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UNDULATORS AS SOURCES OF SYNCHROTRON RADIATION*

Samuel Krinsky
National Synchrotron Light Source
Brookhaven National Laboratory, Upton, N.Y. 11973

Introduction

At the present time the first generation of facilities having electron storage rings designed for and dedicated to synchrotron radiation research are beginning operations in the U.S., Europe and Japan. The use of wigglers and undulators as enhanced sources of synchrotron radiation[1] plays an important role at all these facilities. Moreover, recently there has been much activity in the design of the next generation machines,[2] which will place even greater, and perhaps exclusive, emphasis on the use of wigglers and undulators. The operation of these insertion devices has been made even more attractive by advances in the design and construction of permanent magnet[3] wigglers and undulators. This reliable and economical technology eliminates the need for more complex superconducting magnets, except to achieve very high magnetic fields for the production of hard photons from relatively low energy rings.

It is reasonable to expect that in the coming years the importance of wigglers and undulators in synchrotron radiation research will increase. It is with this in mind that we discuss some of the properties of the radiation from these devices, and some of the considerations which arise in their design and use in storage rings.[4] Space limitations prevent us from describing in detail the properties of both wigglers and undulators.[5] We have, therefore, chosen to emphasize the discussion of undulators, and we have not considered the polarization[6] properties of the radiation. Our emphasis on undulators seems justified since the properties of wiggler radiation[7] are describable in terms of the characteristics of synchrotron radiation from arc sources, which are well-known and well-documented.[8] Undulator radiation, on the other hand, although understood long ago by Motz,[9] is probably less familiar to most of us.

Our paper is organized as follows: First we review the spectral properties of the radiation,[10,11] emphasizing the complementary aspects of time- and frequency-domain analyses. We next study the brightness of the undulator source. Finally, we consider some limitations associated with operating an undulator in a storage ring.

Radiated Spectrum

We consider a wiggler magnet located in the straight section of a storage ring, its axis being parallel to the unperturbed electron motion (z-direction). The magnet produces a vertical magnetic field, B_y, which has a period length λ_o in the z-direction, and to a good approximation is sinusoidal,

$$B_y = B_o \sin(2\pi z/\lambda_o) . \qquad (1)$$

The magnetic field causes an electron to be deflected in the horizontal plane and we denote this deflection x. Measured in units of its rest mass, the electron has energy γ, and we let ρ_o denote the radius of curvature corresponding to the peak field B_o. The angular deflection, $x' = dx/dz$, of the electron is

$$x' = \delta \cos(2\pi z/\lambda_o) , \qquad (2)$$

where the maximum angular deflection δ is given by

$$\delta = \lambda_o/2\pi\rho_o , \qquad (3)$$

and the amplitude of the transverse oscillation is $\rho_o\delta^2$. The magnitude of the electron velocity divided by the speed of light is denoted $\beta = \sqrt{1-\gamma^{-2}}$. Due to the transverse deflection, the average velocity β^* in the z-direction is reduced from the value β, and is approximately

$$\beta^* = \beta(1-\delta^2/4) . \qquad (4)$$

Consider a wavefront radiated in the z-direction by an electron passing through the periodic magnetic field. At time $\lambda_o/\beta^* c$ later, the electron has passed through one period of the magnet, and a second wavefront emitted at this time will follow the first by a time interval

$$T_1 = \lambda_o/\beta^* c - \lambda_o/c . \qquad (5)$$

An observer downstream of the magnet looking in the forward direction sees a radiation spectrum comprised of the fundamental frequency $\omega_1 = 2\pi/T_1$ and its odd harmonics $\omega_k = k\omega_1 (k=1,3,5,...)$. Defining

$$K \equiv \gamma\delta = 0.93\, B_o(T)\lambda_o(cm) , \qquad (6)$$

the fundamental wavelength $\lambda_1 = cT_1$ can be expressed as

$$\lambda_1 = (\lambda_o/2\gamma^2)(1+K^2/2) . \qquad (7)$$

If an observer is looking at radiation emitted at polar angle θ relative to the z-axis, then the time delay between wavefronts emitted before and after an electron has traversed one period of the magnetic field is

$$T_1(\theta) = \lambda_o/\beta^* c - \lambda_o \cos\theta/c , \qquad (8)$$

as illustrated in Fig. 1. One should not interpret Fig. 1. as describing spatial interference, but rather temporal coherence, i.e. a regularity in the time dependence of the electric field. Off the forward direction the radiation spectrum is comprised of both odd and even harmonics of the fundamental frequency. The fundamental wavelength is

$$\lambda_1(\theta) = (\lambda_o/2\gamma^2)(1+K^2/2+\gamma^2\theta^2) . \qquad (9)$$

