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Uniform and Complex Bids for Demand Response and Wind Generation Scheduling in Multi-Period Linked Transmission and Distribution Markets

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Abstract: Power markets serving 70% of US load operate today on period-specific uniform price-quantity bids (UPQBs). However, UPQBs result in a poor representation of utility accruing to many multi period market participants. UPQB can adequately represent utility of consumption only under the restrictive condition that it is additively separable over time. In fact, the additive separability condition is particularly untrue for emerging smart-grid-enabled flexible demands with storage-like characteristics such as EV battery charging and HVAC. We claim that such types of flexible demand exhibit a utility of consumption related to a period-specific state variable – e.g. the battery charge state – whose dynamics are a function of past consumption trajectories. We also claim that wind generation should not be only credited for its energy bids; it should be also charged for the incremental reserves that the Independent System Operator (ISO) must procure to secure system integrity against bids based on volatile wind output forecasts. Appropriate debiting and charging rates equal the market clearing prices resulting from co-optimizing energy and reserve costs. We argue that flexible demand and wind farms that participate in the day-ahead market by submitting UPQBs are motivated to self-dispatch on the basis of energy and reserve clearing price trajectory forecasts. Since actual clearing prices may differ significantly from the forecasts used in the self-dispatch, oscillatory behavior can easily result if actual clearing prices are used as the next forecast. Nevertheless, a smoother price forecast updating process can lead towards Nash Equilibrium. The paper's contribution is the proposal of tractable complex bid rules that (*i*) allow market participants to reveal their true inter temporal utility of consumption and their net revenue from wind generation, and (*ii*) enable the market operator to compute the actual Nash equilibrium in a single solution of the market clearing algorithm. The tractability and reasonableness of the complex bid rules are demonstrated through numerical examples.

I. INTRODUCTION

Power markets introduced in the US in the mid 1990s have transformed successfully the wholesale of high voltage electric power into a competitive sector [15], [19] and serve today 70% of US electricity consumption. Without underestimating the contribution of wholesale markets to productivity growth, we note that it is limited primarily to the generation side while the participation of demand is still minimal. Although market participation of demand connected to the distribution network is practically non-existent, significant steps have been taken recently by progressive market operators to enable demand side participation in the full range of commodities cleared by whole sale markets, including different types of reserves [15],[16], [18]. However, the full potential of demand response has not been realized. This shortcoming is often attributed to the presumption of fundamentally inelastic short term demand. We reject this presumption and claim that the real cause of apparently limited response should be sought in: (*i*) the inability of period specific Uniform Price-Quantity Bid (UPQB) rules in today's multi-period markets [3], [23] to capture inter-temporal demand preferences, and (*ii*) today's average-cost-based distribution charges that hide spatiotemporally varying marginal cost information [3], [23].

Period-specific UPQBs may constitute a poor representation of a market participant's utility in a multi period market. To be sure, a 24 hour day-ahead market participant can express its utility of consumption by hour-specific UPQBs, only under the restrictive assumption that its utility is additively separable over hourly time periods. This assumption is incorrect for flexible demands exhibiting storage-like characteristics like EV battery charging and HVAC. In fact, such flexible demand types are characterized by an inter-temporally coupled utility of consumption which is a function of a state variable – e.g. the battery charge state – whose dynamics may lead to the same state at time t for many *equivalent* consumption trajectories [3], [4], [24]. We also claim that wind generation should not be only credited for its energy bids; it should be also charged for the cost of incremental reserves that the Independent System Operator (ISO) must procure to ensure system security against contingencies that may result from unit commitment and economic dispatch affected by aggressive wind farm bidding [5], [6], [7], [10], [13], [14], [17], [20], [22]. The appropriate credit and charge rates must equal market clearing prices resulting from the co-optimization of energy and reserve procurement costs. We finally claim that it is of paramount importance to reflect spatiotemporally varying distribution network marginal costs in the respective rates charged to distribution network consumers. Distribution costs include local transformer congestion and the lion's share of marginal line losses varying often over a range of 4% to 20% during a 24 hour period [12]. Reflection of these costs on time and location specific transactions should replace the average-cost-based-rates charged today. This will not only improve demand response but will also integrate wholesale and distribution markets improving overall efficiency and social welfare.

In Section II we develop a model of the day-ahead market with several types of participants, some connected at the Transmission and some at the Distribution network, with both transmission line and local/distribution congestion.

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Distribution line losses are modeled explicitly while transmission power flows are approximated by a DC model. Energy balance, reserve requirements – one type only for proof of concept – and T&D congestion are also modeled. Market participant types modeled include: (*i*) conventional non responsive demand, (*ii*) flexible price responsive demand, (*iii*) conventional generation and (*iv*) intermittent wind generation. In Section III, we model the clearing of the day-ahead market when *uniform price quantity bids (UPQBs)* are the only available participation rule, and show that flexible demand and wind farms are motivated to self-dispatch based on forecasted energy and reserve clearing price trajectories. However, since actual clearing prices may differ significantly from the forecasts used in the self-dispatch, oscillatory behavior can easily result if flexible demand and wind re-dispatch use actual clearing prices as the future forecast. Nevertheless, under a smoother price forecast updating process, Nash Equilibrium prices may be approached under steady state. In section IV we propose tractable *complex bid* rules that (*i*) allow market participants to reveal their true inter temporal utility of consumption and the net revenue from wind generation, and (*ii*) enable ISOs to always reach the Nash equilibrium in a single market clearing step. In Section V we demonstrate the tractability and reasonableness of complex bid rules through numerical examples. We conclude in Section VI.

II. MARKET PARTICIPANT REQUIREMENTS COSTS AND BENEFITS

We next introduce variables, capabilities, dynamics, costs and utility of Demand, conventional and flexible, conventional Generation and volatile Wind Generation. We use throughout the following index definitions:

n: index of a bus node in the Transmission network

$n(i)$: location i in the distribution network connected to bus node n.

t: hour t in the 24 hour day-ahead market, $t \in \{1, 2, ..., 24\}$.

