Universal angular magnetoresistance and spin torque in ferromagnetic/normal metal hybrids

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(Received 23 January 2003; published 25 March 2003)

The electrical resistance of ferromagnetic/normal-metal (F/N) heterostructures depends on the nature of the junctions that may be tunnel barriers, point contacts, or intermetallic interfaces. For all junction types, the resistance of disordered $F/N/F$ perpendicular spin valves as a function of the angle between magnetization vectors is shown to obey a simple universal law. The spin-current induced magnetization torque can be measured by the angular magnetoresistance of these spin valves. The results are generalized to arbitrary magnetoelectronic circuits.

DOI: 10.1103/PhysRevB.67.094421 PACS number(s): 72.25.Ba, 75.60.Jk, 75.47.De, 75.75.+a

Magnetoelectronics achieves new functionalities by incorporating ferromagnetic materials into electronic circuits. The giant magnetoresistance, i.e., the dependence of the electrical resistance on the relative orientation of the magnetizations of two ferromagnets in a ferromagnetic/normal/ferromagnetic ($F/N/F$) metal structure or "spin valve," is applied in read heads of high information density magnetic storage systems.[1] Usually, such a device is viewed as a single bit, the magnetization vectors being either parallel or antiparallel. Early seminal contributions by Slonczewski[2] and Berger[3] revealed fundamentally new physics and technological possibilities of noncollinearity, which triggered a large number of experimental and theoretical research. An important example is the nonequilibrium spin-current induced torque (briefly, spin torque), which one ferromagnet can exert on the magnetization vector of a second magnet through a normal metal. This torque can be large enough to dynamically turn magnetizations,[4] which is potentially interesting as a low-power switching mechanism for magnetic random access memories.[5] The spin torque is also essential for magnetic devices such as the spin-flip transistor,[6–8] detection of spin precession,[9] the Gilbert damping of the magnetization dynamics in thin magnetic films,[10] and spin injection induced by ferromagnetic resonance.[11] In this paper, we report a universal analytic formula for the angular magnetoresistance of arbitrary spin valves, which allows direct determination of the spin torque via a one-parameter fit of experimental curves.[12]

Recently, two theoretical approaches have been developed which address charge and spin transport in diffuse noncollinear magnetic hybrid structures. The magnetoelectronic "circuit theory"[6] is based on the division of the system into discrete resistive elements over which the applied potential drops, and low-resistance nodes at quasiequilibrium [as in Fig. 1(a)]. The electrical properties are then governed by generalized Kirchhoff rules in Pauli spin space and can be computed easily. Each resistor is thereby characterized by four material parameters, the spin-up and spin-down conductances $g_{\uparrow(\downarrow)}=\Sigma_{nm}(\delta_{nm}-|r_{nm}^{\uparrow(\downarrow)}|^2)$ as known from the scattering theory of transport,[13] as well as the real and imaginary parts of the "mixing conductance" $g_{\uparrow\downarrow}=\Sigma_{nm}[\delta_{nm}-r_{nm}^{\uparrow}(r_{nm}^{\downarrow})^*]$, where r_{nm}^{s} is the reflection coefficient between nth th and mth transverse modes of an electron with spin s in the normal metal at the contact to a ferromagnet. Waintal *et al.*[14] studied the random-matrix theory of transport in noncollinear magnetic systems as sketched in Fig. 1(b). Their formalism did not require the assumption of highly resistive elements, but the algebra of the 4×4 scattering matrices in spin space seemed so complex that analytical results were obtained in limiting cases only.

Both theories are not valid in the limit of intermetallic interfaces in a diffuse environment [see Fig. 1(c)] like the perpendicular spin valves, studied thoroughly by Pratt and Bass c.s.[15] and others.[16,17] These studies provided a large body of evidence for the two-channel (i.e., spin-up and spin-down) series resistor model and a wealth of accurate transport parameters such as the interface resistances for various material combinations. Transport through transparent interfaces in a diffuse environment has been studied for *collinear* magnetizations by Schep *et al.*[18] Under the condition of isotropy of scattering by disorder, it was found that the bulk resistances, which are proportional to the layer thicknesses, are in series with interface resistances for each spin s,

$$\frac{1}{\tilde{g}_s}=\frac{1}{g_s}-\frac{1}{2}\left(\frac{1}{N_s^F}+\frac{1}{N_N}\right), \tag{1}$$

where N_s^F and N_N are the number of modes of the bulk materials on both sides of the F/N contact. Physically, in Eq. (1) a spurious Sharvin resistance is subtracted from the result of scattering theory. This correction is large for transparent interfaces and essential to obtain agreement between experimental results and first-principles calculations.[18–20]