For a magnet with N periods, the radiated pulse from one electron passing through the device has a time duration of NT_1, hence the pulse contains Nk radiation periods at the k th harmonic frequency ω_k. Consequently, the line width at fixed observation angle θ is

$$\Delta\omega_k/\omega_k = 1/kN . \qquad (10)$$

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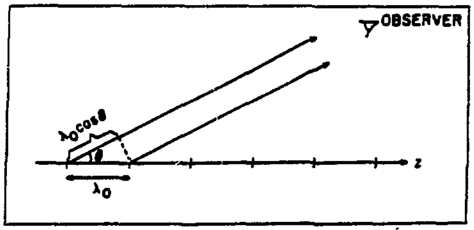

Fig. 1. Path difference vs angle θ.

Due to the dependence of the frequency on observation angle, if the observer accepts radiation in an interval $\Delta\theta$, then the width of the line is broadened relative to its natural value [Eq. (10)]. From Eq. (9), we see that the angular broadening due to accepting radiation in a cone of half-angle $\Delta\theta$ about the forward direction is $\Delta\omega_k/\omega_k = \gamma^2(\Delta\theta)^2/(1+K^2/2)$. This broadening will be small only if

$$\Delta\theta < \sigma'_R \,, \tag{11}$$

where we define

$$\gamma\sigma'_R \equiv \sqrt{(1+K^2/2)/2kN} \,. \tag{12}$$

In the forward direction, radiation from a wiggler is in general characterized by two time scales. The first is the time interval T_1 already discussed in Eq. (5), and the second is the time T_c characteristic of synchrotron radiation,

$$T_c = 4\pi\rho_o/3c\gamma^3 \,. \tag{13}$$

When the magnetic strength parameter $K \gg 1$, then $T_1 \gg T_c$, and the time dependence of the radiated electric field has the form illustrated in Fig. 2a. For high harmonic number k, the frequency spectrum is characterized by the synchrotron radiation cutoff frequency $\omega_c = 2\pi/T_c$.

For a zero-emittance electron beam passing through a wiggler having N periods, the photon flux per unit solid angle emitted in the forward direction at the k th harmonic frequency is[11]

$$[dn(\omega_k)/d\Omega]_{\theta=0} = \alpha N^2\gamma^2 \frac{\Delta\omega}{\omega}\frac{I}{e} F_k(K) \,, \tag{14}$$

expressed in photons/sec, steradian, $\Delta\omega/\omega$, I(Amp). We have denoted the fine structure constant by $\alpha = 1/137$ and the electron charge by $e = 1.6\times10^{-19}$ Coul. The function $F_k(K)$ can be expressed in terms of Bessel functions,[11] and for k odd:

$$F_k(K) = \frac{K^2k^2}{(1+K^2/2)^2}\left[J_{\frac{k+1}{2}}\left(\frac{kK^2}{4+2K^2}\right) - J_{\frac{k-1}{2}}\left(\frac{kK^2}{4+2K^2}\right)\right]^2 . \tag{15}$$

In the limit $K \to \infty$, with k/K^3 held fixed,

$$F_k(K) = (3/\pi^2)(k/k_c)^2\, K^2_{2/3}\,(k/2k_c) \,, \tag{16a}$$

$$k_c = 3K^3/8 \,, \tag{16b}$$

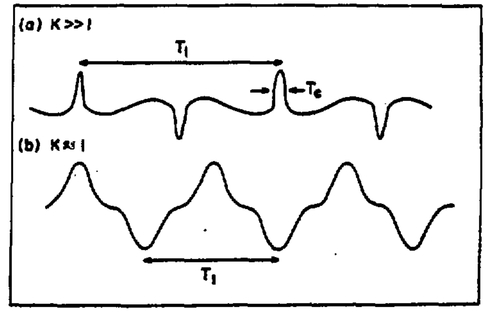

Fig. 2. Electric field vs time for (a) wiggler, K≫1 and (b) undulator K = 1.

and $\omega_c = k_c\omega_1$. Substituting Eq. (16a) into Eq. (14), one finds

$$[dn(\omega_k)/d\Omega]^{\text{wiggler}}_{\theta=0} = (2N)^2[dn(\omega_k)/d\Omega]^{\text{synch.rad.}}_{\theta=0} \tag{17}$$

Hence, radiation in the regieme of $K \gg 1$ and $k \gg 1$ has the spectral properties of normal synchrotron radiation and is called wiggler radiation.