$F_j \in \{HVAC, EV(\tau)\ \tau \in \{1,2,...,24), \text{other}\}$: Flexible demand j which may be any of a number of flexible demand types. Flexible demand types include Plug In Hybrid Electric Vehicles (PHEVs) or purely Electric Vehicles (EVs), Heating Ventilation and Cooling (HVAC), [3], [8], [24] a range of appliances that demand energy rather than energy and hence exhibit storage like characteristics such as hot water heaters, dryers, dehumidifiers, refrigerators and the like which can be equipped with smart interfaces rendering them Grid Friendly Appliances (GFAs), and finally distributed resources such as battery or fly wheel storage, and dimmable lights [4], [24].

R: index indicating reserves offered by a market participant.

x: variable or index associated with the state of a flexible demand and complex bid based market clearing prices.

Without loss of generality, in lieu of the primary, secondary and tertiary reserves transacted in power markets for frequency control, ACE error correction and load following, we model for simplicity of exposition only 5 min regulation service reserves (RSR). RSRs referred to also as secondary reserves is a standby capacity that a market participant offers during a specific hour of the day-ahead market and is compensated/credited for it as soon at the market regulation service clearing price. An accepted and compensated RSR of say, y MW, carries the contractual obligation on the part of the market participant who offered it – whether conventional generation of flexible demand – to respond to market operator requests for an increment or decrement of the participant's generation or consumption. The increment or decrement requested can not exceed y and the response rate can follow a ramp rate of $y/5$ MW per minute [11], [16], [18].

Although we do not model 10 minute or longer load following synchronized reserves, including them would not have a qualitative impact on the model tractability and conclusions presented in this paper.

A. Definitions, Constraints and State Dynamics: Demand

Define:

${}^{c}d^{t}_{n(i)}$ and ${}^{F_j}d^{t}_{n(i)}$: *conventional demand* and *flexible demand* F_j respectively scheduled during hour t in location i of node n. These demands include the distribution network line losses and represent demand at the end of the transmission system.

${}^{F_j,R}d^{t}_{n(i)}$: the part of flexible demand F_j during hour t in distribution network location i of bus node n, which is scheduled as *up and down regulation service*.

$d^{t}_{n(i)} \triangleq \sum_j {}^{F_j}d^{t}_{n(i)}$: total demand during hour t, in location i of node n. Note that more than one flexible demand may exist at $n(i)$, and this is why we sum over j.

$d^{t}_{n} \triangleq \quad_i \quad) + \sum_j {}^{F_j}d^{t}_{n(i)}]$: total demand at node n during hour t.

The flexible demand and the associated regulation service reserves variables defined above must satisfy the following constraints:

$$ {}^{F_j}d^{t}_{n(i)} - {}^{F_j,R}d^{t}_{n(i)} \geq {}^{F_j}\underline{d}^{t}_{n(i)} / m^{t}_{n(i)} \tag{1.1} $$

$$ {}^{F_j}d^{t}_{n(i)} + {}^{F_j,R}d^{t}_{n(i)} \leq {}^{F_j}\overline{d}^{t}_{n(i)} / m^{t}_{n(i)} \tag{1.2} $$

$$ m^{t}_{n(i)} \sum_j [{}^{F_j}d^{t}_{n(i)} + {}^{F_j,R}d^{t}_{n(i)}] \leq \hat{C}^{t}_{n(i)} \tag{1.3} $$

Where

${}^{F_j}\underline{d}^{t}_{n(i)}$, ${}^{F_j}\underline{d}^{t}_{n(i)}$ are minimal and maximal consumption levels that do not include distribution line losses,

$m^{t}_{n(i)}$ the loss factor that converts load at the transmission system bus node n to load net of distribution network line losses at distribution network location i connected to node n[1], and

$\hat{C}^t_{n(i)}$ the maximal available distribution capacity[2] over and above that used to satisfy conventional demand ${}^{c}d^t_{n(i)}$.

We assume here that the distribution assets serving location $n(i)$ have a fixed capacity[3] $\hat{C}^t_{n(i)} + {}^{c}d^t_{n(i)}$.

We next define the State of flexible demand,

${}^{F_j}x^t_{n(i)}$ State of flexible demand j in distribution location i drawing power from node n during hour t.

State dynamics

$$ {}^{F_j}x^{t+1}_{n(i)} = {}^{F_j}f^t_{n(i)}[{}^{F_j}x^t_{n(i)}, {}^{F_j}d^t_{n(i)}, {}^{F_j}\hat{\omega}^t_{n(i)}, m^t_{n(i)}] \qquad (1.4) $$

where ${}^{F_j}\hat{\omega}^t_{n(i)}$ is the best estimate of a random variable that affects the dynamics. Two examples are given below to elaborate the concept and the form of ${}^{F_j}f^t_{n(i)}(.)$:

Example 1. $F_j = EV(\tau)$, where $EV(\tau)$ stands for Electric Vehicles that have plugged in to charge their battery, and intend to depart during hour τ. The state, ${}^{EV(\tau)}x^t_{n(i)}$, represents the uncharged battery capacity, i.e. the energy the battery can still charge at time t.

