student 9	6	4	7	5	5	7	5	5
Student 10	7	7	7	7	7	7	1	1
Student 11	7	2	7	4	5	7	3	2
Student 12	6	1	5	5	5	6	3	3
Student 13	4	4	7	5	5	6	2	2
Student 14	5	4	6	6	6	7	5	6
Student 15	5	5	6	5	4	6	5	7
Student 16	5	4	6	7	3	5	2	1
Student 17	5	1	6	6	2	5	7	7
Student 18	7	5	7	4	7	7	7	7
Student 19	7	4	7	7	5	7	2	1
Student 20	7	1	4	7	7	7	1	1
Student 21	6	5	7	6	6	7	7	7
Student 22	6	5	4	3	2	5	3	4
Student 23	1	1	4	4	4	4	2	5
Student 24	2	4	7	5	6	5	5	4
Student 25	5	2	3	4	1	6	7	7
Student 26	6	4	5	5	1	5	4	4
Student 27	5	1	6	2	6	6	7	7
Student 28	6	4	6	7	6	6	1	1
Student 29	3	4	4	3	3	4	3	4
Student 30	6	5	5	5	4	4	2	1
Student 31	7	6	7	5	6	7	4	5
Student 32	5	4	5	6	6	3	1	7
Student 33	5	3	6	6	5	5	6	5
Student 34	2	2	4	5	4	3	4	2
Student 35	7	6	7	7	1	7	1	4
Student 36	5	4	6	4	5	5	4	3
Student 37	6	6	7	6	7	7	6	5
Student 38	6	4	5	4	7	7	5	6
Student 39	6	2	6	6	4	7	6	7
Student 40	5	4	6	6	3	6	2	4
Student 41	4	5	7	7	5	7	3	5
Student 42	7	4	6	4	6	7	2	5

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 43	4	3	5	5	5	5	2	2
Student 45	5	4	5	5	4	5	6	6
Student 46	6	6	6	5	5	6	3	2
Student 47	5	5	7	7	7	7	3	3

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	MAS_3N	MAS_4PR	MAS_5PR	MAS_6N	MAS_7PR	MAS_8N	MAS_9PR	MAS_10PR	MAS_11N
Student 1	4	3	4	6	2	5	3	3	6
Student 2	7	5	7	7	1	6	7	7	7
Student 3	1	2	3	1	7	1	4	3	2
Student 4	5	6	7	6	2	4	6	6	6
Student 5	7	6	2	7	1	5	3	3	5
Student 6	7	7	7	7	1	3	7	7	7
Student 7	4	3	5	6	2	1	6	6	1
Student 8	5	3	7	3	5	4	7	6	3
Student 9	4	6	6	5	3	4	4	5	4
Student 10	4	1	1	4	4	4	2	1	1
Student 11	4	4	5	4	4	2	5	5	2
Student 12	1	3	5	3	5	3	3	3	3
Student 13	3	2	2	1	7	4	3	3	1
Student 14	5	4	5	7	1	6	6	5	5
Student 15	5	7	1	7	1	6	1	3	7
Student 16	1	2	1	1	7	2	2	1	1
Student 17	6	7	7	7	1	6	7	7	7
Student 18	7	4	7	7	1	7	7	7	7
Student 19	1	1	2	4	4	1	4	4	3
Student 20	1	1	1	1	7	1	1	1	1
Student 21	3	7	7	7	1	5	7	7	7
Student 22	2	3	3	3	5	3	4	3	4
Student 23	1	7	7	3	5	1	7	7	6
Student 24	6	2	1	7	1	4	3	2	3
Student 25	6	7	7	6	2	5	6	7	7
Student 26	4	4	7	2	6	2	4	7	5
Student 27	7	7	7	7	1	7	7	7	7
Student 28	1	1	1	1	7	1	1	1	1
Student 29	4	3	6	4	4	2	7	6	2
Student 30	4	1	2	6	2	5	3	1	2
Student 31	5	4	5	5	3	6	5	5	6
Student 32	7	1	1	1	7	1	2	1	1
Student 33	3	3	7	5	3	5	7	7	5
Student 34	2	2	3	4	4	2	4	6	4
Student 35	1	2	2	1	7	1	5	7	7
Student 36	4	3	3	6	2	2	4	5	6
Student 37	7	3	7	1	7	1	7	7	7
Student 38	4	2	6	1	7	5	4	4	6
Student 39	6	7	7	7	1	6	7	7	7
Student 40	4	4	5	4	4	4	4	7	4
Student 41	4	5	6	3	5	4	6	6	6
Student 42	1	1	2	4	4	1	4	2	1

