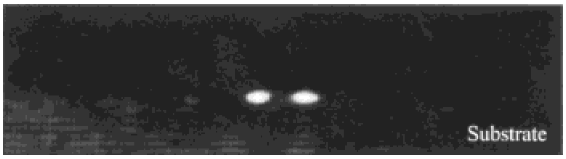

Figure 6 Near optical image coming out from the *Y*-branch waveguide for 1.55 μm light

TABLE 1 Measurement Results of the Propagation Loss in the Straight Reversed-Ridge Waveguide for Different Wavelengths

Wavelength (μm)	Propagation Loss (dB/cm)
0.633	17.7
1.30	4.1
1.55	2.7

waveguide propagation loss decreased significantly with an increase in wavelength from 0.633 to 1.55 μm. This is due partly to the reduction of Rayleigh scattering efficiency as the wavelength increases.

IV. SUMMARY

Single-mode and 6 μm $\times$ 3 μm cross section of PLZT film reversed-ridge waveguides were first fabricated using a sol-gel deposition technique. The sol-gel deposited PLZT was flat across the upper surface of the waveguide over the channel. This kind of waveguide can be developed for a larger cross section single mode of film channel waveguides. Since we etched the ITO spacer film for the waveguide ridge instead of the PLZT, the standard deviation on the horizontal surfaces of the waveguide may be as small as several nanometers by the ITO film deposition process, which is very important for reducing the scattering loss, especially at the beam-splitting step as in a *Y*-branch structure. Consequently, it is possible to obtain propagation losses as low as 2.7 dB/cm at 1.55 μm. The large cross section and ease of fabrication of these waveguides may make ferroelectric thin-film waveguide devices practical for the first time.

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UNIFIED ANALYSIS OF PERFECTLY MATCHED LAYERS USING DIFFERENTIAL FORMS

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ABSTRACT: *The perfectly matched layer (PML) concept is interpreted as a change in the metric of space. By using the language of differential forms applied to the electromagnetic fields and exploring the metric invariance of Maxwell's equations, the various prevalent PML formulations (Maxwellian and non-Maxwellian) are unified. The analysis also reveals that other PML formulations are also possible, embodying the previous PML formulations as special cases.* © 1999 John Wiley & Sons, Inc. Microwave Opt Technol Lett 20: 124–126, 1999.

Key words: *electromagnetic waves; electromagnetic wave absorption; perfectly matched layer*

1. INTRODUCTION

The perfectly matched layer (PML) [1] has been proven to be a highly effective means to truncate the computational domains in electromagnetic (EM) simulations based on differential equation methods. The PML of [1] was shown to be equivalent to an analytical continuation on the coordinate space of Maxwell's equations (MEs) to a complex coordinate space (complex space) [2–5], by which propagating modes are continuously mapped to exponentially decaying modes, allowing for the reflectionless absorption of EM waves. The PML adds degrees of freedom to the MEs in such a way that the fields inside the PML cannot be associated with any possible EM field, resulting in a *non-Maxwellian* (or complex-space) formulation.

In an alternative formulation of the PML [3–6], however, the added degrees of freedom are entirely incorporated into the constitutive parameters, the original form of MEs is retained, and a *Maxwellian* PML is obtained. In such a case, the fields inside the PML can be associated with physical fields in an artificial medium with properly chosen constitutive parameters.

Although both the Maxwellian and non-Maxwellian PML schemes are equivalent in the sense of being reflectionless (in the continuum limit) for all frequencies and angles of incidence, it is of interest to develop frameworks where both PML formulations can be unified, and such that further insight into the PML concept can be obtained [7].

In this work, we interpret the PML as a change in the metric of space, use the language of differential forms (or simply, forms) [8–10], and explore the metric invariance of MEs to develop a concise mathematical framework which properly unifies the non-Maxwellian PML and the Maxwellian PML formulations. As a byproduct of this analysis, it is also revealed that many other PML formulations are also possible, embodying the previous PML formulations as special cases. The convention $e^{-i\omega t}$ is used throughout. For brevity, a basic familiarity with the language of differential forms as applied to the EM fields [8–10] is assumed.

