# Uninformative Advertising as an Invitation to Search

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## Uninformative Advertising as an Invitation to Search Abstract

The choice of content in an advertising message is a critical managerial decision. In this paper, we investigate under what circumstances the firm prefers to include a product attribute-based appeal in its ad (i.e.: "No Hassel Rewards") versus an appeal with no direct information on product attributes (i.e.: "My Life, My Card, American Express.") Since attribute-based messages are meant to inform consumers of the product's high value, here we focus on how the content decision is impacted by the product's quality. One intuitive hypothesis is that the high-quality product would choose to emphasize its attributes. However, the limited bandwidth of advertising media implies that a firm is limited in the number of attributes that it can communicate in its message. Hence, an attribute message may not differentiate a truly excellent product from an average one. We show that there can exist an equilibrium where a high-quality firm chooses to produce messages devoid of any information on product attributes in order to encourage the consumer to engage in search, which is likely to uncover positive information. Hence, an uninformative message can serve as a signal of confidence on the part of the firm. An average firm that imitates this strategy risks to lose its customer in cases when she uncovers negative information as part of the search. Therefore, an average firm may choose to engage in an attribute-based appeal, despite the fact that this perfectly reveals its type. While most of the previous literature has focused on the decision to advertise as a signal of quality, here we show that message content, coupled with consumer search, can also serve as a credible signal of quality.

Keywords: Advertising; advertising content, quality signal, signaling, consumer search.

## 1 Introduction

For the past forty years, economists and marketers have studied how advertising can help consumers learn about products. The information that advertising provides can be direct, such as the existence of the product or its price (for example, see Grossman and Shapiro 1984). Also, the information can sometimes be indirect, where the mere fact that the firm advertises signals an experience good's high quality (see Nelson 1974 and Milgrom and Roberts 1986). The latter is known as the "money burning" theory of advertising.

One of the important and surprising take-aways of the money burning theory is that it is the level of spending that signals the quality of the product, and not the content of the message. That is, content is irrelevant for conveying information on product quality for an experience good. However, a quick look at trade publications such as *Ad Age* and *Ad Week* confirms our intuition as consumers that content is an important driver behind advertising persuasiveness, and one to which firms pay close attention. In this paper, we revisit the result that advertising is irrelevant for signaling quality and investigate whether and how advertising content can convey information on product quality in a rational framework.

In particular, we ask when a firm would choose to mention specific product attributes as opposed to making vague claims in its advertising. We will refer to a campaign that emphasizes product attributes as "attribute-based" advertising. By definition, this type of advertisement contains "hard" information (Tirole 1986) about product benefits and, hence, the claims are credible and verifiable. In contrast we will refer to a campaign that does not emphasize product attributes as "uninformative" advertising.<sup>1</sup>

As an example of attribute-based advertising, consider the credit card issuer Capital One's "What's in Your Wallet?" campaign. One ad featuring the comedian David Spade focused on the difficulty involved in claiming rewards from the competing cards as opposed to the "No Hassle Rewards" card. The ad ended with the statement, "No annual fee! There are no blackout dates on

<sup>&</sup>lt;sup>1</sup>Note that in the advertising literature, authors differentiate between various non-attribute-based appeals, such as feelings-based or emotional appeals or image-based advertisements. In this paper, we group all non-attribute-based forms of advertisements into the "uninformative" rubric. That is, if an advertisement mentions product attributes, we refer to it as attribute-based. If it does not, we refer to it as uninformative.

any airline at any time." In contrast, the 2004 American Express "My Life. My Card." campaign did not directly mention any of the benefits of owning an American Express card, such as the card's excellent rewards program. For example, one ad featured Robert De Niro reciting a "love letter" to New York City. The brand was mentioned only in the closing line, "My life happens here. My card is American Express." Moreover, the practice of avoiding mentioning product attributes is fairly widespread: Abernethy and Butler (1992) find that 37.5% of U.S. TV advertising have no product attribute cues. We will refer to these types of messages as uninformative advertising.<sup>2</sup>

How will these different types of advertising campaigns affect consumers' inference about product quality? What is the relationship between product quality and the firm's decision to make attribute-based claims? One intuitive hypothesis is that the high-quality product would choose to emphasize its product benefits which are, by definition, strong. However, the limited bandwidth of communication inherent in any form of advertising implies that a firm can talk about only a small subset of its product's attributes. It is impossible for the firm to accurately communicate all of the features associated with its product in a 30-second commercial or a print ad (Shapiro 2006, Bhardwaj et al. 2008). Hence, if a firm claims to be good on a few selected attributes, its advertising will be indistinguishable from the advertising of the firm that is *only* good at those attributes. If, on the other hand, the firm makes no attribute-based claims and engages in uninformative advertising, its advertising will be indistinguishable from the advertising of a firm that cannot deliver high quality on any attributes.

For example, Sony Cybershot DSCW300 was ranked number one in *Consumer Reports* in the subcompact digital camera category. This camera is high on quality in many attributes such as high megapixel, image quality, versatality, dynamic range and so on (its webpage displays a list of superior attributes that is two pages long). Clearly, it cannot emphasize all of its attributes in a 30-second commercial. If Sony decides, for example, to emphasize high megapixel or high image quality on its ad, it cannot distinguish itself from Panasonic Lumix DMC-FX35 which has exactly the same megapixel and image stability function but does not perform as well as Sony in

<sup>&</sup>lt;sup>2</sup>In practice, it is not realistic to say that an ad is either attribute-based or uninformative. It is a matter of degree. However, for simplicity, we specify that a firm can choose to have either an "attribute-based" or an "uninformative" ad as an approximation of what is really a continuum.

the versatility dimension.<sup>3</sup> Instead, if Sony chooses to emphasize the versatility dimension, it looks very similar to Nikon Coolpix S220, which is only good at versatility dimension but not any other dimensions.

The argument above highlights the point that the firm may not be able to entirely resolve the uncertainty about its product through advertising alone when there is limited bandwidth in advertising communication. In practice, a consumer who is uncertain about the product's features following exposure to advertising has an additional recourse available to her: she can conduct her own research to discover the product's quality. For example, she can consult her friends, search online reviews, or read *Consumer Reports* before making the purchase decision. Hence, advertising with no information on features may serve as an invitation for the consumer to search on her own. Therefore, the high quality firm may actually prefer to pool with the low quality firm by engaging in uninformative advertising since it is confident that if the consumer chooses to search for additional information, the information uncovered will be positive. In contrast, an average firm that imitates this strategy risks losing its customer in cases when she uncovers negative information as part of her search. Therefore, an average firm may choose to engage in an attribute-based appeal, despite the fact that this perfectly reveals its type. We contrast this to the equilibrium where both the average and the high type engage in attribute-based advertising.

In this paper, we formalize the above argument and develop a framework to analyze simultaneously the choice of advertising content (whether to emphasize an attribute or not) and pricing decisions of a monopolist. Consumers are uncertain about the quality of a product sold by a monopolist. Advertising content can change the prior beliefs of consumers about the quality of the product. Consumers are sophisticated, so that they update their beliefs about the product quality based on observed advertising message and prices. Moreover, they may engage in their own search only if it benefits them. By taking into account the fact that consumers may endogenously engage in their own search, which serves as an additional source of noisy information about product quality, a firm can signal its product quality through its advertising content choice above and beyond the effect of money burning.

The paper is organized in the following manner. In Section 2, we relate our paper to the existing

<sup>&</sup>lt;sup>3</sup>Consumer Reports 2008, Subcompact Digital Camera

literature in economics and marketing. Section 3 presents the model set-up and the results of the main model. We discuss the characteristics and implications of equilibria in more detail in Section 4 and conclude in Section 5.

### 2 Literature Review

The role of advertising in markets has been extensively studied both in marketing and economics (for a comprehensive review on economics literature on advertising, see Bagwell 2007). First, our work relates to the existing models which view advertising as a means of conveying information about the quality of experience goods (Nelson 1974, Kihlstrom and Riordan 1984, Milgrom and Roberts 1986). In these models, the seller of a high quality product optimally spends more resources on advertising since it can derive a greater benefit from advertising through repeat purchase. Hence, advertising becomes a credible signal of quality. These models share the common feature that advertising content plays no role, and the firms signal product quality only indirectly through conspicuous money-burning.

In contrast, our model puts particular emphasis on the role of advertising content (i.e., whether a message is attribute-based or uninformative) as a potential quality signal. In other words, how a firm burns money also matters in signaling product quality. Several previous studies also consider advertising content (Butters 1977, Grossman and Shapiro 1984). These models allow the firm to announce the existence of the product or its price through advertising. Moreover, some studies look at the issue of credibility of price claims in advertising content (Simester 1995, Shin 2005). However, none of these models consider content other than prices as a potential signal on quality. Two recent exceptions to the earlier view of advertising content are Anderson and Renault (2006) and Anand and Shachar (2007). The former looks at the possibility of the firm informing consumers about product attributes through advertising content in a context where consumers are imperfectly informed about product characteristics. Although consumers always learn their true match value before buying, the possibility of a hold-up problem by the firm implies that advertising that provides either match or price information alone is optimal in different cases. Anand and Shachar (2007) show that advertising content can enable quality signaling by influencing out-of-equilibrium beliefs. They present a duopoly model in which advertising content provides direct but noisy information. Moreover, they show that the firms can send information indirectly through their willingness to provide information. Finally, Mayzlin (2006) shows that in equilibrium, a consumer may be influenced by an anonymous online product recommendation, which results from unbiased word of mouth or is manufactured by the firm. Hence, in this work the content of the message is crucial.

This paper models the quality signaling through advertising content choice by explicitly modeling the firm's incentive to reveal or conceal its product information in an advertising. Kuksov (2007) studies the incentives of consumers to reveal or conceal information about themselves to others in the consumer matching context where consumers can communicate with each other using product brands and conversations. He analyzes the value of brand image in social communication over and above cheap talk conversations among consumers. Yoganarasimhan (2009) obtains a similar result that a firm sometimes conceals information about its product to increase its social value. Also, Sun (2009) studies the monopolistic seller's incentive to disclose the horizontal matching attribute of the product.

Formally, the model we present here is most closely related to the literature on counter-signaling (Teoh and Hwang 1991, Feltovich et al. 2002, and Araujo et al. 2008) where signals are not monotonic in sender's type. Contrary to the standard signaling models where high types send a costly signal to separate themselves from the low types, in counter-signaling models high types sometimes avoid the signals that could separate them from the lowest types. In fact, the highesttype senders may be understated rather than overstated in their signaling behavior (Feltovich et People of average abilities, for example, get more education than bright people in al. 2002). labor markets (Hvide 2003). Mediocre firms reveal their favorable earning information while both high quality and low quality firms tend to conceal their earning information in financial market (Teoh and Hwang 1991). Feltovich et al. (2002) formalize this counter-signaling intuition and show that in the presence of a noisy external signal, the high type may pool with the low type, while the medium type prefers to separate. All these counter-signaling models assume that there exists external information other than firm's signal which enables the high type separate from the low type. In this sense, the external information is a prerequisite for the existence of countersignaling. This extra information, therefore, can be regarded as a second signal in the literature on multidimensional signals (Quinzii and Rochet 1985, Engers 1987).

While the model we present is a counter-signaling model in that the high and low types pool together, and an additional source of information is important in enabling separation, there are several important differences between the current work and the previous literature. First and most importantly, in our model the consumer decides whether to search for additional information depending on the prices and the content of the advertising message, while in Feltovich et al. (2002) the player is assumed to *always* receive the second signal. In fact, the cost of search in our model is an important determinant of the type of equilibrium that is played. Second, unlike the earlier papers, we focus on the advertising context. This implies, for example, that we have to include issues such as product price in the model, which were not relevant in the earlier models which were placed in a labor context.

Also, there are several studies which address the similar intuition with the counter-signaling in different context. The pooling behavior of high and low types can be derived from the different desire for conformity (herding behavior) of the mediocre type. In sociology, the idea that conformity is high in the middle yet low at the top and bottom of a status hierarchy is called middle-status conformity (Phillips and Zuckerman 2001). Since people in a high status feel confident in their social acceptace, they dare to deviate from conventional behavior (Hollander 1958). At the same time, low-staus people feel free to defy accepted behavior because they are excluded regardless of their actions. Only the middle class people fear exclusion from the group. Such insecurity experienced by middle-status people fuels conformity in their behavior (Dittes and Kelley 1956).

Substantively, our work contributes to the advertising literature by offering a new perspective on uninformative advertising (such as image advertising) based on a rational model of consumer behavior. There are, of course, a number of alternative explanations in marketing which consider uninformative advertising (in particular, image advertising) as effective persuasion communication from the psychological perspective. In particular, many marketing studies (Carpenter et al. 1994, Brown and Carpenter 2000) show that irrelevant information that arouses an emotionally favorable response sometimes leads to more effective persuasion in consumer choice. These behavioral models emphasize the importance of both cognitive and emotional response to the messages presented in advertising content (Kahneman and Tversky 1982, Scott 1994, Mullainathan and Shleifer 2005, Mullainathan et al. 2007). Many advertising practitioners and scholars, therefore, recognize that products are bought often on the basis of emotional factors, and that emotional appeals can be more effective than rational appeals (Holbrook and O'Shaughnessy 1984, Rosselli et al. 1995, Albers-Miller and Stafford 1999). Since we predict that firms may choose to engage in (non attribute-based) uninformative advertising even in the absence of these psychological forces, our work complements these explanations.

## 3 Model

The game consists of one firm and one consumer. There is an informational asymmetry about the quality of the monopolist's product: the firm knows its product's quality while the consumer must infer the product's quality from signals that she receives from the firm as well as information that she may obtain on her own. To model quality, we use the concept of a discrete match between the product and consumer (Wernerfelt 1994, Godes 2003, Bhardwaj et al 2008). That is, we equate quality with the product's ability to frequently meet the customer's needs, regardless of the exact circumstances.

In particular, suppose that the product consists of two attributes:  $\alpha \in \{A, a\}, \beta \in \{B, b\}$ , where the capital letter stands for higher quality on that dimension. There are two possible states of the world,  $\chi \in \{1, 2\}$ , where either state is equally likely a priori. If  $\chi = 1$ , only attribute  $\alpha$ impacts the customer's experience. Similarly, if  $\chi = 2$ , only attribute  $\beta$  matters. Neither the customer nor the firm can predict the future state of the world. For example, suppose that Bob is considering buying a jogging stroller for his newborn daughter. If he ends up using the stroller mostly for running in his neighborhood, then it would be important for him that the stroller has good shock absorption. However, if he also ends up often driving with his child to the mall, it is important that the stroller be able to fold compactly in order to fit in the trunk of his Jetta. Since this is Bob's first child, he can not accurately predict which mode will be more likely. Similarly, when consumers purchase a personal computer, they do not know whether the CPU speed or the memory is the more important attribute.

The product utilities in the two states of the world are

$$V_{\chi=1} = \left\{ \begin{array}{c} \overline{V} \text{ if } \alpha = A \\ 0 \text{ otherwise} \end{array} \right\}, V_{\chi=2} = \left\{ \begin{array}{c} \overline{V} \text{ if } \beta = B \\ 0 \text{ otherwise} \end{array} \right\}$$

We assume that an attribute is equally likely to be high or low quality, and that there may be correlation between levels of the two attributes:  $P(\beta = B | \alpha = A) = P(\alpha = A | \beta = B) = P(\beta = b | \alpha = a) =$  $P(\alpha = a | \beta = b) = \rho$ , where  $0 < \rho < 1$ . Hence, there are four possible types ( $\theta$ ) of products based on the quality levels of the attributes:  $\theta \in \{H, M_{\alpha}, M_{\beta}, L\} = \{\{A, B\}, \{A, b\}, \{a, B\}, \{a, b\}\}$ , with the a priori probabilities of  $(\frac{\rho}{2}, \frac{1-\rho}{2}, \frac{1-\rho}{2}, \frac{\rho}{2})$  respectively.<sup>4</sup> A priori, *H*-type product delivers utility  $\overline{V}$  to a customer with probability 1, products  $M_{\alpha}$  and  $M_{\beta}$  deliver utility  $\overline{V}$  with probability  $\frac{1}{2}$  and utility of 0 with probability  $\frac{1}{2}$ , and *L* always delivers 0 utility. Hence, all else equal, a consumer would prefer *H* to *M*, and *M* to *L*. Due to this property, *L* type wants to imitate *H* and *M*; *M* type wants to separate itself from *L* and imitate *H*, and *H* wants to separate itself from *M* and *L*.

The firm can communicate to the consumer through advertising. We assume that the cost of advertising is zero.<sup>5</sup> We also assume that the firm must advertise in order to inform the consumer of its product's existence. These two assumptions imply that the firm always chooses to advertise. This allows us to focus on the role of content in advertising above and beyond the well-known effect of money burning where the firm can signal that it is high type by engaging in advertising. More-over, while our model primarily deals with the quality-signaling role of advertising, it is important to note that firms also consider awareness effect of advertising as well in reality.