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 43	3	3	4	2	6	2	4	5	5
Student 45	4	5	6	6	2	5	6	7	5
Student 46	2	4	3	3	5	2	5	5	5
Student 47	5	6	6	6	2	3	6	7	4

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	MAS_12N	MAS_13PR	MAS_14N	ProcedurePurpose	DiscoveryInstruction	RationalIntuitive
Student 1	6	2	7		0	-1
Student 2	6	2	7	-3	3	0
Student 3	1	7	1	3	0	-1
Student 4	5	3	5	0	1	-2
Student 5	1	7	5	3	3	-3
Student 6	5	3	7	0	1	1
Student 7	1	7	7	1	0	1
Student 8	1	7	3	-1	2	-3
Student 9	2	6	4	0	-1	0
Student 10	1	7	1	-2	3	-2
Student 11	3	5	2	0	2	-1
Student 12	1	7	3	1	0	-1
Student 13	4	4	4	1	0	2
Student 14	6	2	7	0	-1	1
Student 15	5	3	3	0	-1	-1
Student 16	1	7	6	0	1	-3
Student 17	3	5	6	-2	1	-3
Student 18	7	1	7	1	2	-2
Student 19	1	7	1	-1	-3	-3
Student 20	1	7	1	-3	0	-3
Student 21	4	4	7	-2	-1	-3
Student 22	1	7	2	-1	1	0
Student 23	1	7	5	-1	1	-3
Student 24	1	7	7	-1	2	-3
Student 25	7	1	7	-1	0	1
Student 26	7	1	1	3	0	-2
Student 27	7	1	7	0	-2	-1
Student 28	1	7	2	2	2	-2
Student 29	2	6	3	1	1	1
Student 30	4	4	2	-3	3	-2
Student 31	1	7	4	-2	-2	1
Student 32	1	7	1	0	-2	1
Student 33	5	3	5	0	0	-2
Student 34	2	6	4	-2	-1	0
Student 35	1	7	1		1	-3
Student 36	1	7	4	0	1	0
Student 37	4	4	7	-3	3	-3
Student 38	1	7	5	-2	0	-1
Student 39	2	6	5	1	-1	-2
Student 40	4	4	3	-3	2	-2
Student 41	3	5	5	2	2	-3
Student 42	1	7	6	-3	1	0

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 43	3	5	3	2	1	1
Student 45	3	5	4	0	1	-1
Student 46	2	6	2	-2	2	-2
Student 47	2	6	6	-1	-1	-3

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	Discrete	Related	Relational	Impersonal	Extensive	Concentrated	Listening	Speaking
Student 1		1		0		1		-2
Student 2		3		-3		3		3
Student 3		3		-1		0		2
Student 4		2		-2		-3		-3
Student 5		3		-3		3		-3
Student 6		1		1		1		1
Student 7		1		1		1		1
Student 8		2		0		1		-3
Student 9		1		-1		0		2
Student 10		3		-3		-3		0
Student 11		1		0		0		0
Student 12		2		-2		0		-3
Student 13		2		0		-3		-2
Student 14		2		1		-1		2
Student 15		1		-1		-1		1
Student 16		2		1		0		-1
Student 17		0		0		0		-2
Student 18		-1		-3		0		0
Student 19		3		-3		-1		-2
Student 20		0		-3		3		-3
Student 21		2		3		-3		3
Student 22		3		-2		0		3
Student 23		1		-1		-1		-1
Student 24		2		0		1		3
Student 25		1		1		-2		-2
Student 26		0		0		3		2
Student 27		-1		1		-2		-2
Student 28		2		-2		-1		0
Student 29		0		1		1		0
Student 30		3		1		2		-2
Student 31		1		-2		-1		3
Student 32		-1		0		-1		-3
Student 33		1		-2		-3		3
Student 34		1		0		0		-2
Student 35		3		3		3		-3
Student 36		0		-1		0		-2
Student 37		3		-3		-3		-3
Student 38		1		-2		0		0
Student 39		2		-1		2		-2
Student 40		2		1		-1		2
Student 41		3		-3		3		-2
Student 42		1		0		1		0

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 43	1	0	0	3
Student 45	-1	0	0	-1
Student 46	-2	2	3	-3
Student 47	1	2	-1	0