2. FORMULATION

Inside the PML, the spatial coordinates are mapped to a complex domain as [2–5]

$$x \to \tilde{x} = \int_0^x s_x(x')\,dx' \tag{1}$$

where $s_x(x) = a_x(x) + i\sigma_x(x)/\omega$ is the complex stretching variables on x, and similarly for y and z. This can be easily interpreted as a change in the metric of space. From the Euclidean metric $(ds)^2 = dxdx + dydy + dzdz$ or

$$[g_{ij}] = [\delta_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

we are led to a modified, complex metric $[\tilde{g}_{ij}(x,y,z)]$ given by $(ds)^2 \to (d\tilde{s})^2 = (s_x)^2 dxdx + (s_y)^2 dydy + (s_z)^2 dzdz$, or

$$[\tilde{g}_{ij}(x,y,z)] = [T_{ij}(x,y,z)] \cdot [\delta_{ij}][T_{ij}(x,y,z)] \tag{3}$$

with

$$[T_{ij}(x,y,z)] = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix}. \tag{4}$$

This is a purely *geometric* interpretation of the PML, meaning that the PML concept does not depend on the particular form of field equations and is applicable to any linear wave phenomena. An interesting property of MEs, however, is that it allows a change in the metric to be entirely translated into a change in the constitutive parameters. This is revealed by writing the MEs in the language of differential forms [8–10]:

$$dE = i\omega B \tag{5a}$$

$$dH = -i\omega D \tag{5b}$$

$$dD = 0 \tag{5c}$$

$$dB = 0 \tag{5d}$$

where E, H are the electric and magnetic field 1-forms, and D, B are the electric and magnetic flux 2-forms. The operator d is the exterior derivative. The MEs as written above are metric invariant (in contrast to the Maxwell equations when written in the vector language), and retain the same form in any coordinate system since d does not depend on a metric [7]. In the differential forms language, the constitutive parameters of a given medium relate the 1-forms E, H to the 2-forms D, B, and are given in terms of so-called *Hodge operators* [8–10], $D = \star_e E$ and $B = \star_h H$. These operators establish a map (isomorphism) between the space of 1-forms as E and H and the space of 2-forms as D and B. The Hodge operators depend on a metric so that all of the information about the metric is contained in the constitutive relations. As a result, a change in the metric properties of space can be entirely incorporated, in a dual formulation, as a modification to the constitutive parameters of the MEs.

A duality relation also exists between the forms E, H, D, and B and the corresponding vector fields $\mathbf{E}$, $\mathbf{H}$, $\mathbf{D}$, and $\mathbf{B}$. Given a general 1-form Ω expanded in terms of the basis of 1-forms dx, dy, dz, its dual vector under the isomorphism governed by a diagonal metric tensor $[g_{ij}] = [g_{ii}\delta_{ij}]$ is given by

$$\Omega_x dx + \Omega_y dy + \Omega_z dz \to \frac{\Omega_x}{\sqrt{g_{11}}}\mathbf{x} + \frac{\Omega_y}{\sqrt{g_{22}}}\mathbf{y} + \frac{\Omega_z}{\sqrt{g_{33}}}\mathbf{z} \tag{6}$$

where $\mathbf{x}, \mathbf{y}, \mathbf{z}$ are the unit vectors. Similarly, in the case of a 2-form Φ expanded in terms of the basis of 2-forms $dxdy, dydz, dzdx$, the isomorphism under $[g_{ii}\delta_{ij}]$ is

$$\Phi_x dydz + \Phi_y dzdx + \Phi_z dxdy \to \frac{\Phi_x}{\sqrt{g_{22}g_{33}}}\mathbf{x} + \frac{\Phi_y}{\sqrt{g_{33}g_{11}}}\mathbf{y} + \frac{\Phi_z}{\sqrt{g_{11}g_{22}}}\mathbf{z}. \tag{7}$$

If we express the electric field 1-form E as $E = E_x dx + E_y dy + E_z dz$, then the operators $\star_e$ act on E to give the electric flux 2-form D as follows:

$$D = \star_e E = \epsilon \frac{\sqrt{g_{22}g_{33}}}{\sqrt{g_{11}}} E_x dydz + \epsilon \frac{\sqrt{g_{33}g_{11}}}{\sqrt{g_{22}}} E_y dzdx + \epsilon \frac{\sqrt{g_{11}g_{22}}}{\sqrt{g_{33}}} E_z dxdy \tag{8}$$

and analogously for the magnetic case. Under the change in the metric $[g_{ij}] \to [\tilde{g}_{ij}]$ inside the PML, the modified MEs read

$$d\tilde{E} = i\omega\tilde{B} \tag{9a}$$

$$d\tilde{H} = -i\omega\tilde{D} \tag{9b}$$

$$d\tilde{D} = 0 \tag{9c}$$

$$d\tilde{B} = 0. \tag{9d}$$

These metric-free equations are the same as (5). However, the new forms $\tilde{E}$, $\tilde{H}$, $\tilde{D}$, and $\tilde{B}$ are related through modified Hodge operators $\tilde{D} = \tilde{\star}_e \tilde{E}$, $\tilde{B} = \tilde{\star}_h \tilde{H}$ defined by the modified metric $[\tilde{g}_{ij}]$. This *uniquely* defines the PML in the differential forms language. The different PML formulations in the vector language arise depending on how we map the forms into corresponding vectors inside the PML, as discussed next.

