#16C

UILU-WRC-74-0088
RESEARCH REPORT NO. 88

UNIT STREAM POWER FOR SEDIMENT TRANSPORT IN NATURAL RIVERS

By CHIH TED YANG and JOHN B. STALL

ILLINOIS STATE WATER SURVEY
URBANA, ILLINOIS

UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN
WATER RESOURCES
CENTER

JULY 1974

WRC RESEARCH REPORT NO. 88

UNIT STREAM POWER FOR SEDIMENT TRANSPORT IN NATURAL RIVERS

by

Chih Ted Yang, Associate Hydrologist

and

John B. Stall, Head, Hydrology Section

Illinois State Water Survey
Urbana, Illinois

FINAL REPORT

Project No. B-075-ILL

UNIT STREAM POWER FOR SEDIMENT TRANSPORT IN NATURAL RIVERS

July 1, 1972 - June 30,1974

The work described in this publication was supported by matching funds of the Illinois State Water Survey and the U. S. Department of the Interior, as authorized under Title 1 of the Water Resources Research Act of 1964, P. L. 88-379

UNIVERSITY OF ILLINOIS
WATER RESOURCES CENTER
2535 Hydrosystems Laboratory
Urbana, Illinois 61801

July 1974

ABSTRACT

UNIT STREAM POWER FOR SEDIMENT TRANSPORT IN NATURAL RIVERS

The relationship between rate of sediment transport and rate of potential energy expenditure has been studied in detail. Unit stream power, defined as the time rate of potential energy expenditure per unit weight of water, is shown to be the dominant factor in the determination of total sediment concentration.

Basic concepts in fluid mechanics and boundary layer theory are used to establish the flow condition at incipient motion. Two equations that provide simple and direct criteria for incipient motion are found. These equations are verified by 153 sets of data independently collected by eight investigators.

A dimensionless unit stream power equation is found for the prediction of total sediment concentration for both laboratory flumes and natural rivers. This equation not only provides a good estimation of the total sediment concentration in an alluvial channel but also correctly reflects the effects of the variations of particle size, water depth, and water temperature on total sediment concentration. Data collected from six natural rivers and more than one thousand sets of laboratory data are used in supporting this equation. Data collected from 17 regular gaging stations on natural rivers also indicate that unit stream power dominates the rate of sediment transport.

REFERENCE: Yang, Chih Ted, and Stall, John B., **UNIT STREAM POWER FOR SEDIMENT TRANSPORT IN NATURAL RIVERS,** University of Illinois Water Resources Center Research Report No. 88.

KEY WORDS: Incipient motion, open channel hydraulics, rivers, sediment transport, unit stream power.

CONTENTS

UNIT STREAM POWER FOR SEDIMENT TRANSPORT IN NATURAL RIVERS

by Chih Ted Yang and John B. Stall

INTRODUCTION

As water flows downgradient, it releases its potential energy to carve its own channel and transport the sediment. Natural channel geometry observed today is the cumulative result of sediment transport in the channel for the past million years. The consistent pattern between channel geometry, drainage area, and flow frequency within a river basin was previously determined by Stall and Fok (1968) and Stall and Yang (1970, 1972). The present study places its emphasis on the process of sediment transport. The cause-and-effect relationship between rate of energy expenditure and rate of sediment transport has been studied in detail. The concept of unit stream power and new criteria for incipient motion are introduced. Theories developed in this study have been verified first by laboratory data and then by application to natural rivers.

Literature Review

The rate of sediment transport by water is related to many variables, such as water discharge, average flow velocity, energy slope, shear stress, bed configuration, intensity of turbulence, particle size, water temperature, etc. The general approach in a study of sediment transport begins with the consideration of the relative importance of these variables and the selection of one or two as the dominant factors governing the rate of sediment transport. Different theories and equations have been developed from assumption of different dominant factors as the independent variable. Most of these equations can be categorized in one of the following forms:

$$q_t = A_1 (Q - Q_{cr})^{B_1} \quad (1)$$

$$q_t = A_2 (V - V_{cr})^{B_2} \quad (2)$$

$$q_t = A_3 (S - S_{cr})^{B_3} \quad (3)$$

$$q_t = A_4 (\tau - \tau_{cr})^{B_4} \quad (4)$$

in which qt is total sediment discharge, and Q, V, S, and T are water discharge, average water velocity, energy slope, and shear stress, respectively. A and B are coefficients, and the subscript, cr, means the critical value at which sediment begins to move. Most available equations were derived under the assumption that there is always a determinate, at least statistically, relationship between sediment discharge and one of the independent variables in the preceding equations. A critical review of these equations by Yang (1972) revealed the nongenerality of these assumptions. Data collected by Guy, Simons, and Richardson (1966) on sediment with median fall diameter of 0.93 mm in a 8-foot wide flume are used here to demonstrate the nongenerality of these assumptions.

Figure 1 shows the hysteretic effect between total sediment discharge and water discharge. Apparently, different total sediment discharges can be transported by the same water discharge, or vice versa. Although qt increases steadily with increasing V in figure 2, it is apparent that for approximately the same value of V the value of qt can differ considerably because of the steepness of this curve. This is partially due to the fact that the experiment can be operated at the same velocity with different slopes, as shown in figure 3. Yet this does not imply that there is a definite relationship between total sediment discharge and slope, as can be seen in figure 4. Different amounts of total sediment discharge can be obtained at the same slope, and different slopes can also produce the same sediment discharge. The relationship between total sediment discharge and shear stress is shown in figure 5. For this set of data, it may be possible for us to define the relationship between qt and r at the median range of sediment discharge. For either higher or lower sediment discharges, the curve in figure 5 becomes vertical. This means that for the same shear stress, numerous values of sediment discharge can be obtained. In view of the preceding facts the generality of an equation which has one of the basic forms shown in equations 1 through 4 is open to question.

ANALYTICAL INVESTIGATIONS

Concept of Unit Stream Power

The only source of energy a unit mass of water in a natural stream can have is its potential energy above a datum. As this unit mass of water flows downgradient, it releases its potential energy to transport sediment and carve its own channel. It is reasonable for us to suspect that the rate of sediment transport is related to the rate of energy expenditure. Let us define the unit stream power as the time rate of potential energy expenditure per unit weight of water in an alluvial channel. This unit stream power can be expressed mathematically in terms of average water velocity, V, and energy slope, S, by

$$\frac{dY}{dt} = \frac{dX}{dt}\frac{dY}{dX} = VS \tag{5}$$

in which Y is the elevation above a datum which also equals the potential energy per unit weight of water above a datum; X is the longitudinal distance; and t is the time. For steady uniform flow, the energy slope can be replaced by water surface slope without introducing any error.

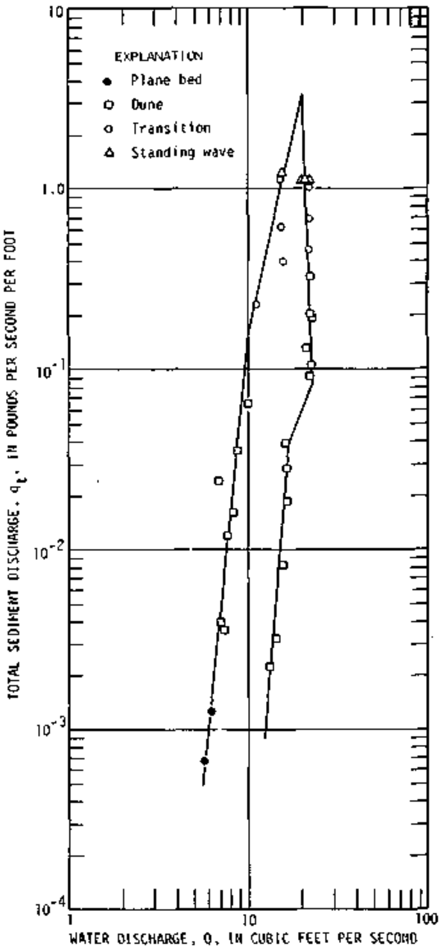

Figure 1. Relationship between total sediment discharge and water discharge for 0.93 mm sand in 8-ft wide flume

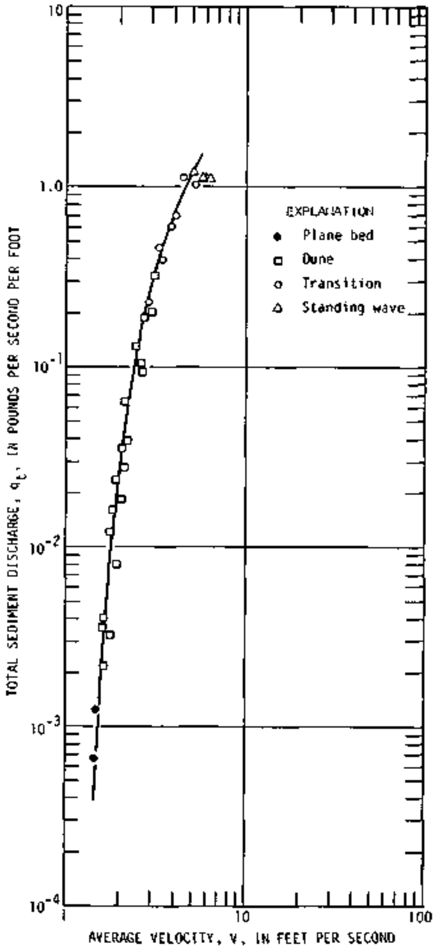

Figure 2. Relationship between total sediment discharge and average water velocity for 0.93 mm sand in 8-ft wide flume

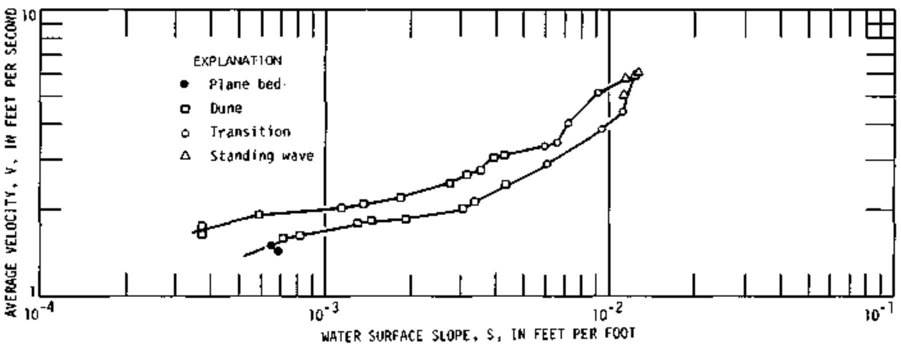

Figure 3. Relationship between average water velocity and water surface slope for 0.93 mm sand in 8-ft wide flume

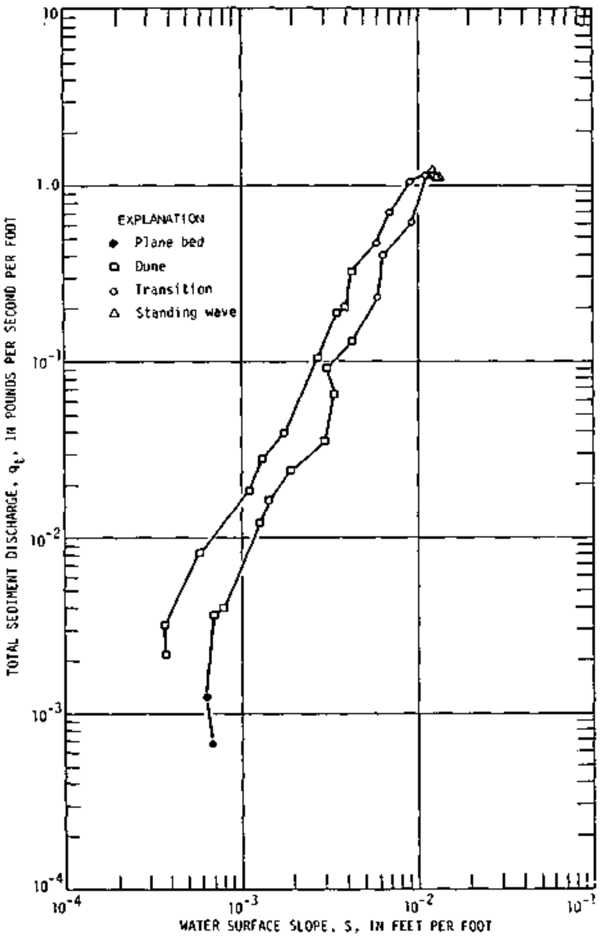

Figure 4. Relationship between total sediment discharge and water surface slope for 0.93 mm sand in 8-ft wide flume

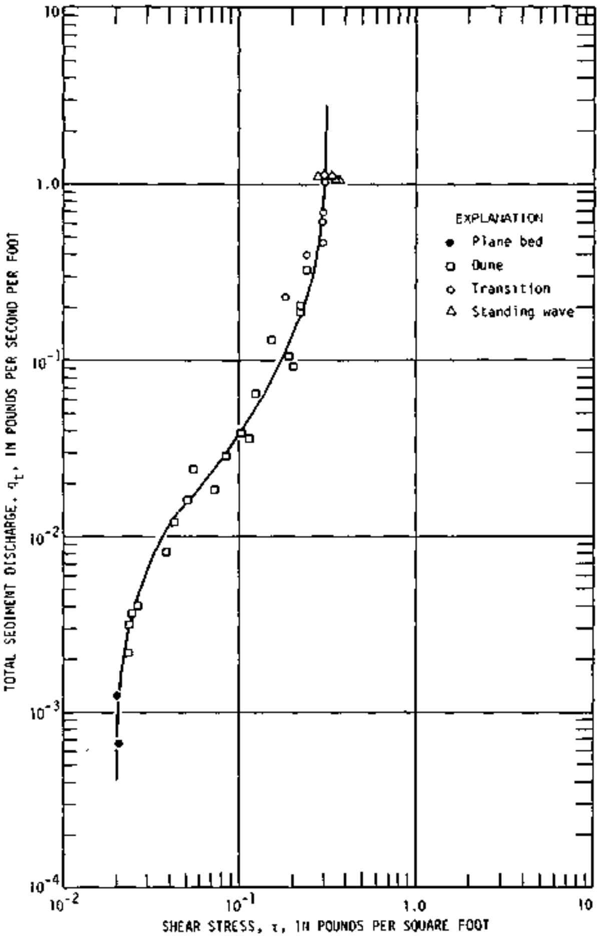

Figure 5. Relationship between total sediment discharge and shear stress for 0.93 mm sand in 8-ft wide flume

The concept of unit stream power was used by Yang (1971a, 1971b, 1971c) to explain the structure of a natural stream network, the formation and behavior of a meandering river and its longitudinal profile, and the formation of riffles and pools. Figure 6 shows an example of meandering channel pattern and location of pools and riffles. All these phenomena involve the movement of sediment, and are the cumulative results of the movement of sediment. If the concept of unit stream power can be used to explain the results of sediment movement, there should be a close relationship between the unit stream power and total sediment concentration or discharge. In order to test the validity of this hypothesis, the data shown in figures 1 through 5 are replotted on figure 7 by using unit stream power as the independent variable. Good correlation between total sediment concentration and effective unit stream power can be seen in figure 7. The relationship between total sediment concentration C_t and effective unit stream power ($VS - V_{cr}S$) can be expressed by the equation

$$\log C_t = A + B \log (VS - V_{cr}S) \tag{6}$$

where $V_{cr}S$ is the critical unit stream power required at incipient motion, and A and B are coefficients. Figure 7 provides us a good starting point for further development of the concept of unit stream power.

