Transactions Letters

Unequal Power Allocation to the Turbo-Encoder Output Bits with Application to CDMA Systems

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***Abstract*—Traditional Turbo-codes with binary phase-shift keying modulation assign equal noise margins to the Turbo-encoder output bits. It is shown that by using unequal power allocation (UPA) among the encoder output streams, one can improve the code performance especially for large block lengths. On the other hand, there has been a growing interest in the application of Turbo-codes for signaling over a code-division multiple-access (CDMA) channel. A usual practice in CDMA systems for matching of rate is based on repeating the encoder output bits. This feature can be used to provide UPA with a negligible increase in the complexity. Simulation results are presented showing a noticeable improvement in the bit-error-rate performance.**

***Index Terms*—CDMA, error correction coding, Turbo-codes.**

I. INTRODUCTION

CONVENTIONAL Turbo-codes [1] assign an equal noise margin to the encoder output bits when binary phase-shift keying (BPSK) modulation is employed. In this article, the problem of "unequal power allocation" (UPA) to Turbo-encoder output bits is considered. This problem was first brought up in [2] for a system of combined Turbo-code and modulation. Later, the problem was addressed for the BPSK case in [3], for very low signal-to-noise ratios (SNR's), where it is shown that more power should be allocated to the systematic bits. Here, however, it is shown that as the interleaver length grows, more power should be allocated to the parity check bits. An abstract describing independent work on the same subject can be found in [4].

The problem is formulated based on the application of Turbo-codes for signaling over a code-division multiple-access (CDMA) channel. A usual practice in CDMA systems for matching of rate is based on repeating the encoder output bits. This feature provides us with a practical method of implementing the UPA with only a negligible increase in the complexity.

II. EFFECT OF UPA ON THE PERFORMANCE OF TURBO-CODES

Consider a Turbo-code of rate $r = 1/3$ with two identical component codes and interleaver length N. To achieve UPA, we consider a CDMA system in which the systematic and the parity bits are repeated k_s and k_p times, respectively. We neglect the effect of the terminating bits and assume that the energy per information bit is equal to E_b. The energies assigned to the systematic and parity bits are then equal to $E_s = (k_s E_b / k_s + 2k_p)$ and $E_p = (k_p E_b / k_s + 2k_p)$, respectively.

Let us define the weight enumerating function (WEF) of the code as

$$A(W, Z) = \sum_{i,j} A_{i,j} W^{i(k_s/k_s+2k_p)} Z^{j(k_p/k_s+2k_p)} \tag{1}$$

where $A_{i,j}$ is the number of codewords having i and j 1's in the systematic and the parity check parts, respectively, and W and Z are dummy variables. For large interleaver lengths, the WEF is replaced with the average WEF (AWEF) as in [5]. The AWEF enumerates the codewords corresponding to every possible permutation in the same form as in (1). In this function the coefficient of each codeword is equal to the expected value of the number of codewords of this form.

With a generalization of the method given in [5], the average performance bound (union bound) for different levels of UPA can be obtained using

$$P_b(k_s, k_p) \le \frac{1}{2} \sum_m \hat{D}_m \operatorname{erfc}\left(\sqrt{\frac{mE_b}{N_0}}\right) \tag{2}$$

where $P_b(k_s, k_p)$ is the probability of bit error and $\hat{D}_m$ is defined as

$$\hat{D}_m \triangleq \sum_{i,j: iE_s + jE_p = mE_b} \frac{i}{N} A_{i,j}. \tag{3}$$

This average bound has proven to provide a good approximation to the true performance of Turbo-codes [5].

The bounds corresponding to Turbo-codes with component codes $(g_{\text{forward}}, g_{\text{feedback}}) = (5, 7)$ and interleaver lengths 20 and 760 are shown in Fig. 1. As can be seen, for $N = 760$, an improvement of about 0.5 in the $\log_{10}$[bit-error rate (BER)] can be achieved by repeating the parity bits seven times more than the systematic bits. Referring to the figure, this corresponds to more than 0.5-dB improvement in the SNR over the case of equal power allocation. For $N = 20$, however, the three lowest bounds, corresponding to $(k_s, k_p) = (1, 1), (4, 3)$ and $(8, 11)$, are almost indistinguishable.

In general, the role of the two parity check bits is not necessarily the same in determining the code performance. However, when a uniform interleaver ([5]) is assumed, these two bits have the same role in the AWEF, and consequently,

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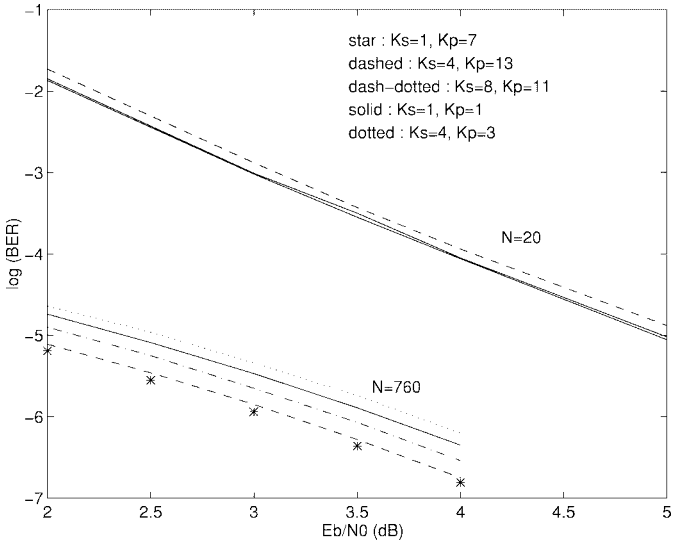

Fig. 1. Average performance bounds for Turbo-codes with interleaver lengths 20 and 760 for different levels of UPA.