The state dynamics are:

$$ {}^{EV(\tau)}x^{t+1}_{n(i)} = {}^{EV(\tau)}x^t_{n(i)} - m^t_{n(i)}\, {}^{EV(\tau)}d^t_{n(i)} + {}^{EV(\tau)}\hat{\omega}^t_{n(i)} \qquad (1.4.\text{EV}) $$

where ${}^{EV(\tau)}\hat{\omega}^t_{n(i)}$ is the estimate of the battery charging demand that will arrive during hour t for departure class τ electric vehicles. Note that the line losses factor $m^t_{n(i)}$ reduces the apparent demand at the transmission system to the charging energy available at the distribution network location i. Assuming, reasonably, ${}^{F_j}\underline{d}^t_{n(i)} = 0$, and letting ${}^{F_j}\underline{d}^t_{n(i)} = \text{ChargingCap}^t$, we have:

$$ {}^{EV(\tau)}d^t_{n(i)} \geq {}^{EV(\tau),R}d^t_{n(i)} \qquad (1.1.\text{EV}) $$

$$ {}^{EV(\tau)}d^t_{n(i)} + {}^{EV(\tau),R}d^t_{n(i)} \leq \text{ChargingCap}^t / m^t_{n(i)} \qquad (1.2.\text{EV}) $$

$$ m^t_{n(i)} \sum_j [{}^{F_j}d^t_{n(i)} + {}^{F_j,R}d^t_{n(i)}] \leq \hat{C}^t_{n(i)} \qquad (1.3.\text{EV}) $$

When inequality (1.2.EV) is often binding, a fact that occurs when regular electrical outlets are used for charging, additional state information on the number of vehicles plugged in during hour t is needed [8]. For simplicity of exposition we assume here that the (1.2EV) is superseded by (1.3.EV) and we do not augment the state variable.

Example 2. $F_{j'}$ *=HVAC*. The state, ${}^{HVAC}x^t_{n(i)}$, represents the inside temperature. The state dynamics during the heating season are:

$$ {}^{HVAC}x^{t+1}_{n(i)} = {}^{HVAC}x^t_{n(i)} + K_Q\, {}^{HVAC}m^t_{n(i)}\, {}^{HVAC}d^t_{n(i)} - K_L({}^{HVAC}x^t_{n(i)} - {}^{HVAC}\hat{\omega}^t_{n(i)}) \qquad (1.4.\text{HVAC}) $$

where ${}^{HVAC}\hat{\omega}^t_{n(i)}$ is the estimate of the external temperature during hour t and K_Q, K_L are constants representing the effective rate of conversion to heat of the energy consumption ${}^{HVAC}d^t_{n(i)}$, and the building cell heat loss rate respectively.

Utility

As already mentioned, flexible demands do not derive utility depending on the energy consumed during a specific hourly period. Instead, the utility incurred during hour t is a function of the state during that hour. We denote the state depended utility by ${}^{F_j}U^t_{n(i)}({}^{F_j}x^t_{n(i)})$. For $F_j = EV(\tau)$ as in example 1, the utility is the sum of the avoided penalty cost associated with a battery state that is not completely charged at the declared time of the EV's departure, plus the marginal cost of the uncharged battery capacity at the end of the day-ahead market horizon. In particular:

$$ {}^{F_j}U^t_{n(i)}({}^{F_j}x^t_{n(i)}) = {}^{EV(\tau)}U^t_{n(i)}({}^{EV(\tau)}x^t_{n(i)}) = -{}^{EV(\tau)}V^{\tau}_{n(i)}\, {}^{EV(\tau)}x^t_{n(i)} 1_{t=\tau} - {}^{EV(24+)}v^{24}_{n(i)}\, {}^{EV(24+)}x^{24}_{n(i)} $$

where:

[1] Although we approximate here the loss factor as an exogenously determined spatiotemporally sensitive constant, under the reasonable assumption that losses occur primarily in the distribution network lines and that wind farm and conventional generation are connected to the high voltage transmission system, it is in fact a function of demand at location i of node n. More specifically, since losses are quadratic in current flow, and hence for all practical purposes quadratic in power flow if voltage levels are fairly constant, the marginal loss factor is linear in power flow, i.e. $m^t_{n(i)} = [1 - \beta_{n(i)}({}^{c}d^t_{n(i)} + \sum_j {}^{F_j}d^t_{n(i)})]$ where $\beta_{n(i)}$ is a constant that depends on location and reflects the distribution line characteristics. Notice that this representation will render the dynamics of ${}^{F_j}x^t_{n(i)}$ quadratic with ${}^{F_j}d^t_{n(i)}\, {}^{F_{j'}}d^t_{n(i)}$, $[{}^{F_j}d^t_{n(i)}]^2$ $j' \neq j$ and ${}^{F_j}x^t_{n(i)}\, {}^{F_j}d^t_{n(i)}$ terms. This transforms the LP problems discussed below into quadratic programming problems that are admittedly harder but still tractable.

[2] To a first approximation, $\hat{C}^t_{n(i)}$ = Cap. Of Distribution network to deliver power at $n(i)$ minus ${}^{c}d^t_{n(i)}$. More precisely, however, the distribution capacity at $n(i)$ depends on t since the hazard rate of key distribution assets, for example transformers, depends on the temperature that they have been for several hours preceding t.

[3] This is a simplified model of distribution congestion which might be better represented by a soft constraint, i.e., a cost representing the acceleration in the rate at which the economic life of distribution assets, most notably transformers, decline as a function of their operating temperature and the number of hours that they have spent already at a high temperature [25]. The model can be improved easily and without compromising tractability by replacing the hard constraint with a cost representing asset life deterioration. This cost would be linear in total consumption at $n(i)$ with time and location dependent parameters provided by the future smart grid cyber physical infrastructure.

${}^{EV(\tau)}V_{n(i)}$ the penalty of releasing a non-fully charged battery[4], and

${}^{EV(24+)}v_{n(i)}^{24}$ a smaller penalty associated with the opportunity cost of uncharged batteries of EVs with a declared departure time at 24^+, i.e., after midnight[5].

For conventional/inflexible demand, we have the usual hour specific utility of consumption, ${}^cU_{n(i)}^t({}^cd_{n(i)}^t) = {}^cu_{n(i)}^t \, {}^cd_{n(i)}^t$, where ${}^cu_{n(i)}^t$ is the value accruing from the satisfaction of an additional unit of conventional demand ${}^cd_{n(i)}^t$. Bidding a high ${}^cu_{n(i)}^t$ allows conventional demand to signal that it is associated with a very high utility rate and is therefore practically inelastic. This is indeed true for most of conventional demand that is indeed inelastic and as a result self-schedules.