Incipient Motion

In figure 7 the critical unit stream power $V_{cr}S$ was determined by trial-and-error to give the best fit between data and equation 6. In order to improve on this trial-and-error fitting, an attempt has been made here to determine the critical velocity V_{cr} at a given flow condition. The forces acting on a spherical sediment particle at the bottom of an open channel are shown in figure 8. F_L, F_D, W_S, F_R, and d are the lift force, drag force, submerged weight of particle, resistance force, and particle diameter, respectively. V is the average flow velocity, V_d is the local velocity at distance d above the bed, and D is the water depth. The incipient motion occurs when $F_D = F_R$. It was shown by Yang (1973) that the critical velocity can be expressed in dimensionless form

$$V_{cr}/\omega = \left\{ \frac{5.75 \left[\log(D/d) - 1\right]}{B'} + 1 \right\} \sqrt{\frac{\psi_1 \psi_2 \psi_3}{\psi_2 + \psi_3}} \tag{7}$$

where ω is the fall velocity of sediment particle, B' is a roughness function, and ψ_1, ψ_2, ψ_3 are coefficients. The value B' depends on whether the flow is in a smooth, transition, or rough regime.

In the hydraulically smooth regime, B' is a function of only the sheer velocity Reynolds number, $U_* d/\nu$, (Schlichting, 1962) and

$$B' = 5.5 + 5.75 \log(U_* d/\nu), \; 0 < (U_* d/\nu) < 5 \tag{8}$$

where U_* is the sheer velocity and ν is the kinematic viscosity. Then equation 7 becomes

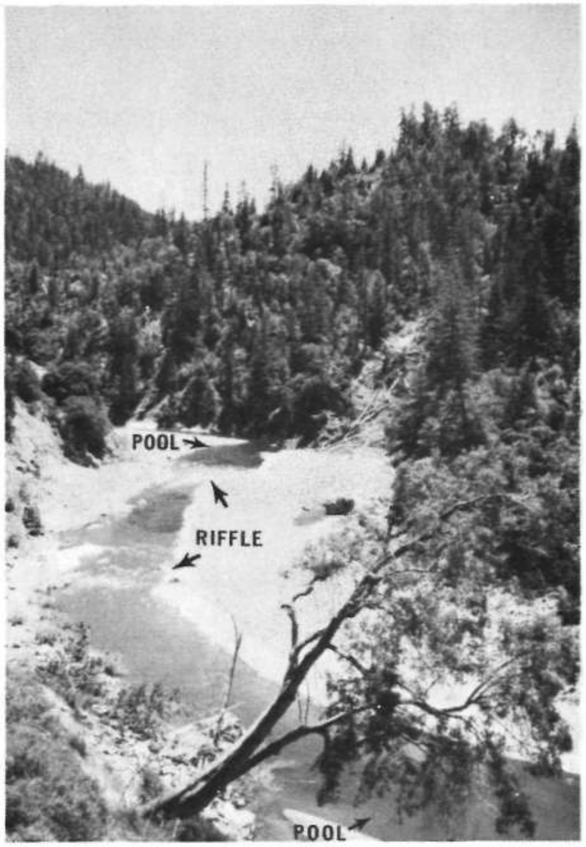

Figure 6. Relationships between pools, riffles, and meanders of a reach of the Blue River in Colorado

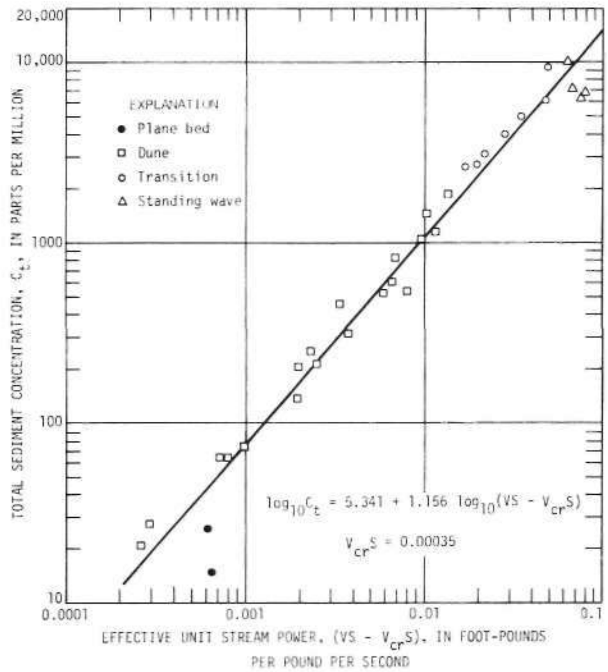

Figure 7. Relationship between total sediment concentration and effective unit stream power for 0.93 mm sand in 8-ft wide flume

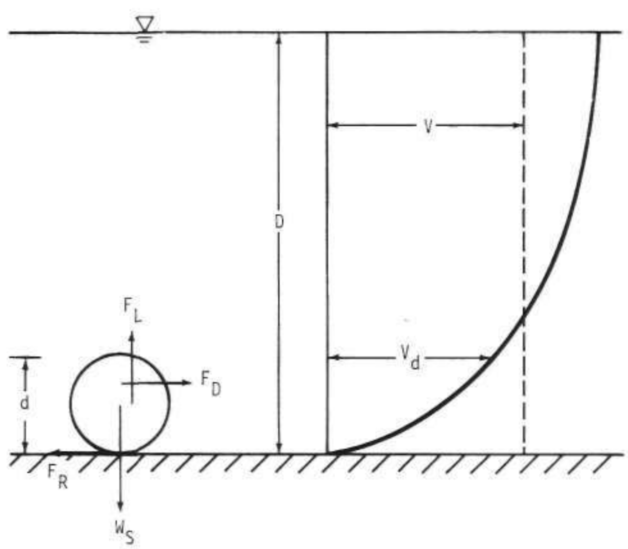

Figure 8. Diagram of forces acting on sediment particle in open channel flow

$$V_{cr}/\omega = \left[\frac{\log(D/d) - 1}{\log(U_* d/\nu) + 0.956} + 1 \right] \sqrt{\frac{\psi_1 \psi_2 \psi_3}{\psi_2 + \psi_3}} \qquad (9)$$

which is a hyperbola on a semilog plot between V_{cr}/ω and $U_* d/\nu$.

In the completely rough regime, B' is not a function of shear velocity Reynolds number (Schlichting, 1962). It can be expressed by

$$B' = 8.5, \qquad (U_* d/\nu) \geqslant 70 \qquad (10)$$

Then equation 7 becomes

$$V_{cr}/\omega = \left[\frac{\log(D/d) - 1}{1.48} + 1 \right] \sqrt{\frac{\psi_1 \psi_2 \psi_3}{\psi_2 + \psi_3}} \qquad (11)$$

Equation 11 is a straight line on a semilog plot between V_{cr}/ω and $U_* d/\nu$.

Yang (1973) used 153 sets of data independently collected by eight investigators to determine the coefficients in equations 9 and 11. These data are plotted on figure 9. These data follow the theoretical equations 9 and 11 very well. The coefficients in equation 9 and 11 can be determined from these data, and the criteria for incipient motion are

$$V_{cr}/\omega = \frac{2.5}{\log(U_* d/\nu) - 0.06} + 0.66 \quad , \; 0 < (U_* d/\nu) < 70 \qquad (12)$$

and

$$V_{cr}/\omega = 2.05, \qquad 70 \leqslant (U_* d/\nu) \qquad (13)$$

Dimensional Analysis

Dimensional analysis, properly applied, is a powerful tool in dealing with a complex problem. The outcome of a dimensional analysis depends on the selection of variables. A meaningful and useful result can be expected only if each variable selected for the analysis has a physical significance pertinent to the problem involved. The variables involved in the determination of total sediment concentration can be described by

$$\phi\,(C_t,\; VS,\; U_*,\; \nu,\; \omega,\; d) = 0 \qquad (14)$$

The variables in equation 14 are total sediment concentration C_t, unit stream power VS, shear velocity U_*, kinematic viscosity ν, fall velocity ω, and particle size d. By the use of Buckingham's π theorem, C_t in equation 14 can be expressed as a function of dimensionless parameters (Yang, 1973), that is

$$C_t = \Phi\,(VS/\omega - V_{cr}S/\omega,\; U_*/\omega,\; \omega d/\nu) \qquad (15)$$

where $(VS/\omega - V_{cr}S/\omega)$ is the dimensionless effective unit stream power, and $\omega d/\upsilon$ is the fall velocity Reynolds number. An analysis of available data indicates that the equation

$$\log C_t = I + J \log (VS/\omega - V_{cr}S/\omega) \qquad (16)$$

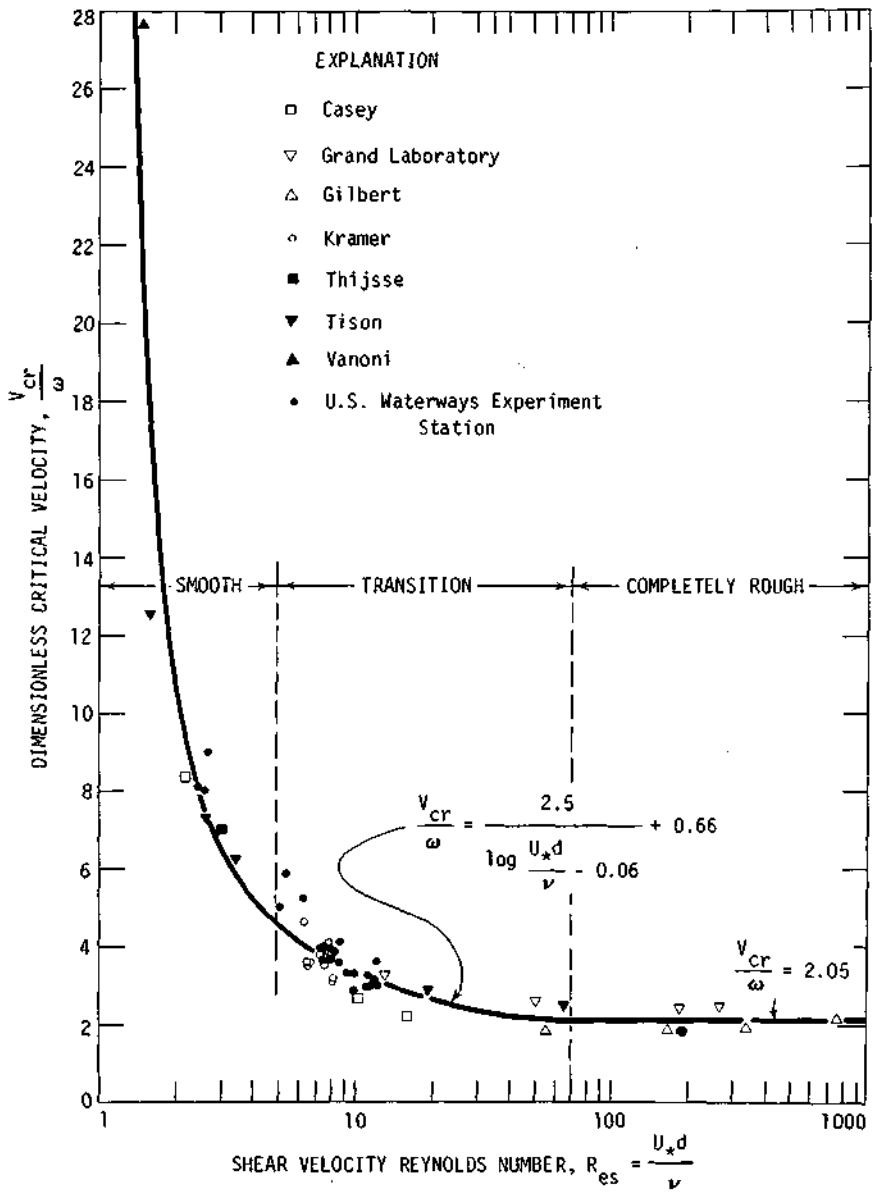

Figure 9. Relationship between dimensionless critical velocity and shear velocity Reynolds number at incipient motion

provides the best correlation between total sediment concentration C_t and dimensionless effective unit stream power (VS/ω — $V_{cr}S/\omega$). *A* comparison between equations 15 and 16 indicates that coefficients I and T should be functions of U_* /ω and $\omega d/v$. The values of I and J must be determined by analyzing actual data.

PRESENTATION OF DATA

The data used herein for testing the validity of the unit stream power equation (equation 6) are summarized in table 1. Altogether, 1225 sets of laboratory flume data were collected by Gilbert (1914), Nomicos (1956), Vanoni and Brooks (1957), Kennedy (1961), Stein (1956), Guy, Simons, and Richardson (1966), Williams (1967), and Schneider (1971). In order to ensure the accuracy of measured data, total sediment concentrations less than 10 ppm by weight were not included in the analysis. Water surface slope was used for the computation of unit stream power. The critical unit stream power VcrS was determined from regression analysis to minimize the deviation of observed data from equation 6. The low values of standard deviation and high values of correlation coefficient shown in table 1 strongly suggest that the total sediment concentration is dominated by the effective unit stream power.

The laboratory data were re-analyzed to determine the coefficients I and J in equation 16. In this analysis, only those data in the sand size range (0.0625 mm < d < 2 mm) were used. The dimensionless critical velocity V_{cr}/ω was calculated by either equation 12 or 13, depending on the value of $U_* d/v$. The particle size, d, is the median sieve diameter of the sediment. Guy, Simons, and Richardson (1966) published their data in terms of fall diameter. The difference between these two measurements of particle size is insignificant when either one is smaller than 0.4 mm. The fall diameter is converted to sieve diameter in accordance with figure 7 of Report 12 of the Inter-Agency Committee on Water Resources (1957). The fall diameters for the coarse sand are also shown in parentheses in the first column of table 2. Since no water temperature measurement was made by Gilbert, a temperature of 20° C is assumed for all Gilbert's data.

The value of I and J in equation 16, and the statistical parameters in terms of logarithmic units for all the data are tabulated in table 2. The low values of standard error of estimate σ and high values of correlation coefficient r indicate good agreement between equation 16 and the measurements. The weighted average value of σ and r, based on the number of data, for the 1093 sets of laboratory data are 0.13 and 0.97, respectively. These results indicate that, basically, equation 16 is correct. Next, the effects of the variations of particle size, water temperature, and water depth on the total sediment concentration were studied by expressing I and J in equation 16 in terms of U_*/Ω and $\omega d/v$.