The firm's action space consists of two possible advertising choices. First, the firm can choose an ad that centers around the product's attributes. Since this advertising contains hard information about specific attributes, we refer to it as "attribute-based" advertising. Here, we impose a truth-telling assumption in that the firm cannot claim to be high quality on an attribute on which it is in fact low quality. While we acknowledge that advertisers often exaggerate their claims, the Federal Trade Commission does require that "advertising be truthful and non-deceptive" and that all claims must have a "reasonable basis."<sup>6</sup>

Due to the limited bandwidth inherent in a communication medium such as TV, we allow the firm to transmit information about only one attribute - either  $\alpha$  or  $\beta$ :  $a = a_j$ , where  $j \in (\alpha, \beta)$ .

<sup>&</sup>lt;sup>4</sup>Note that if  $\rho = 1$  (perfect positive correlation), only  $\{A, B\}$  and  $\{a, b\}$  products exist, and if  $\rho = 2/3$ , all products are equally likely.

<sup>&</sup>lt;sup>5</sup>The results of the model are not qualitatively affected by the presence of an advertising cost, as long as it is not too large.

<sup>&</sup>lt;sup>6</sup>http://www.ftc.gov/bcp/conline/pubs/buspubs/ad-faqs.shtm

In practice, a product contains a large number of features. However, given the constraints on the time available for communication as well as limited consumer resources available to process the advertisement information (Shapiro 2006), the firm is only able to communicate about a small subset of these features (Bhardwaj et al 2008). If the firm chooses to emphasize specific product attributes in its ads, one can think of the set of advertised features as  $\alpha$ , and the set of the unadvertised features as  $\beta$ .

In contrast to attribute-based advertising, a firm can choose not to emphasize any particular attribute:  $a = a_0$ . We refer to this as "uninformative" advertising. These appeals may be image-based or emotional-based. Here we focus on how the absence of hard information may benefit the firm by encouraging the consumer to invest in own information search. Hence, we model the choice between an attribute-based and an uninformative ad. In Table 1 we summarize the possible types and the actions available to them.

	A	A		D 11 4 1	
Product Type	Attribute $\alpha$	Attribute $\beta$	Expected Utility	Possible Ads	Price
L	a	b	0	$a_0$	$p \ge 0$
$M_{lpha}$	A	b	$\overline{rac{V}{2}}$	$a_0, a_{lpha}$	$p \ge 0$
$M_{eta}$	a	В	$rac{\overline{V}}{2}$	$a_0, a_eta$	$p \ge 0$
H	A	В	$\overline{V}$	$a_0, a_lpha, a_eta$	$p \ge 0$

 Table 1: All Types and Possible Actions

We also assume that prices are observable to the consumer at the start of the game:  $p_L, p_M$ and  $p_H$ . Note that price-related information is usually more easily obtainable than information on quality. In out model, the consumer is assumed to observe price information prior to engaging in search (Meurer and Stahl 1994). Otherwise, a hold-up problem can occur (see Wernerfelt 1994).

After the consumer receives the advertising message, she can choose to invest a cost c in order to research the attributes of the product. After incurring this cost of search, consumers obtain extra noisy information about the product quality. This may involve searching for online reviews (Chen and Xie 2005, Chevalier and Mayzlin 2006), observing word-of-mouth (Chen and Xie 2008, Godes and Mayzlin 2004), reading *Consumer Reports*, or doing other types of search activities<sup>7</sup>.

<sup>&</sup>lt;sup>7</sup>Although this additional search can can be noisy because of several factors such as idiosyncratic individual

The binary signal about the product's quality obtained through this search,  $s \in \{\underline{s}, \overline{s}\}$ , is related to its quality level,  $q \in \{L, M, H\}$ , according to the following probabilities:

$$\Pr(\overline{s}|q) = \gamma_q, \text{ where } q \in \{L, M, H\}$$

$$\gamma_L < \gamma_M < \gamma_H.$$
(1)

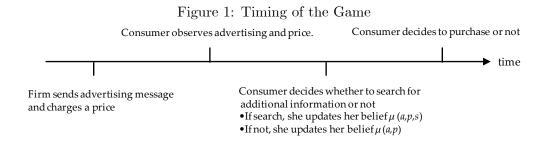
Here  $\overline{s}$  denotes positive news and  $\underline{s}$  denotes negative news. Note that the information the consumer receives through search is potentially richer than the information she can obtain after viewing an ad. The binary signal above can be viewed as a summary of all the attributes.

The firm knows that the consumer can obtain this extra information with the above probabilities, but does not observe what signal the consumer ultimately receives. The signal space of each type has the same support so that no signal is perfectly informative. Also, Equation (1) implies that the higher quality firm is more likely to produce better information. This amounts to a MLRP (Monotone Likelihood Ration Property) assumption over the signal space over types. In other words, positive news ( $\bar{s}$ ) is really "good news" regarding the firm's quality (Milgrom 1981).

After the consumer receives information regarding the product (through either advertising, prices, or own research), she forms a belief on the quality of the product. Here, we signify by  $\Omega$  the consumer's information set, and by  $\mu(\Omega)$  the consumer's belief. In particular,  $\mu(\Omega) =$  $(\mu_L(\Omega), \mu_M(\Omega), \mu_H(\Omega))$ , where  $\mu_L(\Omega) = P(L|\Omega), \mu_M(\Omega) = P(M|\Omega), \mu_H(\Omega) = P(H|\Omega)$ . The consumer's information set ( $\Omega$ ) includes the observation of advertising (a), price (p), and consumer's own search (s) if that takes place. That is, if the consumer performs own search, then  $\Omega = \{a_j, p, s\}$ for a firm that advertises an attribute, and  $\Omega = \{a_o, p, s\}$  for a firm that employs uninformative advertising. If, on the other hand, no consumer search takes place, then  $\Omega = \{a_j, p\}$  for a firm that advertises an attribute, and  $\Omega = \{a_o, p\}$  for a firm that employs image advertising.

The consumer then decides whether to purchase the product at its posted price based on the posterior belief on its quality:  $\mu(a, p, s)$  in the case of consumer research, and  $\mu(a, p)$  in the case of preference, promotional chat generated by firms in online space (Mayzlin 2006), it is by and large informative. For example, when we search for online review at amazon.com for Sony Cybershot DSCW300, it shows that two extreme reviews were most helpful: one most favorable review states that "Great Camera" while the most critical review states that "Worst piece of crap I've ever owned!" Though overall reviews were highly favorable (average 4.9 out of 5 points), it is still not perfect. This example highlight the point that consumer search can be informative but noisy.

no search. We assume that a consumer who is indifferent between purchasing and not purchasing the product chooses to purchase it. The timing of the model can be summarized as follows:



### 3.1 Perfect Bayesian Nash Equilibrium

We start with the consumer's problem and then turn to the firm's strategy. If the equilibrium played is a perfectly separating one, then the consumer can perfectly infer the product's quality from observing the price and advertising. However, if several types pool in equilibrium, the consumer may still be uncertain about the firm's type even after observing the price and advertising. The consumer in turn can either (1) forego search for additional information and make a purchase decision based on her belief,  $\mu(a, p)$ , or (2) search for additional information s at a cost c. In the absence of additional search, the consumer buys the product if and only if  $E(V|\mu) - p \ge 0$ . That is, she buys the product if the prior belief is relatively favorable or the price is relatively low. The consumer will search for additional information if

$$EU(\text{search}) \ge EU(\text{no search}) \equiv \max(0, E(V|\mu) - p).$$
 (2)

Note that the consumer undertakes a costly search only if her decision to purchase would differ depending on the outcome of the signal (i.e., there must be value in the information received). In other words, when a consumer chooses to search, she buys only if the signal is high  $(s = \overline{s})$ . The conditions for when the consumer chooses to search are specified in the following Lemma:

Lemma 1 (Consumer search)

1. If  $E(V|\mu) - p \ge 0$ , the consumer will search for additional information iff

$$c \le \Pr(\underline{s}|\mu)[p - E(V|\mu, \underline{s})] \Leftrightarrow E(V|\mu, \underline{s}) + \frac{c}{\Pr(\underline{s}|\mu)} \le p$$
(3)

2. If  $E(V|\mu) - p < 0$ , the consumer will search for additional information iff

$$c \le \Pr(\overline{s}|\mu)[E(V|\mu,\overline{s}) - p] \Leftrightarrow p \le E(V|\mu,\overline{s}) - \frac{c}{\Pr(\overline{s}|\mu)}$$
(4)

Moreover, when  $p = E(V|\mu)$ ,  $\Pr(\underline{s}|\mu)[p - E(V|\mu, \underline{s})] = \Pr(\overline{s}|\mu)[E(V|\mu, \overline{s}) - p]$ .

### **Proof.** See Appendix

Equations (3) and (4) compare the marginal cost and the marginal benefit of search. The marginal cost of search (the left hand side of equations (3) and (4)) is c. The marginal benefit is represented by the right hand side of these equations and differs depending on the price. If  $E(V|\mu) - p \ge 0$ , the consumer would choose to buy the product based on the prior alone in the absence of an additional signal. Hence, the marginal benefit of search is in preventing purchase in the case when the signal is negative  $(s = \underline{s})$ . On the other hand, when  $E(V|\mu) - p < 0$ , the consumer would not purchase a product in the absence of an additional signal. Therefore, the marginal benefit of search is in enabling the consumer to purchase a product in the case when the signal is negative  $(s = \underline{s})$ . Note that if conditions in Equations (3) and (4) are satisfied, then (2) holds. Hence, Equations (3) and (4) are necessary and sufficient to ensure that search takes place.

One implication of Lemma 1 is that given a belief, the consumer chooses to search for additional information only if the product's price is within a certain range (see Lemma 2 in the technical Appendix for more details). Hence, we can identify the range of prices and beliefs that ensures the existence of consumer search, which plays an important role in our refinement of belief (we will discuss the D1 refinement below in more detail). For example, the Figure 2 below illustrates the search condition for the case when the consumer is not certain whether the firm is type H or type M. In the Figure, the belief  $\mu_H$  (the probability that the product is H-type) is graphed on the x-axis (where  $0 \le \mu_H \le 1$ ).

For a given belief  $(\mu_H)$ , if the price is low enough  $(p < \underline{p}(\mu_H))$ , the consumer prefers to buy the product without further search (see point D in Figure 2). As we mentioned in our discussion of Lemma 1, at relatively low levels of p, i.e.,  $p \leq E(V|\mu)$ , the value of additional search is in preventing purchase when the outcome of search is negative, which in this case is captured by  $p - E(V|\mu, \underline{s})$ . Hence, when p is low, the marginal benefit of search is not high enough to justify its marginal cost. At any point on the convex curve  $p = \underline{p}(\mu_H)$ , the consumer is indifferent between

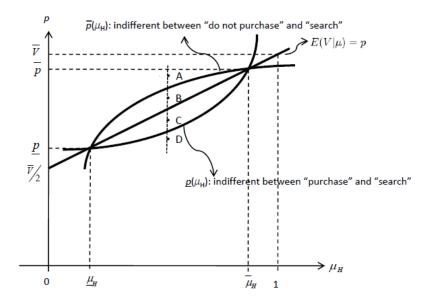


Figure 2: Consumer beliefs and optimal response behaviors

buying without search or engaging in further search. At a higher price  $(\underline{p} < p(\mu_H) < \overline{p})$ , the consumer prefers to search for further information (see points B and C). On the other hand, at any point on the concave curve  $p = \overline{p}(\mu_H)$ , the consumer is indifferent between no purchase and engaging in further search, and at  $p > \overline{p}(\mu_H)$  the price is so high that the consumer surplus obtained in the case when the outcome of search is positive  $(E(V|\mu, \overline{s}) - p)$  is not high enough to justify the cost of search (see point A). As we can see from the Figure, given  $\mu_H$ , the consumer chooses to search for additional information only if  $p \in [\underline{p}(\mu_H), \overline{p}(\mu_H)]$ . Moreover, if the belief is extreme  $(\mu_H < \underline{\mu}_H \text{ or } \mu_H > \overline{\mu}_H)$ , the consumer does not engage in search at any price. However, as the level of uncertainty increases,  $\mu_H \in [\underline{\mu}_H, \overline{\mu}_H]$ , there exists a price range at which search occurs.

We next consider the firm's strategy in more detail. There are a number of equilibria that are possible, ranging from full separation to pooling (see Table 2). For example, in HM, the Hand M types send out the same advertising message and post the same price, while L type differs in at least one of these actions:  $(a_H, p_H) = (a_M, p_M) \equiv (a_{HM}, p_{HM}) \neq (a_L, p_L)$ . This in turn implies that if the consumer observes  $(a_L, p_L)$ , she infers that the product is L-type. On the other hand, if she observes  $(a_{HM}, p_{HM})$ , she is uncertain whether the firm is H-type or M-type. Her decision to search for extra information in this case depends on her prior belief as well as the price p. While the advertising action choice is discrete, the price variable is continuous, which implies that a continuum of prices is possible for each type of equilibrium.

Table 2: All Possible Equilibria					
Equilibrium Type	Description	Notation			
Fully Separating	H, M, L separate	FS			
Semi-Separating	H, M pool	HM			
Semi-Separating	H, L pool	HL			
Semi-Separating	M, L pool	ML			
Semi-Separating	H, M, L pool	HML			

In this setting, we can quickly rule out two potential equilibria: fully-separating equilibrium (FS)and a semi-separating equilibrium where M and L pool (ML). Note that full separation implies that a consumer can simply infer the product's type by examining the prices and the advertising campaign. That is,  $(a_L, p_L) \neq (a_M, p_M) \neq (a_H, p_H)$ . Since search is costly, and the product's type can be perfectly observed, the consumer does not search in this equilibrium. Since L delivers no value to the consumer, it must also be the case that L makes zero profit in this equilibrium. Also, note that if  $p_H > p_M$ , the M type will deviate to the H type's strategy. Similarly, if  $p_H < p_M$ , the H type will deviate to the M type's strategy. This implies that  $p_H = p_M$ . This in turn implies that  $a_H \neq a_M$ . Hence, either H or M must engage in uninformative advertising in FS equilibrium. This of course implies that L type will mimic either H or M's pricing strategy (depending on which type does uninformative advertising). Therefore, it must be the case that either  $(a_L, p_L) = (a_M, p_M)$  or  $(a_L, p_L) = (a_H, p_H)$ . This is a contradiction; a fully separating equilibrium does not exist in our model.

### **Proposition 1** A fully separating equilibrium does not exist.

Similarly, we can show that a semi-separating equilibrium, ML, where the M and L types pool cannot exist. In ML, it must be the case that  $p_L = p_M \equiv p_{ML}$ ,  $a_L = a_M = a_0$ . Note that  $p_{ML} < \frac{\overline{V}}{2}$  since even with search the consumer can not be absolutely certain that the product is not *L* type. However, if *M* type deviates to  $a_j$ ,  $j \in (\alpha, \beta)$ , it can charge at least  $\overline{\frac{V}{2}}$  since an attribute message credibly signals that it is not type *L*. Hence, this equilibrium does not exist.

### **Proposition 2** ML equilibrium does not exist.

The remaining three equilibria candidates (HML, HM, and HL) can be categorized into two types: one in which H separates from M (HL), and one in which H pools with M (HML, and HM).

As is the case for any signaling model, we have to deal with the technical issue of specifying out-of-equilibrium beliefs. There are two main approaches to dealing with this. The first one is to assume a particular set of beliefs following a deviation. While this method is often used, it is vulnerable to the criticism that any specific set of chosen beliefs is, by definition, arbitrary.

The second approach is to start with an unconstrained set of out-of-equilibrium beliefs, but then narrow it using an existing refinement. A number of signaling models employ the Intuitive Criterion (Cho-Kreps 1987) to refine the beliefs. The idea behind this criterion is the following. Suppose that a consumer observes the deviation  $A_1 = (a, p)$ . If type  $\theta'$  makes lower profit in deviation than in equilibrium under *all* possible consumer beliefs, the consumer does not believe that the product could be type  $\theta'$ . However, the Cho-Kreps Intuitive Criterion does not narrow down the beliefs here since in our set-up consumer search is essential in enabling separation between types, and search does not occur at extreme beliefs.

Instead, and following other countersignaling papers (for example, Feltovich et al. 2002), we use the stronger refinement of D1 criterion (Fudenberg and Tirole 1991, p.452) to eliminate unreasonable out-of-equilibrium beliefs. The idea behind the refinement is roughly as follows. Suppose that in deviation  $A_1 = (a, p)$ , type  $\theta'$  makes higher profit than in equilibrium under a strictly bigger set of best responses from the consumer than type  $\theta$  does. In this case D1 requires that the consumer believe that the firm is not type  $\theta$ . We discuss the D1 criterion and a detailed application of it in our model more formally in the Appendix.

### 3.2 High-Type Pools with Medium-Type (HML and HM equilibrium)

First, we consider a full pooling equilibrium (HML). By definition, since L can not engage in attribute advertising, here  $a_L = a_M = a_H = a_0$  and  $p_L = p_M = p_H \equiv \tilde{p}$ .

**Proposition 3** If  $c \leq \frac{\overline{V}(\gamma_H - \gamma_M)}{8}$ , a full pooling equilibrium (HML) without consumer search does not exist.

