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Unification of the Lepton-Quark World by the Gauge Group *SU*(6)

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A unified vectorlike gauge theory for quarks and leptons based on the group $SU(6)$ is presented. All fermions are assigned to the fundamental representation **15** of $SU(6)$. The resulting model is a natural generalization of a model proposed previously by us, and contains eight quark flavors. In the lepton sector, there are two charged heavy leptons and their associated neutrinos, in addition to the known leptons. The Weinberg angle is determined to be $\sin^2\theta_W=3/8$. Brief discussion of the mechanism of the spontaneous breakdown of $SU(6)$ and the fermion mass generation are given.

§ 1. Introduction

Symmetry has played an important role in particle physics since the discovery of neutrons. After the discovery of strange particles, the coexistence of different kinds of iso-multiplets was successfully unified by the group $SU(3)$, which gave important information on the hadron structure.

Presently interesting symmetry is the gauge symmetry of the second kind. The gauge theory of weak and electromagnetic interactions based on the group $SU(2)\times U(1)$[1] now stands on the phenomenological stage. Many experimental results have supplied us information on the form of existence of elementary particles. The experimental suggestion of the existence of heavy leptons[2] may force us to depart from the conventional four-quark-four-lepton scheme.[3] The indications of a right-handed current[4] suggest that the weak and electromagnetic interactions of elementary particles are vectorlike.[5] In the scheme of vectorlike models, parity non-conservation in neutral current interactions[6] requires singlet (or much higher) fermion multiplets of the weak $SU(2)$ in addition to doublet fermions. These situations suggest an existence of a larger group which includes $SU(2)\times U(1)$ as a subgroup.

We have previously considered a vectorlike model of the weak interactions in the framework of $SU(2)\times U(1)$ gauge theory.[7] It has the following fermion structure:

$$\begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L b_L\, t_L : \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L E_L\, M_L\,,$$

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$$\begin{pmatrix} t_\phi \\ b \end{pmatrix}_R \begin{pmatrix} c \\ s \end{pmatrix}_R d_R\, u_{\phi_R} : \begin{pmatrix} \nu_E \\ E \end{pmatrix}_R \begin{pmatrix} \nu_M \\ M \end{pmatrix}_R e_R\, \mu_R\,, \tag{1}$$

where $d_\theta = d\cos\theta + s\sin\theta$, $s_\theta = s\cos\theta - d\sin\theta$, $t_\phi = t\cos\phi + u\sin\phi$ and $u_\phi = u\cos\phi - t\sin\phi$. This model can considerably well explain the various weak phenomena observed recently.[7] We expect that these fermion multiplets are successfully unified by a supermultiplet of some larger group.

Georgi and Glashow[8] have proposed the unified gauge theory of the strong, weak and electromagnetic interactions based on the group $SU(5)$ within the framework of the conventional quartet model. An interesting point of their approach is that the fermion structure in the weak and electromagnetic gauge group $SU(2)\times U(1)$ is deeply correlated to the structure of the strong gauge group, which they took to be unbroken $SU(3)$.[9] In this article we discuss, along their approach, a possibility that the variety of quarks and leptons in (1) is naturally understood by a vectorlike gauge theory based on a simple group which unifies the weak and electromagnetic gauge group $SU(2)\times U(1)$ and the strong gauge group $SU(3)$. We see that the group $SU(6)$ can fairly well reproduce what is required by the present phenomenology. All fermions are assigned to the fundamental representation **15**. It determines the Weinberg angle with $\sin^2\theta_W = 3/8$.

In the next section, we shall study the general structure of the gauge group $SU(n)$. It is shown that the group $SU(6)$ seems to be the most promising one. Further, an explicit tensor representation of the fundamental fermions will be given. Section 3 is devoted to a study of the mechanism of the spontaneous breakdown of $SU(6)$ and the fermion mass generation. Discussion will be given in the final section.

§ 2. The gauge group $SU(6)$ and fundamental fermion multiplet

The unified gauge group should have a subgroup $SU(3)\times SU(2)\times U(1)$. A simple gauge group which contains this subgroup is a unitary group $SU(n)$ with $n \geqq 5$. In this section we attempt to unify all fermions (quarks and leptons) by an irreducible representation of $SU(n)$. In finding an appropriate representation for fermions, we demand that all fermions transform as **1**, **3**, or **3*** under the strong gauge group $SU(3)$. Of course **1** corresponds to leptons, and **3** and **3*** to quarks and anti-quarks, respectively. Further, we demand that all fermions belong to **1** or **2** of the weak $SU(2)$.