2.1. The Maxwellian PML Formulation. In this case, the map from the 1-forms E and H to the corresponding vectors is given by (6) using the Euclidean metric tensors $[g_{ij}] = [\delta_{ij}]$ (natural isomorphism under $[\delta_{ij}]$), i.e.,

$$E = E_x dx + E_y dy + E_z dz \to \mathbf{E}^m = E_x\mathbf{x} + E_y\mathbf{y} + E_z\mathbf{z} \tag{10}$$

and similarly for D and B, but using (7). The superscript "m" denotes the Maxwellian fields. Since (9) above preserves the form of MEs, the resultant system in the vector language also retains the usual form of MEs on the Euclidean metric, with the constitutive relations in the vector language incorporating all of the effects on the change of the metric of space $[g_{ij}] \to [\tilde{g}_{ij}]$ within the PML. Using (3), (4), and (8), they are given by

$$\bar{\epsilon} = \epsilon \frac{s_y s_z}{s_x} \mathbf{xx} + \epsilon \frac{s_z s_x}{s_y} \mathbf{yy} + \epsilon \frac{s_x s_y}{s_z} \mathbf{zz} \tag{11}$$

and analogously for $\bar{\boldsymbol{\mu}}$.

2.2. The Complex-Space (Non-Maxwellian) PML Formulation. In this case, the map from forms to the corresponding dual vectors is given by (6) using the modified complex metric tensor $[\bar{g}_{ij}]$ (natural isomorphism under $[\tilde{g}_{ij}]$):

$$E = E_x dx + E_y dy + E_z dz \to \mathbf{E}^c = \frac{E_x}{s_x}\mathbf{x} + \frac{E_y}{s_y}\mathbf{y} + \frac{E_z}{s_z}\mathbf{z} \tag{12}$$

and analogous relations for the other fields using (7) and duality. The superscript "c" denotes complex-space fields. In this case, the resultant system in the vector language does not retain the form of MEs since the metric factors appearing in the vector language equivalent to (9) are not the usual ones associated with the Euclidean metric, but complex ones associated with the complex metric $[\tilde{g}_{ij}]$ [2–5]. On the other hand, the constitutive relations, when translated to vector language, retain the same form as before the change of the metric $\mathbf{D}^c = \epsilon \mathbf{E}^c$ since complex factors in (8) cancel out. We also see from (10) and (12) that the $\mathbf{E}^m$ and $\mathbf{E}^c$ are related by $\mathbf{E}^m = [T_{ij}] \cdot \mathbf{E}^c$, and similar expressions exist for the other fields.

2.3. Other PML Formulations. This analysis reveals that other choices of metrics are possible to govern the isomorphism between differential forms and vectors, as long as they recover the real metric $[g_{ij}]$ in the physical domain and preserve the perfect matching conditions. An obvious choice is $[\tilde{g}_{ij}]^{\beta}$ with $0 \le \beta \le 1$. The particular choice $\beta = 0$ recovers the Maxwellian PML, and the particular choice $\beta = 1$ recovers the complex-space PML.

The resulting vector fields $\mathbf{E}^{\beta}$ are related to the previous ones through $\mathbf{E}^{\beta} = [T_{ij}]^{1-\beta} \cdot \mathbf{E}^c$. The constitutive tensors are also modified accordingly.

We note that some of the new PMLs may have attractive characteristics for computational purposes. For instance, the choice $\beta = 1/2$ was already used [11] to provide a symmetric modified nabla operator for use in the finite-element method (FEM). This leads to a symmetric FEM matrix, as opposed to the complex-space PML ($\beta = 1$). Moreover, numerical experiments [11] showed that the condition number from the FEM matrix in this formulation is better that in the Maxwellian PML ($\beta = 0$).

3. CONCLUSIONS

The PML is interpreted as a change of the metric of space, and it is shown that differential forms constitute the appropriate language to unify the PML formulations since the metric invariance of MEs is manifested in such a language. Different PML formulations in the vector language are derived from different choices of how to map the *same* form quantities into corresponding vector quantities. Furthermore, this analysis also reveals that many other PML formulations are possible.

The extension of this analysis for other coordinate systems PMLs [2–4] and for PMLs in more general media [5] is also possible, and will be reported elsewhere along with additional issues.

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DESIGN OF A DICHROIC SURFACE FOR DUAL-FREQUENCY RADIOASTRONOMICAL OBSERVATIONS

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ABSTRACT: *The design of a low-loss, frequency-selective surface (FSS) for dual-frequency radioastronomical applications is discussed in*