From equation 15, C_t is related to $\omega d/v$ and U_*/ω. Combinations of different forms of $\omega d/\upsilon$ and U_* /Ω were tried for the determination of I and J in equation 16 and equations 17

Table 1. Laboratory Data and Parameters for Unit Stream Power Equation

Particle size *d(mm)*	Channel width *W(ft)*	Water depth *D(ft)*	Temperature T(°C)	Average velocity *V(ft/sec)*	Water surface slope S x 10^3	Unit stream power *VS(ft-lb/lb-sec)*	Critical unit stream power $V_{cr}S$ *(ft-lb/lb-sec)*	Total sediment concentration C_t*(ppm)*	A	B	Standard deviation **σ**	Correlation coefficient r	Number of data
Gilbert Data (1914)													
0.305	1.32	0.058-0.205		1.44-3.09	2.7-15.5	0.00390-0.0442	0.00002	866-19666	6.129	1.336	0.074	0.976	21
0.305	1.96	0.067-0.283		1.03-3.46	1.8-17.7	0.00238-0.0466	0.00003	400-27552	6.163	1.353	0.068	0.987	33
0.375	0.66	0.037-0.367		1.58-3.81	2.1-27.9	0.00368-0.1128	0.00005	376-35340	5.872	1.301	0.081	0.982	50
0.375	1.00	0.060-0.497		1.06-3.60	1.5-26.3	0.00170-0.0684	0.00013	126-35340	6.002	1.346	0.083	0.991	42
0.375	1.32	0.037-0.289		1.07-3.82	2.5-24.6	0.00302-0.0873	0.00003	506-29515	5.705	1.186	0.084	0.979	51
0.375	1.96	0.044-0.411		1.14-4.21	1.8-16.2	0.00264-0.0623	0.00009	202-24241	5.840	1.246	0.120	0.967	44
0.506	0.44	0.100-0.235		1.31-3.00	6.1-21.6	0.00838-0.0648	0.00003	1216-18829	6.011	1.370	0.101	0.962	15
0.506	0.66	0.050-0.242		1.14-4.45	2.0-25.2	0.00274-0.0997	0.00003	182-23852	5.802	1.276	0.097	0.967	63
0.506	1.00	0.051-0.581		1.13-4.01	1.9-22.4	0.00164-0.0739	0.00020	77-24855	5.880	1.284	0.094	0.980	61
0.506	1.32	0.040-0.409		1.36-4.75	1.6-23.4	0.00218-0.0718	0.00003	226-27379	5.899	1.296	0.074	0.987	46
0.506	1.96	0.060-0.222		1.44-3.92	3.5-20.3	0.00507-0.0725	0.00003	925-22100	5.863	1.266	0.063	0.984	49
0.786	0.66	0.056-0.460		1.66-4.67	1.9-24.7	0.00342-0.0834	0.00003	447-26020	5.770	1.282	0.108	0.975	36
0.786	1.00	0.062-0.491		1.24-4.53	1.8-27.5	0.00223-0.0797	0.00002	195-29515	5.825	1.301	0.073	0.992	53
0.786	1.32	0.087-0.360		1.54-3.66	3.0-19.1	0.00524-0.0604	0.00002	723-24047	5.854	1.310	0.083	0.982	26
1.710	0.66	0.114-0.562		2.21-3.02	5.6-15.6	0.01691-0.0383	0.00005	1389-6018	5.970	1.594	0.119	0.806	12
1.710	1.00	0.081-0.447		1.50-3.93	1.8-22.9	0.00299-0.0756	0.00005	82-16356	5.731	1.384	0.138	0.975	28
3.170	0.66	0.102-0.346		2.14-3.27	9.7-25.1	0.02803-0.0678	0.00002	1781-10288	6.117	1.829	0.088	0.965	10
3.170	1.00	0.078-0.343		2.02-3.48	7.7-25.3	0.01929-0.0580	0.00004	925-8541	6.194	1.888	0.070	0.983	14
3.170	1.32	0.108-0.288		2.33-3.16	7.4-20.7	0.02051-0.0528	0.00002	1251-7008	6.061	1.758	0.052	0.985	12
4.938	0.66	0.158-0.558		2.78-4.79	6.2-27.0	0.01885-0.1150	0.00002	379-10544	5.899	1.877	0.076	0.986	23
4.938	1.00	0.114-0.389		2.54-4.44	6.4-27.4	0.01843-0.0979	0.00003	316-10217	5.972	1.881	0.077	0.985	25
4.938	1.32	0.093-0.297		2.39-4.22	7.1-31.0	0.02173-0.0949	0.00004	433-11975	6.117	2.004	0.085	0.983	21
7.010	0.66	0.167-0.510		2.99-5.08	7.4-29.2	0.02501-0.1285	0.00004	316-10689	5.965	2.133	0.096	0.978	27
Nomicos Data (1956)													
0.152	0.875	0.241	25.0-26.0	0.80-2.66	2.0-3.9	0.00160-0.0104	0.0010	300-5600	6.336	1.142	0.147	0.930	12

(continued)

Table 1 (Concluded)

Particle size *d(mm)*	Channel width *V/(ft)*	Water depth *D(ft)*	Temperature *T(°C)*	Average velocity *V(ft/sec)*	Water surface slope $S \times 10^3$	Unit stream power *VS(ft-lb/lb-sec)*	Critical unit stream power $V_{cr}S$*(ft-lb/lb-sec)*	Total sediment concentration C_t*(ppm)*	A	B	Standard deviation σ	Correlation coefficient r	Number of data
Vanoni and Brooks Data (1957)													
0.137	2.79	0.203-0.553	18.9-27.4	0.77-2.53	0.7-2.8	0.00073-0.0055	0.00022	37-3000	7.975	1.907	0.274	0.902	14
Kennedy Data (1961)													
0.233	0.875	0.147-0.346	24.5-30.1	1.57-3.42	2.6-16.0	0.00502-0.0526	0.00003	730-34700	6.354	1.426	0.125	0.966	14
0.549	0.875	0.074-0.346	24.3-27.0	1.65-4.27	5.5-27.2	0.00924-0.1268	0.00003	1680-35900	5.596	1.190	0.075	0.984	14
0.233	2.79	0.145-0.356	23.0-27.3	1.35-3.45	1.7-22.9	0.00351-0.0753	0.00002	490-58500	6.548	1.518	0.182	0.960	13
Stein Data (1965)													
0.4	4.0	0.59-1.20	20.0-29.0	1.38-5.51	0.61-10.79	0.00084-0.0587	0.00002	93-24260	6.088	1.338	0.059	0.991	42
Guy, Simons, and Richardson CSU Data (1966)													
0.19	8.0	0.49-1.09	12.3-19.7	1.04-4.74	0.43-9.50	0.00056-0.0399	0.00042	29-47300	6.687	1.405	0.169	0.983	29
0.27	8.0	0.45-1.13	10.2-18.5	1.24-4.93	0.46-10.22	0.00057-0.0455	0.00042	12-35800	6.429	1.410	0.128	0.991	18
0.28	8.0	0.30-1.07	10.2-17.6	1.04-4.93	0.45-10.07	0.00060-0.0472	0.00049	12-42400	6.399	1.381	0.103	0.993	33
0.45	8.0	0.19-1.00	9.0-20.0	0.75-6.18	0.39-10.10	0.00045-0.0621	0.00040	10-15100	5.559	1.101	0.170	0.983	34
0.47	8.0	0.30-1.33	10.7-23.5	1.43-5.32	0.42-9.60	0.00067-0.0429	0.00051	23-17700	5.532	1.108	0.133	0.984	50
0.93	8.0	0.38-1.11	16.7-21.7	1.30-6.07	0.37-12.80	0.00061-0.0777	0.00035	15-10200	5.341	1.156	0.142	0.987	32
0.32	2.0	0.54-0.74	7.0-34.3	1.24-5.73	0.86-16.20	0.00107-0.0854	0.00069	55-49300	5.969	1.266	0.150	0.985	29
0.33*	2.0	0.49-0.52	19.8-20.3	1.17-5.93	0.88-11.40	0.00103-0.0676	0.00072	47-18400	5.563	1.104	0.102	0.992	12
0.33**	2.0	0.48-0.53	19.6-24.1	1.06-6.34	0.47-9.80	0.00050-0.0593	0.00047	12-22500	5.617	1.026	0.138	0.990	14
0.54	2.0	0.59-0.89	16.9-25.1	1.43-5.32	0.42-9.60	0.00052-0.1132	0.00032	17-50000	5.497	1.159	0.147	0.975	35
Williams Data (1967)													
1.35	1.0	0.094-0.517	11.9-30.8	1.27-3.49	1.1-22.18	0.00150-0.0668	0.0013	16-15570	5.620	1.285	0.105	0.990	37
Schneider Data (1971)													
0.25	8.0	1.012-2.822	20.4-22.4	1.67-6.45	0.10-4.97	0.00020-0.0320	0.00008	18-17152	5.831	1.127	0.176	0.958	31

• (Uniform)
•* (Graded)

Table 2. Laboratory Data and Parameters for Dimensionless Unit Stream Power Equation

Particle size *d(mm)*	Channel width *Vf(ft)*	Water depth *D(ft)*	Temperature *T(°C)*	Average velocity V*(ft/sec)*	Water surface slope $S \times 10^3$	Total sediment concentration C_t*(ppm)*	I	J	Standard error σ	Corre-lation coefficient r	Number of data	Standard error from Eq. 19
Gilbert Data (1914)												
0.305	1.32	0.058-0.205		1.44-3.09	2.7-15.5	866-19666	4.978	1.215	0.086	0.968	21	0.088
0.305	1.96	0.067-0.283		1.03-3.46	1.8-17.7	400-27552	4.982	1.226	0.083	0.981	33	0.086
0.375	0.66	0.037-0.367		1.58-3.81	2.1-27.9	376-35340	4.926	1.218	0.090	0.978	50	0.120
0.375	1.00	0.060-0.497		1.06-3.60	1.5-26.3	126-35340	4.991	1.234	0.093	0.989	42	0.097
0.375	1.32	0.037-0.289		1.07-3.82	2.5-24.6	506-29515	4.804	1.068	0.092	0.975	51	0.138
0.375	1.96	0.044-0.411		1.14-4.21	1.8-16.2	202-24241	4.907	1.143	0.120	0.966	44	0.136
0.506	0.44	0.100-0.235		1.31-3.00	6.1-21.6	1216-18829	5.118	1.164	0.110	0.955	15	0.175
0.506	0.66	0.050-0.242		1.14-4.45	2.0-25.2	182-23852	4.940	1.093	0.124	0.945	63	0.127
0.506	1.00	0.051-0.581		1.13-4.01	1.9-22.4	77-24855	5.041	1.136	0.104	0.976	61	0.131
0.506	1.32	0.040-0.409		1.36-4.75	1.6-23.4	226-27379	5.084	1.155	0.082	0.984	46	0.131
0.506	1.96	0.060-0.222		1.44-3.92	3.5-20.3	925-22100	5.065	1.134	0.070	0.980	49	0.124
0.786	0.66	0.056-0.460		1.66-4.67	1.9-24.7	447-26020	5.188	1.107	0.139	0.957	36	0.230
0.786	1.00	0.062-0.491		1.24-4.53	1.8-27.5	195-29515	5.160	1.067	0.100	0.984	53	0.232
0.786	1.32	0.087-0.360		1.54-3.66	3.0-19.1	723-24047	5.201	1.107	0.109	0.968	26	0.223
1.710	0.66	0.114-0.562		2.21-3.02	5.6-15.6	1389-6018	5.066	0.945	0.175	0.492	12	0.204
1.710	1.00	0.081-0.447		1.50-3.93	1.8-22.9	82-16356	5.188	0.976	0.202	0.946	28	0.265
Nomicos Data (1956)												
0.152	0.875	0.241	25.0-26.0	0.80-2.66	2.0-3.9	300-5600	5.151	1.384	0.174	0.900	12	0.244
Vanoni and Brooks Data (1957)												
0.137	2.79	0.203-0.553	18.9-27.4	0.77-2.53	0.7-2.8	37-3000	5.725	2.033	0.233	0.930	14	0.311

(continued)

Table 2 (Concluded)

Particle size $d(mm)$	Channel width $V/(ft)$	Water depth $D(ft)$	Temperature $T(°C)$	Average velocity $V(ft/sec)$	Water surface slope $S \times 10^3$	Total sediment concentration $C_t(ppm)$	I	J	Standard error σ	Corre-lation coefficient r	Number of data	Standard error from Eq. 19
Kennedy Data (1961)												
0.233	0.875	0.147-0.346	24.5-30.1	1.57-3.42	2.6-16.0	730-34700	4.983	1.356	0.122	0.968	14	0.155
0.549	0.875	0.074-0.346	24.3-27.0	1.65-4.27	5.5-27.2	1680-35900	4.911	1.068	0.087	0.978	14	0.089
0.233	2.79	0.145-0.356	23.0-27.3	1.35-3.45	1.7-22.9	490-58500	5.085	1.452	0.166	0.966	13	0.197
Stein Data (1965)												
0.4	4.0	0.59-1.20	20.0-29.0	1.38-5.51	0.61-10.79	93-24260	5.085	1.255	0.055	0.992	42	0.089
Guy, Simons, and Richardson Data (1966)												
0.19	8.0	0.49-1.09	12.3-19.7	1.04-4.74	0.43-9.50	29-47300	5.264	1.578	0.191	0.979	29	0.203
0.27	8.0	0.45-1.13	10.2-18.5	1.24-4.93	0.46-10.22	12-35800	5.219	1.517	0.170	0.984	18	0.209
0.28	8.0	0.30-1.07	10.2-17.6	1.04-4.93	0.45-10.07	12-42400	5.265	1.516	0.167	0.982	33	0.190
0.48(0.45)	8.0	0.19-1.00	9.0-20.0	0.75-6.18	0.39-10.10	10-15100	4.951	1.137	0.199	0.977	34	0.235
0.50(0.47)	8.0	0.30-1.33	10.7-23.5	1.43-5.32	0.42-9.60	23-17700	4.978	1.171	0.163	0.976	50	0.144
1.20(0.93)	8.0	0.38-1.11	16.7-21.7	1.30-6.07	0.37-12.80	15-10200	4.999	1.056	0.134	0.988	32	0.131
0.32	2.0	0.54-0.74	7.0-34.3	1.24-5.73	0.86-16.20	55-49300	4.973	1.325	0.187	0.976	29	0.195
0.33 (Uniform)	2.0	0.49-0.52	19.8-20.3	1.17-5.93	0.88-11.40	47-18400	4.790	1.225	0.125	0.989	12	0.197
0.33 (Graded)	2.0	0.48-0.53	19.6-24.1	1.06-6.34	0.47-9.80	12-22500	4.985	1.196	0.260	0.962	14	0.249
0.59(0.54)	2.0	0.59-0.89	16.9-25.1	1.43-5.32	0.42-9.60	17-50000	4.890	1.131	0.155	0.972	35	0.171
Williams Data (1967)												
1.35	1.0	0.094-0.517	11.9-30.8	1.27-3.49	1.1-22.18	16-15570	5.381	1.173	0.240	0.949	37	0.274
Schneider Data (1971)												
0.25	8.0	1.012-2.822	20.4-22.4	1.67-6.45	0.10-4.97	18-17152	4.834	1.160	0.171	0.960	31	0.222

and 18 were selected as the final form to be used.

$$I = a_1 + a_2 \log(\omega d/\nu) + a_3 \log(U_*/\omega) \tag{17}$$

$$J = b_1 + b_2 \log(\omega d/\nu) + b_3 \log(U_*/\omega) \tag{18}$$

in which a_1, a_2, a_3, b_1, b_2, and b_3 are coefficients. These coefficients can be determined by multiple regression analysis. Gilbert's data were not used in this analysis because of the lack of temperature measurement. The multiple regression analysis was made for the remaining 463 sets of laboratory data. The equation thus obtained is

$$\begin{aligned} \log C_t = \; & 5.435 - 0.286 \log(\omega d/\nu) - 0.457 \log(U_*/\omega) \\ & + [1.799 - 0.409 \log(\omega d/\nu) - 0.314 \log(U_*/\omega)] \\ & \times \log(VS/\omega - V_{cr}S/\omega) \end{aligned} \tag{19}$$

Equation 19 is the dimensionless unit stream power equation proposed by Yang (1973) to engineers for their consideration in predicting the total sediment concentration in both laboratory flumes and natural rivers. The dimensionless critical velocity V_{cr}/ω is determined either by equation 12 or 13 depending on the value of the shear velocity Reynolds number $U_* d/\nu$. A comparison of the measured and predicted total sediment concentration is shown in the last column of table 2. The accuracy in predicting the total sediment concentration for such diversified flow and sediment conditions is satisfactory.