The relevant representations which satisfy the above constraints are fundamental representations

$$\boldsymbol{n},\quad [\boldsymbol{n}\times\boldsymbol{n}],\quad [\boldsymbol{n}\times\boldsymbol{n}\times\boldsymbol{n}],\quad \text{etc.,}$$

where [ ] denotes total antisymmetrization. Other representations (totally symmetric or mixed symmetric), such as $\{\boldsymbol{n}\times\boldsymbol{n}\}$, are ruled out because they contain **6**, **8**, etc. of $SU(3)$. The simplest prescription might be to assign all fermions to the lowest representation $\boldsymbol{n}$, but this is impossible because $\boldsymbol{n}$ does not contain $(\mathbf{3},\mathbf{2})$ components of $SU(3)\times SU(2)$ and in consequence it cannot represent the doublet quarks of weak $SU(2)$. Therefore we must examine higher representations such as $[\boldsymbol{n}\times\boldsymbol{n}]$. In general there is a possibility of the coexistence of different kinds of representations such as $\boldsymbol{n}$ and $[\boldsymbol{n}\times\boldsymbol{n}]$. However we demand for simplicity that all fermions belong to the same kind of representation.

Now we investigate the possibility of $[\boldsymbol{n}\times\boldsymbol{n}]$. The numbers of $SU(3)\times SU(2)$ sub-multiplets contained in this multiplet are given as follows:

$$(\mathbf{3},\mathbf{2}):1\,,\qquad (\mathbf{1},\mathbf{2}):n-5\,,$$
$$(\mathbf{3},\mathbf{1}):n-5\,,\quad (\mathbf{1},\mathbf{1}):\tfrac{1}{2}(n-6)(n-5)+1\,.$$
$$(\mathbf{3}^*,\mathbf{1}):1\,,$$

We see that, if we introduce two fermion multiplets of this representation (containing u-quark and c-quark, respectively), we have sufficient number of sub-multiplets to construct a satisfactory vectorlike theory if $n\geqq 6$. Hereafter we discuss the simplest case $n=6$. The supermultiplet $[\mathbf{6}\times\mathbf{6}]=\mathbf{15}$ contains four quark flavors (q_1, q_2, q_3 and q_4) and three leptons (l_1, l_2 and l_3):

$$\psi=\begin{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} & q_3 & q_4{}^C \\ \begin{pmatrix} l_1 \\ l_2 \end{pmatrix} & l_3 & \end{pmatrix}, \tag{2}$$

where the suffix C denotes charge conjugate operation. In the tensor representation of 6×6, these fermions are described as

$$\psi=\begin{pmatrix} 0 & l_3 & l_1 & q_1{}^1 & q_1{}^2 & q_1{}^3 \\ -l_3 & 0 & l_2 & q_2{}^1 & q_2{}^2 & q_2{}^3 \\ -l_1 & -l_2 & 0 & q_3{}^1 & q_3{}^2 & q_3{}^3 \\ -q_1{}^1 & -q_2{}^1 & -q_3{}^1 & 0 & -q_4{}^{3C} & q_4{}^{2C} \\ -q_1{}^2 & -q_2{}^2 & -q_3{}^2 & q_4{}^{3C} & 0 & -q_4{}^{1C} \\ -q_1{}^3 & -q_2{}^3 & -q_3{}^3 & -q_4{}^{2C} & q_4{}^{1C} & 0 \end{pmatrix}, \tag{3}$$

where the upper indices of q's express quark colors. In (3), the subgroup $SU(3)\times SU(2)$ has been taken such that in the sextet representation of $SU(6)$ the first two components correspond to the doublet of weak $SU(2)$ and the last three components to the triplet of strong $SU(3)$.