B. Definitions, Constraints and State Dynamics: Conventional Generation

Define:

$g_{n(\gamma)}^t, {}^Rg_{n(\gamma)}^t$: *Conventional generation* during hour *t* of unit *γ* connected to node *n*. Note that we assume generation is not connected at the distribution network.

${}^Rg_{n(\gamma)}^t$: The part of conventional generation of unit *γ* connected to node *n* that provides *regulation service reserves* during hour *t*.

$g_n^t \triangleq \quad_{\gamma}$: total conventional generation at node *n*.

The conventional generation variables defined above must satisfy the following constraints:

$$g_{n(\gamma)}^t - {}^Rg_{n(\gamma)}^t \geq \underline{g}_{n(\gamma)}^t \qquad (1.5a)$$

$$g_{n(\gamma)}^t + {}^Rg_{n(\gamma)}^t \leq \overline{g}_{n(\gamma)}^t \qquad (1.5b)$$

$${}^Rg_{n(\gamma)}^t \leq 5\,{}^Rramp_{n(\gamma)}^t \qquad (1.6)$$

where

$\underline{g}_{n(\gamma)}^t, \overline{g}_{n(\gamma)}^t$: the technical minimum and generation capacity respectively, and

${}^Rramp_{n(\gamma)}^t$:the per minute ramp up or down rate of generator *γ* connected to node *n*.

Cost

Disregarding start up/shut down costs and minimum up/down times we have variable costs associated with the generation of energy and the response to regulation service requests by the power system/whole sale market operator: $c_{n(\gamma)}^t({}^Eg_{n(\gamma)}^t, {}^Rg_{n(\gamma)}^t) = \overline{c}_{n(\gamma)}^t g_{n(\gamma)}^t + \overline{r}_{n(\gamma)}^t \, {}^Rg_{n(\gamma)}^t$ where $\overline{c}_{n(\gamma)}^t$ is the variable fuel and O&M cost of generator *n(γ)* and $\overline{r}_{n(\gamma)}^t$ is the unit cost bid to the day-ahead market operator by unit *γ* for regulating its generation level between $g_{n(\gamma)}^t - {}^Rg_{n(\gamma)}^t$ and $g_{n(\gamma)}^t + {}^Rg_{n(\gamma)}^t$ in response to potential system operator requests.

C. Definitions, Constraints and State Dynamics: Intermittent Wind Generation

For simplicity we assume all wind farms injecting power into the same node *n* are aggregated into a single bid. We define:

${}^Wg_n^t$: Wind generation during hour *t* at bus *n*.

$${}^W\mathbf{G}^t \triangleq \quad g_2^t,\ldots,{}^Wg_n^t,\ldots,{}^Wg_N^t]$$

M^t : variance covariance matrix of the estimated wind generation vector ${}^W\mathbf{G}^t$.

The variables defined above must satisfy the constraints:

$${}^W\underline{g}_n^t \leq {}^Wg_n^t \leq {}^W\overline{g}_n^t \qquad (1.7)$$

Where Constraint (1.7) states that wind generation during hour *t* must be in the forecasted range of possible wind generation capability during hour *t*.

Subsidies by state governments and preferential treatment by ISOs that intermittent wind generation has been receiving in the US to date are not sustainable with significant wind integration in the future. Studies of such future scenarios indicate that unless potential wind output is severely curtailed, system security will require additional reserves [5], [6], [7], [10], [13], [14], [17], [20], [22]. We describe below a model based on the reasonable assumption that wind generation forecasting organizations are capable of quantifying additional reserve requirements as a function of the day-ahead bids of wind farms in the vicinity of each bus node. In particular we define the additional reserve requirements during hour *t* that are caused by all wind generation bids during hour *t* as:

$${}^WR^t \triangleq \quad {}^t, M^t) \approx \sum_n [{}^Wg_n^t - {}^W\underline{g}_n^t] \frac{\partial \, {}^WR^t}{\partial \, {}^Wg_n^t}(M^t)$$

Note that the additional reserve requirements are approximated by the product of the wind generation bid exceeding the certain minimum generation forecast, ${}^W\underline{g}_n^t$, namely $[{}^Wg_n^t - {}^W\underline{g}_n^t]$, multiplied by the pre-computable marginal contribution to these reserves by wind generation in

[4] For a plug in hybrid electric vehicle the penalty per KWh of uncharged battery capacity would be related to the cost of gasoline needed to drive the equivalent distance. In the rare event that the cost of electricity required to charge the PHEV while it was plugged in exceeds that penalty, it would be more efficient to leave all or part of the battery uncharged. For a purely electric vehicle, the penalty per KWh of uncharged energy would probably be much higher since it has no alternative energy source. We assume that individual EV owners contract with an energy service company or a load aggregator who coordinates EV charging in location *n*(*i*).

[5] A reasonable value for ${}^{EV(24+)}v_{n(i)}^{24}$ is the Lagrange multiplier of ${}^{EV(\tau\geq 1)}x_{n(i)}^1$ in the optimization problems IIIA or IV. The multiplier can be easily estimated in a fast converging iterative process [8]

the vicinity of node n, $\frac{\partial\, {}^{W}R^{t}}{\partial\, {}^{W}g_{n}^{t}}(M^{t})$. We assume that wind generation forecasting entities will be able to quantify this marginal contribution as a function of the variance covariance matrix M^{t} of wind generation located in different nodes.

III. UNIFORM-BID-BASED MARKET PARTICIPATION

As already mentioned Flexible demand and Wind generation do not have an additively separable utility. Therefore, in the event that they must participate with UPQBs, they must first solve an individual optimization problem using expected ISO clearing prices and then make appropriate period specific UPQBs that are in fact self-scheduling bids since prices can be selected (very high or very low) so that the ISO schedules with very high probability the quantities bid by flexible demand participants.