CONSTRAINTS ON TOTAL SEDIMENT CONCENTRATION

The total sediment concentration in a natural river depends not only on its unit stream power but also on the constraints applied to the river. Among other constraints applied to a natural river, particle size, water temperature, and water depth are the three that attract most attention from hydraulic engineers. The effects of the variations of these variables on the total sediment concentration for a given value of unit stream power were studied in accordance with equation 19.

Figure 10 shows the effect of the variation of particle size on the predicted total sediment concentration. This figure shows that the predicted total sediment concentration from equation 19 decreases with increasing particle size for a given unit stream power, water temperature, and water depth. Gilbert's (1914) data shown in figure 11 confirm the result in figure 10.

Figure 12 shows the effect of the variation of water temperature on the predicted total sediment concentration. It indicates that the predicted total sediment concentration decreases with increasing water temperature for higher values of shear velocity Reynolds number R_{es}. As the value of Res decreases, the effect of the change of water temperature on the predicted total sediment concentration tends to reverse. This phenomenon is supported by data collected by Franco (1968) and Taylor and Vanoni (1972). Their results are shown in figure 13.

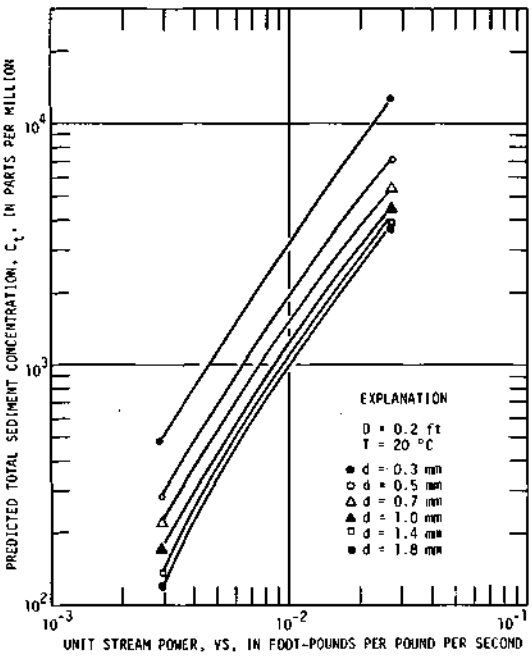

Figure 10. Effect of variation of particle size on predicted total sediment concentration by equation 19

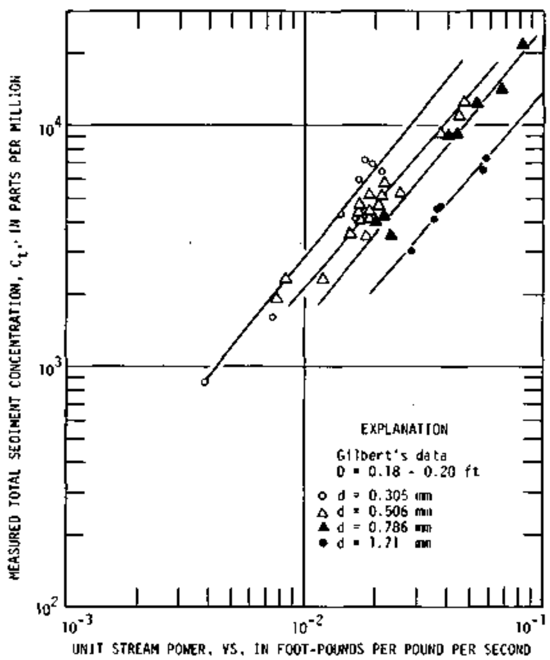

Figure 11. Relationship between measured total sediment concentration and unit stream power for different particle sizes

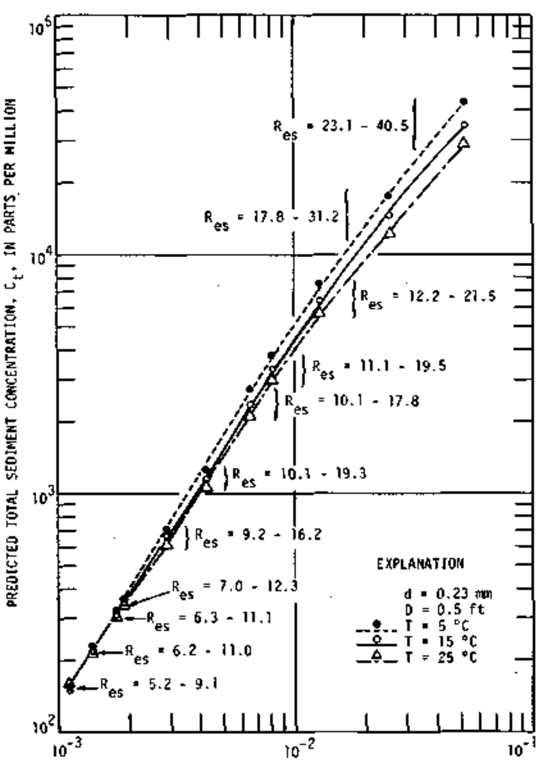

Figure 12. Effect of variation of water temperature on predicted total sediment concentration by equation 19

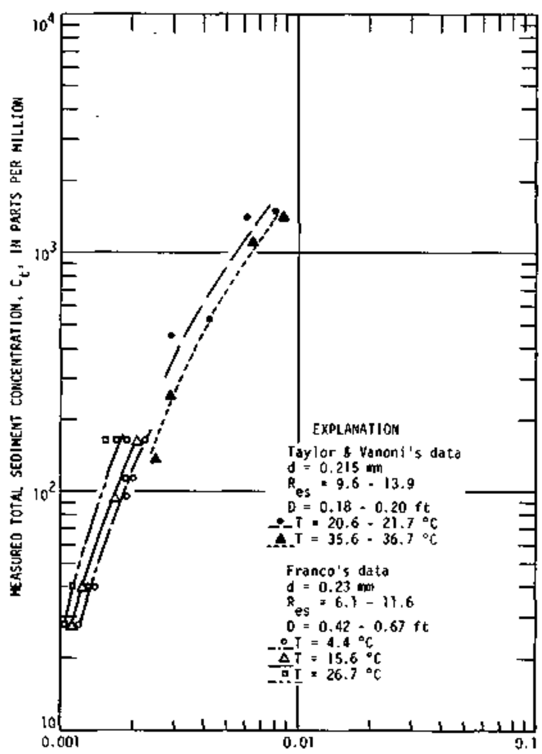

Figure 13. Relationship between measured total sediment concentration and unit stream power at different water temperatures

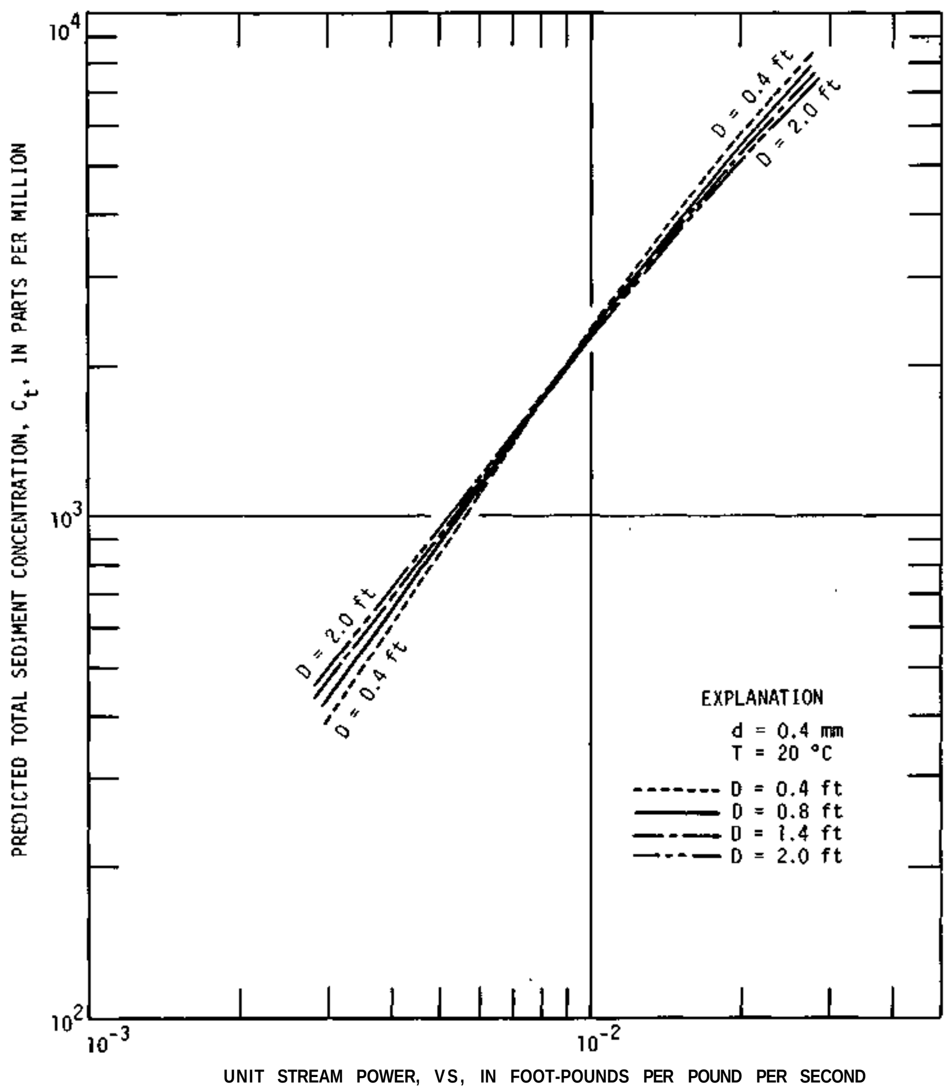

Figure 14. Effect of variation of water depth on total sediment concentration predicted by equation 19

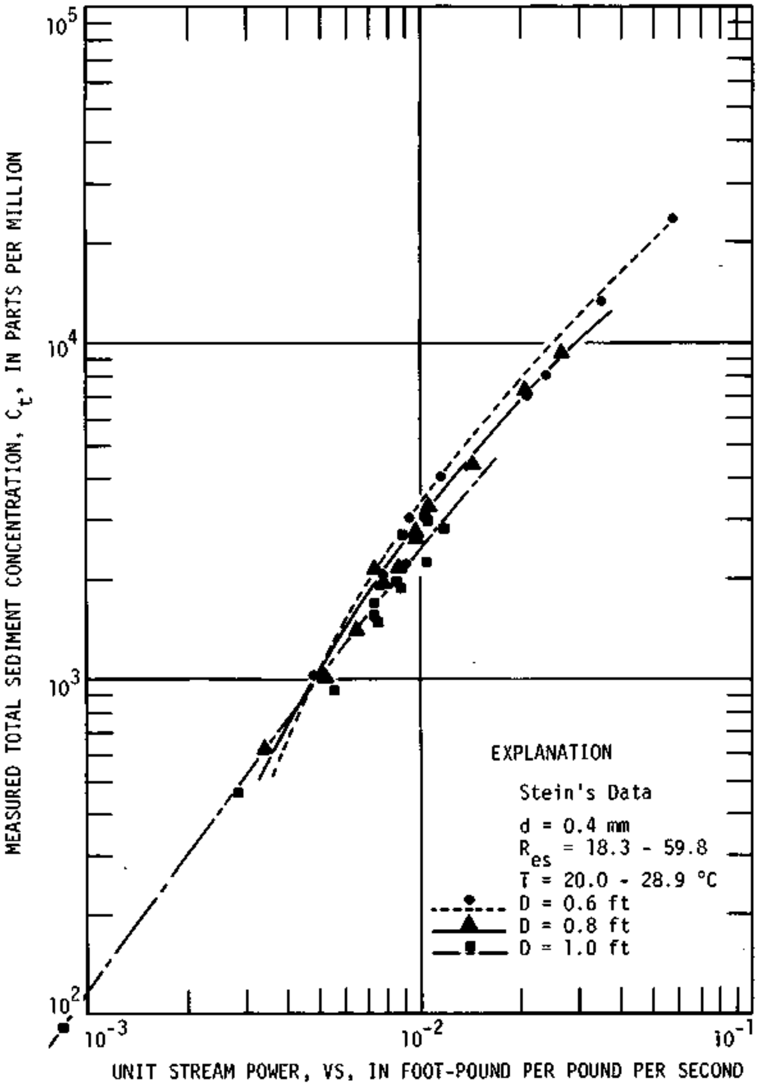

Figure 15. Relationship between measured total sediment concentration and unit stream power at different water depths

Figure 14 shows the effect of the variation of water depth on the predicted total sediment concentration. This figure indicates that total sediment concentration decreases with increasing water depth when the unit stream power is higher than a certain value. When the unit stream power is below a certain value, an increase of water depth causes an increase of the total sediment concentration. Stein's (1965) data shown in figure 15 seem to support this phenomenon.

Figures 10-15 indicate that equation 19 correctly reflects the influence of different constraints on the ability of a natural stream to transport sediment. Further explanations on these phenomena were made by Yang (1972, 1973).

APPLICATIONS TO NATURAL RIVERS

Equation 19 was derived for the calculation of total sediment concentration in the sand size range. To be more specific, since the coefficients in equation 19 were determined from laboratory data with particle size between 0.137 mm and 1.71 mm, this equation is most reliable in predicting total sediment concentration of natural rivers within the same particle size range. The parameters required in using equation 19 are average flow velocity, water surface or energy slope, median particle size, water temperature, and water depth. With a few exceptions, most measurements made from rivers are for suspended sediment concentrations only. The difference between total sediment and suspended sediment concentration varies from station to station. Comparisons were made between measured total sediment concentration and computed concentration by equation 19 as well as by other equations wherever appropriate data were available.