Next we investigate the charge structure of $\mathbf{15}$. $SU(6)$ has thirty-five generators $F_i\,(i=1,\cdots,35)$. For the convention of these generators, we take the straightforward generalization of the familiar $SU(3)$ convention in which F_3, F_8, F_{15}, F_{24} and F_{35} are diagonal. Then the generators which commute with those of both weak $SU(2)$ and strong $SU(3)$ are F_8 and $(1/\sqrt{2})F_{15}+(\sqrt{3/10})F_{24}$

$+(1/\sqrt{5})F_{35}$. Therefore the weak hypercharge Y, which is related to the charge operator Q as

$$Q=F_3+\frac{Y}{2}, \tag{4}$$

is given by the suitable linear combination of these two generators. The corresponding two coefficients determine the full charge structure of the theory. Denoting the charge of q_1 in (2) as Q_q and that of l_1 as Q_l, we get the following expression for Y:

$$Y/2\sqrt{3}=(2Q_q-1)F_8+(Q_q+Q_l-1)\left(\frac{1}{\sqrt{2}}F_{15}+\sqrt{\frac{3}{10}}F_{24}+\frac{1}{\sqrt{5}}F_{35}\right). \tag{5}$$

The charges of fermions in the multiplet (2) are given schematically as

$$\left[\begin{matrix}\begin{pmatrix}Q_q\\ Q_q-1\end{pmatrix} & (-2Q_q+1) & (Q_q+Q_l-1)^C\\ \begin{pmatrix}Q_l\\ Q_l-1\end{pmatrix} & (3Q_q+Q_l-2) & \end{matrix}\right]. \tag{6}$$

Since (suitably normalized) Y is the generator of $U(1)$, Eqs. (4) and (5) give the Weinberg angle:

$$\sin^2\theta_W=\frac{\mathrm{Tr}(F_3^2)}{\mathrm{Tr}(Q^2)}=\frac{1}{1+3(2Q_q-1)^2+3(Q_q+Q_l-1)^2}. \tag{7}$$

Now we discuss the detailed structure of (6). According to the conventional quark model, we take the usual charge assignment for the doublet quarks $(q_1, q_2)^T$, that is, $Q_q=2/3$. As far as the conventional leptons are concerned, there are two possibilities for the value of Q_l due to the equivalence of **2** and **2*** of $SU(2)$.

The first possibility is to take the value $Q_l=0$. Then the doublet leptons such as $(\nu_e, e^-)_L{}^T$ are assigned to the left-handed components of $(l_1, l_2)^T$ as usual. In this case, however, the additional lepton l_3 is neutral and the additional quarks q_3 and q_4 have the same charge $-1/3$. Consequently we cannot incorpolate charged heavy leptons and t-quark to the 15-plet of this charge assignment. Moreover the Weinberg angle given by (7) is $\sin^2\theta_W=3/5$, which seems to be too large compared to the value $\sin^2\theta_W\sim 0.3$ favoured by our previous analysis.[7]

The alternative possibility is to assign the charge conjugate of the lepton doublet, i.e., $(e^+, -\bar{\nu}_e)_R{}^T$, to the right-handed components of $(l_1, l_2)^T$, which requires $Q_l=1$. This assignment, in contrast to the previous case, has quite desirable features. It contains an $SU(2)$ singlet quark with charge 2/3 as well as an additional charged lepton and a singlet quark with charge $-1/3$. This is just what is required by our phenomenological model (1). Furthermore it determines the Weinberg angle with $\sin^2\theta_W=3/8$.

From these successful features, we conclude that the gauge group $SU(6)$ is quite hopeful for the unification of the fundamental interactions of elementary particles. It breaks down to the lower symmetry group $SU(3)\times SU(2)\times U(1)$ specified by the values $Q_q=2/3$ and $Q_l=1$. The theory is vectorlike and all fermions (both the left-handed and the right-handed) belong to the fundamental 15-plet representation of the form

$$\psi=\begin{pmatrix} \begin{pmatrix} p_1 \\ n_1 \end{pmatrix} & n_2 & p_2{}^C \\ \begin{pmatrix} E_1{}^+ \\ N^0 \end{pmatrix} & E_2{}^+ & \end{pmatrix}. \tag{8}$$

In order to complete our phenomenological model (1), we must introduce two additional quarks with charges 2/3 and $-1/3$. Then all quarks and leptons are successfully unified by two 15-plet fermion multiplets. For example, the left- and right-handed components of the supermultiplet containing electron and u-quark are given as

$$\begin{pmatrix} \begin{pmatrix} u \\ d_\theta \end{pmatrix} & b & u_\phi{}^C \\ \begin{pmatrix} E^+ \\ -\bar{\nu}_E \end{pmatrix} & e^+ & \end{pmatrix}_L, \quad \begin{pmatrix} \begin{pmatrix} t_\phi \\ b \end{pmatrix} & d & t^C \\ \begin{pmatrix} e^+ \\ -\bar{\nu}_e \end{pmatrix} & E^+ & \end{pmatrix}_R. \tag{9}$$

The other 15-plet multiplet is the muonic counterpart of (9).