When the field strength parameter $K < 1$, then $T_1 < T_c$, and the time scale T_c characteristic of synchrotron radiation drops out of the problem. In this case one usually speaks of undulator radiation. From Eq. (6) it is seen that undulator radiation is produced by devices which have short periods and low fields. For an undulator with $K = 1$, the time dependence of the electric field in the forward direction has the form illustrated in Fig. 2b. For $K \ll 1$, the electric field has almost a purely sinusoidal time dependence and hence the radiated spectrum is dominated by the line at the fundamental frequency ω_1. When K increases toward unity, the third-harmonic becomes important, corresponding in the time-domain to a sharpening of the peak of the electric field. As K increases toward even higher values, the higher harmonics increase, corresponding to well-defined pulses of width T_c separated by time T_1, as in Fig. 2a. Even for large values of K, however, temporal coherence effects are still significant for the lowest harmonics.

Let us now compare the angle integrated spectrum of undulator radiation with that of synchrotron radiation. For an undulator, the peak value of the integrated spectrum at the fundamental frequency is [photons/sec, $\Delta\omega/\omega$, I(Amp)]:

$$n_u(\omega_1) = \frac{\pi\alpha NK^2}{1+K^2/2}\frac{I}{e}\frac{\Delta\omega}{\omega} \,. \tag{18}$$

Note that although the value of the fundamental frequency ω_1 depends on the electron energy γ, the peak height $n_u(\omega_1)$ is independent of γ. For a bending magnet, the integrated spectrum at the critical frequency ω_c is [photons/sec, $\Delta\omega/\omega$, θ(mrad), I(Amp)]:

$$n_B(\omega_c) = 0.65\,\frac{\sqrt{3}}{2\pi}\,\alpha\gamma\,\frac{\Delta\omega}{\omega}\frac{I}{e}\,\theta\times10^{-3} \,. \tag{19}$$

Here, $n_B(\omega_c)$ is seen to be proportional to the energy γ, in contrast to the case of the undulator.

The ratio of the angle integrated flux at the first-harmonic of an undulator (N,K) to the flux from

1 mrad of an arc source at the critical frequency ω_c is:[12]

$$n_u(\omega_1)/n_B(\omega_c) \approx 17500\ NK^2/[\gamma(1+K^2/2)] \ , \qquad (20)$$

for the same bandwidth $\Delta\omega/\omega$. As an example take $N = 100$, $K = 1$, $\gamma = 5000$, then the flux radiated by the undulator is about 240 times that radiated by 1 mrad of the arc source.

The total radiated power from a wiggler or undulator is given by

$$P(kW) = 1.9\times10^{-9}\ N\gamma^2 K^2 I(Amp)/\lambda_o(cm) \ . \qquad (21)$$

An important advantage of an undulator over a wiggler is the reduced radiated power. As an example consider producing radiation of wavelength $\lambda = 1$Å from an undulator with $N = 100$, $\lambda_o = 2$ cm, $K = 1/2$ operating at $\gamma = 10{,}000$. The total radiated power is 2.4kW/Amp and the angle integrated photon flux [see Fig. 3.] at the spectral peak is 3×10^{16}ph/sec, 1%, Amp. We compare this with producing 1Å radiation from a wiggler with $N = 10$, $\lambda_o = 14$, $K = 20$ operating at $\gamma = 6850$. The total radiated power is 25 kW/Amp and the integrated photon flux is 10^{16}photons/sec, 1%, mrad θ, Amp.

In Figs. 3-5 we plot the angle integrated spectrum of radiation in a cone of specified half-angle $\Delta\theta$, for three different undulators operating at three different electron energies. The undulators of Figs. 3-5 deliver hard x-rays, soft x-rays, and VUV-radiation, respectively.