Total Sediment Discharge

A search revealed that measured data on total sediment concentrations were available on four rivers. They are the Niobrara River near Cody, Neb. (Colby and Hembree, 1955), Mountain Creek at Greenville, S. C. (Einstein, 1944), West Goose Creek at Oxford, Miss. (Einstein, 1944), and Middle Loup River at Dunning, Neb. (Hubbell and Matejka, 1959). Vanoni, Brooks, and Kennedy (1960) made comparisons between the measured total sediment load and the computed sediment load by using different equations for the first three stations. Their results are shown in figures 16, 17, and 18. Total sediment concentrations at these three stations were calculated by equation 19. When these calculated total sediment concentrations were multiplied by their corresponding water discharge and then plotted on figures 16, 17, and 18, they agreed very well with the measurements. Figure 19 shows the comparison between the measured and calculated total sediment discharges of the Middle Loup River. The total sediment discharges computed by equation 19 agree very well with the measurements. Most of

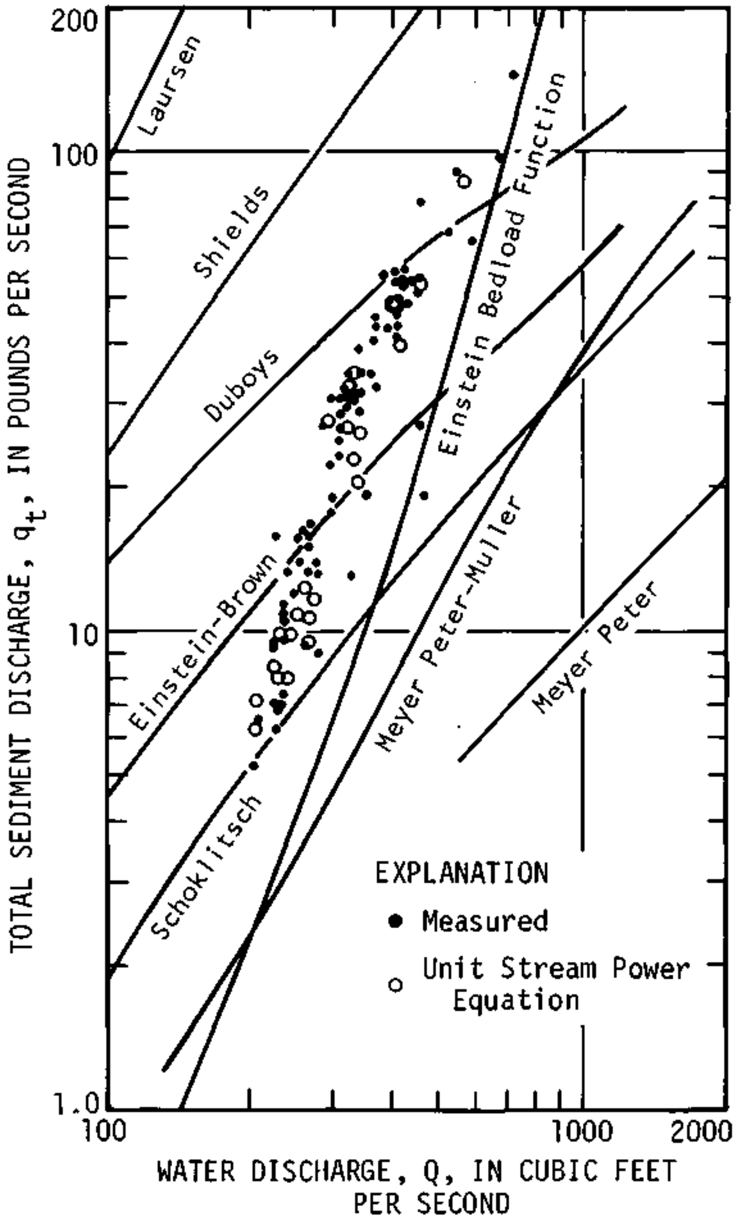

Figure 16. Measured total sediment discharge for Niobrara River near Cody, Neb compared with that computed by Vanoni et al. using various sediment transport equations, and with that computed by equation 19

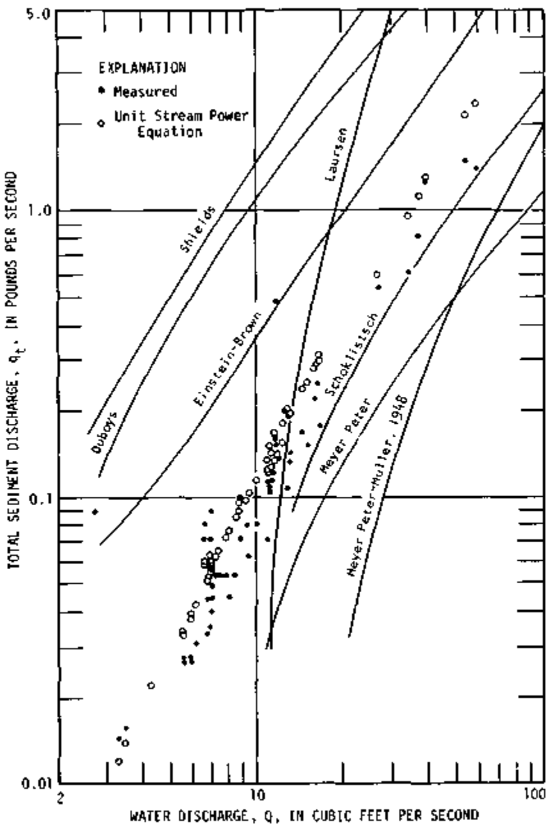

Figure 17. Measured total sediment discharge for Mountain Creek at Greenville, S. C, compared with that computed by Vanoni et al. using various sediment transport equations, and with that computed by equation 19

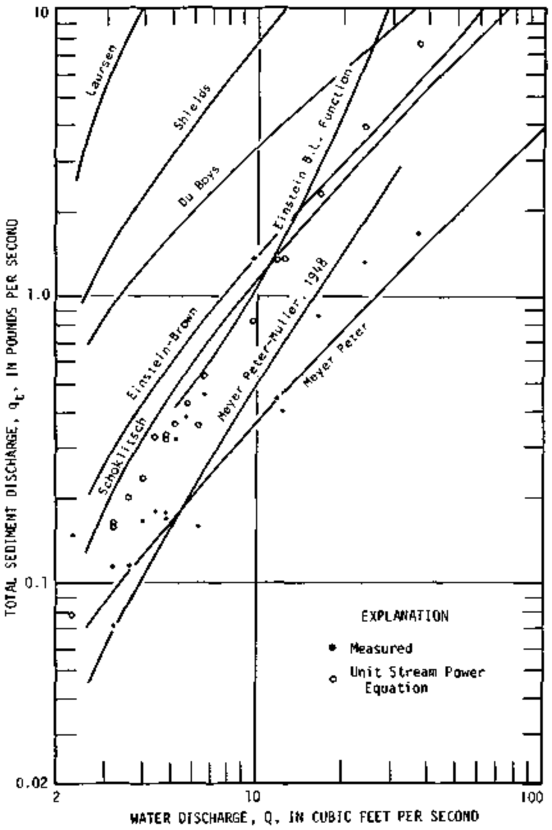

Figure 18. Measured total sediment discharge for the West Goose Creek at Oxford, Miss., compared with that computed by Vanoni et al. using various sediment transport equations, and with that computed by equation 19

them are within the narrow range defined by the two dashed lines. With the exception of the modified Einstein method, the other equations cannot be used to predict the total sediment discharge in the Middle Loup River with confidence.

Comparisons made in figure 16, 17, 18, and 19 indicate that equation 19 is the best equation for the prediction of total sediment discharge in natural rivers. This equation can be used by engineers with confidence when there is no significant amount of wash load and the bed material is in the sand range.

Bed Material Discharge

Under ordinary conditions, only the suspended sediment concentration can be measured in a natural river. The sediment transported in suspension includes those with particle size within the range of the channel bed composition, and those which are finer in size. Wash load is defined as that part of the sediment load which consists of grain sizes finer than those of the bed (Einstein, 1950). Bed material discharge equals the product of water discharge and the difference between total suspended concentration and the suspended concentration with particle size in the range of wash load. Comparisons were made between measured bed material load and total sediment discharge computed by equation 19 at two stations.

The average bed material size distributions from the Mississippi River at St. Louis, Mo., and the Rio Grande River near Bernalillo, N. M., are shown in figure 20. On the average, only 0.05 percent of the bed material from these two stations is finer than 0.125 mm. Thus the bed material load for these two stations is defined as the product of water discharge and the portions of suspended concentration with particle size coarser than 0.125 mm. Comparison between total sediment discharge computed by equation 19 and the measured bed material discharge from the Mississippi River at St. Louis, Mo., is shown in figure 21. This figure also shows the computed sediment discharges made by Jordan (1965) with other well-known equations. A comparison among the calculated results from different equations indicates that equation 19 is the best equation for the prediction of bed material discharge in the Mississippi River at St. Louis. Figure 22 shows similar comparisons made for Sections A2 and F of the Rio Grande River near Bernalillo, N. M. The sediment discharges computed by the modified Einstein method and Laursen's equation were done by Nordin (1964). Figure 22 also indicates that equation 19 is superior to other equations in predicting the bed material discharge in a natural river.

Total Suspended Sediment Discharge

Throughout the United States, the U. S. Geological Survey has carried out over a long period of time a program of gaging flow in streams. During the past few decades, routine measurements have also been made of the suspended sediment concentration at selected stream gaging stations. Figure 23 is a picture taken in the summer of 1972 at USGS station 11-4770,

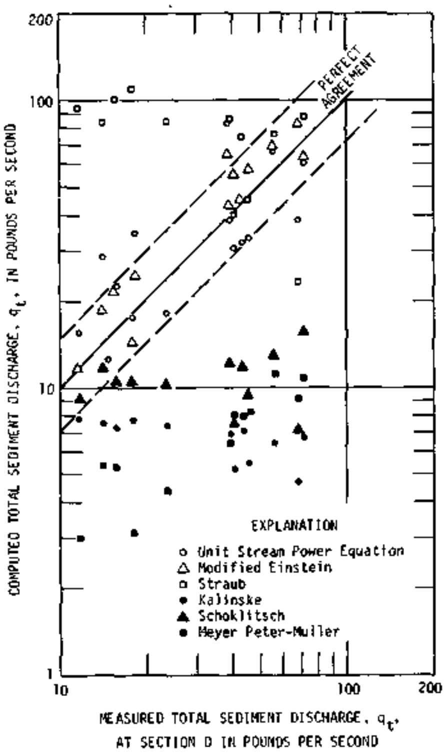

Figure 19. Comparison between the measured total sediment discharge of the Middle Loup River at Dunning, Neb., and the results computed by different equations

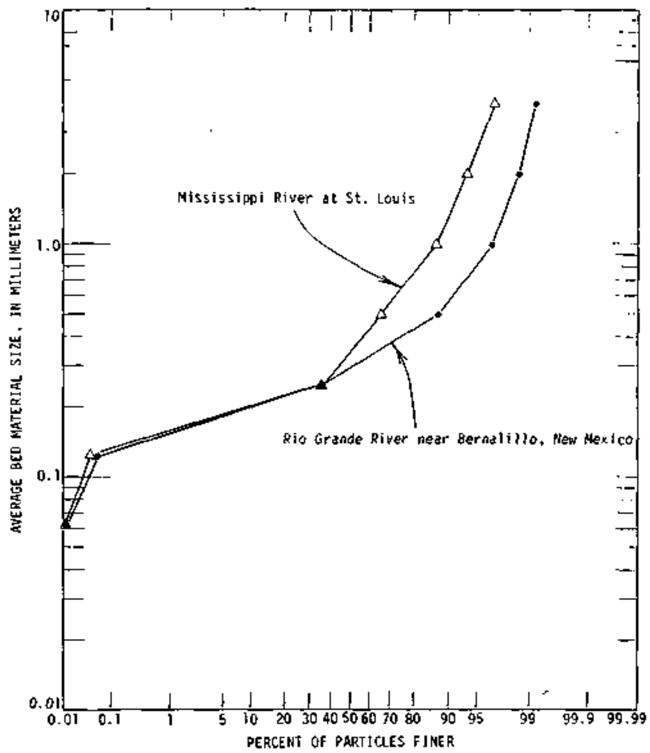

Figure 20. Size distribution of bed material for the Mississippi River at St. Louis, Mo., and the Rio Grande River near Bernalillo, N. M.

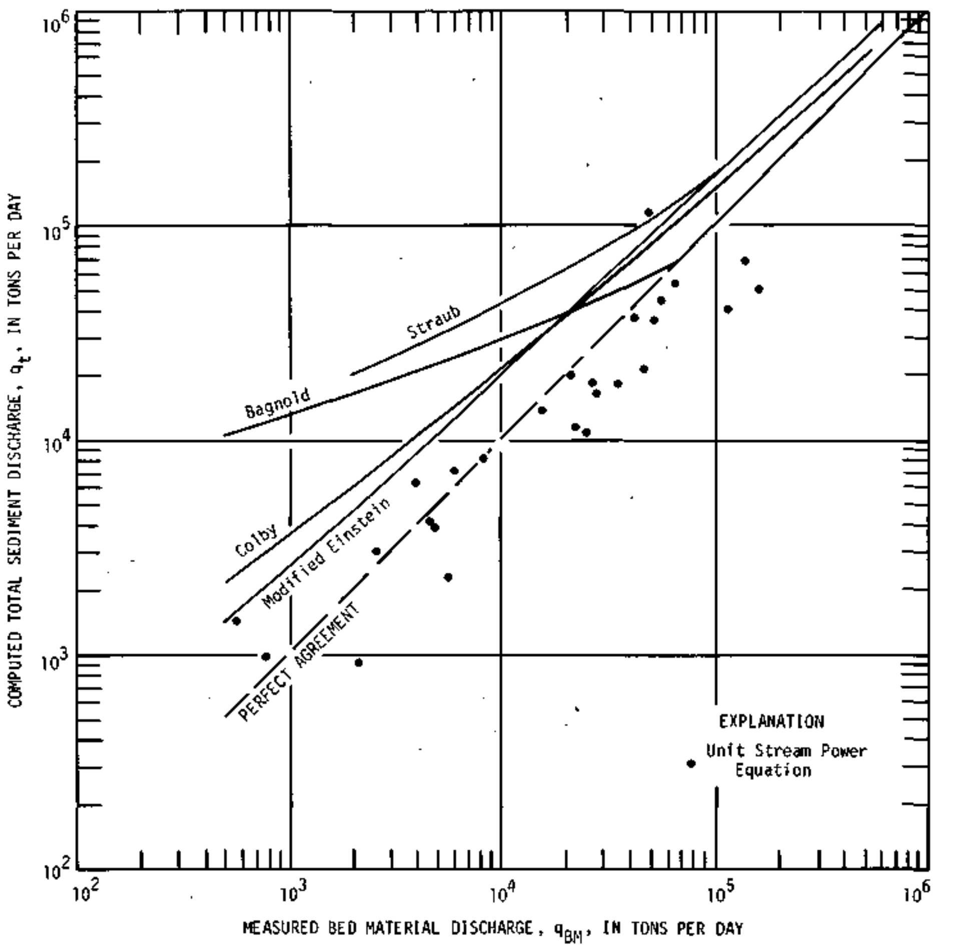

Figure 21. Comparison between the measured bed material discharge of the Mississippi River at St. Louis, Mo., and the results computed by different equations