§ 3. Breakdown of $SU(6)$ to $SU(3)\times SU(2)\times U(1)$ and fermion mass generation

The group $SU(6)$ has various desirable features for the framework of the unified gauge theory as discussed in the previous section. In order to confirm these discussions, we must next check that this symmetry can really be broken to the level of the required subgroup $SU(3)\times SU(2)\times U(1)$ and that the required mixing patterns and masses of the 15-plet fermions shown in (9) can be really reproduced by suitable Yukawa interactions. In this section we discuss these problems.

The group $SU(6)$ has thirty-five gauge fields $A_i{}^\mu$ related to the corresponding generators F_i. Eight of these fields mediate the strong interactions and four the weak and electromagnetic interactions. The others may mediate the interactions which link leptons and quarks or doublet fermions and singlet ones of weak $SU(2)$. Of course these unfamiliar interactions should be suppressed by the large masses of relevant vector bosons which are generated by the spontaneous breakdown of the symmetry $SU(6)$ to $SU(3)\times SU(2)\times U(1)$ associated by the Higgs mechanism.[10)]

The structure of the subgroup $SU(3)\times SU(2)\times U(1)$ is related to the charge assignment of fermions by (5). We have discussed two cases for the value of Q_l. In the case of $Q_l=1$, the fundamental sextet has a neutral $(\mathbf{1},\mathbf{1})$ component of $SU(3)\times SU(2)$. This means that the group $SU(3)\times SU(2)\times U(1)$ can be included in the subgroup $SU(5)$ of $SU(6)$. This is the reason why we have the same Weinberg angle ($\sin^2\theta_W=3/8$) as was predicted by the $SU(5)$ model of Georgi and Glashow.[10] On the other hand, in the case of $Q_l=0$, the group $SU(3)\times SU(2)\times U(1)$ cannot be included in $SU(5)$.

Here, we discuss the mechanism of the spontaneous breakdown of $SU(6)$ to $SU(3)\times SU(2)\times U(1)$ in the case of $Q_l=1$. In order to get this breakdown, we must introduce a sextet scalar multiplet χ_α and a 35-plet scalar multiplet ϕ_i. For the vacuum expectation value of χ fields, we take the following form corresponding to the previous convention:

$$\langle\chi_\alpha\rangle=\left(0\ 0\ \frac{u}{\sqrt{2}}\ 0\ 0\ 0\right)^{\mathrm{T}}. \tag{10}$$

Vacuum expectation values for ϕ fields are taken to be

$$\langle\phi\rangle=\frac{u'}{2}\frac{Y}{2} \tag{11}$$

in the matrix representation. By these expectation values, twenty-three gauge fields acquire masses as follows:

$$\begin{aligned}
&A_4\quad A_5\quad A_6\quad A_7\ :\sqrt{\left(\frac{gu}{2}\right)^2+\frac{1}{4}\left(\frac{gu'}{2}\right)^2},\\
&\left.\begin{matrix}A_9 & A_{10} & A_{11} & A_{12}\\ A_{16} & A_{17} & A_{18} & A_{19}\\ A_{25} & A_{26} & A_{27} & A_{28}\end{matrix}\right\}:\frac{5}{6}\left|\frac{gu'}{2}\right|,\\
&\left.\begin{matrix}A_{13} & A_{14}\\ A_{20} & A_{21}\\ A_{29} & A_{30}\end{matrix}\right\}:\sqrt{\left(\frac{gu}{2}\right)^2+\frac{1}{9}\left(\frac{gu'}{2}\right)^2},\\
&-\frac{2}{\sqrt{5}}A_8+\frac{1}{\sqrt{10}}A_{15}+\frac{\sqrt{3}}{5\sqrt{2}}A_{24}+\frac{1}{5}A_{35}:\sqrt{\frac{5}{3}}\left|\frac{gu}{2}\right|,
\end{aligned} \tag{12}$$

where g is a universal gauge coupling constant of $SU(6)$. These massive vector bosons mediate the interactions which link the following fermions:

$$\begin{aligned}
&A_4\quad A_5\ :\ E_2-N,\quad p_1-n_2,\\
&A_6\quad A_7\ :\ E_2-E_1,\quad n_1-n_2,\\
&\left.\begin{matrix}A_9 & A_{10}\\ A_{16} & A_{17}\\ A_{25} & A_{26}\end{matrix}\right\}:\ E_2-n_1,\quad E_1-n_2,\quad p_1-p_2{}^C,
\end{aligned}$$

$$\left.\begin{matrix} A_{11} & A_{12} \\ A_{18} & A_{19} \\ A_{27} & A_{28} \end{matrix}\right\} : \ E_2 - p_1 , \quad N - n_2 , \quad n_1 - p_2{}^C ,$$

$$\left.\begin{matrix} A_{13} & A_{14} \\ A_{20} & A_{21} \\ A_{29} & A_{30} \end{matrix}\right\} : \ E_1 - p_1 , \quad N - n_1 , \quad n_2 - p_2{}^C .$$

At present, we have no information about the relative magnitudes of the expectation values u and u'. If one of them is much larger than the other, then there may exist the hierarchy in these superweak interactions. For example, if $|u| \gg |u'|$, the first stage of the breakdown is caused by $\langle \chi \rangle$, which reduces $SU(6)$ to $SU(5)$, then successively $\langle \phi \rangle$ reduces $SU(5)$ to $SU(3) \times SU(2) \times U(1)$.

The final stage of the breakdown of $SU(2) \times U(1)$ to $U(1)$ is induced by the nonvanishing vacuum expectation values of $SU(2)$ doublet or triplet (and of course $SU(3)$ singlet) components of the adequate scalar multiplets. In general, these vacuum expectation values introduce a mixing between the ordinary neutral weak boson and the superheavy boson. But the effect may be negligible due to the extremely large mass of the latter.

Next we study the gross features of masses and mixing patterns of fermions. Masses of fermions are generated in general by both Yukawa couplings of scalar particles and gauge invariant mass terms. The latter, however, may be inadequate from the consideration of masslessness of neutrinos. We introduce scalar (φ_i) and pseudoscalar (φ_i') multiplets of the 35-plet representation which couple to the 15-plet fermion multiplet ψ as follows:

$$-f\overline{\psi}\frac{\Lambda_i}{2}\varphi_i\psi + if'\overline{\psi}\frac{\Lambda_i}{2}\gamma_5\varphi_i'\psi , \tag{13}$$

where $\Lambda_i/2$ is the generator of $SU(6)$ in the 15-plet representation and ψ is given as (8). The vacuum expectation values of the $SU(2)$ doublet component of these fields $\langle\varphi_6\rangle$ and $\langle\varphi_7'\rangle$ give the following zeroth order fermion mass terms:

$$\begin{aligned} \mathcal{L}_{\text{mass}} = &-\frac{1}{2}(\overline{E}_1{}^+, \overline{E}_2{}^+)_R \begin{pmatrix} 0 & f\langle\varphi_6\rangle - f'\langle\varphi_7'\rangle \\ f\langle\varphi_6\rangle + f'\langle\varphi_7'\rangle & 0 \end{pmatrix} \begin{pmatrix} E_1{}^+ \\ E_2{}^+ \end{pmatrix}_L + \text{h.c.} \\ &-\frac{1}{2}(\overline{n}_1, \overline{n}_2)_R \begin{pmatrix} 0 & f\langle\varphi_6\rangle + f'\langle\varphi_7'\rangle \\ f\langle\varphi_6\rangle - f'\langle\varphi_7'\rangle & 0 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}_L + \text{h.c.} \end{aligned} \tag{14}$$

These mass terms are diagonalized by the following transformations:

$$\begin{pmatrix} E_1{}^+ \\ E_2{}^+ \end{pmatrix}_L = \begin{pmatrix} E^+ \\ e^+ \end{pmatrix}_L , \quad \begin{pmatrix} E_1{}^+ \\ E_2{}^+ \end{pmatrix}_R = \begin{pmatrix} e^+ \\ E^+ \end{pmatrix}_R , \quad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}_L = \begin{pmatrix} d \\ b \end{pmatrix}_L , \quad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}_R = \begin{pmatrix} b \\ d \end{pmatrix}_R , \tag{15}$$

giving

$$\mathcal{L}_{\text{mass}} = -\frac{f\langle\varphi_6\rangle + f'\langle\varphi_7'\rangle}{2}\{\bar{E}^+E^+ + \bar{b}b\}$$
$$-\frac{f\langle\varphi_6\rangle - f'\langle\varphi_7'\rangle}{2}\{\bar{e}^+e^+ + \bar{d}d\}. \tag{16}$$