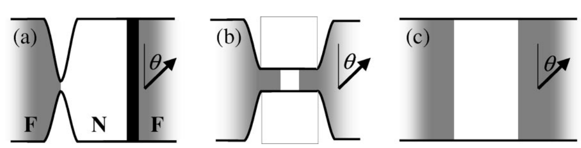

FIG. 1. Different realizations of perpendicular spin valves. (a) Highly resistive junctions such as point contacts and tunneling barriers limit the conductance. (b) Spin valve in a geometrical constriction amenable to the scattering theory of transport. (c) Magnetic multilayers with transparent interfaces. θ is the angle between magnetization directions.

In exchange-biased spin valves, it is possible to measure the electric resistance as a function of the angle between magnetizations, which has been analyzed experimentally and theoretically.[12,21,22] The present study has been motivated by Pratt's observation that experimental magnetoresistance curves[12] could accurately be fitted by the form[6]

$$\frac{R(\theta)-R(0)}{R(\pi)-R(0)}=\frac{1-\cos\theta}{\chi(1+\cos\theta)+2} \tag{2}$$

with one free parameter χ that is given by circuit theory

$$\chi=\frac{1}{1-p^2}\frac{|\eta|^2}{\mathrm{Re}\,\eta}-1 \tag{3}$$

in terms of the normalized mixing conductance $\eta=2g_{\uparrow\downarrow}/g$, the polarization $p=(g_\uparrow-g_\downarrow)/g$, and the average conductance $g=g_\uparrow+g_\downarrow$. This was surprising, since the circuit theory, as mentioned above, was not designed for metallic multilayers, and, indeed, the numerical value of fitted parameters did not make sense, also after including effects of bulk scattering in the ferromagnetic layers.[23]

In the following we develop a theory of transport in disordered magnetoelectronic circuits and devices in the diffuse regime, which unifies and extends previous theoretical approaches. We find simple analytical results with parameters that are accessible to realistic electronic-structure calculations. The angular magnetoresistance for perpendicular spin valves has the universal form [Eq. (2)] in agreement with measurements,[12] and is used to determine the mixing conductance and spin torque. The theory is valid under two conditions: (i) the system should be diffuse, i.e., the elastic mean free path ℓ (including scattering at interfaces) should be smaller than typical sample scales and (ii) the ferromagnetic elements should have an exchange splitting Δ, which is large enough that the magnetic coherence length $\ell_c=\hbar/\sqrt{2m\Delta}<\min(\ell d_F)$, where d_F is the thickness of the ferromagnetic layer. These conditions are usually fulfilled in transition-metal systems: Deviations from diffuse behavior, such as quantum-size effects and breakdown of the series resistor model, are small or controversial,[20,24] whereas the magnetic coherence length is of the same order as the lattice constant in high-T_c transition-metal ferromagnets.[10,25] We obtain identical results by two methods: The first one is a combination of the Boltzmann-like method of Schep *et al.*[18] for collinear systems and the random-matrix theory of Waintal *et al.*[14] The second one is an extension of magnetoelectronic circuit theory[6] to arbitrary resistors.

Let us consider planar spin-valve structures as shown in Fig. 1. We assume the existence of a distribution function at a certain position x in the sample (a "node"), which in spin-polarized systems has eight elements $f^{\pm}_{ss'}(x)$. We arrange them into a 4×1 vector $\vec{f}^{\pm}=(f^{\pm}_{\uparrow\uparrow},f^{\pm}_{\uparrow\downarrow},f^{\pm}_{\downarrow\uparrow},f^{\pm}_{\downarrow\downarrow})^T$ as well as into a 2×2 matrix, denoted by a hat:

$$\hat{f}^{\pm}(x)=\begin{pmatrix} f^{\pm}_{\uparrow\uparrow}(x) & f^{\pm}_{\downarrow\uparrow}(x)\\ f^{\pm}_{\uparrow\downarrow}(x) & f^{\pm}_{\downarrow\downarrow}(x)\end{pmatrix}. \tag{4}$$

