

Series in Real Analysis Volume 6

**UNIQUENESS AND
NONUNIQUENESS CRITERIA
FOR ORDINARY
DIFFERENTIAL EQUATIONS**

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