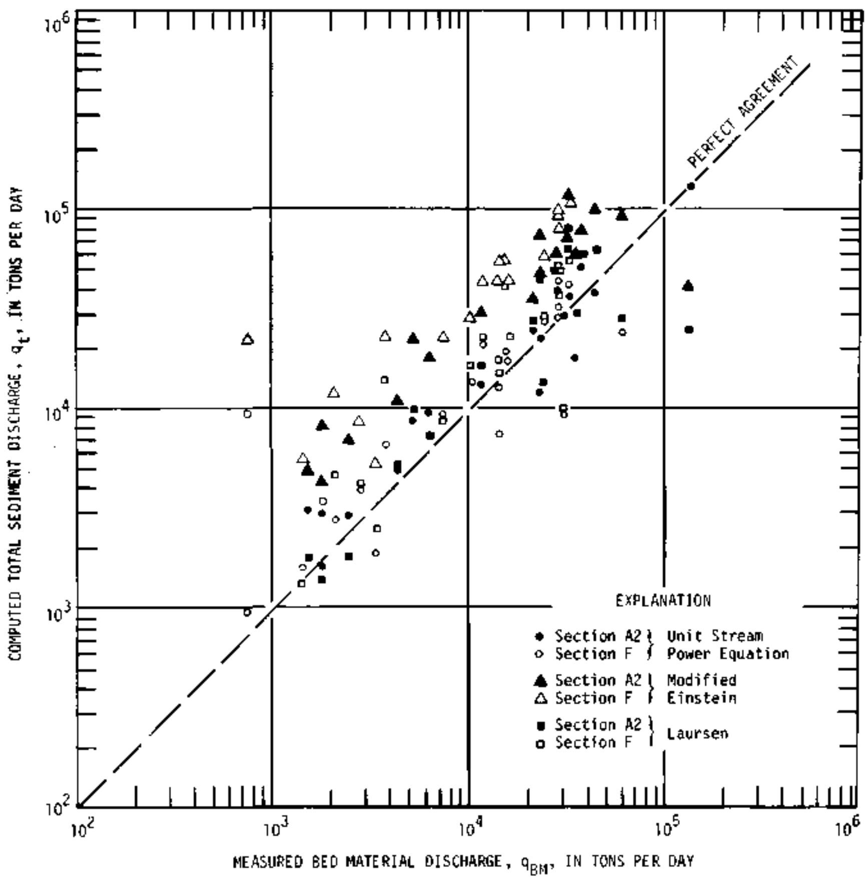

Figure 22. Comparison between the measured bed material discharge of the Rio Grande River near Bernalillo, N. M., and the results computed by different equations

Figure 23. Eel River at Scotia, Calif.

Eel River at Scotia, Calif. Suspended sediment concentration data collected at all gaging stations are published by the USGS in *Water Resources Data, Part 2 Water Quality Records* once a year for each state. In order to verify equation 19 against measured sediment discharge at these stations, data on the instantaneous value of average velocity, slope, bed material size distribution, suspended material size distribution, water temperature, and average water depth are essential. With a few exceptions, these data are not available. Only the daily water discharge and daily total suspended sediment concentration data are published in the USGS *Water Quality Records,* Under this condition, equation 19 cannot be used, and we must use the unit stream power equation in its primitive form, that is, equation 6. Because of the lack of daily measurements of velocity and slope, reasonable assumptions and procedures were developed in this study to estimate their values.

When the U. S. Geological Survey personnel make a water discharge measurement, flow velocities across the channel are measured. These local velocities and their average value at a given discharge are recorded on USGS Form 9-275. Information on water stage, channel width, cross-sectional area and shape, and average depth corresponding to each measured discharge can also be found in Form 9-275. The daily water discharge data published by the USGS is based on the stage-discharge rating curve developed for each station. Only those stations with a reliable stage-discharge relationship were used in this study. On the basis of Form 9-275, the relationship between average velocity and water discharge was plotted as shown in figure 24 for the case of Eel River at Scotia, Calif. A velocity-discharge rating curve was made for each selected station.

For the case shown in figure 24, the velocity-discharge relationship can be well defined by the rating curve when the water discharge is greater than 1000 cfs. Only daily suspended sediment concentrations collected at Scotia corresponding to daily discharges greater than 1000 cfs were considered. The daily average velocities at Scotia corresponding to the published daily water discharge data can be read from the rating curve in figure 24. The same procedure was applied to other stations for the determination of daily average velocity.

The water surface slope of a river at a given station varies from time to time. In general, water surface slope is steeper at a rising stage than at a falling stage. Because no slope measurement was made at regular gaging stations, we had to determine the slope from topographic maps. The slope thus determined is a constant for each station. A sample hydrograph for water discharge and suspended sediment concentration for the Eel River at Scotia, Calif., is shown in figure 25. In order to use a constant slope at a given station, only the daily measurements obtained during the common recession periods for both water discharge and suspended sediment concentrations were used. Data obtained from the first day of a recession were not used in order to increase the accuracy of the assumption that slope does not change with respect to time. Examples of the durations thus selected for data use in our study are shown in figure 25.

The velocity determined from a velocity-discharge rating curve and the slope determined from topographic maps were used to verify the relationship shown in equation 6 between unit stream power and total suspended sediment concentration for the selected periods at each selected

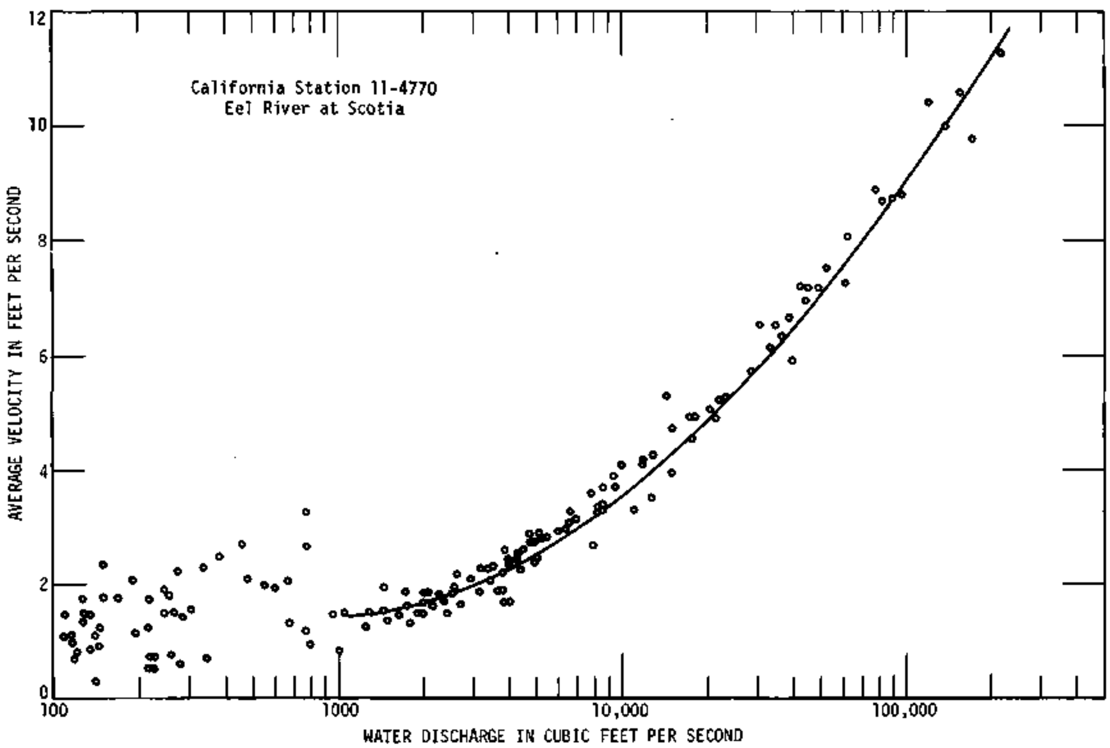

Figure 24. Relationship between average water velocity and discharge of the Eel River at Scotia, Calif.

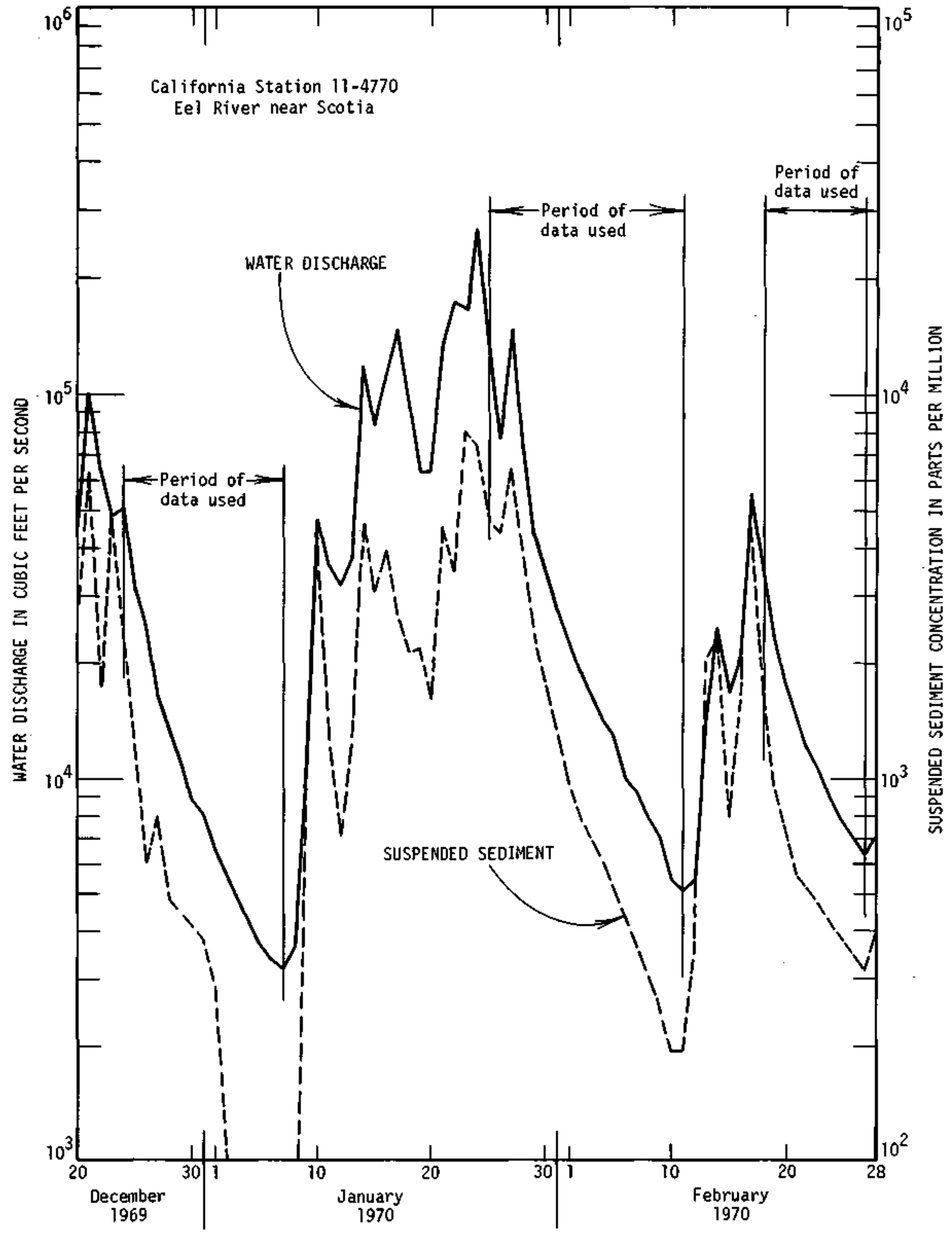

Figure 25. Hydrographs for water discharge and suspended sediment concentration for the Eel River at Scotia, Calif.

station. Table 3 summarizes the results obtained from 17 USGS gaging stations. The first 12 stations showed good correlation between effective unit stream power ($VS - V_{Cr}S$) and total suspended sediment concentration C_s. Their correlation coefficients are greater than 0.77. The last 5 stations in table 3 showed fair correlations between effective unit stream power and total suspended sediment concentration. Their correlation coefficients have values between 0.67 and 0.74. The results shown in table 3 are plotted on figures 26 through 30. The straight lines on these figures represent the regression equations which have the form shown in equation 6 and the A and B values shown in table 3.

The total suspended sediment concentration published by the USGS for regular gaging stations includes suspended bed material concentration and wash load concentration. Because of the lack of information on bed material size and suspended material size distributions, wash load concentrations cannot be determined. The amount of wash load transported in a river has little to do with the hydraulics of the river; it depends mainly on the availability from the watershed. Thus an increase of wash load will cause a decrease in the accuracy of predicting total sediment concentration by the unit stream power approach. In general, wash load increases with decreasing particle size. Thus the correlation between total suspended sediment concentration and effective unit stream power decreases with decreasing particle size. Figures 31 and 32 show some of the suspended particle size distributions obtained from the Eel River at Scotia, Calif., and the Arkansas River at Arkansas City, Kans., respectively. Because particle size at Scotia is coarser than that at Arkansas City, the correlation between C_s and ($VS - VcrS$) for Scotia in figure 26 is better than that for Arkansas City in figure 29. The former has a correlation coefficient of 0.917, while the latter has a value of 0.711 as shown in table 3.

It is unfortunate that only a few river stations in the United States have the complete information required by equation 19. Because of this lack of information the results shown in table 3 and plotted on figures 26 through 30 cannot be used directly to verify the accuracy of equation 19 in predicting the total sediment concentration. However, these results do indicate that total sediment concentration in a river is dominated by its unit stream power.