We describe below two individual decision problems and the ISO decision problem.

A. The Flexible Demand Decision Problem

Given joint probability distributions of ISO energy and regulation service reserve clearing prices ${}^{F_j,n(i)}h_{{}^{E}\boldsymbol{\lambda}_n,{}^{R}\boldsymbol{\lambda}_n}(\lambda_n^E,\lambda_n^R \mid {}^{c}d_{n(i)}^{t}, {}^{F_j}d_{n(i)}^{t}, {}^{F_j,R}d_{n(i)}^{t}, \sum_{j'\neq j} {}^{F_{j'}}d_{n(i)}^{t},$ $\sum_{j'\neq j} {}^{F_{j'},R}d_{n(i)}^{t}, {}^{W}g_n^t\ \forall t,i,n)$
where
${}^{E}\boldsymbol{\lambda}_n = [{}^{E}\lambda_n^1, {}^{E}\lambda_n^2, ..., {}^{E}\lambda_n^{24}],\ {}^{R}\boldsymbol{\lambda}_n = [{}^{R}\lambda_n^1, {}^{R}\lambda_n^2, ..., {}^{R}\lambda_n^{24}]$,
each Load Aggregator (LA) representing flexible demand F_j at node n location i, $\forall t \in \{1,2,...,24\}$ will solve problem IIIA below:

$$\min_{{}^{F_j}d_{n(i)}^{t},\, {}^{F_j,R}d_{n(i)}^{t}, \forall j,t} \sum_{j,t} \{ \underset{\lambda_n^E,\lambda_n^R}{E}[{}^{E}\lambda_n^t\, {}^{F_j}d_{n(i)}^{t} - {}^{R}\lambda_n^t\, {}^{F_j,R}d_{n(i)}^{t}] - {}^{F_j}U_{n(i)}^{t}({}^{F_j}x_{n(i)}^{t})\}$$

Subject to 1.1, 1.2, 1.3, 1.4 $\forall t, j$.

The optimal solution of problem IIIA, ${}^{F_j}d_{n(i)}^{t*}, {}^{F_j,R}d_{n(i)}^{t*}$ can then form a price quantity bid paired with very high energy prices and very low – even negative – regulation service cost, so that the ISO selects them regardless of other participant bids. This is equivalent to the LA actually self-dispatching by self-scheduling ${}^{F_j}d_{n(i)}^{t*}, {}^{F_j,R}d_{n(i)}^{t*}$ in the day-ahead market.

Inspecting problem IIIA it is obvious that the full joint p.d.f. of clearing prices is superfluous. Instead, their mean values, ${}^{E}\overline{\boldsymbol{\lambda}} = [{}^{E}\overline{\lambda}_n^1, {}^{E}\overline{\lambda}_n^2, ..., {}^{E}\overline{\lambda}_n^{24} : n \in \{1,2,...,n,...N\}]$ and ${}^{R}\overline{\boldsymbol{\lambda}} = [{}^{R}\overline{\lambda}_n^1, {}^{R}\overline{\lambda}_n^2, ..., {}^{R}\overline{\lambda}_n^{24} : n \in \{1,2,...,n,...N\}]$ are sufficient. These mean values can be either estimated conditional upon onerous system information including ${}^{c}d_{n(i)}^{t}, {}^{F_j}d_{n(i)}^{t}, {}^{F_j,R}d_{n(i)}^{t}, \sum_{j'\neq j} {}^{F_{j'}}d_{n(i)}^{t}, \sum_{j'\neq j} {}^{F_{j'},R}d_{n(i)}^{t}, {}^{W}g_n^t$ $\forall t,i,n$, or, set equal to observed clearing prices on a similar previous day. In Section IIIC below we discuss whether these mean value estimates, used by flexible demand and wind farm participants to essentially self-dispatch, converge for all $F_j, n(i)$ and ${}^{W}g_n^t$ to a unique Nash Equilibrium, ${}^{E}\overline{\boldsymbol{\lambda}}, {}^{R}\overline{\boldsymbol{\lambda}}$.

Proposition 1: The solution to problem IIIA will schedule consumption and reserve offers to hours $t < \tau$ following a merit order according to the magnitude of ${}^{R}\lambda_n^t - {}^{E}\lambda_n^t$ modified only by binding local capacity constraints.

Proof: By inspection of problem IIIA.

B. The Wind Generation Decision Problem

For the same reason discussed above, the expected clearing prices, ${}^{E}\overline{\boldsymbol{\lambda}}, {}^{R}\overline{\boldsymbol{\lambda}}$ are sufficient for wind farms at node n to solve their individual self-dispatch problem IIIB:

$$\max_{{}^{W}g_n^t, \forall t} \{\sum_t {}^{E}\overline{\lambda}_n^t\ {}^{W}g_n^t - {}^{R}\overline{\lambda}_n^t[{}^{W}g_n^t - {}^{W}\underline{g}_n^t]\frac{\partial\, {}^{W}R^t}{\partial\, {}^{W}g_n^t}(M^t)\}$$

Subject to 1.7 $\forall t$

Note that once, ${}^{W}g_n^{t*}$, the optimal values to problem IIIB are selected, the wind farms at node n can bid a zero (or negative) energy cost to make sure that their bids will be accepted by the ISO. Again, this is equivalent to the wind farm actually self-scheduling in the day-ahead market. The issue of whether conditional clearing price means will converge for all $F_j, n(i)$ and ${}^{W}g_n^t$ to the same Nash Equilibrium ${}^{E}\overline{\boldsymbol{\lambda}}, {}^{R}\overline{\boldsymbol{\lambda}}$, is discussed in Section IIIC.