The superscript denotes that the distribution is in general anisotropic in reciprocal space, + for right moving and − for left moving, indicating that, in contrast to Refs. 6 and 14, the current density in the nodes is not negligible. The distribution functions at different nodes are matched via boundary conditions

$$\vec{f}^{+}(x_B)=\check{T}_{A\to B}\vec{f}^{+}(x_A)+\check{R}_{B\to B}\vec{f}^{-}(x_B), \tag{5a}$$

$$\vec{f}^{-}(x_A)=\check{R}_{A\to A}\vec{f}^{+}(x_A)+\check{T}_{B\to A}\vec{f}^{-}(x_B), \tag{5b}$$

where the 4×4 transmission and reflection probability matrices (indicated by the caret) have elements like[14]

$$[\check{T}_{A\to B}]_{ij}=\frac{1}{N_i^B}\sum_{nm}(\vec{t}_{nm}^{\,A\to B})_i(\vec{t}_{nm}^{\,A\to B})_j^{\dagger}, \tag{6}$$

where $N_i^B=N_\uparrow^B(\delta_{i,1}+\delta_{i,2})+N_\downarrow^B(\delta_{i,3}+\delta_{i,4})$, N_s^B is the number of modes for spin s in B, and $\vec{t}_{nm}^{\,A\to B}$ is a vector of the transmission coefficients in spin space.

Let us calculate the electrical charge current in a symmetric two-terminal spin valve with relative magnetization angle θ (Fig. 1). x_L and x_R are within left and right ferromagnets at a distance from the interface equal to the spin-diffusion length in the ferromagnet $\ell_{sd}^F\gg\ell_c$, and thus define the magnetically active region. In the coordinate systems defined by the magnetization directions, the transverse components of the spin accumulation in the ferromagnets vanish[6,25] and the distributions in the magnets depend on the local spin-current densities γ_s and (spin-independent) chemical potentials μ only:

$$\vec{f}^{\pm}(x)=((\pm\gamma_\uparrow+\mu)(x),0,0,(\pm\gamma_\downarrow+\mu)(x)). \tag{7}$$

In symmetric junctions the spin current is symmetric as well, $\gamma_s(x_L)=\gamma_s(x_R)$. The charge current $i_c=(e/h)\Sigma_s N_s^F\gamma_s$ divided by the chemical potential drop equals the electrical conductance $G=ei_c/\Delta\mu$. Equations (5) and (7) then lead to

$$G=\frac{2e^2}{h}\sum_{\substack{i=1,4\\ j=1,4}}\{N_i^F[\check{1}-\check{T}_{L\to R}+\check{R}_{R\to R}]^{-1}\check{T}_{L\to R}\}_{ij}. \tag{8}$$

In principle, the matrices $\check{T}$ and $\check{R}$ do not need to be approximate.

In dirty systems, more nodes may be introduced at convenient locations in the sample and Eqs. (5) imply that total transport probability matrices can be composed in terms of those of individual elements by semiclassical concatenation rules.[26] For instance, the transmission through a $F(0)/N/F(\theta)$ double heterojunction as in Fig. 1 (without bulk scattering) takes the form:

$$\check{T}(\theta)\equiv\check{T}_{N\to F}(\theta)[\check{1}-\check{R}_{N\to N}(0)\check{R}_{N\to N}(\theta)]^{-1}\check{T}_{F\to N}(0). \tag{9}$$

These rules have been derived from the (phase-coherent) scattering theory by averaging over random matrices[14] and found to be valid to leading order in N_N^{-1}, where N_N is the number of transport channels in the normal metal. Bulk impurity scattering can be represented by diagonal matrices[14,18]

$$(\check{T}_B)_{ss'} = \left(1 + \frac{1}{N_s^B} + \frac{e^2}{h}\frac{\rho_s^B d_B}{A_B}\right)^{-1}\delta_{ss'}, \tag{10}$$

where ρ_s^B, d_B, A_B are the single-spin bulk resistivities, thickness, and cross section of the bulk material B, respectively.