Brightness of the Undulator Source

In the preceeding section we have considered the angle integrated flux (photons/sec, unit bandwidth) as in Eqs. (18) and (19), and the flux per unit solid angle (photons/sec, steradian, unit bandwidth) as in Eqs. (14) and (17). For experiments which require a photon beam with small angular deviation incident upon a small sample, the true figure of merit of a source is its brightness, i.e. the flux per unit phase space volume (photons/sec, steradian, mm^2, unit bandwidth). The brightness of an undulator can be 10,000 times that of an arc source.

To understand the phase space of the undulator source, it is necessary to consider the phase space distribution in the electron beam. The transverse dimensions and the angular spread of the electron beam are determined by the emittances ε_x, ε_y of the storage ring and the local values of the betatron functions $\beta_x(s)$ and $\beta_y(s)$, where x denotes the radial direction and y the vertical. We suppose the undulator to be situated in a ring insertion, within which the betatron functions have the form (when ε, $\beta(s)$ or σ appear without a subscript, the equation is to be understood as holding separately for x and y):

$$\beta(s) = \beta^* + s^2/\beta^* \ , \qquad (22)$$

where s is the distance from the insertion center $s = 0$. The electron beam size is given by $\sigma(s) = \sqrt{\varepsilon\beta(s)}$ and the angular spread $\sigma'(s) = \sqrt{\varepsilon/\beta^*} \equiv \sigma'$. At the insertion center,

$$\sigma\sigma' = \varepsilon \ , \qquad (23)$$

and off center $\sigma(s) = \sqrt{\sigma^2 + (\sigma' s)^2}$.

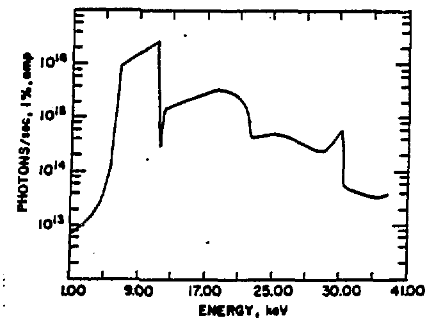

Fig. 3. Undulator spectrum for $\lambda_o = 2$ cm, $N = 100$, $K = 0.5$, $\gamma = 10000$ and $\gamma\Delta\theta = 1$.

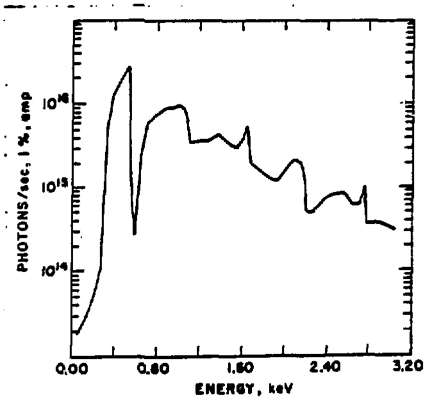

Fig. 4. Undulator spectrum for $\lambda_o = 7.5$ cm, $N = 40$, $K = 1$, $\gamma = 5000$ and $\gamma\Delta\theta = 1$.

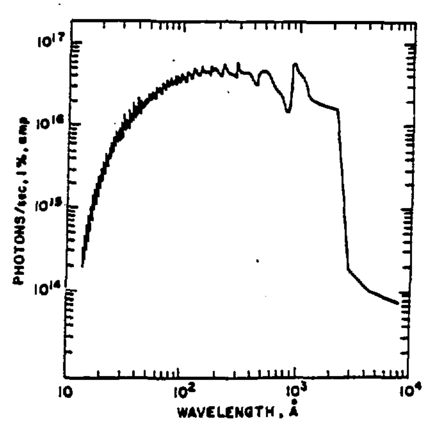

Fig. 5. Undulator spectrum for $\lambda_o = 6.5$ cm, $N = 38$, $K = 3$, $\gamma = 1470$, and $\gamma\Delta\theta = 3$.

It is interesting to note that Eq. (14) can be rewritten in the form

$$[dn(\omega_k)/d\Omega]_{\theta=0} = n_k/2\pi\sigma_R'^2 , \quad (24)$$

where σ_R' was defined earlier in Eq. (12) and

$$n_k = \pi\alpha N(1+K^2/2)\frac{1}{k}F_k(K)\frac{\Delta\omega}{\omega}\frac{I}{e} \quad (25)$$

is the contribution from the k th harmonic to the angle integrated photon flux at frequency ω_k, expressed in [photons/sec, $\Delta\omega/\omega$, Amp]. For small K, n_k is approximately equal to the total angle integrated flux at frequency ω_k, but for larger K, harmonics higher than the k th also contribute. In the case of the first harmonic, k = 1, and small K, $F_1(K) = K^2/(1+K^2/2)^2$, so Eq. (25) is seen to reduce to Eq. (18) given earlier.