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	StudentTeacher
Student 1	2
Student 2	0
Student 3	-1
Student 4	3
Student 5	0
Student 6	1
Student 7	1
Student 8	1
Student 9	2
Student 10	0
Student 11	2
Student 12	1
Student 13	0
Student 14	0
Student 15	2
Student 16	0
Student 17	1
Student 18	3
Student 19	0
Student 20	3
Student 21	3
Student 22	0
Student 23	2
Student 24	1
Student 25	-1
Student 26	3
Student 27	0
Student 28	-3
Student 29	0
Student 30	1
Student 31	0
Student 32	3
Student 33	0
Student 34	-3
Student 35	0
Student 36	0
Student 37	3
Student 38	3
Student 39	2
Student 40	0
Student 41	0
Student 42	0

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 43	2
Student 45	1
Student 46	3
Student 47	2

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	Age	Gender	Ethnicity	ATMI_1V	ATMI_2V	ATMI_3E	ATMI_4V	ATMI_5V	ATMI_6V	ATMI_7V
Student 48	45	0	3	6	5	6	6	7	5	6
Student 49	25	0	3	7	5	4	6	6	3	7
Student 50	21	0	3	7	6	7	7	7	7	6
Student 51	20	0	2	7	6	5	5	7	7	7
Student 52	25	0	3	5	5	3	6	5	4	4
Student 53	52	1	3	7	6	7	6	4	6	7
Student 54	19	1	3	5	5	1	5	3	3	4
Student 55	19	0	3	6	6	4	5	6	6	3
Student 56	22	1	3	3	3	4	5	3	4	6
Student 57	19	1	3	7	7	4	6	6	7	7
Student 58	37	0	6	7	6	5	7	6	6	7
Student 59	19	1	3	5	5	2	7	5	1	7
Student 60	19	1	1	6	7	5	7	7	6	5
Student 61	19	1	5	7	7	6	7	6	6	7
Student 61	19	1	5	7	7	6	7	6	6	7
Student 62	20	0	3	6	6	4	5	5	5	5
Student 63	32	1	3	7	6	7	7	7	6	7
Student 65	19	1	2	7	7	6	7	7	7	7
Student 66	32	1	3	6	6	5	5	6	6	7
Student 67	20	1	2	6	6	6	6	6	5	6
Student 68	24	0	3	7	6	5	6	7	7	6
Student 69	18	0	2	7	7	5	5	4	4	4
Student 70	40	1	3	6	5	3	6	5	5	5
Student 71	22	0	1	6	6	4	7	7	6	6
Student 73			6	5	7	3	7	6	5	7
Student 74	20	1	3	6	5	2	7	6	6	6
Student 75	29	1	1	7	6	4	7	7	4	4
Student 76	20	0	3	2	1	1	4	3	2	3
Student 77	23	1	1	7	7	2	7	7	6	6
Student 78	40	1	3	7	7	5	7	7	7	7
Student 80	18	1	3	4	6	6	5	6	5	5
Student 81	20	0	3	2	1	1	7	5	1	1
Student 82	19	0	3	7	7	1	1	3	7	7
Student 83	28	1	3	7	7	7	6	4	5	4
Student 84	27	0	3	6	7	7	7	7	6	6
Student 85	52	1	3	6	6	5	5	7	6	6
Student 86	23	0	3	2	2	1	5	3	2	1
Student 87	55	0	3	7	7	6	7	7	6	7
Student 88	36	1	1	7	7	6	7	7	7	7
Student 89	27	1	3	7	7	7	7	7	7	7
Student 90	35	1	3	7	7	7	7	7	6	7
Student 91	30	0	2	6	7	5	5	5	5	4