SUMMARY AND CONCLUSIONS

Two closely related subjects, incipient motion and sediment transport, have been discussed in this report. The criteria for incipient motion were developed from boundary layer theory and other basic concepts in fluid mechanics. The dimensionless equation 19 for the prediction of total sediment concentration in natural rivers was developed from the concept of unit stream power and dimensional analysis. An exhaustive search of published related data was made to support the theories found in this study. This study has reached the following conclusions:

1) Equations 12 and 13 are the new criteria for incipient motion found in this study. These equations were supported by 153 sets of data independently collected by eight investiga-

Table 3. Summary of Field Data and Parameters from Regular Gaging Stations

Station No.	Station Name	Drainage area (sq mi)	Period of data (water year)	Number of data	Water discharge *Q(cfs)*	Total suspended sediment concentration C_s*(ppm)*	Average velocity *V(ft/sec)*	Slope S	A	B	Critical unit stream power $V_{cr}S$ *(ft-lb/lb-sec)*	Correlation coefficient r	Standard deviation σ
11-4770	Eel River at Scotia, Calif.	3,113	1964-1970	647	1010-150000	8.0-11400	1.40-10.36	0.00056	11.465	3.292	0.00002	0.917	0.281
11-4750	Eel River at Ft. Seward, Calif.	2,107	1966-1970	345	1020-90000	8.0-5110	1.08-10.58	0.00092	8.700	2.443	0.00003	0.911	0.242
11-4610	Russian River near Ukiah, Calif.	99.7	1964-1968	188	96-4470	4.0-1900	1.51-5.69	0.002462	10.828	4.144	0.00002	0.907	0.237
11-4630	Russian River near Cloverdale, Calif.	503	1964-1968	185	526-6410	10.0-772	2.82-6.09	0.002062	10.849	4.258	0.00004	0.887	0.184
11-4670	Russian River near Guerneville, Calif.	1,340	1965-1972	188	1500-55500	12.0-880	0.89-5.64	0.000703	7.988	2.102	0.00002	0.896	0.187
14-0185	Walla Walla River near Touchet, Wash.	1,657	1964-1970	388	200-5730	12.0-5250	1.28-6.31	0.00139	11.050	3.606	0.00003	0.877	0.223
14-0335	Umatilla River near Umatilla, Ore.	2,290	1964-1970	211	112-4460	9.0-1830	1.15-8.28	0.008767	6.843	3.242	0.00002	0.927	0.201
14-0480	John Day River at McDonald Ferry, Ore.	7,580	1965-1970	172	1180-13500	13.0-2000	1.54-6.13	0.001427	8.333	2.706	0.00003	0.806	0.275
9-4485	Gila River at Safford Valley, Ariz.	7,896	1968	77	158-6240	29.0-8100	1.68-7.05	0.002128	10.498	3.630	0.00005	0.978	0.133
5-4874.7	South River near Ackworth, Iowa	460	1964-1967 (Corps of Engrs.)	108	150-8270	153-13410	1.63-4.44	0.000947	12.052	3.310	0.00004	0.804	0.270
4-1255.1	Pine River near Wellston, Mich.	265	1966-1970	296	274-1060	40-610	1.66-3.08	0.000625	11.100	3.107	0.00003	0.774	0.115
6-8702	Smoky Hill River at New Cambria, Kans.	11,730	1964-1968	255	62-3540	12-3800	0.88-3.45	0.000272	10.625	2.445	0.00003	0.770	0.295
6-8545	Republican River at Scandia, Kans.	22,903	1967-1969	190	107-7590	37-4700	1.13-3.46	0.0007034	11.408	3.013	0.00005	0.672	0.276
7-1465	Arkansas River at Arkansas City, Kans.	43,713	1964-1971	452	1010-24500	53-4300	1.76-4.17	0.000641	12.385	3.381	0.00002	0.711	0.249
3-2345	Scioto River at Higby, Ohio	5,131	1964-1968	207	1290-29100	9-630	1.32-3.57	0.000306	8.460	2.079	0.00002	0.719	0.198
9-2610	Green River near Jensen, Utah	25,400	1964-1970	257	1330-18400	28-5000	1.79-6.22	0.001286	8.559	2.546	0.00004	0.728	0.303
3-3285	Eel River near Logansport, Ind.	789	1969-1972	230	570-6090	15-1050	1.11-5.43	0.000658	6.464	1.570	0.00002	0.739	0.251

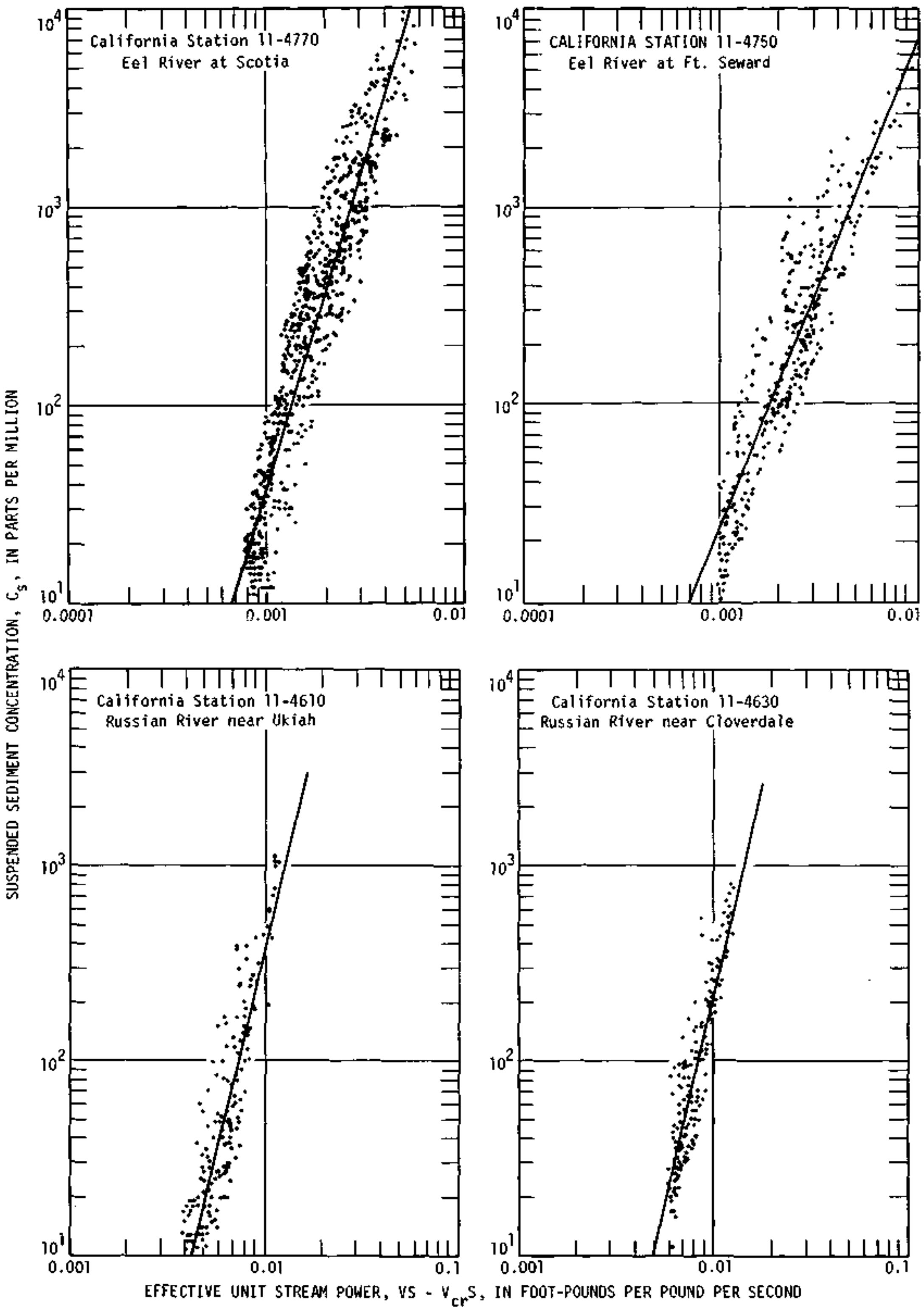

Figure 26. Relation between measured suspended sediment concentration and effective unit stream power for four stations

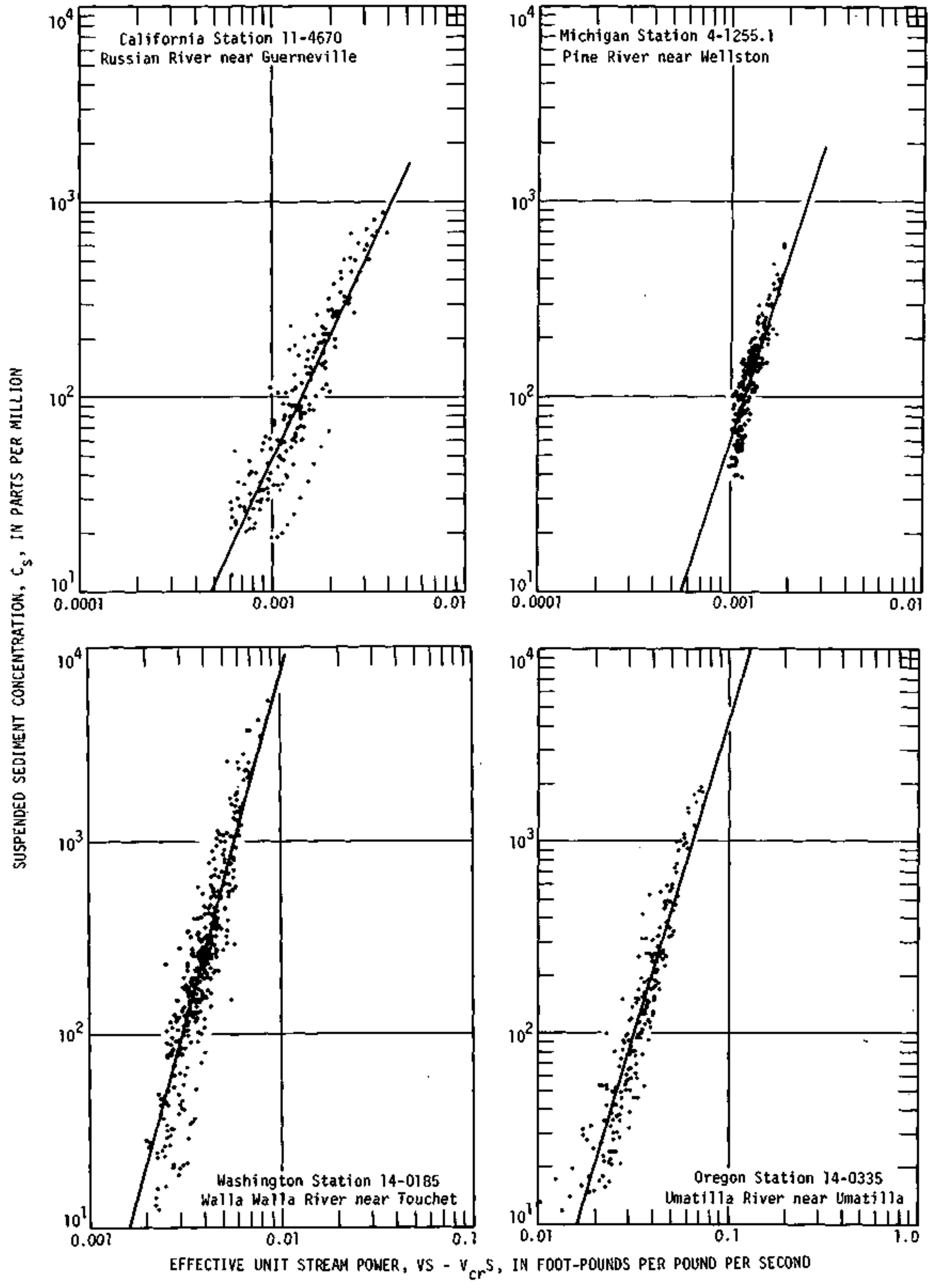

Figure 27. Relationship between measured suspended sediment concentration and effective unit stream power for four stations

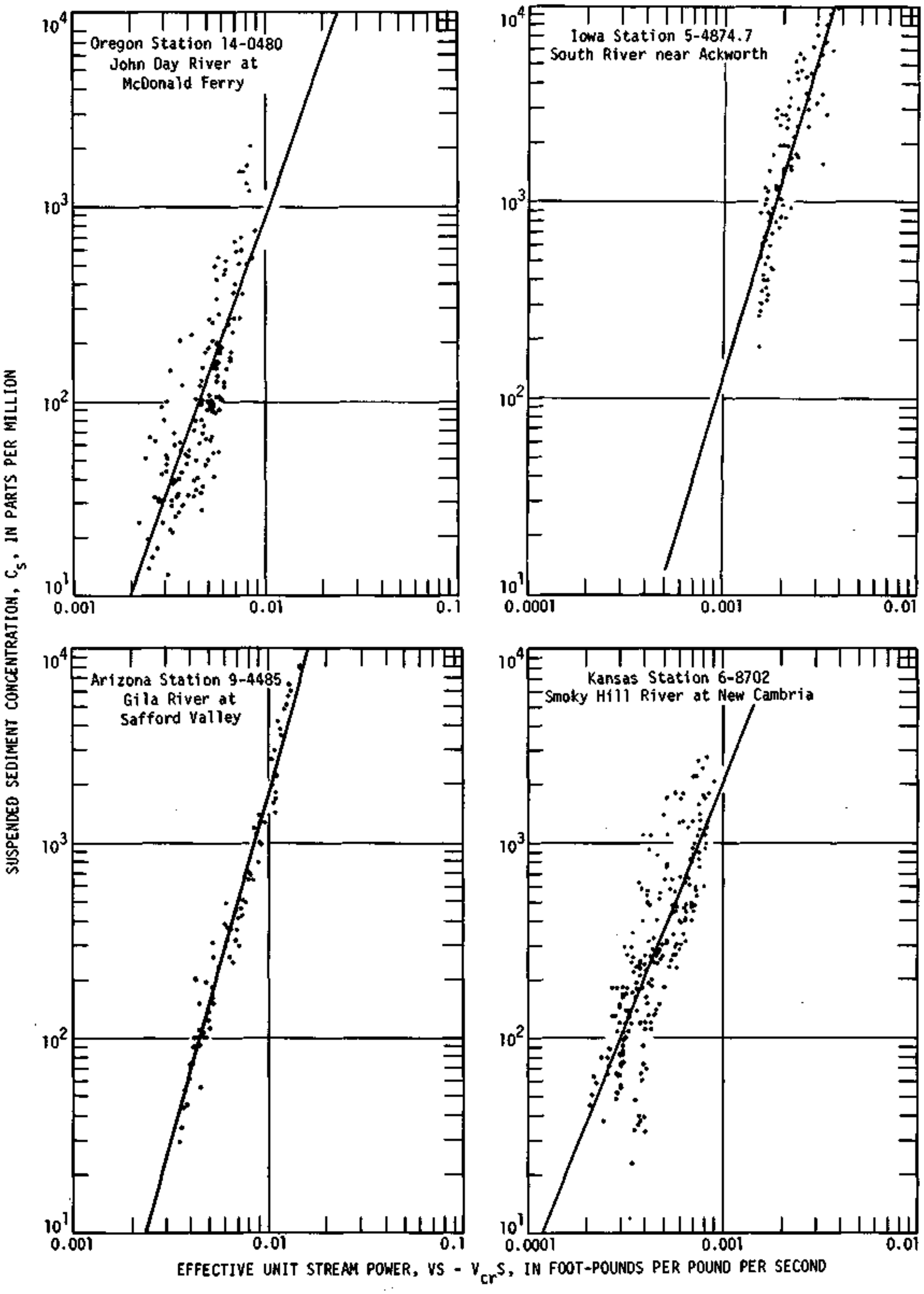

Figure 28. Relationship between measured suspended sediment concentration and effective unit stream power for four stations