**Proof.** Suppose that there exists an HML equilibrium, where all types pool on image advertising and price,  $(a_0, p_{HML}^{ns})$ , and the consumer does not search. In equilibrium, the beliefs following  $(a_0, p_{HML}^{ns})$  must be equal to the a priori beliefs, which are  $\mu_H^0 = \frac{\rho}{2}$ ,  $\mu_M^0 = 1 - \rho$ ,  $\mu_L^0 = \frac{\rho}{2}$ . According to Lemma 2 in the Appendix, the maximum price that the firms can charge in equilibrium such that the consumer does not search (given these beliefs) is  $\underline{p}_{HML}^0 = \underline{p}^j(\rho) = \frac{(1-\gamma_H)\rho\overline{V}+(1-\gamma_M)(1-\rho)\overline{V}+2c}{(2-\gamma_H-\gamma_L)\rho+2(1-\gamma_M)(1-\rho)} < \frac{\overline{V}}{2}$ . This in turn implies that H and M types would prefer to deviate to  $(a_j, p^{dev} = \frac{\overline{V}}{2})$ , which credibly signals that the firm is not L type and hence results in purchase. This destroys this potential equilibrium. **Q.E.D.**<sup>8</sup>

**Proposition 4** A full pooling equilibrium HML with consumer search exists and survives the D1 refinement if

 The search cost (c) and the pooling price for H and M (p<sup>\*</sup><sub>HML</sub>) are such that the consumer chooses to search after observing (a<sub>0</sub>, p<sup>\*</sup><sub>HML</sub>):

$$c \leq \frac{V(\gamma_{H} - \gamma_{M})}{8} \quad and \quad p_{HML} \in [\underline{p}_{HML}^{0}, \overline{p}_{HML}^{0}],$$
  
where  $\underline{p}_{HML}^{0} = \frac{\overline{V}(1 - \gamma_{H})\underline{\rho}_{2}^{2} + \overline{\underline{V}}(1 - \gamma_{M})(1 - \rho) + c}{(1 - \gamma_{H})\underline{\rho}_{2}^{2} + (1 - \gamma_{M})(1 - \rho) + (1 - \gamma_{L})\underline{\rho}_{2}^{2}} \quad and \quad \overline{p}_{HML}^{0} = \frac{\overline{V}\gamma_{H}\underline{\rho}_{2}^{2} + \overline{\underline{V}}\gamma_{M}(1 - \rho) - c}{\gamma_{H}\underline{\rho}_{2}^{2} + \gamma_{M}(1 - \rho) + \gamma_{L}\underline{\rho}_{2}^{2}}$ 

2.  $\gamma_M$  is high enough such that M prefers to pool with  $H: \gamma_M p_{HML}^* > \frac{\overline{V}}{2}$ .

In this equilibrium,  $\Pi^*(H) = \gamma_H p_{HML}^* > \Pi^*(M) = \gamma_M p_{HML}^* > \Pi^*(L) = \gamma_L p_{HML}^*$ .

**Proof.** See Appendix for proof

<sup>8</sup>We can show, under D1, that even under the most general priors,  $\mu_H^0 = \alpha$ ,  $\mu_M^0 = \beta$ ,  $\mu_L^0 = 1 - \alpha - \beta$ , this equilibrium does not exist. We adopt this less general proof mainly for brevity.

The first condition ensures that the consumer searches for additional information in equilibrium. In particular, this condition states that 1) the search cost must be low enough relative to the benefit that can be obtained through seeking additional information, 2) bounds are set on the price: if the price is too low, the consumer prefers to buy without search, and if the price is too high, the consumer will not buy even with search.

The second condition guarantees that both H and M prefer to play the equilibrium strategy  $(\Pi^*(H) = \gamma_H p_{HML}^*, \Pi^*(M) = \gamma_M p_{HML})$  to the optimal deviation. Of course, the optimal deviation is a function of the off-equilibrium beliefs. Here we only allow those beliefs that are consistent with D1. To illustrate the mechanism through which D1 constrains the beliefs, consider the consumer inference following the deviation  $A_1 = (a_j, p^{dev})$ . By engaging in attribute-based advertising the firm credibly signals that its product is *not* type L. However, both types H and M are capable of engaging in attribute-based advertising. This is where we use the D1 criterion to refine the beliefs.

D1 states that if in  $A_1 = (a_j, p^{dev})$ , type M makes higher profit than in equilibrium under a strictly bigger set of best responses from the consumer than type H, the consumer believes that the product must be type M. In particular, consider a deviation to a lower price than in equilibrium:  $\gamma_M p_{HML}^* < p^{dev} < \gamma_H p_{HML}^*$ . For illustration purposes, consider the pure strategy best responses only.<sup>9</sup> If the consumer's best response is to search at  $p^{dev}$ , both types are worse off in deviation than under equilibrium since  $p^{dev} < p_{HML}^*$ . Of course, if the consumer's best response is to not purchase, both types are also worse off in deviation. Finally, if the best response is to buy without search, M-type is better off in deviation than in equilibrium ( $\Pi^*(M) = \gamma_M p_{HML} > p^{dev}$ ), while H-type is worse off in deviation ( $\Pi^*(H) = \gamma_H p_{HML}^* < p^{dev}$ ). Hence, the consumer concludes that M is more likely to benefit from the deviation than H since M is better off in deviation under a strictly bigger set of responses. Hence, in this price range, D1 constrains the off-equilibrium belief to be  $\mu_M(A_1) = 1$ .<sup>10</sup> One example of an off-equilibrium belief which is consistent with the properties described above is  $\mu_L = 1$  for all  $(a_0, p \neq p_{HML}^*)$  and  $\mu_M = 1$  for all  $(a_j, p)$  (see the Appendix for

<sup>&</sup>lt;sup>9</sup>In the formal proof in the Appendix, we consider the generalized consumer response, which is a mixture between these three actions.

<sup>&</sup>lt;sup>10</sup>Note that D1 may not always constrain the off-equilibrium belief. For example, we can show that for  $p^{dev} > p_{HML}^*$ , D1 has "no bite."

more detailed proof). Given this, the optimal deviation from equilibrium,  $A_1 = (a_j, \frac{V}{2})$ , yields a maximum profit of  $\frac{\overline{V}}{2}$  (the maximum price that M can charge).

Next, we consider the HM equilibrium, where H and M types pool on attribute advertising (otherwise, L type would prefer to deviate) and price:  $(a_j, p_{HM}^*)$ , and L separates and makes zero profit. Here, both H and M types choose to emphasize their strong attribute. Hence, this is an intuitive equilibrium in that the firm which has anything positive to say about its product chooses to do so. As before, we can also show that search is essential for the existence of this equilibrium:

**Proposition 5** A semi-separating HM equilibrium without consumer does not survive the D1 refinement if

$$c \le \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$$

**Proof.** See Appendix.

To illustrate why HM equilibrium without search does not survive D1, we outline a proof by contradiction (see Appendix for the complete proof). In particular, consider a deviation  $A_1 = (a_j, p^{dev} > p_{HM}^*)$  from the equilibrium  $(a_j, p_{HM}^*)$ . It is clear that the two types may benefit differentially from deviation which is followed by consumer search since in equilibrium no search occurs and  $\gamma_H > \gamma_M$ . In fact, when  $c \leq \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$ , we can show that there exists a deviation  $A_1 = (a_j, p^{dev} > p_{HM}^*)$  and a set of beliefs where the consumer's best response is to search, which would benefit H-type but not M-type. Since H is better off in deviation under a bigger set of responses, the refined belief is  $\mu_H(A_1) = 1$ : the consumer buys without search following  $A_1$ . Since  $p_{HM}^* < p^{dev}$ , both types prefer the deviation to the equilibrium strategy. Therefore, the equilibrium cannot be sustained.

In Proposition 6 below, we identify the conditions for the existence of HM equilibrium with consumer search.

**Proposition 6** An HM equilibrium, where the H-type and the M-type pool on  $(a_j, p_{HM}^*)$ , and the consumer searches after observing  $(a_j, p_{HM}^*)$ , exists and survives D1 refinement if

1. The search cost (c) and the pooling price for H and M  $(p_{HM}^*)$  are such that the consumer chooses to search after observing  $(a_j, p_{HM}^*)$ :

$$c \leq \frac{V}{2}\rho(1-\rho)(\gamma_H - \gamma_M) \ and \ p_{HM}^* \in [\underline{p}_{HM}, \overline{p}_{HM}],$$

where 
$$\underline{p}_{HM} = \frac{(1-\gamma_H)\rho\overline{V} + (1-\gamma_M)(1-\rho)\overline{\frac{V}{2}} + c}{(1-\gamma_H)\rho + (1-\gamma_M)(1-\rho)}, \ \overline{p}_{HM} = \frac{\gamma_H\rho\overline{V} + \gamma_M(1-\rho)\overline{\frac{V}{2}} - c}{\gamma_H\rho + \gamma_M(1-\rho)}.$$

2.  $\gamma_M$  is high enough such that M prefers to pool with  $H: \gamma_M p_{HM}^* > \frac{\overline{V}}{2}$ .

In this equilibrium,  $\Pi^*(H) = \gamma_H p_{HM}^* > \Pi^*(M) = \gamma_M p_{HM}^* > \Pi^*(L) = 0.$ 

### **Proof.** See Appendix.

We first turn to the condition on the search cost:  $c \leq \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$ . Why does  $\rho$  play a role in the decision to search? Recall that  $\rho$  is the correlation between attributes, which in this equilibrium translates to a correlation between types following  $a_j$  since  $P(H|a_j) = P(\beta = B|\alpha = A) = \rho$ . Therefore, if  $\rho$  is either close to 1 or to 0, there is little remaining uncertainty on whether the firm is *H*-type or *M*-type following  $a_j$ , which in turn implies that search would not be optimal in equilibrium.

Note that while the exact expressions for the conditions on the search cost as well as the price bounds differ across HM and HML, the basic equilibrium structure is identical for these two equilibria since in both of these equilibria H and M types pool, and the consumer searches for additional information. The fact that they pool on different actions changes the details of the proof, but does not impact the core results: (1) the cost search is small compared to  $\gamma_H - \gamma_M$ , (2) the price is in a certain range which ensures that the consumer searches, and (3)  $\gamma_M$  is high enough.

The last condition is especially significant: it highlights the fact that in the equilibria where Hand M types pool, the probability that M receives a positive signal following search must be high enough so that M is willing to pool with H in price and advertising. Another way of looking at this is the following: by pooling with a higher type and charging a high price, as is the case for HM and HML, M-type loses control over the consumer's final inference since in both of these equilibria the consumer chooses to search for additional information. On the other hand, in the HLequilibrium, which we discuss below, by revealing its type, M faces lower risk since the consumer has no uncertainty. This decrease in uncertainty, however, comes with a lower upside potential since in this case M cannot charge more than  $\frac{\overline{V}}{2}$ . The amount of risk that M faces – summarized by  $\gamma_M$ , where a higher  $\gamma_M$  entails a lower risk – determines whether a pooling equilibrium with Hand M exists.

### **3.3** High-Type Separates from Medium-Type (*HL* equilibrium)

Next, we consider the equilibrium that is the core of this paper: HL equilibrium. In this equilibrium, H and L types pool on uninformative advertising and price  $(a_H = a_L = a_0, p_H = p_L \equiv p_{HL}^*)$ , whereas the M type engages in attribute-based advertising and, therefore, perfectly reveals own type to the consumer  $(a_M = a_j, p_M = \frac{\overline{V}}{2})$ . HL is a counter-signaling equilibrium in that the high and low types pool together on the same action (Feltovich et al. 2002). In particular, the type with the most to say (H type) chooses a message devoid of any information on product attributes. As before, we first show that an HL equilibrium without search does not exist:

**Proposition 7** If  $c \leq \frac{\overline{V}(\gamma_H - \gamma_M)}{8}\overline{V}$ , a semi-separating HL equilibrium without consumer search does not exist.

**Proof.** See Appendix.

The logic here is similar to Proposition 3. The maximum price that the H and L-type can charge in equilibrium such that the consumer chooses not to search is strictly less than  $\frac{\overline{V}}{2}$  if  $c \leq \frac{\overline{V}(\gamma_H - \gamma_M)}{8}\overline{V}$ . Hence, H-type would prefer to deviate to M's strategy, which of course destroys this potential equilibrium.

**Proposition 8** An HL equilibrium, where the H and L type pool on  $(a_0, p_{HL}^*)$ , M type separates on  $(a_j, \frac{\overline{V}}{2})$ , and the consumer searches after observing  $(a_0, p_{HL}^*)$ , exists and survives the D1 refinement if:

$$\begin{array}{rcl} c & \leq & \displaystyle \frac{\overline{V}\left(\gamma_{H}-\gamma_{M}\right)}{8} & and \\ \\ \displaystyle \frac{\left(\gamma_{H}-\gamma_{M}\right)\overline{p}_{j}+\frac{\overline{V}}{2}(1-\gamma_{H})}{\gamma_{H}(1-\gamma_{M})} & \leq & p_{HL}^{*}<\min\left\{\frac{\gamma_{H}\overline{V}-2c}{\gamma_{H}+\gamma_{L}}, \frac{\overline{V}}{2\gamma_{M}}\right\} \\ \\ where \ \overline{p}^{j} = \frac{3}{4}\overline{V} + \frac{\sqrt{\frac{\overline{V}^{2}}{4}(\gamma_{H}-\gamma_{M})^{2}-2\overline{V}c(\gamma_{H}-\gamma_{M})}}{2(\gamma_{H}-\gamma_{M})}. \\ \\ In \ this \ equilibrium, \ \Pi^{*}(H) = \gamma_{H}p_{HL}^{*} > \Pi^{*}(M) = \frac{\overline{V}}{2} > \Pi^{*}(L) = \gamma_{L}p_{HL}^{*}. \end{array}$$

### **Proof.** See Appendix

As before, this equilibrium requires that the search cost be low relative to the potential information value of an additional signal:  $c \leq \frac{\overline{V}}{8}(\gamma_H - \gamma_M)$ . In addition, the price must be in the appropriate range:  $p_{HL}^* \in [\underline{p}_{HL} = \frac{(1-\gamma_H)\overline{V}+2c}{2-\gamma_H-\gamma_L}, \overline{p}_{HL} = \frac{\gamma_H\overline{V}-2c}{\gamma_H+\gamma_L}]$  (see Lemma 2 in Appendix). Once again, we apply D1 to refine the out-of-equilibrium beliefs. The structure of the derivation is somewhat different here since in this equilibrium *H*-type and *M*-type choose different actions. In particular, we find that if  $p_{HL}^*$  is low enough, then there exists a deviation  $A_1 = (a_j, p^{dev} > p_{HL}^*)$ such that  $\mu_H(A_1) = 1$ . This of course would destroy *HL*. Hence, in order to rule this out, we need the additional constraint that  $p_{HL}^* \ge \frac{(\gamma_H - \gamma_M)\overline{p}_j + \overline{\frac{V}{2}}(1 - \gamma_H)}{\gamma_H(1 - \gamma_M)}$ . Assuming that this constraint holds, we can show that the out-of-equilibrium beliefs  $\mu_L = 1$  for all  $(a_0, p \neq p_{HL}^*)$  and  $\mu_H = 0$  for all  $(a_j, p \neq \overline{\frac{V}{2}})$  are consistent with the requirements imposed by D1.

Finally, as before, we need to ensure that all types prefer the equilibrium strategy to the optimal deviation. Given the assumed out-of-equilibrium beliefs, this reduces to the following conditions:  $\Pi^*(a_0, p_{HL}^*|q = H) = \gamma_H p_{HL}^* > Max_{A_1} \Pi(A_1|q = H) = \frac{\overline{V}}{2}, \text{ and } \Pi^*(a_j, p_M|q = M) = \frac{\overline{V}}{2} > Max_{A_1} \Pi(A_1|q = M) = \gamma_M p_{HL}^*.$ Since not all of the conditions are binding, the constraints reduce to the ones given in Proposition 8.

In contrast to the findings in Proposition 4 and Proposition 6, for HL equilibrium to exist, it must be the case that  $\gamma_H$  is *large* enough and  $\gamma_M$  is *small* enough. HL exists if H-type prefers to pool with L-type on uninformative advertising rather than pursue an attribute-based strategy which perfectly signals that the firm is *not* L type. Note that since the signals associated with each type are noisy, after an uninformative ad and own search, the consumer may mistake an Htype firm for an L type. However, the risk H bears by pooling with L must be relatively small ( $\gamma_H$ is *large* enough) such that H type prefers this to the certain outcome of pretending to be M type by engaging in an attribute-based ad. In other words, after the consumer obtains a signal, the probability that she will misjudge H as L type is low. This is the source of H's confidence. On the other hand, M type prefers to separate itself from L type than pool with it. This can happen only if the external signal cannot effectively separate between M and L types (in other words,  $\gamma_M$ is small). Hence, M lacks H's confidence and prefers not to mimic H type because the probability that it may be misjudged as L type is too high. That is, while H-type is willing to relinquish control in its communication strategy (by engaging in uninformative advertising with an uncertain outcome following consumer search), the M-type prefers the lower risk attribute-based strategy.