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 92	23	0	4	7	7	7	7	7	7	1
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UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	ATMI_8V	ATMI_9CR	ATMI_10CR	ATMI_11CR	ATMI_12CR	ATMI_13CR	ATMI_14CR
Student 48	7	2	2	2	5	4	4
Student 49	7	5	7	6	4	7	5
Student 50	7	4	1	7	7	7	7
Student 51	6	1	2	3	5	4	3
Student 52	4	4	5	6	5	5	7
Student 53	7	4	7	7	7	1	1
Student 54	1	1	2	2	1	1	1
Student 55	5	5	5	6	6	6	4
Student 56	5	4	5	3	3	4	2
Student 57	6	1	2	1	2	2	1
Student 58	6	6	6	6	6	6	7
Student 59	5	3	7	4	4	6	3
Student 60	5	7	7	7	7	7	7
Student 61	7	7	7	6	7	7	7
Student 61	7	7	7	6	7	7	7
Student 62	5	3	3	3	4	4	3
Student 63	7	1	3	1	1	2	3
Student 65	5	3	5	6	7	7	7
Student 66	5	3	3	3	6	5	5
Student 67	6	3	6	4	6	6	4
Student 68	5	4	4	4	4	4	3
Student 69	4	4	1	4	5	4	4
Student 70	5	5	6	6	6	6	6
Student 71	7	4	4	4	4	4	4
Student 73	6	1	3	3	3	4	2
Student 74	5	1	1	1	1	2	2
Student 75	7	2	2	2	3	3	4
Student 76	4	1	3	2	4	1	1
Student 77	6	6	6	5	5	5	4
Student 78	7	3	2	1	3	3	2
Student 80	3	1	3	3	3	1	2
Student 81	1	1	7	7	7	7	1
Student 82	7	1	5	1	1	1	1
Student 83	4	2	3	2	2	2	1
Student 84	7	6	6	6	7	7	6
Student 85	7	7	7	7	7	7	7
Student 86	2	1	1	2	2	3	3
Student 87	7	3	5	7	7	7	7
Student 88	6	7	3	6	7	7	7
Student 89	7	2	3	3	3	5	3
Student 90	7	1	1	1	1	1	1
Student 91	5	4	1	1	2	2	2

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 92	7	2	2	2	3	3	3
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UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	ATMI_15CR	ATMI_16C	ATMI_17C	ATMI_18C	ATMI_19C	ATMI_20CR	ATMI_21CR	ATMI_22C
Student 48	4	4	3	5	2	4	4	5
Student 49	7	4	6	6	6	6	4	6
Student 50	7	7	6	6	6	7	4	7
Student 51	4	4	2	2	2	4	3	1
Student 52	6	3	3	4	3	5	5	4
Student 53	1	1	1	1	2	1	1	1
Student 54	2	1	1	3	4	2	2	1
Student 55	6	6	4	6	6	6	6	6
Student 56	3	7	7	4	5	5	4	3
Student 57	1	4	4	5	5	3	2	5
Student 58	7	4	5	5	6	6	6	5
Student 59	4	5	2	4	5	7	6	5
Student 60	7	7	7	6	6	7	7	7
Student 61	7	6	6	6	7	6	7	6
Student 61	7	6	6	6	7	6	7	6
Student 62	5	3	3	4	3	5	4	4
Student 63	3	1	1	2	1	3	2	2
Student 65	7	6	6	5	7	6	6	6
Student 66	5	5	3	3	4	3	3	3
Student 67	5	4	6	6	6	6	6	6
Student 68	3	3	3	4	5	5	4	4
Student 69	4	7	7	7	7	4	4	7
Student 70	6	6	5	5	5	6	6	5
Student 71	4	6	5	6	5	6	6	5
Student 73	4	2	1	3	1	3	4	2
Student 74	3	1	1	6	5	4	1	1
Student 75	4	4	4	6	5	5	5	5
Student 76	4	4	2	1	5	3	4	2
Student 77	4	2	2	2	6	6	5	2
Student 78	1	6	5	4	5	4	2	4
Student 80	2	2	1	2	2	2	3	1
Student 81	7	7	5	6	7	7	7	5
Student 82	5	5	7	7	5	1	3	6
Student 83	2	1	1	4	2	4	3	1
Student 84	7	5	6	6	6	6	7	6
Student 85	7	7	6	6	6	7	6	6
Student 86	3	1	1	1	2	6	6	1
Student 87	7	6	5	3	5	7	7	4
Student 88	7	7	7	6	7	7	7	6
Student 89	3	3	4	5	5	6	4	5
Student 90	1	1	1	1	1	1	1	1
Student 91	2	3	3	4	6	4	3	2