Proposition 2:Problem IIIB exhibits the bang-bang solution:

$${}^{W}g_n^{t*} = {}^{W}\underline{g}_n^t \text{ for } {}^{E}\lambda_n^{t,k} < {}^{R}\lambda_n^{t,k}\frac{\partial^{W}R^t}{\partial^{W}g_n^t}(M^t)$$

$${}^{W}g_n^{t*} = {}^{W}\overline{g}_n^t \text{ for } {}^{E}\lambda_n^{t,k} > {}^{R}\lambda_n^{t,k}\frac{\partial^{W}R^t}{\partial^{W}g_n^t}(M^t) \text{ and}$$

Indifference/singularity, ${}^{W}\underline{g}_n^t \le {}^{W}g_n^{t*} \le {}^{W}\overline{g}_n^t$,

when equality ${}^{E}\lambda_n^{t,k} = {}^{R}\lambda_n^{t,k}\frac{\partial^{W}R^t}{\partial^{W}g_n^t}(M^t)$ holds.

Proof: By inspection of problem IIIB.

C. The ISO Day-Ahead Market Clearing Problem under Uniform Bid Rules

Given self-scheduled flexible demand and wind quantity bids, ${}^{F_j}d_{n(i)}^{t*}, {}^{F_j,R}d_{n(i)}^{t*}$ and ${}^{W}g_n^{t*}$ obtained by the solution to the individual optimization problems IIIA and IIIB, the ISO will proceed to schedule conventional generation and demand by solving problem IIIC below:

$$\max_{{}^{c}d^{t}_{n(i)},\, g^{t}_{n(\gamma)},\, {}^{R}g^{t}_{n(\gamma)},\forall t} \sum_{t,i,\gamma}[{}^{c}u^{t}_{n(i)}\,{}^{c}d^{t}_{n(i)} - \bar{c}^{t}_{n(\gamma)} g^{t}_{n(\gamma)} - \bar{r}^{t}_{n(\gamma)}\,{}^{R}g^{t}_{n(\gamma)}]$$

subject to

-Energy Balance Constraints that yield energy clearing prices under the uniform bid rules, ${}^{E,u}\lambda^{t}_{n}$,

$$\sum_{n(\gamma)} g^{t}_{n(\gamma)} + \sum_{n} {}^{W}g^{t*}_{n} - \sum_{n(i)} {}^{F_j}d^{t*}_{n(i)} - \sum_{n(i)} {}^{c}d^{t}_{n(i)} = 0,\ \forall t \qquad \Rightarrow {}^{E,u}\lambda^{t}_{n} \qquad (2.1u)$$

-Regulation Reserve Requirements Constraints[6] that yield reserve clearing prices under the uniform bid rules ${}^{R,u}\lambda^{t}_{n}$

$$\sum_{n(\gamma)} {}^{R}g^{t}_{n(\gamma)} + \sum_{n(i)} {}^{F_j,R}d^{t*}_{n(i)} \geq {}^{c}R^{t} + \sum_{n}[{}^{W}g^{t*}_{n} - {}^{W}\underline{g}^{t}_{n}]\frac{\partial^{W}R^{t}}{\partial^{W}g^{t}_{n}}(M^{t}),\ \forall t \ \Rightarrow {}^{R,u}\lambda^{t}_{n} \qquad (2.2u)$$

-Conventional Generation capacity and ramp constraints

1.5a, 1.5b, 1.6 $\forall t$ (2.3)

-and Line Flow constraints

$$\underline{z}^{t} \leq H^{t}P^{t} \leq \bar{z}^{t} \quad \forall t \qquad (2.4)$$

where $\underline{z}^{t}, \bar{z}^{t}$ are Kx1 vectors of positive and negative line flow constraints, H^{t} the KxN line flow distribution matrix, $P^{t} \triangleq \quad ...P^{t}_{n},...,P^{t}_{N}]'$ with $P^{t}_{n} = g^{t}_{n} + {}^{W}g^{t}_{n} - d^{t}_{n}$

Discussion of Clearing Price Convergence

Consider the iterative solution of problems IIIA, IIIB, IIIC described below:

(*i*) sub-problems IIIA and IIIB solve for ${}^{F_j}d^{t*}_{n(i)}, {}^{F_j,R}d^{t*}_{n(i)}$ and ${}^{W}g^{t*}_{n}$ setting ${}^{F_j,E}\bar{\boldsymbol{\lambda}}_{n}, {}^{F_j,R}\bar{\boldsymbol{\lambda}}_{n}$ equal to clearing prices ${}^{E,u}\lambda^{t}_{n}, {}^{R,u}\lambda^{t}_{n}$ obtained from the most recent solution of problem IIIC.

(*ii*) IIIC is solved again with ${}^{F_j}d^{t*}_{n(i)}, {}^{F_j,R}d^{t*}_{n(i)}$ and ${}^{W}g^{t*}_{n}$ as inputs to obtain new clearing prices ${}^{E,u}\lambda^{t}_{n}, {}^{R,u}\lambda^{t}_{n}$

(*iii*) steps above are repeated with the new price estimates.

Propositions 1 and 2 imply that flexible demand loads will be scheduled predominantly in hours $t^{low,k}$ with low ${}^{E}\lambda^{t,k}_{n}$ and high ${}^{R}\lambda^{t,k}_{n}$, while ${}^{W}g^{t*}_{n}$ will be negatively correlated and scheduled in hours $t^{high,k}$ with high ${}^{E}\lambda^{t,k}$ and low ${}^{R}\lambda^{t,k}$. Each one of the above will result in new clearing prices from the solution of problem IIIC that will tend to switch the sets of $t^{low,k}$ and $t^{high,k}$ and $t^{low,k+1}$, $t^{high,k+1}$ hours. Moreover, this switching of low to high and high to low price hour sets will be reinforced by the negative correlation between flexible demand loads and wind schedules and result in *oscillatory behavior*.