Moreover, when  $\gamma_H$  is large and  $\gamma_M$  is low, HL is the *only* equilibrium that survives the D1 refinement.

**Proposition 9** *HL* equilibrium is unique when  $\gamma_H$  is sufficiently large and  $\gamma_M$  is sufficiently small such that,

1. The conditions for HL hold (see Proposition 8)

2. 
$$c < \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$$

3. 
$$\gamma_M < \frac{\overline{V}}{2\overline{p}_{HM}}$$
.

Moreover, this region is non-empty.

### **Proof.** See Appendix.

When would we expect to observe high  $\gamma_H$  and low  $\gamma_M$ , the prerequisites for observing HL? Note that these parameters represent the probability of positive news following search by the consumer. One factor that may moderate the relative size of these parameters is the propensity to discuss negative experience with others. For example, Godes and Wojnicki (2009) show that experts may be less likely to share their negative experiences since it sends a negative signal about their ability to choose a high-quality product. Hence, we would expect that in a category with a higher proportion of experts, word of mouth may be skewed to be more positive. While this would have little effect on word of mouth generated for high-quality products since all consumers would have a positive experience with them ( $\gamma_H$  is high), this would affect on the distribution of reviews of the average product which is expected to receive mixed reviews. That is, a category with fewer experts (which is usually the case for a new product category) would have a lower  $\gamma_M$ . Similarly, the average price level of the category may impact the level of  $\gamma_M$ . That is, a bad experience with a car may prompt an instant posting on a blog, whereas a negative experience with a movie may not inspire as much outrage. Again, this would imply that a higher average category price would lead to a lower  $\gamma_M$ . Finally, in a category where consumers have high expectation about product quality, a product which is high-quality on some attributes but not others (M type) is more likely to have a lower  $\gamma_M$ .

In summary, we show that in the case of limited bandwidth, there are two types of equilibria that are possible: one in which H and M types pool (HML, HM), and another one in which Hand M types separate (HL). In the latter, the superior product (H) prefers to pool with the terrible product (L) in its communication strategy in order to distinguish itself from the mediocre product (M). The mediocre product prefers to perfectly separate itself from the terrible product rather than risk being confused with it. Our findings emphasize the importance of modeling the decision to search (with its costs and benefits) since search is crucial in enabling this separation across neighboring types. A superior firm chooses uninformative advertising since it is confident that consumers will realize its high quality through their own search. for information. When the firm is not confident about its quality, which is the case for the mediocre firm, it prefers to make a product claim in order to separate itself from the terrible firm.

## 4 Signaling Quality in a 3-Type v. 2-Type Model

How do we reconcile our finding that advertising content can signal quality with the earlier intuition that content is irrelevant for signaling quality (Bagwell 2007, Milgrom and Roberts 1986)? There are several salient differences between the assumptions of our model and the earlier moneyburning models. In this Section, we contrast our results to the results in a model where only 2 types are assumed, as is the case in the money-burning models.

First, consider a modified model where only two types are possible: H and L (or M and L). This is also equivalent to assuming that  $\rho = 1$  (or  $\rho = 0$ ). Clearly, in this model, advertising content can signal quality since perfect separation is possible if type H (or M) engages in attribute-based advertising. However, a more interesting and realistic case is one in which types H and M are present. That is, there does not exist an advertising message that perfectly separates all types. We again use D1 to refine the out-of-equilibrium beliefs.

**Proposition 10** When only H and M types are present in the market, only pooling equilibria exist and survive D1. Moreover, if  $\gamma_M$  is sufficiently low, no pure strategy equilibrium survives D1.

### **Proof.** See Appendix $\blacksquare$

Note that in a pooling equilibrium advertising content by definition cannot signal quality. Hence, the assumption that three types as opposed to two types are present is important for our result. This may explain why our result was not obtained in earlier models which have focused on two types only.

Also, consider the externalities that the lower-type firms exert on the higher-type firms. In

particular, in most signaling models, the presence of the low type implies a negative externality on the high type. That is, the high type may need to engage in costly signaling in order to separate itself from the low type. This intuition occurs in our main model as well since the presence of M and L imply that H makes less profit compared to the case where H is the only possible type. However, as we see from Proposition 10, if  $\gamma_M$  is low, no pure strategy equilibrium exists in the 2-types (H and M) model. On the other hand, in the 3-types model, HL equilibrium exists and is unique (if we assume that the conditions outlined in Proposition 9 hold). Hence, the presence of L-type here helps H communicate its quality through advertising content by separating itself from M. In fact, L serves as a credible threat to keep M from imitating H-type.

Finally, note that in the 2-types model the firm may be able to signal its quality through moneyburning (Milgrom and Roberts 1986), a possibility that we do not consider in our model since our focus is on content-signaling. In our model the H-type is able to avoid spending extra resources on money-burning by choosing an advertising strategy that motivates the consumer to engage in own search.

### 5 Conclusion

In this research, we show that advertising content – whether the advertisement is uninformative or attribute-based – can be a credible quality signal under the realistic assumptions of (1) limited bandwidth of communication inherent in advertising, and (2) the possibility of consumer search following the consumer's exposure to the advertisement. We show that this desire to signal one's quality may result in the surprising phenomenon that a firm with the most to say may choose not to make any "hard" claims at all. This withholding strategy may be rational in that vague claims can be made by either the superior or the terrible products, which necessitates a search for further information on the part of the consumer. In our opening example, American Express Card is confident that a consumer who engages in own search will find out about its superior service<sup>11</sup>. This confidence allows it to engage in uninformative advertising in favor of making any hard claims.

<sup>&</sup>lt;sup>11</sup>2008 Credit Card Satisfaction Study (J.D.Power, http://www.jdpower.com/finance/articles/2008-Credit-Card-Satisfaction-Study) shows that Amex is ranked at number 1 while Capital one is ranked number 12 in customer satisfaction index.

Capital One, on the other hand, which is weaker on some attributes, is not confident that search will distinguish it from a truly terrible product and does not want to undertake the risk of search. Instead, it chooses to emphasize one of its attributes in order to separate itself from a truly terrible product. In conclusion, the combination of advertising content and consumer search enables the firm to signal its quality even in the absence of any money-burning effect. The consumer search (which is determined endogenously in the model) is crucial in enabling this type of equilibrium. While most of the previous literature has focused on the decision to advertise (the mere fact that the firm is willing to burn its money) as a signal of quality, we show that the message content, coupled with consumer search, can also serve as a credible signal of quality.

There can be, of course, a lot of different explanations for the existence and effectiveness of uninformative advertising (in particular, image advertising), and we do not wish to claim that our explanation is the only possible theory for this phenomenon. Nevertheless, we offer a novel explanation for uninformative advertising, one that to our knowledge is the first one that assumes consumer rationality.

## Appendix 1

### Proof of Lemma 1

The consumer will search if and only if  $EU(\text{search}) = \Pr(\overline{s}|\mu)[E(V|\mu,\overline{s}) - p] - c \ge EU(\text{nonsearch}) = \max(0, E(V|\mu) - p)$ . Therefore,

1) If  $E(V|\mu) - p \ge 0$ , then  $EU(\text{search}) \ge EU(\text{no search})$  iff

$$\Pr(\overline{s}|\mu)[E(V|\mu,\overline{s}) - p] - c \ge E(V|\mu) - p$$

$$\Leftrightarrow \quad \Pr(\overline{s}|\mu)E(V|\mu,\overline{s}) - \Pr(\overline{s}|\mu)p - c \ge \Pr(\overline{s}|\mu)E(V|\mu,\overline{s}) + \Pr(\underline{s}|\mu)E(V|\mu,\underline{s}) - p$$

$$\Leftrightarrow \quad c \le \Pr(\underline{s}|\mu)[p - E(V|\mu,\underline{s})] \equiv g$$
(5)

2) If  $E(V|\mu) - p < 0$ , then  $EU(\text{search}) \ge EU(\text{no search})$  if

$$\Pr(\overline{s}|\mu)[E(V|\mu,\overline{s}) - p] - c \ge 0 \Leftrightarrow c \le \Pr(\overline{s}|\mu)[E(V|\mu,\overline{s}) - p] \equiv f$$
(6)

Next we show that f = g at  $p = E(V|\mu)$ 

$$f - g = \Pr(\overline{s}|\mu)[E(V|\mu,\overline{s}) - p] - \Pr(\underline{s})[p - E(V|\mu,\underline{s})]$$

$$= \Pr(\overline{s}|\mu)E(V|\mu,\overline{s}) - \Pr(\overline{s}|\mu)p - \Pr(\underline{s}|\mu)p + \Pr(\underline{s}|\mu)E(V|\mu,\underline{s})$$

$$= \Pr(\overline{s}|\mu)E(V|\mu,\overline{s}) + \Pr(\underline{s}|\mu)E(V|\mu,\underline{s}) - p = E(V|\mu) - p = 0$$

$$(7)$$

This completes the proof. **Q.E.D.** 

### **D1** Refinement

We apply D1 (Fudenberg and Tirole 1991) to eliminate unreasonable out of equilibrium beliefs. The idea behind the refinement is the following. Suppose that in deviation  $A_1 = (a, p)$ , type  $\theta'$ makes higher profit than in equilibrium under a bigger set of best responses from the consumer than type  $\theta$ . Then we can impose the restriction on the consumer's beliefs that type  $\theta'$  is infinitely more likely to deviate to  $A_1$  than type  $\theta$ .

More formally, following Fudenberg and Tirole (1991, p 452), we define  $\Pi^*(\theta)$  to be the equilibrium profit of type  $\theta$ . We also define the set of mixed strategy best responses of the consumer,  $\alpha_2$ ( $\alpha_2 = \{\alpha_{21}, \alpha_{22}, \alpha_{23}\} = \{\Pr(\text{purchase without search}), \Pr(\text{no purchase}), \Pr(\text{search})\})$  to a deviation by the firm,  $A_1 = (a, p)$ , such that type  $\theta$  strictly prefers  $A_1$  to the equilibrium strategy:

$$D(\theta, A_1) =$$

$$\{\alpha_2 \in MBR(\mu(A_1), A_1) \text{ s.t. } \Pi^*(\theta) < \Pi(A_1, \alpha_2, \theta) | \mu_H(A_1) + \mu_{M_e}(A_1) + \mu_{M_e}(A_1) + \mu_L(A_1) = 1\}$$
(8)

Note that the consumer's best response depends on her belief,  $\mu(A_1) = (\mu_H(A_1), \mu_{M_{\alpha}}(A_1), \mu_{M_{\beta}}(A_1), \mu_L(A_1))$ .

Similarly, we define a set of consumer's best responses such that the firm is indifferent between deviating and playing the equilibrium strategy.

$$D^{0}(\theta, A_{1}) =$$

$$\{\alpha_{2} \in MBR(\mu(A_{1}), A_{1}) \text{ s.t. } \Pi^{*}(\theta) = \Pi(A_{1}, \alpha_{2}, \theta) | \mu_{H}(A_{1}) + \mu_{M_{\alpha}}(A_{1}) + \mu_{M_{\beta}}(A_{1}) + \mu_{L}(A_{1}) = 1\}$$
(9)

The criterion D1 puts zero probability on type  $\theta$  if there exists another type  $\theta'$  such that

$$D(\theta, A_1) \cup D^0(\theta, A_1) \subset D(\theta', A_1).$$

$$\tag{10}$$

Using Lemma 1, below we derive the set of consumer's mixed best responses,  $MBR(\mu(A_1), A_1)$ :

- 1. If  $E(V|\mu(A_1)) p > 0$ ,
  - (a) Consumer will search:  $\alpha_2 = \{0, 0, 1\}$ , if  $c < \Pr(\underline{s}|\mu(A_1))[p E(V|\mu(A_1), \underline{s})]$
  - (b) Consumer will purchase without search:  $\alpha_2 = \{1, 0, 0\}$ , if  $c > \Pr(\underline{s}|\mu(A_1))[p E(V|\mu(A_1), \underline{s})]$
  - (c) Consumer mixes between search and purchase without search:  $\alpha_2 = \{\alpha_{21}, 0, 1 \alpha_{21}\}$ , if  $c = \Pr(\underline{s}|\mu(A_1))[p - E(V|\mu(A_1), \underline{s})]$
- 2. If  $E(V|\mu(A_1)) p < 0$ ,
  - (a) Consumer will search:  $\alpha_2 = \{0, 0, 1\}$ , if  $c < \Pr(\overline{s}|\mu(A_1))[E(V|\mu(A_1), \overline{s}) p]$
  - (b) Consumer will not purchase:  $\alpha_2 = \{0, 1, 0\}$ , if  $c > \Pr(\overline{s}|\mu(A_1))[E(V|\mu(A_1), \overline{s}) p]$
  - (c) Consumer mixes between search and no purchase:  $\alpha_2 = \{0, \alpha_{22}, 1 \alpha_{22}\}, \text{ if } c = \Pr(\overline{s}|\mu(A_1))[E(V|\mu(A_1), \overline{s}) p]$
- 3. If  $E(V|\mu(A_1)) p = 0$  and  $c = \Pr(\underline{s}|\mu(A_1))[E(V|\mu(A_1)) E(V|\mu(A_1), \underline{s})]$ , consumer chooses either  $\alpha_2 = \{0, \alpha_{22}, 1 - \alpha_{22}\}$  or  $\alpha_2 = \{\alpha_{21}, 0, 1 - \alpha_{21}\}.$

Note that  $\alpha_2 = \{\alpha_{21}, 1 - \alpha_{21}, 0\} \notin MBR(\mu(A_1), A_1)$  since we assume that if the consumer is indifferent between purchasing the product and no purchase, she chooses to purchase it.

### Bounds on prices and beliefs for consumer search

Next, using the results above, we derive explicit bounds on prices and beliefs such that the consumer searches as a best response to  $A_1$ .

**Lemma 2** Assume that  $c \leq \frac{\overline{V}(\gamma_H - \gamma_M)}{8}$ .

1. Consider the case where the firm engages in attribute-based advertising,  $A_1 = (a_j, p)$  and the consumer's belief is  $\mu^j = (0, \mu_M^j, \mu_H^j)$ . There exists a consumer belief under which search is a best response for the consumer when  $p \in [\underline{p}^j, \overline{p}^j]$ , where  $\underline{p}^j = \frac{3}{4}\overline{V} - \frac{\sqrt{\overline{V}^2}(\gamma_H - \gamma_M)^2 - 2\overline{V}c(\gamma_H - \gamma_M)}{2(\gamma_H - \gamma_M)}$  $\geq \overline{V}_2, \ \overline{p}^j = \frac{3}{4}\overline{V} + \frac{\sqrt{\overline{V}^2}(\gamma_H - \gamma_M)^2 - 2\overline{V}c(\gamma_H - \gamma_M)}{2(\gamma_H - \gamma_M)} \leq \overline{V}.$ 

$$\begin{split} &Moreover, for \ a \ given \ \mu_{H}^{j}, \ consumer \ chooses \ to \ search \ iff \ \underline{p}^{j}(\mu_{H}^{j}) = \frac{\mu_{H}^{j}(1-\gamma_{H})\overline{V} + (1-\mu_{H}^{j})(1-\gamma_{M})\frac{V}{2} + c}{\mu_{H}^{j}(1-\gamma_{H}) + (1-\mu_{H}^{j})(1-\gamma_{M})} \\ &\leq p \leq \frac{\mu_{H}^{j}\gamma_{H}\overline{V} + (1-\mu_{H}^{j})\gamma_{M}\frac{\overline{V}}{2} - c}{\mu_{H}^{j}\gamma_{H} + (1-\mu_{H}^{j})\gamma_{M}} = \overline{p}^{j}(\mu_{H}^{j}). \end{split}$$

2. Consider the case where the firm engages in uninformative advertising,  $A_1 = (a_0, p)$  and the consumer's belief is  $\mu^0 = (\mu_L^0, \mu_M^0, \mu_H^0)$ , where  $\mu_L^0 \leq \widehat{\mu_L} = \frac{1}{2}(1 + \sqrt{1 - \frac{4c}{\overline{V}(\gamma_H - \gamma_L)}})$ . When  $p \in [\underline{p}^0, \overline{p}^0]$ , there exists a consumer belief  $(\mu^0)$ , under which search is a best response for the consumer, where  $\underline{p}^0 \equiv \min_{0 \leq \mu_L^0 \leq \widehat{\mu}_L} \underline{p}^0(\mu_L^0) \leq \underline{p}^j$ ,  $\overline{p}^0 \equiv \max_{0 \leq \mu_L^0 \leq \widehat{\mu}_L} \overline{p}^0(\mu_L^0) \geq \overline{p}^j$ . Moreover, for a given  $\mu^0$ , consumer chooses to search iff  $\underline{p}^0(\mu^0) = \frac{\mu_H^0(1 - \gamma_H)\overline{V} + \mu_M^0(1 - \gamma_M)\overline{V} + c}{\mu_H^0(1 - \gamma_H) + \mu_M^0(1 - \gamma_M) + \mu_L^0(1 - \gamma_L)} \leq p \leq \frac{\mu_H^0\gamma_H\overline{V} + \mu_M^0\gamma_M\frac{\overline{V}}{2} - c}{\mu_H^0\gamma_H + \mu_M^0\gamma_M + \mu_L^0\gamma_L} = \overline{p}^0(\mu^0)$ .