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 92	3	1	2	4	4	3	3	5
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UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	ATMI_23M	ATMI_24E	ATMI_25ER	ATMI_26E	ATMI_27E	ATMI_28MR	ATMI_29E	ATMI_30E
Student 48	5	6	6	4	7	2	4	4
Student 49	2	5	5	4	7	6	5	2
Student 50	7	7	7	7	7	7	7	6
Student 51	1	3	3	3	2	7	2	2
Student 52	3	3	5	3	6	5	3	3
Student 53	6	7	6	6	1	6	5	2
Student 54	1	1	2	1	1	7	1	1
Student 55	6	4	2	4	3	6	4	2
Student 56	2	5	4	2	1	2	7	6
Student 57	3	2	2	4	1	4	1	2
Student 58	7	6	5	5	4	7	5	4
Student 59	3	3	2	3	3	4	3	3
Student 60	7	6	6	4	7	6	7	4
Student 61	6	7	7	6	7	7	7	7
Student 61	6	7	7	6	7	7	7	7
Student 62	4	2	4	3	5	5	3	3
Student 63	1	2	6	3	1	1	2	1
Student 65	7	6	4	5	7	4	5	5
Student 66	4	5	6	5	4	5	4	3
Student 67	6	4	6	6	6	6	6	4
Student 68	7	5	4	4	4	4	4	4
Student 69	7	4	4	7	7	1	4	4
Student 70	5	5	5	3	1	3	5	2
Student 71	6	5	4	5	4	4	5	4
Student 73	2	1	4	5	1	5	2	1
Student 74	4	4	4	1	5	4	2	5
Student 75	6	2	4	4	2	7	4	4
Student 76	1	1	1	1	1	1	1	2
Student 77	6	2	5	4	2	3	2	2
Student 78	7	7	6	6	7	3	7	4
Student 80	1	1	2	2	1	2	1	2
Student 81	3	1	1	1	1	7	1	1
Student 82	1	1	7	1	1	7	1	1
Student 83	1	1	4	2	1	4	1	4
Student 84	7	5	6	6	5	6	6	4
Student 85	6	6	6	6	7	6	6	4
Student 86	1	1	1	2	1	1	1	1
Student 87	6	4	7	6	1	4	4	2
Student 88	7	7	7	6	7	6	7	7
Student 89	7	6	3	5	7	4	5	3
Student 90	4	6	4	1	1	4	4	1
Student 91	4	4	4	4	2	6	3	3

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 92	7	7	5	5	5	3	1	1
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UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	ATMI_31E	ATMI_32M	ATMI_33M	ATMI_34M	ATMI_35V	ATMI_36V	ATMI_37E	ATMI_38E
Student 48	4	1	1	3	1	4	6	6
Student 49	4	1	1	3	4	4	2	6
Student 50	6	4	4	7	6	7	1	5
Student 51	5	1	1	2	6	5	4	5
Student 52	3	3	3	4	4	7	4	5
Student 53	5	1	1	2	3	2	4	7
Student 54	1	1	1	1	1	1	1	6
Student 55	2	4	2	4	3	4	6	6
Student 56	4	1	1	2	1	3	5	1
Student 57	2	2	2	3	5	4	4	4
Student 58	5	5	5	5	7	7	6	6
Student 59	3	1	1	1	4	3	3	6
Student 60	5	4	5	6	6	5	6	7
Student 61	7	6	6	7	6	7	6	7
Student 61	7	6	6	7	6	7	6	7
Student 62	4	5	5	3	6	3	3	3
Student 63	7	1	1	5	4	5	5	5
Student 65	3	1	4	4	5	6	6	4
Student 66	5	3	5	3	5	5	4	3
Student 67	6	4	4	6	5	6	6	6
Student 68	4	3	2	3	3	5	4	5
Student 69								
Student 70	5	1	1	2	3	3	3	5
Student 71	4	2	5	5	4	5	6	6
Student 73	5	1	1	4	4	5	4	7
Student 74	6	5	2	2	4	4	4	4
Student 75	6	2	7	7	7	7	7	6
Student 76	1	1	1	1	1	2	1	4
Student 77	6	2	2	5	4	5	3	3
Student 78	7	1	1	3	7	7	7	4
Student 80	2	1	2	1	3	4	5	4
Student 81	1	1	1	1	1	1	7	7
Student 82	3	2	5	4	7	6	3	5
Student 83	4	4	1	4	3	4	5	7
Student 84	5	6	7	6	6	6	5	6
Student 85	6	4	4	5	5	6	6	6
Student 86	1	1	1	1	3	3	1	1
Student 87	6	4	4	6	7	7	7	4
Student 88	7	6	5	6	6	6	6	7
Student 89	5	1	4	4	6	7	7	6
Student 90	4	1	1	1	1	6	1	1
Student 91	4	3	3	2	3	5	3	3