The problem can be simplified by transformations into the coordinate systems defined by the magnetization directions of the ferromagnets. In terms of the spin rotation

$$\hat{U} = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \tag{11}$$

and projection matrices ($s = \pm 1$)

$$\hat{u}_s(\theta) = \frac{1}{2}\begin{pmatrix} 1+s\cos\theta & s\sin\theta \\ s\sin\theta & 1-s\cos\theta \end{pmatrix}, \tag{12}$$

the interface scattering matrices (omitting the mode indices for simplicity) are transformed as[6] $t_{ss'}^{F\to N} = U_{ss'}t_{s'}^{cF}$, $t_{ss'}^{N\to F} = t_s^{cN}U_{ss'}^{\dagger}$, $\hat{r}_{N\to N} = \Sigma_s \hat{u}_s r_s^{cN}$, and $r_{ss'}^{F\to F} = r_s^{cF}\delta_{ss'}$, where the superscript c indicates that the matrices should be evaluated in the reference frame of the local magnetization and spin-flip scattering in the contacts has been disregarded.

The angular magnetoresistance can now be evaluated analytically for our spin valve in terms of the three interface conductances $g_\uparrow$, $g_\downarrow$, $g_{\downarrow\uparrow}$ defined above, the bulk number of modes N_s^F, N^N, and (single-spin) bulk resistances ρ_s^F, ρ^N, whereas the magnetization angle and layer thicknesses are the variables. Surprisingly, the form Eq. (2) is recovered, but with renormalized parameters. The spin-dependent interface conductances are identical to Eq. (1), whereas, including also bulk scattering in one-half of the spacer thickness $d/2$,

$$\frac{1}{\tilde{g}_{\uparrow\downarrow}} = \frac{1}{g_{\uparrow\downarrow}} + \frac{1}{2}\left(\frac{e^2}{h}\frac{\rho_N d_N}{A_N} - \frac{1}{N_N}\right). \tag{13}$$

By letting $N_s^F \to \infty$ we are in the regime of Ref. 14. The circuit theory is recovered when, additionally, $N_N \to \infty$. The bare mixing conductance is bounded not only from below $\mathrm{Re}g_{\uparrow\downarrow} \geqslant g/2$,[6] but also from above $|g_{\uparrow\downarrow}|^2/\mathrm{Re}g_{\uparrow\downarrow} \leqslant 2N_N$. The polarization and relative mixing conductances are also renormalized, with $0 < |\tilde{\eta}| < \infty$.

It is not obvious how these results should be generalized to more complicated circuits and devices and to the presence of spin-flip scattering in the normal metal. The magnetoelectronic circuit theory[6] does not suffer from these drawbacks. In the following, we demonstrate that above results can be obtained with less effort, proving that with the renormalization of the transport parameters by subtracting Sharvin resistances, circuit theory remains valid for arbitrary contacts. To this end we construct the fictitious circuit depicted in Fig. 2. Consider a junction that in conventional circuit theory is

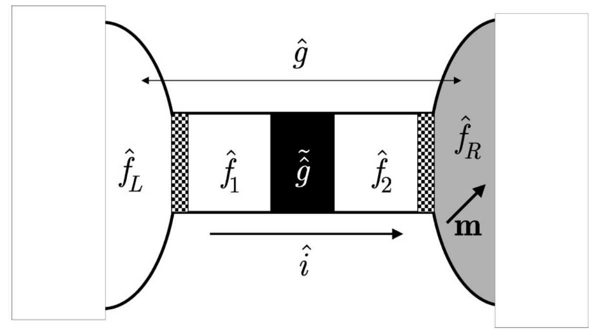

FIG. 2. Fictitious device that illustrates the generalization of circuit theory to transparent resistive elements as discussed in the text.

characterized by a matrix conductance $\hat{g}$, leading to a matrix current $\hat{\imath}$ when the normal and ferromagnetic distributions $\hat{f}_L$ and $\hat{f}_R$ are not equal. When the distributions of the nodes are isotropic, we know from circuit theory that

$$\hat{\imath} = \sum_{ss'} (\hat{g})_{ss'}\hat{u}_s(\hat{f}_L - \hat{f}_R)\hat{u}_{s'}, \tag{14}$$

where the projection matrices $\hat{u}_s$ are defined in Eq. (12) and $(\hat{g})_{ss} = g_s$, $(\hat{g})_{s,-s} = g_{s,-s}$. Introducing lead conductances, which modify the distributions $\hat{f}_L \to \hat{f}_1$ and $\hat{f}_2 \leftarrow \hat{f}_R$, respectively, we may define a (renormalized) conductance matrix $\tilde{\hat{g}}$, which causes an identical current $\hat{\imath}$ for the reduced (matrix) potential drop:

$$\hat{\imath} = \sum_{ss'} (\tilde{\hat{g}})_{ss'}\hat{u}_s(\hat{f}_1 - \hat{f}_2)\hat{u}_{s'}. \tag{15}$$