**Proof.** See the Technical Appendix.

### HML Equilibrium

#### **Proof of Proposition 4.**

Before we turn to the equilibrium conditions, we first examine the restrictions on out-ofequilibrium beliefs that are imposed by D1:

**Lemma 3** The out-of-equilibrium beliefs that are consistent with D1 have the following properties:

1. When the consumer observes the deviation  $A_1 = \{a_j, p^{dev}\}$ , where  $\max(\gamma_M p^*_{HML}, \underline{p}^j) < p^{dev} < \min(p^*_{HML}, \overline{p}^j)$  or  $\gamma_M p^*_{HML} < p^{dev} < \min(\gamma_H p^*_{HML}, \underline{p}^j)$ , the consumer forms the off-equilibrium belief that  $\mu_M(A_1) = 1$ .

- 2. When the consumer observes the deviation  $A_1 = \{a_0, p^{dev}\},\$ 
  - (a) if  $\max(\gamma_L p_{HML}^*, \underline{p}^0) < p^{dev} < \min(p_{HML}^*, \overline{p}^0)$  or  $\gamma_L p_{HML}^* < p^{dev} < \min(\gamma_M p_{HML}^*, \underline{p}^0)$ , the consumer forms the off-equilibrium belief that  $\mu_L(A_1) = 1$ ,
  - (b) if  $\gamma_M p_{HML}^* < p^{dev} < \min(\gamma_H p_{HML}^*, \underline{p}^0)$ , the consumer forms the off-equilibrium belief that  $\mu_H(A_1) = 0$ .

**Proof.** We consider the *HML* equilibrium with search: all firms choose  $(a_0, p_{HML}^*)$  and the consumer searches in equilibrium. We first derive the restrictions that D1 imposes on the out-of-equilibrium beliefs following the deviation  $A_1$ . The consumer's mixed best response has 2 possible forms: (1)  $\alpha_2 = \{0, \alpha_{22}, 1-\alpha_{22}\}$  (mixing between no purchase and search), and (2)  $\alpha_2 = \{\alpha_{21}, 0, 1-\alpha_{21}\}$  (mixing between purchase and search). In the first case,  $\Pi(A_1, \alpha_2, H) = (1 - \alpha_{22})\gamma_H p^{dev}$  and  $\Pi(A_1, \alpha_2, M) = (1 - \alpha_{22})\gamma_M p^{dev}$ ; in the second case,  $\Pi(A_1, \alpha_2, H) = (\alpha_{21} + (1 - \alpha_{21})\gamma_H) p^{dev}$  and  $\Pi(A_1, \alpha_2, M) = (\alpha_{21} + (1 - \alpha_{21})\gamma_M) p^{dev}$ . Also, of course,  $\Pi^*(H) = \gamma_H p_{HML}^*$  and  $\Pi^*(M) = \gamma_M p_{HML}^*$ .

Let us first define the sets

$$D^{0}(H, A_{1}) = X_{H}^{0} \cup Y_{H}^{0} = \left\{ \left(0, \alpha_{22}, 1 - \alpha_{22}\right) \mid \alpha_{22} = \frac{p^{dev} - p_{HML}^{*}}{p^{dev}} \right\} \cup \left\{ \left(\alpha_{21}, 0, 1 - \alpha_{21}\right) \mid \alpha_{21} = \frac{\gamma_{H} \left(p_{HML}^{*} - p^{dev}\right)}{(1 - \gamma_{H})p^{dev}} \right\},$$

$$D(H, A_1) = X_H \cup Y_H = \left\{ (0, \alpha_{22}, 1 - \alpha_{22}) \mid \alpha_{22} < \frac{p^{dev} - p^*_{HML}}{p^{dev}} \right\} \cup \left\{ (\alpha_{21}, 0, 1 - \alpha_{21}) \mid \alpha_{21} > \frac{\gamma_H \left( p^*_{HML} - p^{dev} \right)}{(1 - \gamma_H) p^{dev}} \right\},$$

$$D^{0}(M, A_{1}) = X_{M}^{0} \cup Y_{M}^{0} = X_{H}^{0} \cup Y_{M}^{0} = \left\{ (0, \alpha_{22}, 1 - \alpha_{22}) \mid \alpha_{22} = \frac{p^{dev} - p_{HML}^{*}}{p^{dev}} \right\} \cup \left\{ (\alpha_{21}, 0, 1 - \alpha_{21}) \mid \alpha_{21} = \frac{\gamma_{M} \left( p_{HML}^{*} - p^{dev} \right)}{(1 - \gamma_{M}) p^{dev}} \right\},$$

$$D(M, A_1) = X_M \cup Y_M = X_H \cup Y_M = \left\{ (0, \alpha_{22}, 1 - \alpha_{22}) \mid \alpha_{22} < \frac{p^{dev} - p^*_{HML}}{p^{dev}} \right\} \cup \left\{ (\alpha_{21}, 0, 1 - \alpha_{21}) \mid \alpha_{21} > \frac{\gamma_M \left( p^*_{HML} - p^{dev} \right)}{(1 - \gamma_M) p^{dev}} \right\},$$

$$D^{0}(L, A_{1}) = X_{L}^{0} \cup Y_{L}^{0} = X_{H}^{0} \cup Y_{L}^{0} = \left\{ (0, \alpha_{22}, 1 - \alpha_{22}) \mid \alpha_{22} = \frac{p^{dev} - p_{HML}^{*}}{p^{dev}} \right\} \cup \left\{ (\alpha_{21}, 0, 1 - \alpha_{21}) \mid \alpha_{21} = \frac{\gamma_{L} \left( p_{HML}^{*} - p^{dev} \right)}{(1 - \gamma_{L}) p^{dev}} \right\},$$
$$D(L, A_{1}) = X_{L} \cup Y_{L} = X_{H} \cup Y_{L} = \left\{ (0, \alpha_{22}, 1 - \alpha_{22}) \mid \alpha_{22} < \frac{p^{dev} - p_{HML}^{*}}{p^{dev}} \right\} \cup \left\{ (\alpha_{21}, 0, 1 - \alpha_{21}) \mid \alpha_{21} > \frac{\gamma_{L} \left( p_{HML}^{*} - p^{dev} \right)}{(1 - \gamma_{L}) p^{dev}} \right\},$$

One can easily note that  $X_L^0 = X_M^0 = X_H^0$ , and  $X_L = X_M = X_H$ . Hence,

$$D^{0}(H, A_{1}) \cup D(H, A_{1}) = \widehat{X_{H}} \cup \widehat{Y_{H}} = (X_{H}^{0} \cup X_{H}) \cup (Y_{H}^{0} \cup Y_{H}) =$$
(11)  
$$\left\{ (0, \alpha_{22}, 1 - \alpha_{22}) \mid \alpha_{22} \leq \frac{p^{dev} - p_{HML}}{p^{dev}} \right\} \cup \left\{ (\alpha_{21}, 0, 1 - \alpha_{21}) \mid \alpha_{21} \geq \frac{\gamma_{H} \left( p_{HML} - p^{dev} \right)}{(1 - \gamma_{H}) p^{dev}} \right\}$$

$$D^{0}(M, A_{1}) \cup D(M, A_{1}) = \widehat{X_{H}} \cup \widehat{Y_{M}} = \left(X_{H}^{0} \cup X_{H}\right) \cup \left(Y_{M}^{0} \cup Y_{M}\right) =$$

$$\left\{ \left(0, \alpha_{22}, 1 - \alpha_{22}\right) \mid \alpha_{22} \le \frac{p^{dev} - p_{HML}^{*}}{p^{dev}} \right\} \cup \left\{ \left(\alpha_{21}, 0, 1 - \alpha_{21}\right) \mid \alpha_{21} \ge \frac{\gamma_{M} \left(p_{HML}^{*} - p^{dev}\right)}{(1 - \gamma_{M})p^{dev}} \right\},$$
(12)

$$D^{0}(L,A_{1}) \cup D(L,A_{1}) = \widehat{X_{H}} \cup \widehat{Y_{L}} = \left(X_{H}^{0} \cup X_{H}\right) \cup \left(Y_{L}^{0} \cup Y_{L}\right) =$$

$$\left\{ \left(0,\alpha_{22},1-\alpha_{22}\right) \mid \alpha_{22} \leq \frac{p^{dev} - p_{HML}^{*}}{p^{dev}} \right\} \cup \left\{ \left(\alpha_{21},0,1-\alpha_{21}\right) \mid \alpha_{21} \geq \frac{\gamma_{L}\left(p_{HML}^{*} - p^{dev}\right)}{(1-\gamma_{L})p^{dev}} \right\}.$$
(13)

Note that  $0 \leq \alpha_{2k} \leq 1$  (where  $k \in \{1, 2, 3\}$ ), and hence some of the sets may be empty depending on the value of  $p^{dev}$  relative to  $p^*_{HML}$ .

- 1. Consider a deviation to a price such that the consumer chooses not to purchase at any offequilibrium belief:  $A_1 = (a_j, p^{dev})$  where  $p^{dev} > \overline{p}^j$  or  $A_1 = (a_0, p^{dev})$  where  $p^{dev} > \overline{p}^0$ . Here, none of the types are better off than in equilibrium, which implies that D1 does not apply.
- 2. Next, consider a deviation to a price such that the consumer chooses to purchase without search at any off-equilibrium belief:  $A_1 = (a_j, p^{dev})$  where  $p^{dev} < \underline{p}^j$  or  $A_1 = (a_0, p^{dev})$  where  $p^{dev} < \underline{p}^0$ , i.e.,  $\alpha_{21} = 1$  and  $X_i = \emptyset$  for all  $i \in \{L, M, H\}$ . Here, a type i is (weakly) better off in deviation than in equilibrium if  $p^{dev} \ge \gamma_i p^*_{HML}$  (we can also see this from examining the Y sets). Hence, D1 rules out type j if  $\gamma_i p^*_{HML} < p^{dev} < \gamma_j p^*_{HML}$ . Therefore, if  $A_1 = (a_0, p^{dev})$ , for all  $p^{dev} \in (\gamma_M p^*_{HML}, \min(\gamma_H p^*_{HML}, \underline{p}^j))$ , D1 imposes that  $\mu_H = 0$ . Similarly, if  $A_1 = (a_0, p^{dev})$ , for all  $p^{dev} \in (\gamma_L p^*_{HML}, \min(\gamma_M p^*_{HML}, \underline{p}^0))$ ,  $\mu_L = 1$ , and  $\mu_H = 0$  for all  $p^{dev} \in (\gamma_M p^*_{HML}, \min(\gamma_H p^*_{HML}, \underline{p}^0))$ .

3. Consider the deviation  $A_1 = (a_j, p^{dev})$ , where  $\underline{p}^j \leq p^{dev} \leq \overline{p}^j$  (so that there exists a belief where search is the best response based on Lemma 2).

(1) Let us first identify the conditions under which  $D^0(H, A_1) \cup D(H, A_1) \subset D(M, A_1)$  holds. This condition is equivalent to  $(\widehat{X_H} \cup \widehat{Y_H}) \subset (X_H \cup Y_M)$  using the notation defined above. This would, of course, imply that D1 imposes the out-of-equilibrium belief  $\mu_M(A_1) = 1$ .

- (a) Note that  $X_H \subseteq \widehat{X_H}$ . Hence, it must be the case that  $X_H = \widehat{X_H}$ . Otherwise,  $\left(\widehat{X_H} \cup \widehat{Y_H}\right) \subset (X_H \cup Y_M)$  would not hold. This, in turn, implies that  $X_M = \widehat{X_H}$ . The condition  $X_M = \widehat{X_H}$  can hold if (1)  $X_M = \widehat{X_H} = \emptyset$ , which implies that  $\alpha_{22} < \frac{p^{dev} p_{HML}}{p^{dev}} < 0 \Leftrightarrow p^{dev} < p^*_{HML}$ , or (2)  $\frac{p^{dev} p^*_{HML}}{p^{dev}} > 1 \Leftrightarrow p^*_{HML} < 0$ . Hence, for  $X_H = \widehat{X_H}$  to hold under a non-negative equilibrium price, it must be the case that  $p^{dev} < p^*_{HML}$ .
- (b) Note that  $\widehat{Y_H} \subseteq Y_M$  since  $\gamma_M < \gamma_H$ . For the condition  $\left(\widehat{X_H} \cup \widehat{Y_H}\right) \subset (X_H \cup Y_M)$  to hold, the range of prices must be such that  $\widehat{Y_H} \subset Y_M$  since we just determined that  $X_H = \widehat{X_H}$ . Hence, the additional conditions that are needed are that (1)  $\frac{\gamma_M(p_{HML}^* - p^{dev})}{(1 - \gamma_M)p^{dev}} < 1$  $(Y_M \text{ is non-empty}) \Leftrightarrow \gamma_M p_{HML}^* < p^{dev}$ , and (2)  $\underline{p}^j \leq p^{dev} \leq \overline{p}^j$  (search is best response based on Lemma 2).

Hence, it must be the case that when  $\max(\gamma_M p_{HML}^*, \underline{p}^j) < p^{dev} < \min(p_{HML}^*, \overline{p}^j)$ , D1 imposes the belief that  $\mu_M(A_1) = 1$ .

(2) Next, let's look at the conditions under which  $D^0(M, A_1) \cup D(M, A_1) \subset D(H, A_1)$  or  $\left(\widehat{X_H} \cup \widehat{Y_M}\right) \subset (X_H \cup Y_H)$ , which would impose the belief that  $\mu_H(A_1) = 1$ . As we noted before,  $X_H \subseteq \widehat{X_H}$ , and we can see that  $Y_H \subset \widehat{Y_M}$  since  $\gamma_M < \gamma_H$ , which rules out the case  $D^0(M, A_1) \cup D(M, A_1) \subset D(H, A_1)$ .

4. Consider the deviation  $A_1 = (a_0, p^{dev})$  where  $\underline{p}^0 \leq p^{dev} \leq \overline{p}^0$  (so that there exists a belief where search is the best response based on Lemma 2). Using the same techniques as before, we can show that for  $p^{dev} < p_{HML}^*$ ,  $\widehat{X_H} = \widehat{X_M} = \widehat{X_L} = \emptyset$ , and  $\frac{\gamma_L(p_{HML}^* - p^{dev})}{(1 - \gamma_L)p^{dev}} < \frac{\gamma_M(p_{HML}^* - p^{dev})}{(1 - \gamma_H)p^{dev}} < \frac{\gamma_L(p_{HML}^* - p^{dev})}{(1 - \gamma_H)p^{dev}}$ . Moreover, we can show that  $\frac{\gamma_L(p_{HML}^* - p^{dev})}{(1 - \gamma_L)p^{dev}} < 1$  if  $p^{dev} > \gamma_L p_{HML}^*$ . This implies that  $\widehat{Y_H} \subset Y_L$  and  $\widehat{Y_M} \subset Y_L$  if  $max(\underline{p}^0, \gamma_L p_{HML}^*) < p^{dev} < \min(p_{HML}^*, \overline{p}^0)$ ; therefore, D1 implies that  $\mu_L = 1$  in this region. As before, if  $p^{dev} > p_{HML}^*$ ,  $\widehat{X_H} \notin X_M, \widehat{X_M} \notin X_H$ , etc., which implies that D1 does not apply. One example of an off-equilibrium belief which is consistent with the properties described above is  $\mu_L = 1$  for all  $(a_0, p \neq p_{HML}^*)$  and  $\mu_M = 1$  for all  $(a_j, p)$ .

From now on, we assume that the off-equilibrium beliefs are:  $\mu_L = 1$  for all  $(a_0, p \neq p_{HML}^*)$  and  $\mu_M = 1$  for all  $(a_j, p)$ , which we just showed are consistent with D1.

Next we show that if  $c \leq \frac{\overline{V}(\gamma_H - \gamma_M)}{8}$ ,  $\gamma_M p_{HML}^* \geq \frac{\overline{V}}{2}$ , and  $p_{HML}^* \in [\underline{p}_{HML}^0, \overline{p}_{HML}^0]$ , there exists HML equilibrium with search. According to Lemma 2, in order for the consumer to search in equilibrium, it must be the case that  $c \leq \frac{\overline{V}(\gamma_H - \gamma_M)}{8}$  and  $p_{HML}^* \in [\underline{p}_{HML}^0, \overline{p}_{HML}^0]$ , where  $\underline{p}_{HML}^0 = \underline{p}^j(\rho) = \frac{(1 - \gamma_H)\rho\overline{V} + (1 - \gamma_M)(1 - \rho)\overline{V} + 2c}{(2 - \gamma_H - \gamma_L)\rho + 2(1 - \gamma_M)(1 - \rho)} < \frac{\overline{V}}{2}$  and  $\overline{p}_{HML}^0 = \overline{p}^j(\rho) = \frac{\rho\gamma_H\overline{V} + (1 - \rho)\gamma_M\overline{V} - 2c}{\rho(\gamma_H + \gamma_L) + 2(1 - \rho)\gamma_M} > \frac{\overline{V}}{2}$ .