Such oscillatory behavior has been observed and described for EV loads by Zhongjing et al, [26]. Zhongjing et al have also shown that the oscillations can be damped and made to converge by smoothing the evolution of prices. For example, using the assignment ${}^{E}\lambda^{t,k+1}_{n} := {}^{E}\lambda^{t,k}_{n} + fraction^{k}({}^{E}\lambda^{t,k+1}_{n} - {}^{E}\lambda^{t,k}_{n})$ and similarly for ${}^{R}\lambda^{t,k+1}_{n}$ with $fraction^{k}$ an appropriately diminishing step size with unbounded sum. The iterations described above can converge to a fixed price vector for most flexible demand types. We noticed, however, that according to proposition 2, when ${}^{E}\lambda^{t,k}_{n} = {}^{R}\lambda^{t,k}_{n}\frac{\partial^{W}R^{t}}{\partial^{W}g^{t}_{n}}(M^{t})$ and stepwise supply curves are employed, the individual wind generation scheduling optimization problem IIIB becomes singular and can not select a unique solution, since any convex combination of minimum and maximum wind generation is consistent with optimality. Although this is not a significant problem when each wind farm is relatively small or stepwise supply curves are regularized to remove first derivative discontinuities, it may be an issue in the absence of regularization and large wind farms. We discuss this further in the numerical results section.

Proposition 3. Under mild convexity and continuity assumptions[7], and steady state conditions across day ahead market clearing, the Nash Equilibrium fixed price vectors, ${}^{E}\bar{\boldsymbol{\lambda}}$ and ${}^{R}\bar{\boldsymbol{\lambda}}$, exist and are solution of the iterative finite step size gradient like diminishing step size algorithm described above. Moreover, the asymptotic mean of consecutive, similar-day clearing price samples converges to the Nash Equilibrium as well. More specifically, given the random sample of energy clearing price vectors ${}^{E}\boldsymbol{\lambda}^{1}, {}^{E}\boldsymbol{\lambda}^{2},...{}^{E}\boldsymbol{\lambda}^{k}, {}^{E}\boldsymbol{\lambda}^{k+1}, {}^{E}\boldsymbol{\lambda}^{k+2},...{}^{E}\boldsymbol{\lambda}^{K}$,

$${}^{E}\bar{\boldsymbol{\lambda}}_{K} = \frac{\sum_{k=1}^{K} {}^{E}\boldsymbol{\lambda}^{k}}{K} \underset{K\to\infty}{\to} {}^{E}\bar{\boldsymbol{\lambda}}, \text{ and similarly } {}^{R}\bar{\boldsymbol{\lambda}}_{K} \underset{K\to\infty}{\to} {}^{R}\bar{\boldsymbol{\lambda}}.$$

Proof: We claim that the gradient algorithm convergence to the Nash Equilibrium vector of prices, and that the Nash Equilibrium exists under mild conditions on convexity of costs in problem IIIC. Detailed arguments are not included due to space limitations. They follow arguments in [21] and results in [23]. Moreover, we also claim, again without detailed presentation arguments due to space limitation, that the Nash price equilibrium is the vector of the means of the

[6] ${}^{c}R^{t}$ is the contingency planning reserve requirement during hour t disregarding wind bid related requirements.

[7] Regularization can easily be introduced to insure continuity and strict convexity in cases of step-wise demand and supply functions.

associated density functions, $\forall t, i, n$, related to the potential function in dynamic games with many players [1], [2]. Following arguments in [1], [2] the price means can be shown to be the asymptotic sample average defined above.

The next section shows that the Nash equilibrium prices can be obtained by the market operator if it clears the market on the basis of complex bids by flexible demand and intermittent generation from wind farms.

IV. COMPLEX-BID-BASED MARKET PARTICIPATION

The day-ahead market clearing problem under complex bids by flexible demand and intermittent generation wind farms is the solution to problem IV below:

$$\max_{{}^{c}d^{t}_{n(i)}, g^{t}_{n(\gamma)}, {}^{R}g^{t}_{n(\gamma)}, {}^{F_j}d^{t}_{n(i)}, {}^{F_j,R}d^{t}_{n(i)}, {}^{W}g^{t}_{n(\gamma)} \forall t,\gamma,i,\tau} \sum_{t,\gamma,t}[{}^{c}u^{t}_{n(i)}\,{}^{c}d^{t}_{n(i)}$$
$$-\overline{c}^{t}_{n(\gamma)}g^{t}_{n(\gamma)} - \overline{r}^{t}_{n(\gamma)}\,{}^{R}g^{t}_{n(\gamma)} + {}^{F_j}U^{t}_{n(i)}({}^{F_j}x^{t}_{n(i)})]$$

Subject to: Energy balance constraints yielding complex-bid-based energy clearing prices denoted by ${}^{E,x}\lambda^{t}_{n}$

$$\sum_{n(\gamma)} g^{t}_{n(\gamma)} + \sum_{n} {}^{W}g^{t}_{n} - \sum_{n(i)} {}^{F_j}d^{t}_{n(i)} - \sum_{n(i)} {}^{c}d^{t}_{n(i)} = 0, \quad \forall t \qquad \Rightarrow {}^{E,x}\lambda^{t}_{n} \tag{2.1x}$$

Regulation Reserve Constraints yielding complex-bid-based reserves clearing prices denoted by ${}^{R,x}\lambda^{t}_{n}$

$$\sum_{n(\gamma)} {}^{R}g^{t}_{n(\gamma)} + \sum_{n(i)} {}^{F_j,R}d^{t}_{n(i)} \geq {}^{c}R^{t} + \sum_{n}[{}^{W}g^{t}_{n} - {}^{W}\underline{g}^{t}_{n}]\frac{\partial\, {}^{W}R^{t}}{\partial\, {}^{W}g^{t}_{n}}(M^{t}), \forall t \Rightarrow {}^{R,x}\lambda^{t}_{n} \tag{2.2x}$$

Conventional Generation capacity and ramp constraints
1.5a, 1.5b an 1.6 $\forall t$, (2.3)

Line Flow constraints as defined in section IIIC,

$$\underline{z}^{t} \leq H^{t}P^{t} \leq \overline{z}^{t} \quad \forall t \tag{2.4}$$

and the individual optimization constraints of problems IIIA and IIIB,

1.1, 1.2, 1.3, 1.4 and 1.7 $\forall t, j$ (2.5x)

Proposition 4. Under mild convexity conditions, the solution to problem IV yields a Nash equilibrium that optimizes the sum of participant objective functions, and, as such, no participant benefits from making an incremental move away from it. This market clearing solution coincides with and is obtained more robustly and reliably than the gradient algorithm discussed under section IIIC and proposition 3.