The mixing pattern (15) is just what is required in (9) if we neglect the Cabibbo angle. Further, (16) predicts that if E^+ is heavier than e^+, then b-quark is heavier than d-quark. Next we discuss the Yukawa coupling of 15-plet scalar (ω') and pseudoscalar (ω) multiplets of the form

$$-\frac{h}{32}\varepsilon_{ijpqkl}\bar{\psi}_{ij}\omega_{pq}\psi^C_{kl} + \text{h.c.}$$
$$+\frac{h'}{32}\varepsilon_{ijpqkl}\bar{\psi}_{ij}\gamma_5\omega'_{pq}\psi^C_{kl} + \text{h.c.}, \tag{17}$$

where ψ, ω and ω' are represented by the tensor form (3) and ε is a totally antisymmetric tensor. Nonzero values of $(p, q) = (2, 3)$ and $(3, 2)$ components of ω and ω', which are the only neutral components in **15**, give the mass term

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2}(\bar{p}_2, \bar{p}_1)_R \begin{pmatrix} 0 & h\langle\omega_{23}\rangle - h'\langle\omega'_{23}\rangle \\ h\langle\omega_{23}\rangle + h'\langle\omega'_{23}\rangle & 0 \end{pmatrix} \begin{pmatrix} p_2 \\ p_1 \end{pmatrix}_L + \text{h.c.}$$
$$= -\frac{h\langle\omega_{23}\rangle + h'\langle\omega'_{23}\rangle}{2}\bar{t}t - \frac{h\langle\omega_{23}\rangle - h'\langle\omega'_{23}\rangle}{2}\bar{u}u, \tag{18}$$

where

$$\begin{pmatrix} p_2 \\ p_1 \end{pmatrix}_L = \begin{pmatrix} t \\ u \end{pmatrix}_L, \quad \begin{pmatrix} p_2 \\ p_1 \end{pmatrix}_R = \begin{pmatrix} u \\ t \end{pmatrix}_R. \tag{19}$$

This gives the mass difference between t-quark and u-quark. In order to get more detailed mixing patterns and mass differences of fermions, we must discuss more carefully by introducing larger multiplets of scalar fields, but we see that the above simple mechanism can well reproduce the gross features of fermion masses and mixing patterns.

§ 4. Discussion

We have discussed the unified gauge theory of the strong, weak and electromagnetic interactions based on the simple gauge group $SU(6)$. By this unification the numbers of quarks and leptons and their transformation properties in the weak gauge group $SU(2) \times U(1)$ are strongly restricted by the strong gauge group $SU(3)$. We found that two fermion multiplets of the fundamental representation **15** can successfully unify quarks and leptons which are required by the present phenomenology, and that these multiplets can consistently construct a vectorlike theory. The present model predicts eight quark flavors[11] in nature. The simple Yukawa interactions with 35-plet and 15-plet scalar and pseudoscalar particles reproduce the gross features of the required masses and mixing patterns of fermions. The Weinberg angle is determined to be $\sin^2\theta_W = 3/8$. By the renormalization effects, this value may become somewhat smaller.[12]

From these successes, we conclude that our model is quite promising as a unified theory of the fundamental interactions of elementary particles.

Here we comment on some further problems in our model. In discussing the spontaneous breakdown of the symmetry, we have given the suitable vacuum expectation values to each scalar field in order to get the required group $SU(3) \times SU(2) \times U(1)$. However these expectation values should be determined by the structure of the potential of scalar fields. It will be interesting to investigate what types of potential give rise to these expectation values. The next problem which is also concerned with the potential is the masslessness of neutrinos. In general in the vectorlike theories, neutrinos acquire masses due to the Yukawa interactions even if there are no gauge invariant mass terms, and then in order to preserve their masslessness we must require the relevant vacuum expectation values to vanish. These should be clarified by the structure of the potential.

Finally we make a comment on the right-handed doublet $(c, s)_R$. Our phenomenological vectorlike model (1) contains this doublet due to the constraint of six quark model. The present model contains eight quarks and therefore this constraint is released by the additional mixing freedom. If the 15-plet muonic counterpart has a similar mixing pattern to that of the electronic multiplet (9), then s_R may become an $SU(2)$ singlet and the above doublet may not exist.

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