The *H*-type can deviate on both advertising and price. If *H* deviates on price alone  $(a_0, p' \neq p_{HML}^*)$ , the consumer believes that the firm is *L* type, which can not be profitable. If *H* deviates on advertising, the consumer believes that the firm is *M* type, which yields a maximum profit of  $\frac{\overline{V}}{2}$ . Hence, in order to ensure that *H* type does not deviate from the equilibrium strategy, it must be the case that:

$$\Pi^*(a_0, p_{HML}^*|q = H) = \gamma_H p_{HML}^* > Max_p \Pi(a_j, p|q = H) = \frac{V}{2}$$
(14)

Similarly, for M-type not to deviate from the equilibrium, it must be the case that:

$$\Pi^*(a_0, p_{HML}^*|q = M) = \gamma_M p_{HML}^* > Max_p \Pi(a_j, p|q = M) = \frac{\overline{V}}{2}$$
(15)

The *L* type by definition can not deviate on advertising. A deviation on price can yield a maximum profit of 0. In order to prevent a deviation from the equilibrium, it must be the case that  $\Pi^*(a_0, p_{HML}^*|q = L) = \gamma_L p_{HML}^* > 0$ , which is trivially satisfied. Since  $\Pi^*(a_0, p_{HML}^*|q = H) > \Pi^*(a_0, p_{HML}^*|q = M)$ , the non-deviation condition for *H*-type is not binding if the non-deviation condition for *M*-type holds. This completes the proof of Proposition **4. Q.E.D.** 

### HM Equilibrium

### **Proof of Proposition 5**

We present a proof by contradiction. Suppose that there exists an HM equilibrium where both types pool on  $(a_j, p_{HM}^{ns})$ , and the consumer buys without search.

1. First, we show that  $p_{HM}^{ns} < \overline{p}^j$  if  $c < \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$ , where  $\overline{p}^j$  is the upper bound in price where consumers may choose to search. Note that in equilibrium, the a priori probabilities

on the two types  $\{H, M_j\}$  are  $(\rho, 1 - \rho)$ . Given this, and using Lemma 2, we can show that the highest price that the firm can charge so that the consumer chooses not to search in equilibrium is  $\max(p_{HM}^{ns}) = \frac{(1-\gamma_H)\rho\overline{V} + (1-\gamma_M)(1-\rho)\overline{\frac{V}{2}} + c}{\rho(1-\gamma_H) + (1-\rho)(1-\gamma_M)} = \underline{p}^j(\rho)$ . We can also show that  $p_{HM}^{ns} \leq \underline{p}^j(\rho) < \overline{p}^j$  as long as  $c < \overline{\frac{V}{2}}\rho(1-\rho)(\gamma_H - \gamma_M)$ .

2. Next we derive the restrictions that D1 imposes on the out-of-equilibrium beliefs following the deviation  $A_1 = (a_j, p^{dev})$  where  $p^{dev} \in (p_{HM}^{ns}, \overline{p}^j]$ . From 1 above, as long as  $c < \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$ , we know that there exists a belief for which search is the best response since  $p^{dev} \leq \overline{p}^j$ . Therefore, the consumer's mixed best response has 2 possible forms: (1)  $\alpha_2 = \{0, \alpha_{22}, 1 - \alpha_{22}\}$  and (2)  $\alpha_2 = \{\alpha_{21}, 0, 1 - \alpha_{21}\}$ . In the first case,  $\Pi(A_1, \alpha_2, H) = (1 - \alpha_{22})\gamma_H p^{dev}$  and  $\Pi(A_1, \alpha_2, M) = (1 - \alpha_{22})\gamma_M p^{dev}$ ; in the second case,  $\Pi(A_1, \alpha_2, H) = (\alpha_{21} + (1 - \alpha_{21})\gamma_H) p^{dev}$  and  $\Pi(A_1, \alpha_2, M) = (\alpha_{21} + (1 - \alpha_{21})\gamma_M) p^{dev}$ . Also, of course,  $\Pi^*(H) = p_{HM}^{ns}$  and  $\Pi^*(M) = p_{HM}^{ns}$ .

This yields the sets,

$$D^{0}(M, A_{1}) \cup D(M, A_{1}) = \widehat{X_{M}} \cup \widehat{Y_{M}} =$$

$$\left\{ (0, \alpha_{22}, 1 - \alpha_{22}) \mid \alpha_{22} \leq \frac{\gamma_{M} p^{dev} - p_{HM}^{ns}}{\gamma_{M} p^{dev}} \right\} \cup \left\{ (\alpha_{21}, 0, 1 - \alpha_{21}) \mid \alpha_{21} \geq \frac{p_{HM}^{ns} - \gamma_{M} p^{dev}}{(1 - \gamma_{M}) p^{dev}} \right\}$$
(16)

$$D(H, A_1) = X_H \cup Y_H =$$

$$\left\{ (0, \alpha_{22}, 1 - \alpha_{22}) \mid \alpha_{22} < \frac{\gamma_H p^{dev} - p_{HM}^{ns}}{\gamma_H p^{dev}} \right\} \cup \left\{ (\alpha_{21}, 0, 1 - \alpha_{21}) \mid \alpha_{21} > \frac{p_{HM}^{ns} - \gamma_H p^{dev}}{(1 - \gamma_H) p^{dev}} \right\}$$
(17)

We can show that since  $\gamma_H > \gamma_M$ ,  $\frac{\gamma_M p^{dev} - p_{HM}^{ns}}{\gamma_M p^{dev}} < \frac{\gamma_H p^{dev} - p_{HM}^{ns}}{\gamma_H p^{dev}}$ , which implies that  $\widehat{X_M} \subseteq X_H$ . In addition, since  $\frac{\gamma_M p^{dev} - p_{HM}^{ns}}{\gamma_M p^{dev}} < 1 \Leftrightarrow p_{HM}^{ns} > 0$ , it must be the case that  $\widehat{X_M} \subset X_H$ . Finally, we can similarly show that  $\frac{p_{HM}^{ns} - \gamma_M p^{dev}}{(1 - \gamma_M) p^{dev}} > \frac{p_{HM}^{ns} - \gamma_H p^{dev}}{(1 - \gamma_H) p^{dev}}$ , which implies that  $\widehat{Y_M} \subseteq Y_H$ . Hence, we have shown that  $(\widehat{X_M} \cup \widehat{Y_M}) \subset (X_H \cup Y_H)$ , which implies that  $D^0(M, A_1) \cup D(M, A_1) \subset D(H, A_1)$  for all  $A_1 = (a_j, p^{dev})$  where  $p^{dev} \in (p_{HM}^{ns}, \overline{p}^j]$ . This implies that D1 constrains the belief to be  $\mu_H^j = 1$  following  $A_1 = (a_j, p^{dev})$ , where  $p_{HM}^{ns} < p^{dev} \leq \overline{p}^j$ , which implies that both H and M types prefer to deviate to  $A_1$ , which, in turns, destroys this equilibrium. **Q.E.D.** 

### **Proof of Proposition 6**

We consider the HM equilibrium where H and M pool on  $(a_j, p_{HM}^*)$ , and the consumer searches. Before we turn to the equilibrium conditions, we first examine the restrictions on the out-of-equilibrium beliefs that are imposed by D1: Lemma 4 The out-of-equilibrium beliefs that are consistent with D1 have the following properties:

- 1. When the consumer observes the deviation  $A_1 = (a_j, p^{dev} \neq p_{HM}^*)$ , where  $\max(\gamma_M p_{HM}^*, \underline{p}^j) < p^{dev} < \min(p_{HM}^*, \overline{p}^j)$  or  $\gamma_M p_{HM}^* < p^{dev} < \min(\gamma_H p_{HM}^*, \underline{p}^j)$ , she forms the belief  $\mu_M(A_1) = 1$ .
- 2. When the consumer observes the deviation  $A_1 = (a_0, p^{dev} \neq 0)$ , where  $0 < p^{dev} \leq \overline{p}^0$ , she forms the belief  $\mu_L(A_1) = 1$ .

**Proof.** We follow the same logic as we do in the Proof for Lemma 3. For example,  $D^0(H, A_1) \cup D(H, A_1)$  and  $D^0(M, A_1) \cup D(M, A_1)$  are the same as equations (11) and (12). The only difference is that L type in equilibrium earns 0 profit. Hence,

$$D^{0}(L, A_{1}) = X_{L}^{0} \cup Y_{L}^{0} = \{(0, \alpha_{22}, 1 - \alpha_{22}) \mid \alpha_{22} = 1\} \cup \emptyset,$$
  
$$D(L, A_{1}) = X_{L} \cup Y_{L} = \{(0, \alpha_{22}, 1 - \alpha_{22}) \mid \alpha_{22} < 1\} \cup \{(\alpha_{21}, 0, 1 - \alpha_{21}) \mid 0 \le \alpha_{21} \le 1\}.$$

- 1. Consider a deviation to a price such that. the consumer chooses not to purchase at any off-equilibrium belief:  $A_1 = (a_j, p^{dev})$  where  $p^{dev} > \overline{p}^j$  or  $A_1 = (a_0, p^{dev})$  where  $p^{dev} > \overline{p}^0$ . Here D1 does not apply.
- 2. Next, consider a deviation to a price such that. the consumer chooses to purchase without search at any off-equilibrium belief:  $A_1 = (a_j, p^{dev})$  where  $p^{dev} < \underline{p}^j$  or  $A_1 = (a_0, p^{dev})$  where  $p^{dev} < \underline{p}^0$ ; i.e.,  $\alpha_{21} = 1$ . Therefore, if  $A_1 = (a_j, p^{dev})$  where  $p^{dev} \in (\gamma_M p_{HM}^*, \min(\gamma_H p_{HM}^*, \underline{p}^j))$ , D1 imposes that  $\mu_H = 0$ . Similarly, if  $A_1 = (a_0, p^{dev})$  where  $p^{dev} \in (0, \min(\gamma_M p_{HM}^*, \underline{p}^0))$ ,  $\mu_L = 1$ , and for all  $p^{dev} \in (\gamma_M p_{HM}^*, \min(\gamma_H p_{HM}^*, \underline{p}^0))$ ,  $\mu_H = 0$ .
- 3. If the deviation is  $A_1 = (a_j, p^{dev} \neq p_{HM}^*)$  and  $\underline{p}^j \leq p^{dev} \leq \overline{p}^j$ , the proof is identical to the Proof for Lemma 3: D1 imposes the belief that  $\mu_M(A_1) = 1$  if  $\max(\gamma_M p_{HM}^*, \underline{p}^j) < p^{dev} < \min(p_{HM}^*, \overline{p}^j)$ . Furthermore, we can see that  $\underline{p}^j \leq \gamma_M p_{HM}^*$  since  $p_{HM}^* \in [\underline{p}_{HM}, \overline{p}_{HM}]$ , where  $\overline{p}_{HM} = \overline{p}^j(\rho) \leq \overline{p}^j$  from Lemma 2. Also, we can show that  $p_{HM}^* \leq \overline{p}^j$ . This is so because M must prefer its equilibrium strategy to an optimal deviation, in equilibrium  $\gamma_M p_{HM}^* > Max_{A_1}\Pi(A_1|q = M)$ . In particular, a firm can deviate and charge a price  $p \leq \underline{p}^j$ such that the consumer chooses to purchase without search. Hence,  $Max_{A_1}\Pi(A_1|q = M) \geq \underline{p}^j$  and, therefore,  $\gamma_M p_{HM}^* \geq \underline{p}^j$ . Therefore, D1 imposes the belief that  $\mu_M(A_1) = 1$  if  $\gamma_M p_{HM}^* < p^{dev} < p_{HM}^*$ .

4. Consider a deviation strategy of  $A_1 = (a_0, p^{dev} < \overline{p}^0)$ , where  $\underline{p}^0 \leq p^{dev} \leq \overline{p}^0$ . First, note that the consumer's response  $(0, \alpha_{22} = 1, 0) \notin D^0(H, A_1) \cup D(H, A_1)$  and  $(0, \alpha_{22} = 1, 0) \notin D^0(M, A_1) \cup D(M, A_1)$ . This is true since in equilibrium the H and M types make non-zero profit. Hence,  $\widehat{X_H} \subset X_L$  and  $\widehat{X_M} \subset X_L$ . Moreover,  $\widehat{Y_H} \subset Y_M$  and  $\widehat{Y_M} \subset Y_L$  since  $\gamma_L < \gamma_M < \gamma_H$ . This implies that  $D^0(H, A_1) \cup D(H, A_1) \subset D(L, A_1)$  and  $D^0(M, A_1) \cup D(M, A_1) \subset D(L, A_1)$ . Note that if  $p^{dev} > \overline{p}^0$ , the consumer's best response is no purchase, i.e.,  $\alpha_{22} = 1$ , which in turn would imply that  $D(L, A_1)$  is an empty set or that D1 does not apply. Hence, if  $A_1 = (a_0, p^{dev} < \overline{p}^0)$ , D1 implies that  $\mu_L(A_1) = 1$ .

One example of an off-equilibrium belief which is consistent with the properties described above is  $\mu_L = 1$  for all  $(a_0, p)$  and  $\mu_M = 1$  for all  $(a_j, p \neq p^*_{HM})$ .

From now on, we assume that the off-equilibrium beliefs are:  $\mu_L = 1$  for all  $(a_0, p)$  and  $\mu_M = 1$  for all  $(a_j, p \neq p_{HM}^*)$ , which we just showed are consistent with D1.

Next, we show that the HM equilibrium with search exists if  $c < \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$  and  $Max\left[\frac{\overline{V}}{2\gamma_M}, \underline{p}_{HM}\right] < p_{HM}^* \leq \overline{p}_{HM}.$ 

On the equilibrium path, the probability that the firm is type H is  $\rho$ , and the probability that it is type M is  $1 - \rho$ . As we can see from Lemma 2, search will not occur at *any* price unless  $\underline{\mu} \leq \rho \leq \overline{\mu}$  or  $c \leq \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$  (see the Technical Appendix for the proof of Lemma 2). In addition, in order for the consumer to engage in search at the equilibrium price, the price must be such that  $p_{HM}^* \in [\underline{p}_{HM}, \overline{p}_{HM}]$ , where  $\overline{p}_{HM} = \frac{\gamma_H \rho \overline{V} + \gamma_M (1-\rho) \overline{V} - c}{\gamma_H \rho + \gamma_M (1-\rho)} \equiv \overline{p}^j(\rho)$  and  $\underline{p}_{HM} = \frac{(1-\gamma_H)\rho \overline{V} + (1-\gamma_M)(1-\rho)\overline{V} + c}{(1-\gamma_H)\rho + (1-\gamma_M)(1-\rho)} \equiv \underline{p}^j(\rho)$  (see Lemma 2). Hence, when  $c < \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$ and  $p_{HM}^* \in [\underline{p}_{HM}, \overline{p}_{HM}]$ , the consumer chooses to search along the equilibrium path.

In order for the equilibrium to hold, all types must prefer the equilibrium profits to the optimal deviation. Given the assumed out-of-equilibrium beliefs, this reduces to the following conditions:

$$\Pi^{*}(a_{j}, p_{HM}|q = H) = \gamma_{H}p_{HM}^{*} > Max_{A_{1}}\Pi(A_{1}|q = H) = \frac{V}{2}$$
$$\Pi^{*}(a_{j}, p_{HM}|q = M) = \gamma_{M}p_{HM}^{*} > Max_{A_{1}}\Pi(A_{1}|q = M) = \frac{\overline{V}}{2}$$

This of course reduces to the non-deviation condition on M-type:  $\gamma_M p_{HM}^* > \overline{\frac{V}{2}}$  or  $p_{HM}^* > \frac{\overline{V}}{2\gamma_M}$ . This along with the condition that  $p_{HM}^* \in [\underline{p}_{HM}, \overline{p}_{HM}]$  results in the necessary condition for existence:  $Max \left[ \frac{\overline{V}}{2\gamma_M}, \underline{p}_{HM} \right] < p_{HM}^* \leq \overline{p}_{HM}$ . This completes the proof of Proposition 6. Q.E.D.