Proof: Skipping the detailed arguments due to lack of space, we provide the following proof steps: (*i*) We introduce the socialized costs of non-wind related reserve requirements as a fixed average payment by conventional demand and write a corresponding individual optimization problem, we then (*ii*) use appropriate Lagrange multipliers to append constraints to the objective function of problem IV, and finally (*iii*) derive a version of problem IV which optimizes the sum of the individual market participant optimization objective functions. Using results in [23] we conclude that the solution of problem IV coincides with the Nash equilibrium in the multiple market participant game. To guarantee that the optimal solution to IV exists, we need costs to be convex and benefits concave. This condition is satisfied in our formulation for a given commitment of conventional generating units with the mild additional assumption that the wind farms reserves requirements functions is not strictly concave in wind bids, so that the we can guarantee convexity of net costs.

V. NUMERICAL RESULTS

We employ a three bus system. Each bus feeds three distinct distribution network locations each with conventional and flexible EV charging demands. One location is predominantly commercial with EV departure times in the evening and two predominantly residential with departure times in the morning. 15 conventional generating units with variable costs ranging from \$20/MWh to \$100/MWh feed the transmission busses and a single wind generation farm is located at each bus. To demonstrate the ability of complex bid based markets to schedule wind generation optimally and uniquely under the singularity condition discussed in section IIIB, we employ identical wind farm marginal contributions to reserve requirements.

Figures 1 presents 24 hour trajectories of total (i.e. summed over all bus nodes and distribution locations) flexible demand consumption and reserve offers (MW axis), as well as energy and reserve clearing prices (same for all nodes since no line flow constraint is binding in the reported scenario). Figure 2 presents total wind farm generation as scheduled by the solution of problem IV.

It is interesting to note that wind generation is scheduled at its maximum potential output when energy prices are higher than reserve prices. The rest of the hours, the singularity condition holds and wind is scheduled in-between its maximum and minimum generation potential.

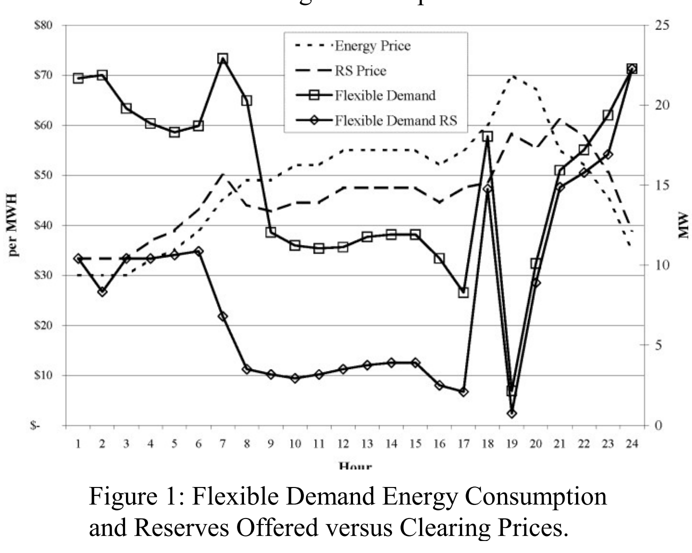

Figure 1: Flexible Demand Energy Consumption and Reserves Offered versus Clearing Prices.

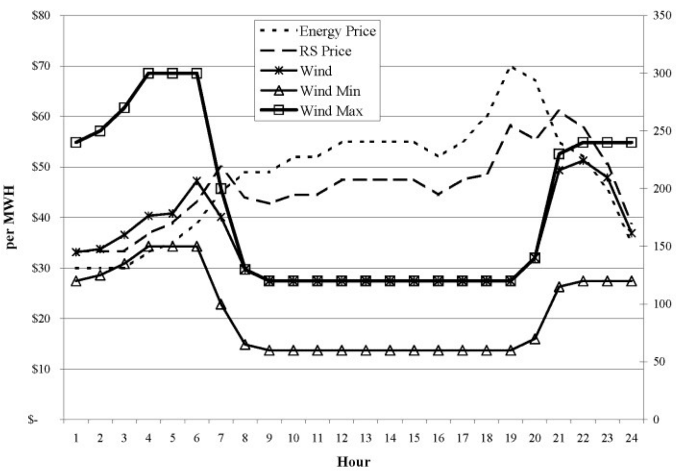

Figure 2: Wind Bids versus Clearing Prices

VI. CONCLUSION

We claim that the Nash Equilibrium clearing prices can be calculated by solving a single optimization problem described in Section III.C which clears the day-ahead market with flexible demand and wind farm market participants expressing their utility through complex bids that we define next in section III.C.

We propose complex bids to power markets by flexible demand and intermittent generation wind farm participants. While complex bids do not compromise the tractability of clearing multi period day-ahead markets they reflect the true utility, costs and capabilities of flexible demand and wind farm market participants. We show that current practice requiring all market participants to participate through UPQBs is equivalent to forcing them to pretend that they have consumption utility that is additively separable. To make this translation from the actually non-additive to a fictitious additive utility, participants use estimated market clearing prices to optimize their preferred energy consumption, reserves and wind generation bids and associate with them extreme price bids so as to essentially self-schedule. This paper proves and demonstrates with numerical examples that complex bids are superior in terms of both maximizing social welfare and stabilizing power markets.

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