When the lead conductances are now chosen to be twice the Sharvin conductances, and using (matrix) current conservation

$$\hat{\imath} = 2N_N(\hat{f}_L - \hat{f}_1) \tag{16}$$

$$= \sum_s 2N_s^F \hat{u}_s(\hat{f}_2 - \hat{f}_R)\hat{u}_s, \tag{17}$$

straightforward matrix algebra leads to the result that $\tilde{\hat{g}}$ is identical to the renormalized interface conductances found above [Eqs. (1) and, without the bulk term, Eq. (13)]. By replacing $\hat{g}$ by $\tilde{\hat{g}}$ we not only recover results for the spin valve obtained above, but we can now use the renormalized parameters also for circuits with arbitrary complexity and transparency of the contacts. Also spin-flip scattering in N can be included;[6] it does not affect the form of Eq. (2) either, but only reduces the parameter $\tilde{\chi}$.

Experimental values for the parameters for Cu/Permalloy (Py) spin valves are $\tilde{\chi} = 1.2$ and $\tilde{p} = 0.6$.[12] Disregarding a very small imaginary component of the mixing conductance,[8] using the known values for the bulk resistivities, the theoretical Sharvin conductance for Cu ($0.55\times10^{15}\Omega^{-1}$ m^{-2}/ spin,[18]) and the spin-flip length of Py as the effective thickness of the ferromagnet [$\ell_{sd}^F = 5$ nm (Ref.

15)], we arrive at the bare Cu/Py interface mixing conductance $G_{\uparrow\downarrow}=0.39(3)\times 10^{15}\Omega^{-1}\ \mathrm{m}^{-2}$, which is close to that of Co/Cu.[8]

The spin torque on a ferromagnet[2,14] equals the spin current through the interface with vector component normal to the magnetization direction and its evaluation is closely related to the charge conductance.[6,14] An analytical expression for the spin valve reads ($\tilde{\eta}$ assumed real)[8]

$$L(\theta)=\frac{\tilde{p}\tilde{g}}{2}\frac{\tilde{\eta}\sin\theta}{(\tilde{\eta}-1)\cos\theta+1+\tilde{\eta}}\frac{\Delta\mu}{4\pi}, \tag{18}$$

in terms of parameters that can be measured as well as computed from first principles. Previous results[2,14] are recovered in the limit that $\tilde{\eta}\rightarrow 2$ and $\tilde{p}\rightarrow 1$. Note that we cannot meaningfully compare our results with those of Zhang *et al.*,[27] which are derived for weak ferromagnets and are not applicable to transition metals considered here.[25] By the generalized circuit theory, it is now straightforward to compute the torque on the base contact of the spin-flip transistor with antiparallel source-drain magnetizations (three identical contacts, but disregarding spin-flip scattering in the base).[8] Interestingly, it is larger and has a symmetric and flatter dependence on the angle of the base magnetization direction θ:

$$L_b(\theta)=\frac{\tilde{p}\tilde{g}\tilde{\eta}\sin\theta}{(1-\tilde{\eta})\cos^2\theta+2+\tilde{\eta}}\frac{\Delta\mu}{4\pi}.$$

In conclusion, we reported analytical results for the angular magnetoresistance of arbitrary spin valves, which, by comparison with experiments,[12] leads to a value for the mixing conductance and spin torque for the Cu/Py interface of $G_{\uparrow\downarrow}=0.39(3)\times 10^{15}\ \Omega^{-1}\ \mathrm{m}^{-2}$. The associated generalization of magnetoelectronic circuit theory opens the way to engineer materials and device configurations to optimize switching properties of magnetic random access memories.

We would like to thank Bill Pratt for attracting our attention to the problem and sharing unpublished experimental data. We acknowledge discussions with Paul Kelly, Ke Xia, Jack Bass, Bert Halperin, Yuli Nazarov, as well as support by FOM, the Schlumberger Foundation, DARPA Grant No. MDA 972-01-1-0024, NSF Grant No. DMR 99-81283 and the NEDO joint research program ''Nano-Scale Magnetoelectronics.'' G.E.W.B. is grateful for the hospitality of Dr. Y. Hirayama and his group at the NTT Basic Research Laboratories.

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