Inspection of Eq. (24) suggests that σ_R' is the rms angular spread of the undulator radiation with frequency ω_k. Following the approach of Green[13] in his treatment of synchrotron radiation from a bending magnet, we would expect that in the presence of angular spread σ_x', σ_y' in the electron beam, Eq. (24) should be replaced by

$$[dn(\omega_k)/d\Omega]_{\theta=0} = n_k/(2\pi\Sigma_x'\Sigma_y') , \quad (26)$$

where

$$\Sigma_{x,y}' = \sqrt{\sigma_R'^2 + \sigma_{x,y}'^2} . \quad (27)$$

The subscript x,y means that the equation holds independently for x and y. The effective size $\Sigma_{x,y}$ of the undulator source depends upon its length $L = N\lambda_o$, and again following Green,[13] we assume

$$\Sigma_{x,y} = \sqrt{\sigma_{x,y}^2 + (\sigma_R'^2 + \sigma_{x,y}'^2)\,L^2/4} . \quad (28)$$

The brightness B_k of the undulator at frequency ω_k is

$$B_k = \frac{1}{2\pi\Sigma_x\Sigma_y}[dn(\omega_k)/d\Omega]_{\theta=0} , \quad (29)$$

which can be rewritten using Eq. (26) as

$$B_k = n_k/(4\pi^2\Sigma_x\Sigma_x'\Sigma_y\Sigma_y') . \quad (30)$$

It is interesting to note that

$$\Sigma_{x,y}\,\Sigma_{x,y}' \geq (\sigma_{x,y}^2 + L^2\sigma_R'^2/4)^{1/2}\Sigma_{x,y}' \geq \epsilon_{x,y} + \lambda_k/2 , \quad (31)$$

where the emittances $\epsilon_{x,y}$ were defined in Eq. (23), and we have used the expression

$$\lambda_k = \sigma_R'^2\,L = (\lambda_o/2k\gamma^2)(1+K^2/2) , \quad (32)$$

for the wavelength λ_k of the k th harmonic. The inequality (31) provides a diffraction limit on the phase space of the photon source, and implies an upper bound on the brightness,

$$B_k \leq n_k/\pi^2\lambda_k^2 . \quad (33)$$

In Eq. (33), the right hand side will be expressed in photons/sec, steradian, mm^2, $\Delta\omega/\omega$ if Eq. (25) is used for n_k and λ_k is expressed in mm.

For fixed values of the undulator parameters λ_o, K, $L = N\lambda_o$, and the ring parameters γ and ϵ, the source brightness at wavelength λ can be optimized by the choice of β^*. Defining the scaled parameters

$$b = \beta^*/L \text{ and } E = \epsilon/\lambda , \quad (34)$$

the phase space area of the source can be written

$$\Sigma\Sigma' = \lambda\sqrt{(1+E/b)(Eb+E/4b+1/4)} , \quad (35)$$

where we have used Eqs. (27) and (28) together with $b = \sigma^2/\epsilon L$. The condition for the function in the square root to be a minimum is found by differentiating with respect to b for fixed E, and we obtain $2b^3-b-E = 0$.

For E = 1, the optimum value of the betafunction is given by b = 1, hence $(\Sigma\Sigma')_{min} = \sqrt{3}\epsilon$, slightly larger than the lower bound of 1.5 ϵ which follows from (31). On the other hand, when E >> 1, then the optimum value is $b = (E/2)^{1/3}$, and the corresponding minimum phase space area is $(\Sigma\Sigma')_{min} = \epsilon$. In this case, when $\epsilon >> \lambda$, the phase space area $\Sigma\Sigma'$ is a slowly varying function of β^* in the neighborhood of the minimum. This weak dependence on β^* is demonstrated by noting that:

$$\Sigma\Sigma' = \sqrt{2}\,\epsilon \quad \text{for } b = 1/2 , \quad (36a)$$

$$\Sigma\Sigma' = \epsilon \quad \text{for } b = (E/2)^{1/3} , \quad (36b)$$

$$\Sigma\Sigma' = \sqrt{2}\,\epsilon \quad \text{for } b = E. \quad (36c)$$

We conclude that the brightness is quite insensitive to the value of β^* when β^* lies in the interval