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 92	4	1	1	4	4	5	5	4
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UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	ATMI_39V	ATMI_40C	ATTLS_1C	ATTLS_2S	ATTLS_3S	ATTLS_4C	ATTLS_5C	ATTLS_6S
Student 48	4	6	5	7	4	4	5	7
Student 49	7	7	4	1	6	5	7	7
Student 50	7	6	4	1	4	7	1	7
Student 51	7	1	1	3	5	4	3	6
Student 52	5	5	5	3	6	3	4	6
Student 53	4	4	6	1	2	3	7	7
Student 54	1	1	4	1	4	6	6	7
Student 55	5	6	5	6	2	4	5	7
Student 56	2	4						
Student 57	6	5	5	5	4	5	3	4
Student 58	6	5						
Student 59	3	6	6	5	4	7	5	3
Student 60	6	7	5	3	4	6	7	7
Student 61	7	7	5	5	6	6	7	6
Student 61	7	7	5	5	6	6	7	6
Student 62	7	4	3	3	3	5	3	3
Student 63	5	2	5	5	3	7	6	6
Student 65	7	5	5	7	4	4	7	7
Student 66	7	3	4	3	5	5	5	5
Student 67	6	6	6	6	5	4	6	7
Student 68	4	4						
Student 69			4	4	4	4	4	4
Student 70	5	5	5	5	5	6	6	3
Student 71	7	6	6	4	4	5	7	7
Student 73	5	2	5	3	3	4	6	7
Student 74	7	5	3	4	5	3	2	3
Student 75	5	5	4	2	4	4	6	5
Student 76	3	1	4	1	4	4	6	4
Student 77	6	5						
Student 78	7	4	7	5	7	7	7	4
Student 80	3	2	3	1	5	6	6	7
Student 81	5	6	7	5	1	6	7	6
Student 82	7	3						
Student 83	3	4						
Student 84	6	6						
Student 85	7	1	6	4	3	6	6	5
Student 86	2	2	4	2	4	4	4	6
Student 87	4	5	7	7	7	4	2	7
Student 88	7	6	5	5	5	6	7	6
Student 89	7	5	6	4	5	5	7	5
Student 90	7	1	4	6	4	7	1	7
Student 91	4	4	5	2	3	4	6	6

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 92 7 5 4 1 7 5 2 7

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	ATTLS_7S	ATTLS_8C	ATTLS_9S	ATTLS_10C	ATTLS_11S	ATTLS_12S	ATTLS_13S	ATTLS_14C
Student 48	4	5	4	6	1	4	3	4
Student 49	6	6	4	5	2	4	2	5
Student 50	7	7	5		1	1	1	1
Student 51	4	6	3	5	1	2	2	3
Student 52	4	4	3	4	4	5	4	6
Student 53	1	6	1	1	1	4	2	7
Student 54	1	7	6	6	1	4	2	4
Student 55	2	3	6	4	2	2	2	4
Student 56								
Student 57	5	3	4	4	3	5	4	7
Student 58								
Student 59	4	6	3	5	2	1	5	6
Student 60	4	7	6	4	2	5	4	1
Student 61	4	6	5	6	4	6	5	7
Student 61	4	6	5	6	4	6	5	7
Student 62	3	4	3	5	3	3	3	4
Student 63	7	6	3	5	2	4	3	5
Student 65	5	5	5	5	3	5	5	7
Student 66	3	5	4	5	4	5	3	5
Student 67	4	6	5	4	5	5	4	6
Student 68								
Student 69	4	4	4	4	4	4	4	4
Student 70	3	3	3	5	2	3	3	6
Student 71	6	6	7	6	4	6	6	6
Student 73	3	6	5	6	2	4	5	4
Student 74	3	3	3	4	5	3	3	3
Student 75	1	4	6	6	1	6	4	5
Student 76	6	6	6	6	4	4	4	4
Student 77								
Student 78	4	7	7	7	4	7	7	7
Student 80	5	6	5	4	2	4	5	6
Student 81	7	6	7	7	3	5	3	6
Student 82								
Student 83								
Student 84								
Student 85	4	6	5	4	4	4	4	6
Student 86	4	5	4	4	4	5	5	1
Student 87	7	4	7	4	4	7	5	1
Student 88	5	6	6	5	3	5	4	6
Student 89	5	3	3	4	1	3	5	4
Student 90	5	2	7	4	4	7	4	6
Student 91	5	4	6	5	4	6	6	5