### **HL Equilibrium**

### **Proof of Proposition 7**

We show this result by contradiction. Suppose that there exists an HL equilibrium without consumer search:  $(a_H = a_L = a_0, p_H = p_L \equiv p_{HL}^{ns}, a_M = a_j, p_M = \frac{\overline{V}}{2})$ . Note that in equilibrium, given a priori beliefs, the belief following  $(a_0, p_{HL})$  must be  $\mu_H^0 = \frac{1}{2}, \ \mu_L^0 = \frac{1}{2}$ . Hence, applying Lemma 2, we know that  $p_{HL}^{ns} \leq \underline{p}^0 = \frac{\frac{1}{2}(1-\gamma_H)\overline{V}+c}{\frac{1}{2}(1-\gamma_H)+\frac{1}{2}(1-\gamma_L)}$ . Note that  $\underline{p}^0 < \overline{\underline{V}}$  as long as  $c < \frac{\overline{V}(\gamma_H - \gamma_L)}{4}$ . Hence, this implies that if  $c < \frac{\overline{V}(\gamma_H - \gamma_L)}{4}, \ p_{HL}^{ns} < \frac{\overline{V}}{2}$  and H-type prefers to deviate to M's strategy, which would destroy the proposed equilibrium. Finally, note that  $\frac{\overline{V}(\gamma_H - \gamma_L)}{4} > \frac{\overline{V}(\gamma_H - \gamma_M)}{8}$ . Hence, for  $c < \frac{\overline{V}(\gamma_H - \gamma_M)}{8}$ , this equilibrium similarly does not exist. **Q.E.D.** 

### **Proof of Proposition 8**

We consider the HL equilibrium where H and L pool on  $(a_0, p_{HL}^*)$ , and the consumer searches following  $(a_0, p_{HL}^*)$ . M, on the other hand, truthfully reveals its type and separates on  $(a_j, \frac{\overline{V}}{2})$ . We examine the restrictions on the out-of-equilibrium beliefs that are imposed by D1. First, we assume that  $p_{HL}^* < \frac{\overline{V}}{2\gamma_M}$ . We will return to this assumption below and confirm that it is indeed the case in equilibrium.

**Lemma 5** Suppose that  $p_{HL}^* < \frac{\overline{V}}{2\gamma_M}$ . D1 imposes the following constraints on out-of-equilibrium beliefs:

1. If the consumer observes  $A_1 = (a_j, p^{dev})$ ,

$$(a) when \frac{\overline{V}}{2} < p^{dev} < \min(\gamma_H p_{HL}^*, \underline{p}^j), \ \mu_H(A_1) = 0,$$

$$(b) if \frac{(\gamma_H - \gamma_M)\overline{p}_j + \frac{\overline{V}}{2}(1 - \gamma_H)}{\gamma_H(1 - \gamma_M)} \le p_{HL}^*, \ when \ \underline{p}^j \le p^{dev} \le \overline{p}^j = \min(\frac{\gamma_H \left(1 - \gamma_M\right) p_{HL}^* - \frac{\overline{V}}{2}(1 - \gamma_H)}{\gamma_H - \gamma_M}, \overline{p}^j),$$

$$\mu_H(A_1) = 0,$$

$$(c) if \frac{(\gamma_H - \gamma_M)\overline{p}_j + \frac{\overline{V}}{2}(1 - \gamma_H)}{\gamma_H(1 - \gamma_M)} > p_{HL}^*, \ when \ \max(\underline{p}^j, \frac{\gamma_H \left(1 - \gamma_M\right) p_{HL}^* - \frac{\overline{V}}{2}(1 - \gamma_H)}{\gamma_H - \gamma_M}) < p^{dev} \le \overline{p}^j, \ \mu_H(A_1) = 1.$$

2. If the consumer observes the deviation  $A_1 = (a_0, p^{dev})$ ,

(a) when 
$$\gamma_L p_{HL}^* < p^{dev} < \min(\gamma_H p_{HL}^*, \underline{p}^0), \ \mu_H(A_1) = 0,$$
  
(b) when  $\gamma_M p_{HL}^* < \overline{\frac{V}{2}}, and \ max(\underline{p}^0, \gamma_L p_{HL}^*) < p^{dev} < \min(p_{HL}^*, \overline{p}^0), \ \mu_L(A_1) = 1,$ 

(c) when 
$$\gamma_M p_{HL}^* < \frac{\overline{V}}{2}$$
, and  $p_{HL}^* < p^{dev} < \overline{p}^0$ ,  $\mu_M(A_1) = 0$ .

**Proof.** Here, we have

$$\begin{split} D^{0}(H,A_{1}) \cup D(H,A_{1}) &= \widehat{X_{H}} \cup \widehat{Y_{H}} = \\ &\left\{ (0,\alpha_{22},1-\alpha_{22}) \mid \alpha_{22} \leq \frac{p^{dev} - p_{HL}^{*}}{p^{dev}} \right\} \cup \left\{ (\alpha_{21},0,1-\alpha_{21}) \mid \alpha_{21} \geq \frac{\gamma_{H}\left(p_{HL}^{*} - p^{dev}\right)}{(1-\gamma_{H})p^{dev}} \right\} \\ &D^{0}(M,A_{1}) \cup D(M,A_{1}) = \widehat{X_{M}} \cup \widehat{Y_{M}} = \\ &\left\{ (0,\alpha_{22},1-\alpha_{22}) \mid \alpha_{22} \leq \frac{\gamma_{M}p^{dev} - \frac{\overline{V}}{2}}{\gamma_{M}p^{dev}} \right\} \cup \left\{ (\alpha_{21},0,1-\alpha_{21}) \mid \alpha_{21} \geq \frac{\overline{V}}{2} - \gamma_{M}p^{dev}}{(1-\gamma_{M})p^{dev}} \right\} \\ &D^{0}(L,A_{1}) \cup D(L,A_{1}) = \widehat{X_{L}} \cup \widehat{Y_{L}} = \\ &\left\{ (0,\alpha_{22},1-\alpha_{22}) \mid \alpha_{22} \leq \frac{p^{dev} - p_{HL}^{*}}{p^{dev}} \right\} \cup \left\{ (\alpha_{21},0,1-\alpha_{21}) \mid \alpha_{21} \geq \frac{\gamma_{L}\left(p_{HL}^{*} - p^{dev}\right)}{(1-\gamma_{L})p^{dev}} \right\} \end{split}$$

- 1. Consider a deviation to a price such that the consumer chooses not to purchase at any offequilibrium belief:  $A_1 = (a_j, p^{dev})$  where  $p^{dev} > \overline{p}^j$  or  $A_1 = (a_0, p^{dev})$  where  $p^{dev} > \overline{p}^0$ ; i.e.,  $\alpha_{22} = 1$ . Here, D1 does not apply.
- 2. Next, consider a deviation to a price such that the consumer chooses to purchase without search at any off-equilibrium belief:  $A_1 = (a_j, p^{dev})$  where  $p^{dev} < \underline{p}^j$  or  $A_1 = (a_0, p^{dev})$  where  $p^{dev} < \underline{p}^0$ ; i.e.,  $\alpha_{21} = 1$ . Therefore, D1 imposes that  $\mu_H(A_1) = 0$  if  $A_1 = (a_j, p^{dev})$ , for all  $\overline{\underline{V}} \leq p^{dev} < \min(\gamma_H p^*_{HL}, \underline{p}^j)$ . Similarly, if  $A_1 = (a_0, p^{dev})$ , for all  $\gamma_L p^*_{HL} \leq p^{dev} < \min(\gamma_H p^*_{HL}, \underline{p}^0)$ ,  $\mu_H(A_1) = 0$ .
- 3. Consider  $A_1 = (a_j, p^{dev})$ , and  $\underline{p}^j \le p^{dev} \le \overline{p}^j$ .
  - (a) Assume that  $\frac{(\gamma_H \gamma_M)\overline{p}_j + \overline{\underline{V}}_2(1 \gamma_H)}{\gamma_H(1 \gamma_M)} \leq p_{HL}^* \Leftrightarrow \overline{p}_j \leq \frac{\gamma_H(1 \gamma_M)p_{HL}^* \overline{\underline{V}}_2(1 \gamma_H)}{\gamma_H \gamma_M}$ . If  $p^{dev} < \frac{\gamma_H(1 \gamma_M)p_{HL}^* \overline{\underline{V}}_2(1 \gamma_H)}{\gamma_H \gamma_M}$ , we can show, using simple calculus, that  $\frac{\gamma_H(p_{HL}^* p^{dev})}{(1 \gamma_H)p^{dev}} > \frac{\gamma_M p^{dev} \overline{\underline{V}}_2}{(1 \gamma_M)p^{dev}}$ , which implies that  $\widehat{Y_H} \subset Y_M$ . Also, we can see that  $\frac{\gamma_H(1 \gamma_M)p_{HL}^* \overline{\underline{V}}_2(1 \gamma_H)}{\gamma_H \gamma_M} < p_{HL}^*$  as long as  $p_{HL}^* < \frac{\overline{V}}{2\gamma_M}$ . Hence, we have  $p^{dev} < \frac{\gamma_H(1 \gamma_M)p_{HL}^* \overline{\underline{V}}_2(1 \gamma_H)}{\gamma_H \gamma_M} < p_{HL}^*$  here. This in turn implies that  $X_M = \widehat{X_H} = \emptyset$ . Therefore, for  $\underline{p}^j < p^{dev} < \min(\frac{\gamma_H(1 \gamma_M)p_{HL}^* \overline{\underline{V}}_2(1 \gamma_H)}{\gamma_H \gamma_M}, \overline{p}^j)$ , D1 constrains the belief to be  $\mu_H = 0$  following  $A_1$ . Of course, since  $\overline{p}_j \leq \frac{\gamma_H(1 \gamma_M)p_{HL}^* \overline{\underline{V}}_2(1 \gamma_H)}{\gamma_H \gamma_M}$ ,
  - (b) Second, consider  $\frac{(\gamma_H \gamma_M)\overline{p}_j + \overline{\frac{V}{2}}(1 \gamma_H)}{\gamma_H(1 \gamma_M)} > p_{HL}^* \Leftrightarrow \overline{p}_j > \frac{\gamma_H(1 \gamma_M)p_{HL}^* \overline{\frac{V}{2}}(1 \gamma_H)}{\gamma_H \gamma_M}$ . Then, there exists an interval such that  $\frac{\gamma_H(1 - \gamma_M)p_{HL}^* - \overline{\frac{V}{2}}(1 - \gamma_H)}{\gamma_H - \gamma_M} \leq p^{dev} < \min(\overline{p}^j, p_{HL}^*)$ . Using

the same argument as in (a) above, we can show that here  $\widehat{Y_M} \subset Y_H$ , and  $X_H = \widehat{X_M} = \varnothing$ . Hence, as long as  $\max(\underline{p}^j, \frac{\gamma_H(1-\gamma_M)p_{HL}^* - \overline{\underline{\nabla}}(1-\gamma_H)}{\gamma_H - \gamma_M}) < p^{dev} < \min(\overline{p}^j, p_{HL}^*)$ , D1 constrains the belief to be  $\mu_H = 1$  following  $A_1$ . Next, consider  $p^{dev} \ge p_{HL}^*$ . We can see that when  $p_{HL}^* < \frac{\overline{\nabla}}{2\gamma_M}, \frac{\gamma_M p^{dev} - \overline{\underline{\nabla}}}{\gamma_M p^{dev}} < \frac{p^{dev} - p_{HL}^*}{p^{dev}} < 1$ , which implies that  $\widehat{X_M} \subset X_H$ . Also, we know that in this region  $\widehat{Y_M} \subset Y_H$ . Hence, D1 implies that  $\mu_H = 1$  following  $p^{dev}$ , where  $\max(p_{HL}^*, \underline{p}^j) < p^{dev} < \overline{p}^j$ . In summary, D1 implies that for  $\max(\underline{p}^j, \frac{\gamma_H(1-\gamma_M)p_{HL}^* - \overline{\underline{\nabla}}(1-\gamma_H)}{\gamma_H - \gamma_M}) < p^{dev} \le \overline{p}^j, \mu_H = 1$ .

4. Consider  $A_1 = (a_0, p^{dev} \neq p^*_{HL})$  and  $\underline{p}^0 \leq p^{dev} \leq \overline{p}^0$ .

- (a) Consider  $p^{dev} < p_{HL}^*$ , which implies that  $X_H = X_M = X_L = \varnothing$ . Also,  $\frac{\gamma_L(p_{HL}^* p^{dev})}{(1 \gamma_L)p^{dev}} < \frac{\gamma_H(p_{HL}^* p^{dev})}{(1 \gamma_L)p^{dev}}$ ,  $\frac{\gamma_L(p_{HL}^* p^{dev})}{(1 \gamma_L)p^{dev}} < 1$  if  $p^{dev} > \gamma_L p_{HL}^*$ , and  $\frac{\gamma_L(p_{HL}^* p^{dev})}{(1 \gamma_L)p^{dev}} < \frac{\overline{\Sigma} \gamma_M p^{dev}}{(1 \gamma_M)p^{dev}}$  if  $p^{dev} < \frac{\overline{\Sigma} (1 \gamma_L) \gamma_L (1 \gamma_M) p_{HL}^*}{\gamma_M \gamma_L}$ . Moreover, we can see that when  $\gamma_M p_{HL}^* < \overline{\Sigma}$ ,  $p_{HL}^* < \frac{\overline{\Sigma} (1 \gamma_L) \gamma_L (1 \gamma_M) p_{HL}^*}{\gamma_M \gamma_L}$ . Hence, when  $\gamma_M p_{HL}^* < \overline{\Sigma}$ ,  $\widehat{Y_H} \subset Y_L$  and  $\widehat{Y_M} \subset Y_L$  if  $max(\underline{p}^0, \gamma_L p_{HL}^*) < p^{dev} < p^{dev} < \min(p_{HL}^*, \overline{p}^0)$ , which implies that D1 constrains the belief to be  $\mu_L = 1$  following  $A_1 = (a_0, p^{dev})$ .
- (b) Second, consider  $p^{dev} > p_{HL}^*$ , which implies that  $\widehat{X_H} = \widehat{X_L} \neq \emptyset$  and  $Y_H = Y_L = \{ \forall \alpha_{21} \in [0,1] \}$ . Also, if  $\gamma_M p_{HL}^* < \frac{\overline{V}}{2}$ ,  $\frac{\gamma_M p^{dev} \frac{\overline{V}}{2}}{(1 \gamma_M) p^{dev}} < \frac{p^{dev} p_{HL}^*}{p^{dev}}$ , which implies that  $\widehat{X_M} \subset X_L$  and  $\widehat{X_M} \subset X_H$ . Hence,  $D^0(M, A_1) \cup D(M, A_1) \subset D(L, A_1)$  and  $D^0(M, A_1) \cup D(M, A_1) \subset D(H, A_1)$ , which implies that D1 constraints the belief to be  $\mu_M = 0$  following  $A_1 = (a_0, p^{dev})$ .

Given the out-of-equilibrium beliefs which are consistent with D1, if  $p_{HL}^* < \frac{(\gamma_H - \gamma_M)\bar{p}_j + \overline{\underline{V}}_2(1 - \gamma_H)}{\gamma_H(1 - \gamma_M)}$ , there always exists a profitable deviation under D1. To show this, consider  $A_1 = (a_j, p^{dev})$ where  $\frac{\gamma_H(1 - \gamma_M)p_{HL}^* - \overline{\underline{V}}_2(1 - \gamma_H)}{\gamma_H - \gamma_M} < p^{dev} \leq \overline{p}^j$ . Based on Lemma 5 1-(c),  $\mu_H = 1$ : consumer buys the product without search. Both H and M types prefer to deviate to  $A_1$ , which, in turn, destroys this equilibrium. Hence, for the HL equilibrium to exist, it must be the case that  $\frac{(\gamma_H - \gamma_M)\overline{p}_j + \overline{\underline{V}}_2(1 - \gamma_H)}{\gamma_H(1 - \gamma_M)} \leq p_{HL}^*$ . When  $\frac{(\gamma_H - \gamma_M)\overline{p}_j + \overline{\underline{V}}_2(1 - \gamma_H)}{\gamma_H(1 - \gamma_M)} < p_{HL}^* < \frac{\overline{V}}{2\gamma_M}$ , one example of an off-equilibrium belief which is consistent with the properties described above is  $\mu_L = 1$  for all  $(a_0, p \neq p_{HL}^*)$  and  $\mu_H = 0$  for all  $(a_j, p \neq \overline{\underline{V}})$ . This is the belief that we assume to demonstrate existence below.