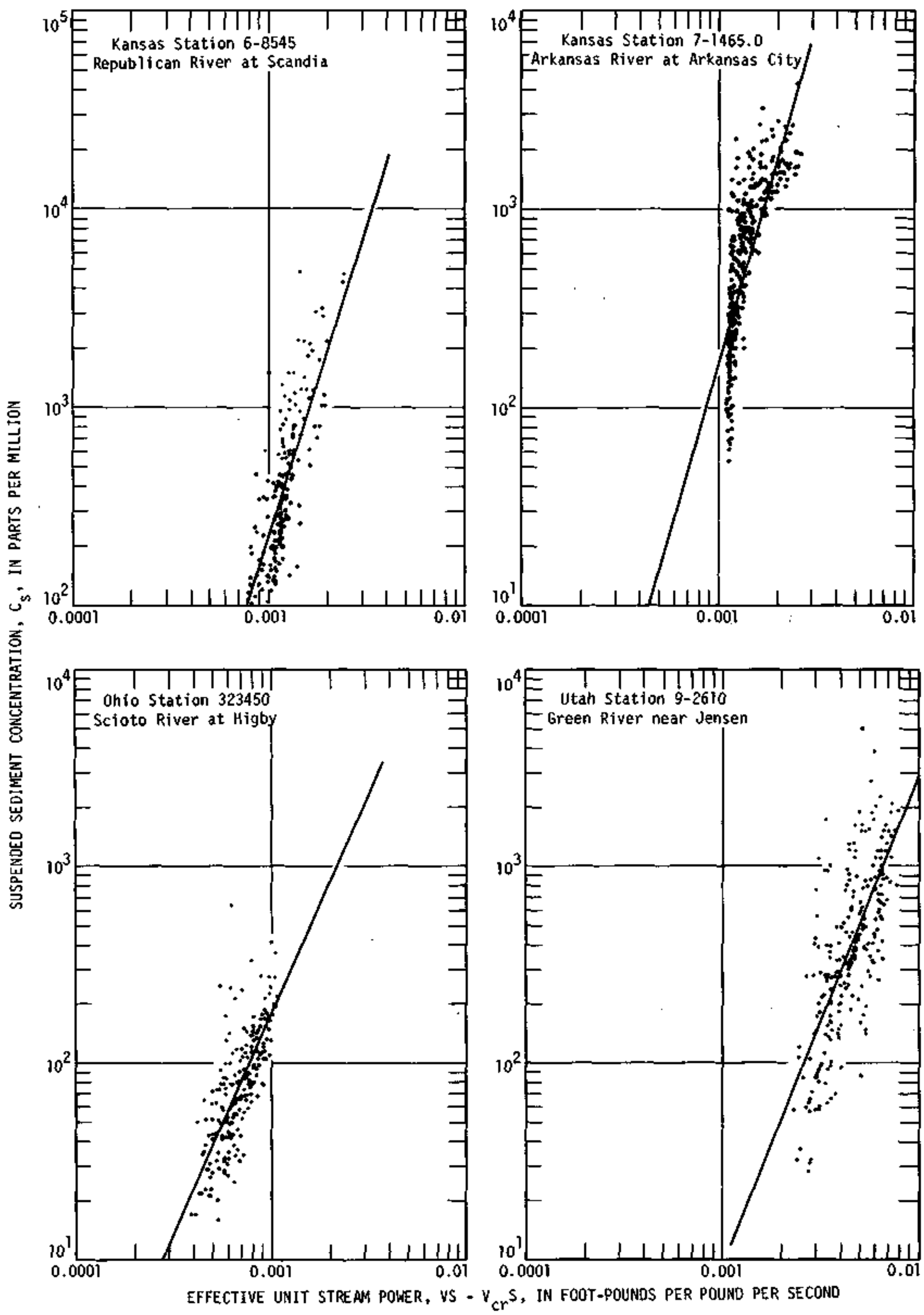

Figure 29. Relationship between measured suspended sediment concentration and effective unit stream power for four stations

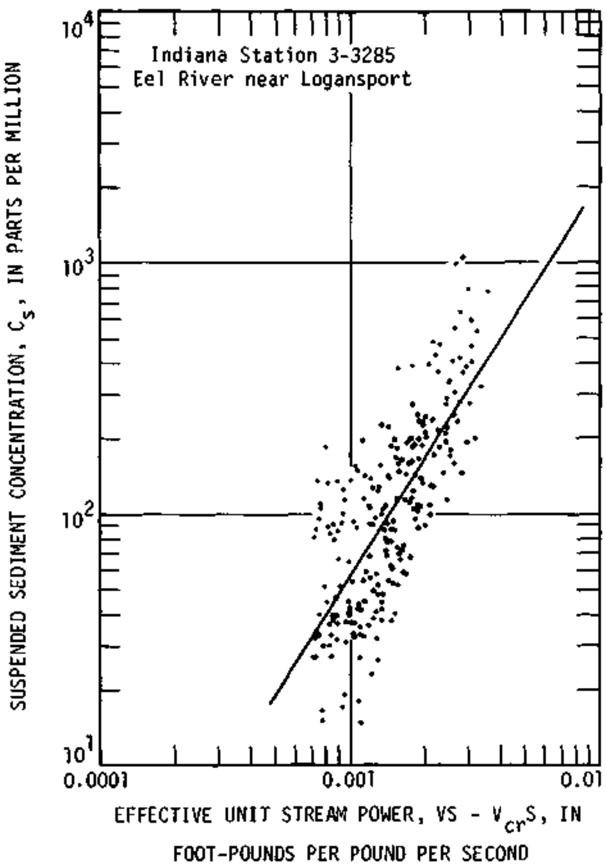

Figure 30. Relationship between measured suspended sediment concentration and effective unit stream power for one station

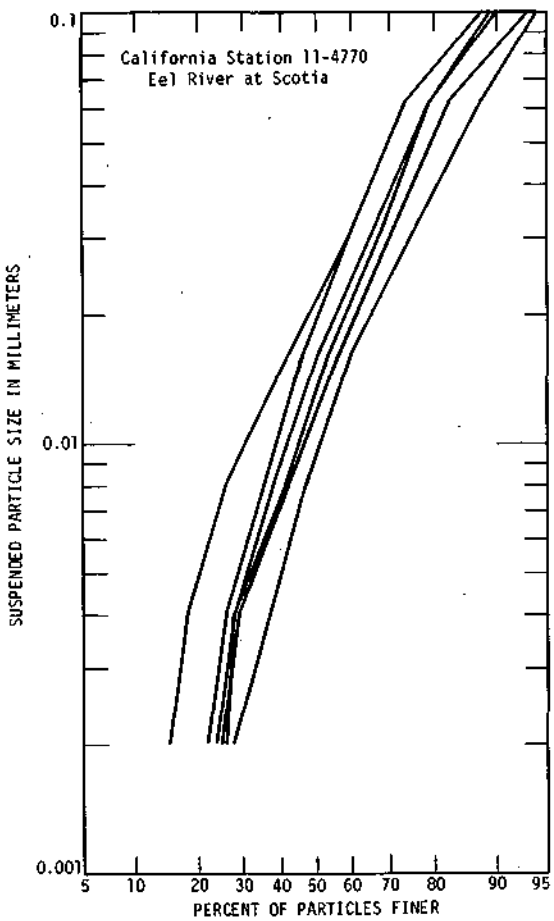

Figure 31. Suspended sediment size distribution for the Eel River at Scotia, Calif.

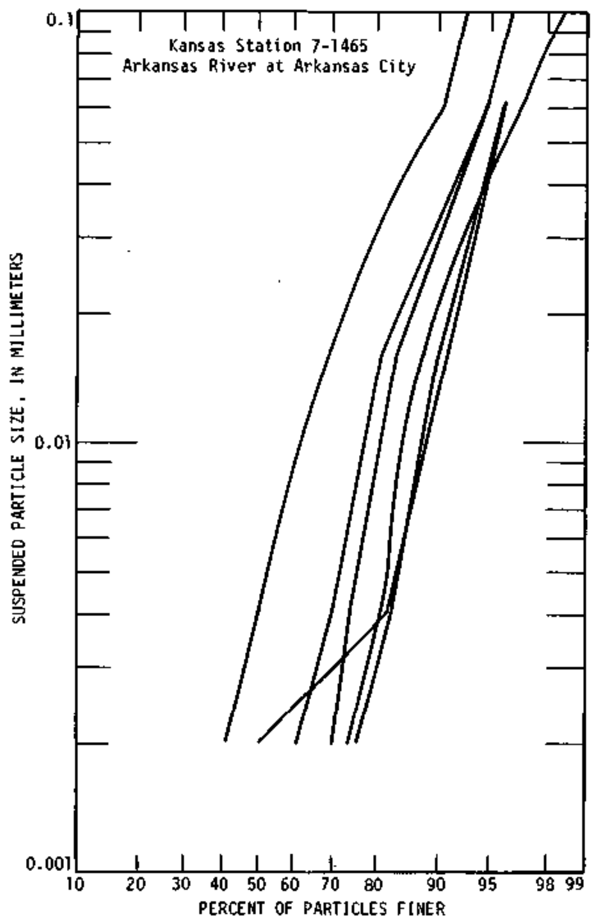

Figure 32. Suspended sediment size distribution for the Arkansas River at Arkansas City, Kans.

tors. These data cover the hydraulically smooth, transition, and completely rough flow regimes. These two equations can also be considered as scour criteria for stable channel design and other related engineering design.

2) Equation 19 is the dimensionless unit stream power equation the authors would like to propose to engineers for their consideration in predicting total sediment concentration in natural rivers. If a significant amount of wash load exists in a river, it must be excluded before equation 19 can be applied.

3) The accuracy of equation 19 was verified by the use of 1093 sets of laboratory data collected by different investigators. The low values of standard error shown in table 2 indicate that this equation can be used with confidence in predicting total sediment concentration.

4) Total sediment concentrations collected from four natural rivers were used to verify the accuracy of equation 19. Comparisons between the measured and predicted sediment discharges at these stations with the use of different equations indicate that equation 19 is the most accurate one.

5) Comparisons between sediment discharges calculated by different equations and the measured bed material load from two natural rivers demonstrate that equation 19 is superior to other published equations.

6) Suspended sediment concentrations collected for 17 regular USGS gaging stations indicate that suspended sediment concentration in natural rivers is dominated by the unit stream power of the river.

ACKNOWLEDGMENTS

The research described in this report has been carried out by the authors as a part of their regular work at the Illinois State Water Survey under the general direction of Dr. William C. Ackermann, Chief. The work was supported, in part, by funds provided by the U. S. Department of the Interior as authorized under the Water Resources Act of 1964, Public Law 88-379.

The U. S. Geological Survey and the U. S. Army Corps of Engineers provided the hydraulic and sediment data at gaging stations. The authors have benefitted from discussions with Paul C. Benedict, William M. Brown, III, and Thomas Maddock, Jr., of the USGS; and Frank Mark and Brien R. Winkley of the Army Corps of Engineers. These discussions were very helpful in understanding the problems of sediment transport in natural rivers.

Illustrations for the report were done by the graphic art unit of the Illinois State Water Survey under the supervision of John W. Brother, Jr. The report was edited by J. Loreena Ivens. Engineering assistants who worked part time on this project while graduate students at the University of Illinois in Civil Engineering were Jahangir Azma, Mu Jin Chung, Ahmet Suha Sevuk, and Yung-yuan Shen. Carl G. Lonnquist, James C. Neill, and Robert A. Sinclair offered considerable assistance in computer analysis.

REFERENCES

Colby, B. R., and C. H. Hembree. 1955. *Computation of total sediment discharge, Niobrara River near Cody, Nebraska.* U. S . Geological Survey Water Supply Paper 1357.

Einstein, H. A. 1944. *Bed-load transportation in Mountain Creek.* U. S. Department of Agriculture, Soil Conservation Service, SCS-TP-55.

Einstein, H. A. 1950. *The bed-load function for sediment transportation in open channel flows.* U. S. Soil Conservation Service, Technical Bulletin 1026.

Franco, J. J. 1968. *Effect of water temperature on bed-material movement.* ASCE Journal of the Waterway and Harbors Division, V.94(WW 3):343-352.

Gilbert, K. G. 1914. *The transportation of debris by running water.* U. S. Geological Survey Professional Paper 86.

Guy, H. P., D. B. Simons, and E. V. Richardson. 1966. *Summary of alluvial channel data from flume experiment, 1956-1961.* U. S. Geological Survey Professional Paper 462-I.

Hubbell, D. W., and D. Q. Matejka. 1959. *Investigations of sediment transportation, Middle Loup River at Dunning, Nebraska.* U. S. Geological Survey Water Supply Paper 1476.

Inter-Agency Committee on Water Resources, Subcommittee on Sedimentation. 1957. *Some fundamentals of particle size analysis.* St. Anthony Falls Hydraulic Laboratory, Minneapolis, Minn.

Jordan, P. R. 1965. *Fluvial sediment of the Mississippi River at St. Louis, Missouri.* U.S. Geological Survey Water Supply Paper 1802.

Kennedy, J. F. 1961. *Stationary waves and antidunes in alluvial channels.* California Institute of Technology, Pasadena, W. M. Keck Laboratory of Hydraulics and Water Resources Report No. KH-R-2.

Nomicos, G. N. 1956. *Effects of sediment load on the velocity field and friction factor of turbulent flow in an open channel.* Ph.D. thesis presented to the California Institute of Technology, Pasadena.

Nordin, C. F., Jr. 1964. *Aspects of flow resistance and sediment transport, Rio Grande near Bernalillo, New Mexico.* U. S. Geological Survey Water Supply Paper 1498-H.

Schlichting, H. 1962. *Boundary layer theory.* McGraw-Hill Book Co., Inc., New York, 4th ed., pp. 509-526.

Schneider, V. R. 1971. Personal communication. Data collected from an 8-foot wide flume at Colorado State University, Fort Collins.

Stall, J. B., and Y. S. Fok. 1968. *Hydraulic geometry of Illinois streams.* University of Illinois Water Resources Center Research Report 15, Urbana.

Stall, J. B., and C. T. Yang. 1970. *Hydraulic geometry of 12 selected stream systems of the United States.* University of Illinois Water Resources Center Research Report 32, Urbana.

Stall, J. B., and C. T. Yang. 1972. *Hydraulic geometry and low streamflow regimen.* University of Illinois Water Resources Center Research Report 54, Urbana.

Stein, R. A. 1965. *Laboratory studies of total load and apparent bed-load.* Journal of Geophysical Research, V. 70(8):1831-1842.

Taylor, B. D., and V. A. Vanoni. 1972. *Temperature effects in low-transport, flat-bed flow.* ASCE Journal of the Hydraulics Division, V. 98(HY 8): 1427-1455.

Vanoni, V. A., and N. H. Brooks. 1957. *Laboratory studies of the roughness and suspended load of alluvial streams.* California Institute of Technology, Pasadena, Sedimentation Laboratory MRD Sediment Series No. 11, pp. 35-36.

Vanoni, V. A., N. H. Brooks, and J. F. Kennedy. 1960. *Lecture notes of sediment transportation and channel stability.* California Institute of Technology, Pasadena, W. M. Keck Laboratory of Hydraulics and Water Resources, Report KH-RI.

Williams, G. P. 1967. *Flume experiments on the transport of a coarse sand.* U. S. Geological Survey Professional Paper 562-B.

Yang, C. T. 1971a. *Potential energy and stream morphology.* Water Resources Research, V. 7(2): 311-322.

Yang, C. T. 1971b. *On river meanders.* Journal of Hydrology, V. 13(3):231-253.

Yang, C. T. 1971c. *Formation of riffles and pools.* Water Resources Research, V. 7(6):1567-1574.

Yang, C. T. 1972. *Unit stream power and sediment transport.* ASCE Journal of the Hydraulics Division, V. 98(HY 10): 1805-1826.

Yang, C. T. 1973. *Incipient motion and sediment transport.* ASCE Journal of the Hydraulics Division, V. 99(HY 10): 1679-1704.