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 92 6 4 5 1 5 6 1 7

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	ATTLS_15C	ATTLS_16S	ATTLS_17C	ATTLS_18S	ATTLS_19C	ATTLS_20C	MAS_1PR	MAS_2N
Student 48	6	1	4	5	3	5	4	4
Student 49	6	2	3	6	6	7	2	2
Student 50	1	1	1	1	4	4	2	4
Student 51	5	5	5	6	6	3	7	7
Student 52	5	3	5	5	4	4	3	2
Student 53	7	4	7	6	4	4	7	3
Student 54	5	4	5	5	4	7	6	7
Student 55	5	2	5	5	4	5	2	3
Student 56								
Student 57	7	6	5	4	4	4	4	4
Student 58								
Student 59	7	3	7	5	4	7	6	5
Student 60	5	5	6	5	4	7	1	1
Student 61	7	6	7	6	7	6	1	1
Student 61	7	6	7	6	7	6	1	1
Student 62	5	5	3	3	3	5	5	3
Student 63	6	3	7	5	6	7	7	7
Student 65	7	3	7	7	5	6	2	5
Student 66	5	3	5	5	5	5	3	4
Student 67	6	6	6	4	6	6	2	4
Student 68								
Student 69	4	4	4	4	4	4	1	4
Student 70	6	3	6	6	5	6	3	2
Student 71	5	4	6	6	4	6	3	6
Student 73	6	3	7	5	4	7		
Student 74	1	3	3	2	2	3	5	2
Student 75	6	3	6	4	5	6	5	5
Student 76	6	3	6	3	6	6	5	7
Student 77								
Student 78	7	7	7	7	7	7	1	4
Student 80	6	5	6	7	6	6	6	6
Student 81	7	7	7	7	7	7	3	7
Student 82								
Student 83								
Student 84								
Student 85	6	4	7	5	5	6	2	2
Student 86	1	2	4	4	1	4	7	1
Student 87	4	1	5	2	4	4	4	2
Student 88	6	3	7	6	7	6	1	1
Student 89	6	6	5	6	5	6	4	5
Student 90	6	7	5	4	2	4	7	7
Student 91	6	4	4	4	6	5	3	3

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 92	7	7	7	6	7	7	5	5
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UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	MAS_3N	MAS_4PR	MAS_5PR	MAS_6N	MAS_7PR	MAS_8N	MAS_9PR	MAS_10PR	MAS_11N
Student 48	4	1	3	7	1	3	4	5	5
Student 49	3	4	4	5	3	3	5	7	5
Student 50	4	1	2	7	1	1	2	1	1
Student 51	4	3	7	4	4	5	6	7	7
Student 52	1	3	5	5	3	3	5	5	3
Student 53	5	2	4	7	1	3	5	7	4
Student 54	5	7	7	6	2	6	7	7	7
Student 55	3	5	5	3	5	1	6	6	3
Student 56									
Student 57	4	4	5	7	1	3	6	6	6
Student 58									
Student 59	2	4	7	1	7	3	7	7	4
Student 60	1	3	1	2	6	1	4	3	1
Student 61	2	1	1	2	6	2	2	1	1
Student 61	2	1	1	2	6	2	2	1	1
Student 62	4	3	5	7	1	2	7	4	4
Student 63	6	3	7	7	1	5	7	7	7
Student 65	4	3	5	5	3	3	3	3	3
Student 66	5	3	4	5	3	5	5	5	4
Student 67	4	1	2	4	4	4	4	4	5
Student 68									
Student 69	4	1	1	7	1	7	4	4	3
Student 70	2	3	5	3	5	3	5	5	3
Student 71	4	3	6	3	5	4	4	5	4
Student 73									
Student 74	5	4	7	7	1	6	6	5	5
Student 75	5	3	6	5	3	4	5	6	4
Student 76	4	7	7	4	4	4	6	7	7
Student 77									
Student 78	6	1	2	7	1	7	3	2	5
Student 80	2	7	7	7	1	5	7	7	6
Student 81	4	7	7	5	3	1	7	7	1
Student 82									
Student 83									
Student 84									
Student 85	1	2	2	2	6	1	4	4	2
Student 86	6	7	7	5	3	5	7	7	7
Student 87	1	1	4	4	4	1	7	7	5
Student 88	1	1	1	2	6	1	2	1	1
Student 89	4	2	5	5	3	5	5	6	5
Student 90	7	3	7	7	1	6	7	7	7
Student 91	4	4	5	6	2	6	7	7	6