Next, we show that the *HL* equilibrium with search exists if  $c < \frac{\overline{V}(\gamma_H - \gamma_M)}{8}$  and  $\frac{\overline{V}}{2\gamma_H} \le p_{HM}^* < p_{HM}^*$  $\min\left\{\frac{\gamma_H \overline{V} - 2c}{\gamma_H + \gamma_L}, \frac{\overline{V}}{2\gamma_M}\right\}.$  As we can see from Lemma 2, we know that in order for the consumer to search in equilibrium, it must be the case that  $c \leq \frac{\overline{V}(\gamma_H - \gamma_M)}{8}$  and  $p_{HL} \in [\underline{p}^0(\mu_L), \overline{p}^0(\mu_L)]$ , where  $\overline{p}^0(\mu_L) = \frac{\mu_H^0 \gamma_H \overline{\nabla} + \mu_M^0 \gamma_M \frac{\overline{\nabla}}{2} - c}{\mu_H^0 \gamma_H + \mu_M^0 \gamma_M + \mu_L^0 \gamma_L}$  and  $\underline{p}^0(\mu_L) = \frac{\mu_H^0 (1 - \gamma_H) \overline{\nabla} + \mu_M^0 (1 - \gamma_M) \frac{\overline{\nabla}}{2} + c}{\mu_H^0 (1 - \gamma_H) + \mu_M^0 (1 - \gamma_M) + \mu_L^0 (1 - \gamma_L)}$ . In addition, on the equilibrium path, the probability that the firm is type H and L when  $(a_0, p_{HL}^*)$  are  $\frac{1}{2}$ , and  $\frac{1}{2}$ . Hence,  $\overline{p}^0(\frac{1}{2}) = \frac{\gamma_H \overline{V} - 2c}{\gamma_H + \gamma_L} > \frac{1}{2} \overline{V}$  and  $\underline{p}^0(\frac{1}{2}) = \frac{(1 - \gamma_H)\overline{V} + 2c}{2 - \gamma_H - \gamma_L} < \frac{\overline{V}}{2}$  since  $c \leq \frac{\overline{V}(\gamma_H - \gamma_M)}{8}$ . Hence, the price should be  $p_{HL}^* \in [\frac{(1-\gamma_H)\overline{V}+2c}{2-\gamma_H-\gamma_L}, \frac{\gamma_H\overline{V}-2c}{\gamma_H+\gamma_I}].$ 

In order for the equilibrium to hold, all types must prefer the equilibrium profits to the Given the assumed out-of-equilibrium beliefs, this reduces to the following optimal deviation. conditions:

$$\Pi^{*}(a_{0}, p_{HL}^{*}|q = H) = \gamma_{H}p_{HL}^{*} > Max_{A_{1}}\Pi(A_{1}|q = H) = \frac{V}{2}$$

$$\Pi^{*}(a_{j}, p_{M}|q = M) = \frac{\overline{V}}{2} > Max_{A_{1}}\Pi(A_{1}|q = M) = \gamma_{M}p_{HL}^{*}$$
(18)

The *H* type can deviate on both advertising and price. If he deviates on price alone  $(a_0, p' \neq p_{HL})$ , the consumer believes that he is L type. Hence, he can not earn a higher profit than in equilibrium. Also, if he deviates on advertising  $a = a_j$ , the consumer believes that he is M type. Hence, the maximal price that he can charge is  $p = \frac{\overline{V}}{2}$ . Similarly, the maximum profit that the M type can obtain from deviation is to deviate on advertising and charging  $p_{HL}^*$ . Finally, the L type by definition can not deviate on advertising. A deviation on price only yields a maximum profit of 0 under the off-equilibrium beliefs. In order to prevent a deviation from equilibrium, it must be the case that  $\Pi^*(a_0, p_{HL}|q = L) = \gamma_L p_{HL}^* > 0$ , which is trivially satisfied.

From the equation (18) and the search condition that  $p_{HL}^* \in \left[\frac{(1-\gamma_H)\overline{V}+2c}{2-\gamma_H-\gamma_L}, \frac{\gamma_H\overline{V}-2c}{\gamma_H+\gamma_L}\right]$ , we can see that the equilibrium price must be  $\max\left\{\frac{\overline{V}}{2\gamma_{H}}, \frac{(1-\gamma_{H})\overline{V}+2c}{2-\gamma_{H}-\gamma_{L}}\right\} \le p_{HL}^{*} < \min\left\{\frac{\gamma_{H}\overline{V}-2c}{\gamma_{H}+\gamma_{L}}, \frac{\overline{V}}{2\gamma_{M}}\right\}$ . Also note that  $\frac{(1-\gamma_{H})\overline{V}+2c}{2-\gamma_{H}-\gamma_{L}} < \frac{\overline{V}}{2} < \frac{\overline{V}}{2\gamma_{H}}$ . Hence,  $\max\left\{\frac{(1-\gamma_{H})\overline{V}+2c}{2-\gamma_{H}-\gamma_{L}}, \frac{\overline{V}}{2\gamma_{H}}\right\} = \frac{\overline{V}}{2\gamma_{H}}$ . Combined with the condition from D1,  $\frac{(\gamma_{H}-\gamma_{M})\overline{p}_{j}+\frac{\overline{V}}{2}(1-\gamma_{H})}{\gamma_{H}(1-\gamma_{M})} \le p_{HL}^{*}$ , the equilibrium price must be  $\max\left\{\frac{\overline{V}}{2\gamma_{H}}, \frac{(\gamma_{H}-\gamma_{M})\overline{p}_{j}+\frac{\overline{V}}{2}(1-\gamma_{H})}{\gamma_{H}(1-\gamma_{M})}\right\} \le p_{HL}^{*}$ . We can also show that  $\overline{p}_{j} \ge \frac{\overline{V}}{2}$  implies that  $\overline{V} = (\gamma_{H}-\gamma_{V})\overline{p}_{J}+\frac{\overline{V}}{2}(1-\gamma_{V})}$ .

 $\frac{\overline{V}}{2\gamma_H} \leq \frac{(\gamma_H - \gamma_M)\overline{p}_j + \frac{\overline{V}}{2}(1 - \gamma_H)}{\gamma_H(1 - \gamma_M)}.$  Therefore, the necessary conditions for the existence of the *HL* 

equilibrium under D1 are the following:

$$c < \frac{V(\gamma_H - \gamma_M)}{8} \text{ and}$$
$$\frac{(\gamma_H - \gamma_M)\overline{p}_j + \frac{\overline{V}}{2}(1 - \gamma_H)}{\gamma_H(1 - \gamma_M)} \leq p_{HL}^* < \min\left\{\frac{\gamma_H\overline{V} - 2c}{\gamma_H + \gamma_L}, \frac{\overline{V}}{2\gamma_M}\right\}.$$

Q.E.D.

### **Proof of Proposition 9**

The first condition ensures that HL exists (see Proposition 8). The remaining equilibria that survive the D1 refinement are HM with search and HML with search (see Proposition 6 and Proposition 4). We turn to HM first. Note that in order for the consumer to search in equilibrium,  $p_{HM}^* \leq \overline{p}_{HM}$  when  $c < \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$  (see Lemma 2). Suppose that  $\gamma_M \overline{p}_{HM} < \frac{\overline{V}}{2}$ , which implies that  $\gamma_M p_{HM}^* < \frac{\overline{V}}{2}$ . Consider a deviation by M to  $A_1 = \left(a_j, p^{dev} = \frac{\overline{V}}{2}\right)$ . The consumer is willing to purchase the product with no additional search (see Lemma 2). Hence, this implies that M prefers to deviate, which destroys this equilibrium. Hence, we demonstrated that HM does not exist if  $\gamma_M \overline{p}_{HM} < \frac{\overline{V}}{2}$ . Similarly, we can show that HML does not exist if  $\gamma_M \overline{p}_{HML}^0 < \frac{\overline{V}}{2}$ . Therefore, HM and HML do not exist if  $\gamma_M \cdot \max\left[\overline{p}_{HML}^0, \overline{p}_{HM}\right] < \frac{\overline{V}}{2}$ . Finally, using algebra, we can show that  $\overline{p}_{HM} > \overline{p}_{HML}^0$ , which reduces the sufficient "non-existence" condition to  $\gamma_M \cdot \overline{p}_{HM} < \frac{\overline{V}}{2}$ . To demonstrate that this region is non-empty, consider the following example:  $\gamma_H = 0.9$ ,  $\gamma_M = 0.5$ ,  $\gamma_L = 0.1, \overline{V} = 100, c = 5, \rho = \frac{2}{3}$ , and  $p_{HL}^* = [77.491, 80]$ . Here HL is the only equilibrium that survives D1. **Q.E.D.** 

### **Proof of Proposition 10**

First, we consider a separating equilibrium, where H's and M's equilibrium actions are respectively  $(a_H, p_H)$  and  $(a_M, p_M)$ , where  $(a_H, p_H) \neq (a_M, p_M)$ . Note that an equilibrium cannot be sustained unless  $p_H = p_M = \frac{\overline{V}}{2}$  since (1)  $p_H \neq p_M$  would induce one of the types to dviate, (2) a consumer would not pay more than  $\frac{\overline{V}}{2}$  for M-type, and (3) if the price is  $p < \frac{\overline{V}}{2}$ , a deviation to  $\frac{\overline{V}}{2}$ dominates the equilibrium. Next, using the same techniques as in the Proof of Proposition 5, we can show that D1 constrains  $\mu_H(A_1 = (a, p^{dev} = \overline{p}^j)) = 1$ . Since  $\overline{p}^j > \frac{\overline{V}}{2}$ , this would destroy this equilibrium.

Second, we consider a pooling equilibrium without search. Using the same techniques as in the

Proof of Proposition 5, we can show that the equilibrium does not survive the D1 refinement as long as  $c < \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$ .

Next, we consider a pooling equilibrium with search. Note that both types can pool on either  $a_j$  or  $a_0$ . Using a very similar derivation to the one shown in the Proof of Proposition 5, we can show that this equilibrium survives D1 as long as  $c \leq \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$ ,  $p_{HM}^* \in [\underline{p}_{HM}, \overline{p}_{HM}]$  where  $\underline{p}_{HM} = \frac{(1-\gamma_H)\rho\overline{V} + (1-\gamma_M)(1-\rho)\overline{\frac{V}{2}} + c}{(1-\gamma_H)\rho + (1-\gamma_M)(1-\rho)}$  and  $\overline{p}_{HM} = \frac{\gamma_H\rho\overline{V} + \gamma_M(1-\rho)\overline{\frac{V}{2}} - c}{\gamma_H\rho + \gamma_M(1-\rho)}$ , and  $\gamma_M p_{HM}^* > \frac{\overline{V}}{2}$ .

Finally, showing the same reasoning as in the Proof of Proposition 9, we can show that if  $\gamma_M \overline{p}_{HM} < \frac{\overline{V}}{2}$ , no pooling equilibrium can exist. **Q.E.D.** 

## References

- Abernethy, Avery M. and Daniel D. Butler (1992), "Advertising Information: Services versus Products," *Journal of Retailing*, 68 (4), 398-419.
- [2] Albers-Miller, Nancy D., and Marla Royne Stafford (1999), "An International Analysis of Emotional and Rational Appeals in Service vs Goods Advertising," *Journal of Consumer Marketing*, 16 (1), 42-57.
- [3] Anand, Bharat, and Ron Shachar (2007), "(Noisy) Communication," Quantitative Marketing and Economics, 5, 211-237.
- [4] Anderson, Simon P., and Regis Renault (2006), "Advertising Content," American Economic Review, 96 (1), 93-113.
- [5] Araujo, Aloisio, Daniel Gottlieb, and Humberto Moreira (2008), "A Model of Mixed Signals with Applications to Countersignaling and the GEC," RAND Journal of Economics, forthcoming.
- [6] Bagwell, Kyle (2007), "The Economic Analysis of Advertising," Handbook of Industrial Organization, Vol. 3, North-Holland: Amsterdam, 1701-1844.
- [7] Bhardwaj, Pradeep, Yuxin Chen, and David Godes (2007), "Buyer-Initiated versus Seller-Initiated Information Revelation," Harvard University working paper.
- [8] Butters, Gerard R. (1977), "Equilibrium Distributions of Sales and Advertising Prices," *Review of Economic Studies*, 44 (3), 465-491.
- [9] Carpenter, Gregory, Rashi Glazer, and Kent Nakamoto (1994), "Meaningful Brands from Meaningless Differentiation: The Dependence on Irrelevant Attributes," *Journal of Marketing Research*, 31, 339-350.
- [10] Chen, Yubo, and Jinhong Xie (2005), "Third-Party Product Review and Firm Marketing Strategy," *Marketing Science*, 24 (2), 218-240.

- [11] Chen, Yubo, and Jinhong Xie (2008), "Online Consumer Review: Word-of-Mouth as A New Element of Marketing Communication Mix," *Management Science*, 54 (3), 477-491.
- [12] Chevalier, Judith, and Dina Mayzlin (2006), "The Effect of Word of Mouth on Sales: Online Book Reviews," *Journal of Marketing Research*, 43 (3), 345-354.
- [13] Cho, In-Koo, and David M. Kreps (1987), "Signaling Games and Stable Equilibria," Quarterly Journal of Economics, 102 (May), 179-221.
- [14] Dittes, James E., and Harold H. Kelley (1956), "Effects of Different Conditions of Acceptance upon Conformity to Group Norms," *Journal of Abnormal and Social Psychology*, 53 (1), 100-107.
- [15] Engers, Maxim (1987), "Signalling with Many Signals," *Econometrica*, 55 (3), 663-674.
- [16] Feltovich, Nick, Richmond Harbaugh, and Ted To (2002), "Too Cool for School? Signalling and Countersignalling," RAND Journal of Economics, 33 (4), 630-649.
- [17] Fudenberg, Drew, and Jean Tirole (1991), *Game Theory*, Cambridge, MA, MIT Press.
- [18] Godes, David (2003), "In the Eye of the Beholder: An Analysis of the Relative Value of a Top Sales Rep Across Firms and Product," *Marketing Science*, 22 (2), 161-187.
- [19] Godes, David, and Dina Mayzlin (2003), "Using Online Conversations to Study Word-of-Mouth Communication," *Marketing Science*, 23 (4), 545-560.
- [20] Godes, David, and Andrea Wojnicki (2009), "Word-of-Mouth as Self Enhancement," HBS working paper.
- [21] Grossman, Gene M., and Carl Shapiro (1984), "Informative Advertising with Differentiated Products," *Review of Economic Studies*, 51 (1), 63-81.
- [22] Holbrook, Morris B., and John O'Shaughnessy (1984), "The Role of Emotion in Advertising," Psychology and Marketing, 1 (2), 45-64.
- [23] Hollander, E. P. (1958), "Conformity, Status, and Idiosyncrasy Credit," *Psychological Review*, 65 (2), 117-127.

- [24] Hvide, Hans K. (2003), "Education and the Allocation of Talent," *Journal of Labor Economics*, 21 (4), 945-976.
- [25] Kahneman, Daniel, and Amos Tversky (1982), Judgement Under Uncertainty: Heuristics and Biases, New York, Cambridge University Press.
- [26] Kuksov, Dmitri (2007), "Brand Value in Social Interaction," Management Science, 53 (10), 1634-1644.
- [27] McCarthy, Michael (2005), "Capital One's 'What's in your wallet?' ads filling airwaves," USA Today, March 13.
- [28] Mayzlin, Dina (2006), "Promotional Chat on the Internet," Marketing Science, 25 (2), 155-163.
- [29] Meurer, Michael and Stahl, Dale O., II.(1994), "Informative Advertising and Product Match," International Journal of Industrial Organization, 12 (1), 1-19.
- [30] Milgrom, Paul (1982), "Good News and Bad News: Representation Theorems and Applications," Bell Journal of Economics, 12, 380-391.
- [31] Milgrom, Paul and John Roberts (1986), "Price and Advertising Signals of Product Quality," Journal of Political Economy, 94 (August), 796-821.
- [32] Mullainathan, Sendhil, and Andrei Shleifer (2005), "Persuasion in Finance," Harvard University working paper.
- [33] Mullainathan, Sendhil, Joshua Schwartzstein, and Andrei Shleifer (2007), "Coarse Thinking and Persuasion," Harvard University *working paper*.
- [34] Nelson, Phillip (1974), "Advertising as Information," Journal of Political Economy, 78, 311-329.
- [35] Phillips, Damn J. and Ezra W. Zuckerman (2001), "Middle-Status Conformity: Theoretical Restatement and Empirical Demonstration in Two Markets," *American Journal of Sociology*, 107 (2), 379-429.

- [36] Quinzii, Martine, and Jean-Charles Rochet (1985), "Multidimensional Signalling," Journal of Mathematical Economics, 14, 261-284.
- [37] Rosselli, Francine, John J. Skelly, and Diane M. Mackie (1995), "Processing Rational and Emotional Messages: The Cognitive and Affective Mediation of Persuasion," *Journal of Experimental Social Psychology*, 31, 163-190.
- [38] Scott, Linda M.(1994), "Images in Advertising: The Need for a Theory of Visual Rhetoric," Journal of Consumer Research, 21 (3), 461-490.
- [39] Shapiro, Jesse M. (2006), "A Memory-Jamming Theory of Advertising," University of Chicago working paper.
- [40] Shin, Jiwoong (2005), "The Role of Selling Costs in Signaling Price Image," Journal of Marketing Research, 32 (August), 302-312.
- [41] Simester, Duncan (1995), "Signalling Price Image Using Advertised Prices," Marketing Science, 14 (Summer), 166-188.
- [42] Sun, Monic (2009), "Disclosing Multiple Product Attributes," Journal of Economics & Management Strategy, forthcoming.
- [43] Teoh, Siew H., and Chuan Y. Hwang (1991), "Nondisclosure and Adverse Disclosure as Signals of Firm Value," *Review of Financial Studies*, 4(2), 283-313.
- [44] Tirole, Jean (1986), "Hierarchies and Bureaucracies: On the Role of Collusion in Organizations," Journal of Law, Economics, & Organization, 2 (2), 181-214.
- [45] Wernerfelt, Birger (1994), "On the function of sales assistance," Marketing Science, 13 (Winter) 68-82.
- [46] Yoganarasimhan, Hema (2009), "Cloak or Flaunt? The Firm's Fashion Dilemma," UC Davis, working paper.