$$\epsilon/\lambda > \beta^*/L > 1/2 . \quad (37)$$

Taking the pessimistic value, $\Sigma\,\Sigma' = \sqrt{2}\,\epsilon$, we obtain the following estimate for the brightness at the first-harmonic frequency with K < 1, which should be useful for design purposes:

$$B_1 = \frac{1.8\times10^{12}}{\epsilon_x\,\epsilon_y}\frac{NK^2}{1+K^2/2} , \quad (38)$$

in units of photons/sec, 0.1%, $mrad^2$, mm^2, Amp, when $\epsilon_{x,y}$ are in mm-mrad. Clearly the key to achieving high source brightness is to obtain small electron beam emittances.

When an undulator is operated in an insertion having $\beta_{x,y}^*/L = \epsilon_{x,y}/\lambda$, the radiated wavelength is correlated with observation angle, as described in Eq. (9). The bandwidth $\Delta\lambda/\lambda$ is on the order of that given in Eq. (10), and hence certain experiments not requiring very narrow bandwidths can benefit from the elimination of the need for a monochromator. On the other hand, when one operates the undulator in an insertion having smaller $\beta_{x,y}^*$, such that the angular spread in the electron beam is on the order of $1/\gamma$, then the radiated wavelength is not correlated with observation angle. The spectrum seen by an observer accepting radiation into only a very small solid angle will be proportional to a partially angle-integrated spectrum, since the spread in directions of the average velocity vectors of the different electrons produces essentially the same effect as does a spread in observation angles. One can still take advantage of the high brightness by using a monochromator.

Let us conclude this section by recalling that the brightness is a key parameter in determining the utility of a source for holography. If the source size is characterized by $\Sigma_{x,y}$ then only radiation within a solid angle,[14]

$$\Delta\Omega = \lambda^2/4\Sigma_x\Sigma_y \ , \qquad (39)$$

is spatially coherent. From Eq. (29) we see that the coherent flux[14] within this solid angle is

$$n_c = B(\lambda)\lambda^2 \ , \qquad (40)$$

showing the importance of high brightness $B(\lambda)$ and the difficulty of holography at short wavelengths. In Eq. (40), if $B(\lambda)$ is given in photon/sec, 0.1%, $mrad^2$, mm^2, Amp, then λ should be in μm.

Limitations Associated with Storage Ring Operation

As noted earlier, Eq. (38), the key to achieving high source brightness is to obtain small electron beam emittances. A storage ring lattice structure designed to produce low emittance electron beams was proposed by R. Chasman and G. K. Green,[15] and has been implemented at the NSLS. The lattice is comprised of achromatic bends separated by insertions where one locates wigglers and undulators. For such a lattice with M achromatic bends, and hence M insertions, the lowest possible emittance is

$$\epsilon_x^{min} = (7.7 \times 10^{-13} \text{ m-rad})\ \gamma^2/M^3 \ . \qquad (41)$$

In order to avoid undesirably large values of the betatron functions, practical designs seem to yield emittances about two times greater than this lower bound.

For a given machine, the emittances depend quadratically on energy, $\epsilon_{x,y} \sim \gamma^2$, so at the cost of softening the radiation the brightness can be increased by operating at lower energy. Since the photon energy from an undulator is proportional to γ^2/λ_0, the only way to obtain hard photons at smaller γ is to reduce the undulator period.[16] However, there is a practical limitation on how small λ_0 can be made without having K become unacceptably small. Very small K values are not desirable since Eq. (38) shows that the brightness at ω_1 is proportional to K^2. If G is the full gap of the undulator magnet, one finds that the peak field varies as $B_0 \sim \exp(-\pi G/\lambda_0)$, hence B_0 decreases rapidly with λ_0 unless the gap G is also reduced. The minimum gap allowed by the operation of the storage ring thus limits how small λ_0 can be made. A fixed gap is restricted by the aperture required during injection. If the gap is variable, i.e. large at injection and narrowed after the beam is stored, then the minimum gap is limited by quantum lifetime effects, transverse resistive wall instabilities and gas scattering. Small values of the vertical betafunction ($\beta_y \approx L/2$) will reduce the seriousness of all these effects and allow a smaller vertical gap. Instabilities and gas scattering are less troublesome at higher energy, hence smaller gaps may be expected to be achieved at higher electron energies.

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