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 92 6 3 6 7 1 4 7 7 5

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	MAS_12N	MAS_13PR	MAS_14N	ProcedurePurpose	DiscoveryInstruction	RationalIntuitive
Student 48	1	7	3	-1	1	-2
Student 49	1	7	2	0	3	-3
Student 50	1	7	1	1	-3	-1
Student 51	1	7	6	-2	1	-3
Student 52	1	7	1	0	1	-3
Student 53	1	7	1	1	3	-2
Student 54	4	4	5	-1	3	-1
Student 55	1	7	2	-2	-2	-2
Student 56				1	3	2
Student 57	5	3	4	1	-2	3
Student 58				1	-1	-3
Student 59	2	6	1	0	2	-3
Student 60	1	7	1	0	3	-3
Student 61	1	7	6	1	1	-1
Student 61	1	7	6	1	1	-1
Student 62	5	3		0	1	0
Student 63	6	2	4	-3	0	-3
Student 65	4	4	3	-2	0	-3
Student 66	1	7	3	0	0	-1
Student 67	4	4	4	1	1	-1
Student 68				0	0	-1
Student 69	4	4	4	1	1	-1
Student 70	2	6	2	0	0	0
Student 71	1	7	4	2	2	1
Student 73				0	2	-2
Student 74	2	6	1	2	2	2
Student 75	1	7	2	1	0	1
Student 76	2	6	4			
Student 77				0	2	-3
Student 78	6	2	3	2	2	2
Student 80	2	6	5	-1	1	-2
Student 81	1	7	1	2	3	-3
Student 82				0	-3	-3
Student 83				-3	3	-3
Student 84				0	0	-3
Student 85	2	6	2	2	-1	-1
Student 86	2	6	6	-3	3	-3
Student 87	1	7	1	0	-1	-3
Student 88	1	7	2	-3	-3	0
Student 89	2	6	3	-2	1	2
Student 90	4	4	7	0	3	-3
Student 91	5	3	3	-1	1	-2

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 92

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UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Participant	DiscreteRelated	RelationallImpersonal	ExtensiveConcentrated	ListeningSpeaking
Student 48	2	-2	0	-3
Student 49	1	-2	-3	-2
Student 50	3	-1	2	0
Student 51	2	0	-2	-2
Student 52	1	0	0	0
Student 53	-3	3	3	0
Student 54	-1	2	-3	-3
Student 55	1	2	-2	-2
Student 56	0	0	-2	-3
Student 57	3	0	-3	3
Student 58	2	-2	-2	0
Student 59	1	0	3	-3
Student 60	3	-3	-3	2
Student 61	2	-1	-1	2
Student 61	2	-1	-1	2
Student 62	1	-1	1	-1
Student 63	3	-3	0	-3
Student 65	1	-2	2	0
Student 66	1	0	0	0
Student 67	1	-1	-1	-2
Student 68	0	0	0	0
Student 69	1	-2	3	3
Student 70	2	0	0	-2
Student 71	2	0	1	0
Student 73	0	1	1	0
Student 74	1	1	0	1
Student 75	1	1	1	2
Student 76				
Student 77	2	-1	1	-3
Student 78	2	2	2	2
Student 80	0	0	-1	-2
Student 81	2	-3	0	-3
Student 82	3	3	0	-3
Student 83	3	-2	-1	-3
Student 84	2	-1	0	0
Student 85	2	-1	1	2
Student 86	3	-3	-3	0
Student 87	1	1	0	0
Student 88	0	-1	-1	-3
Student 89	2	-1	2	-3
Student 90	3	2	-2	-3
Student 91	0	1	-3	-2

UNDERSTANDING STUDENT MATHEMATICAL WAYS OF KNOWING

Student 92

3

-2

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Participant	StudentTeacher
Student 48	3
Student 49	3
Student 50	2
Student 51	2
Student 52	-2
Student 53	1
Student 54	3
Student 55	-2
Student 56	-2
Student 57	1
Student 58	2
Student 59	3
Student 60	3
Student 61	2
Student 61	2
Student 62	0
Student 63	0
Student 65	0
Student 66	1
Student 67	-2
Student 68	0
Student 69	3
Student 70	0
Student 71	1
Student 73	2
Student 74	3
Student 75	2
Student 76	
Student 77	2
Student 78	3
Student 80	1
Student 81	0
Student 82	-3
Student 83	3
Student 84	2
Student 85	2
Student 86	2
Student 87	1
Student 88	-2
Student 89	-1
Student 90	3
Student 91	2

Student 92

3