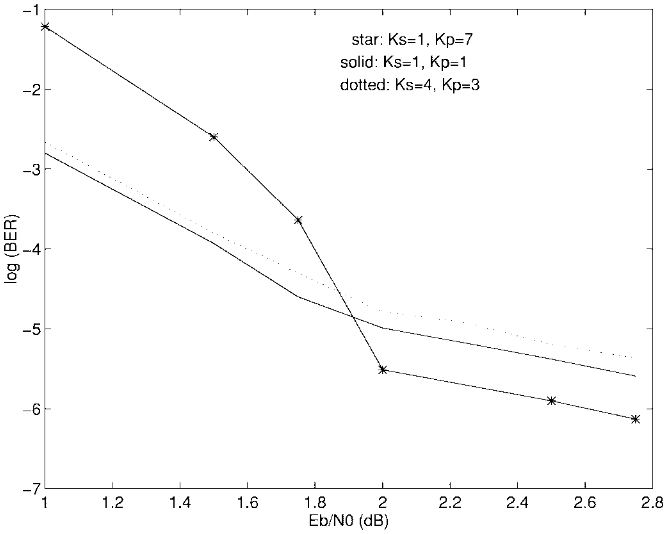

Fig. 3. Simulation results corresponding to Turbo-codes with $N = 760$ in the waterfall region.

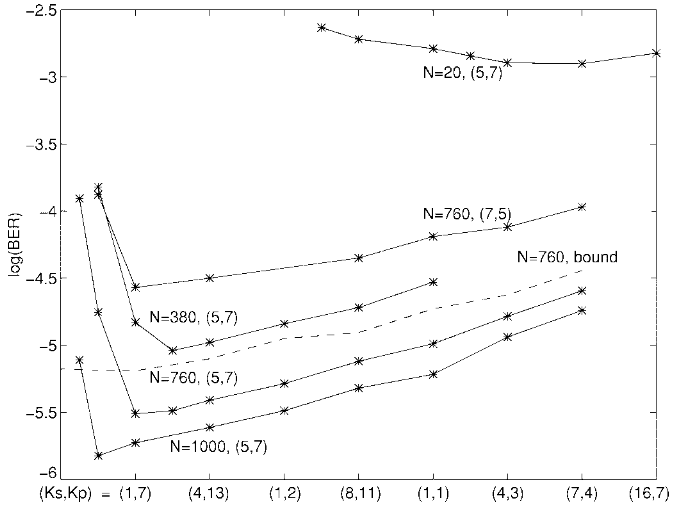

Fig. 2. Simulation results corresponding to Turbo-codes with $N = 380, 760$, and $1000, E_b/N_0 = 2$ (dB), and $N = 20$, $E_b/N_0 = 3$ (dB).

in the code performance. For this reason, we have limited our discussions to the case that the two parity check bits have the same noise margins.

III. SIMULATION RESULTS AND DISCUSSIONS

Simulations are performed for Turbo-codes of rate $r = 1/3$, employing two identical (5, 7) recursive systematic convolutional (RSC) codes for signaling over an additive white Gaussian noise channel. The decoding is performed using the BCJR algorithm with proper modifications reflecting the effect of the unequal noise margins. Fig. 2 shows the BER performance for different values of k_s and k_p and several interleaver lengths. As can be seen, the effect of UPA is negligible for a short interleaver length $(N = 20)$. For larger interleaver lengths, better performance is achieved when the systematic bits are protected less than the parity check bits, and this effect gets stronger as the block length increases. Referring to Fig. 2, we also observe that the effect of UPA is less for the code with nonprimitive feedback polynomial, i.e., (7, 5), as compared to (5, 7).

It is also observed that as k_p/k_s increases, there is a sudden rise in the BER of the simulation results. This degradation is due to the suboptimum decoding method employed. In this case, the extrinsic information passed to the second component decoder becomes very unreliable, and consequently, the iterative decoding procedure does not converge.

In order to observe the effect of UPA for SNR's below the error floor of the performance curves, simulation results for this region are shown in Fig. 3. As can be seen, allocating more power to the parity bits results in better performance after the waterfall region, and also for $k_s = 1, k_p = 7$, the curve is much more steep in the waterfall region. It can be concluded that employing UPA can be most beneficial when Turbo-codes are operating at the beginning of the error floor region.

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