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UNIVERSAL LENGTH AND VALIDITY OF SPECIAL RELATIVITY*

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ABSTRACT

We suggest the reexamination of the validity of the Lorentz invariance at the small distance of the order of 10^{-16} cm in the light of high energy available at CERN and NAL. Using the modified weak interaction Lagrangian with a noncausal form factor we evaluate the muon decay lifetime including the effect of the radiative corrections as a function of the particle energy and characteristic length. A possible link between the universal length and a violation of the Lorentz invariance will be discussed.

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In recent papers[1] it has been pointed out that the validity of special relativity at small distances can be tested by means of a lifetime study of unstable particles in flight. The starting point of the suggested test is based on the construction of a weak interaction Lagrangian with a noncausal form factor. The possible violation of the Lorentz invariance would then appear in the form of a change in lifetime of the unstable particles. This test has been performed in terms of the cosmic ray experiment, the high energy available at the time, by Dardo et al.[2] It turned out, however, that the result of the experiment could not be conclusive because the errors in the experimental data were too large.

The purpose of this note is to propose the reexamination of the validity of the Lorentz invariance in terms of the formalism suggested above but including the effect of radiative corrections. The energies available in high energy laboratories at present enable one to carry out the proposed test. The second point we would like to report in this note is that we have evaluated a characteristic length 0.69×10^{-16} cm by using the nonlocal field formulation[3] of the CP violation with the latest observed CP violating parameters. The relation between the characteristic length and a possible violation of the Lorentz invariance will be discussed in the last part of the paper.

In the following we shall show explicitly the velocity dependence of the decay lifetime of muon which is a measure of a possible violation of Lorentz invariance. The suggested weak interaction Lagrangian contains a noncausal form factor which permits interaction between simultaneous space-time events in the laboratory frame when their spatial distance is less than the characteristic length a_0. In practice, a usual local V-A coupling weak interaction Lagrangian[5] has been modified by introducing a noncausal form factor:

$$\overleftarrow{L}_{int} = \frac{g}{4\sqrt{2}} \int d^4x\, \bar{\psi}_e(x)\,(1-\gamma_5)\,\gamma_\lambda(1+\gamma_5)\,\psi_\mu(x)$$

$$\cdot \int d^4x'\, G(x-x')\, \psi_{\nu_\mu}(x')\,(1-\gamma_5)\,\gamma_\lambda(1+\gamma_5)\,\psi_{\nu_e}(x') \tag{1}$$

The G(x) in Eq. (1) is a distribution function

$$G(x) = \left(\frac{4}{3}\,\pi a_0^3\right)^{-1} \delta(x_0)\ \theta\left(a_0^2 - |\vec{x}|^2\right) \tag{2}$$

with a step function θ. One notes that the modified weak interaction Lagrangian reduces the usual local Lagrangian for $a_0 \to 0$ because G(x) becomes then $\delta(x)$. The distribution function plays the role of "shifting apart" the fields over a spatial extention of a_0 at the instant $(x_0 = 0)$ of the interactions.

After the usual evaluation, one obtained the decay lifetime of the muon in flight (by restoring c and h)

$$\tau = \tau_0\left(1 - \frac{v^2}{c^2}\right)^{-1/2}\left[1 + \frac{E_\mu^2}{\left(3.23\,\lambda\!\!\!^{-}_\mu\,\frac{m_\mu c^2}{a_0}\,\mathrm{GeV}\right)^2}\right] \tag{3}$$

$\tau_0 = 192\,\pi^3/g^2 m_\mu^5$ is the muon decay lifetime at the rest frame and E_μ and $\lambda\!\!\!^{-}_\mu$ are the kinetic energy and Compton wave length respectively of the muon. The equation can be rewritten in the simplified form

$$\tau = \tau_0\left(1 - \frac{v^2}{c^2}\right)^{-1/2}(1+\kappa) \tag{4}$$

with

$$\kappa = \left[E_\mu / E(a_0)\right]^2 \tag{5}$$

where $E(a_0)$ is the characteristic energy 6.37×10^{-14} cm/a_0 GeV. The κ is responsible for a possible deviation from the usual Lorentz invariance and is a function of the energy E_μ and the characteristic length a_0.

Now we shall re-evaluate the lifetime of the muon decay above by including the effect of the radiative correction. This can be done by using the Fierz transformed Lagrangian (6) (as in Eq. (1)) and the Landau gauge in which all the integrals are calculated to be finite without going through the process of regularization of conventional calculations.

$$L_{int}(x) = \frac{g}{\sqrt{2}} \bar{\psi}_e \gamma_\lambda (1+\gamma_5) \psi_\mu \bar{\psi}_\nu \gamma_\lambda (1+\gamma_5) \psi_\nu \tag{6}$$

Note that the neutrino pair plays the role of "external source" while the muon is transformed into electron (analogous to electrodynamics), i.e., the change in mass of the spinor field with the external neutral current. The contribution of the effect of the radiative corrections of the order α to the lifetime of the muon is evaluated to be

$$\Delta\tau = \left(\frac{192\pi^3}{g^2 m_\mu^5}\right)\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \frac{\alpha}{2\pi}\left(\pi^2 - \frac{25}{4}\right) \quad . \tag{7}$$

Thus the lifetime of the muon with the effects of both the noncausal and radiative corrections is found to be

$$\tau = \frac{192\pi^3}{g^2 m_\mu^5}\left(1 - \frac{v^2}{c^2}\right)^{-1/2}\left[1 + \frac{\alpha}{2\pi}\left(\pi^2 - \frac{25}{4}\right)\right] \cdot \left[1 + \frac{E_\mu^2}{\left(3.23 \lambda\!\!\!^{-}_\mu \frac{m_\mu c^2}{a_0} \text{ GeV}\right)^2}\right] \tag{8}$$

where both effects tend to suppress the decay rate (or lengthen the lifetime). In the figure we have shown the measure of the violation of the Lorentz invariance κ as a function of muon energy for a number of fixed characteristic lengths a_0. As we have shown in the figure for the muon energy of 200 GeV^2 and higher, one may observe a measureable effect (of broken Lorentz invariance) for an assumed characteristic length of the order of 0.5×10^{-16} cm (as an example) and smaller lengths. If we assume the characteristic length to be a universal length with a magnitude of $a_0 = 0.69 \times 10^{-16}$ cm (the reason for suggesting this value will be explained in a later part of this paper), there will be a 2.6% deviation in the lifetime measurement from the

$$\tau = \tau_0 \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \left[1 + \frac{\alpha}{2\pi}\left(\pi^2 - \frac{25}{4}\right)\right]$$

with the muon energy of 150 GeV. With muon energy of 400 GeV, one should be able to observe a deviation of about 14.3%.

The works described in the preceeding pages were merely an attempt to justify the belief[5-10] that a Minkowskian metric structure for space-time in a region of the order of 10^{-16} cm may have a small scale inhomogeniety and nonisotropy (nonlocality of field). At this point, we should like to discuss such a nonlocal field aspect of discrete symmetry breaking mechanism in kaon decay.[3, 10]

In the study of CP violation in the neutral kaon decay process, we have found that such a violation may possibly take place at the small distances of the order of 10^{-16} cm. In other terms, such a violation of discrete symmetry in decay process is a characteristic of the nonlocal behavior of the interaction. In this paper, we have used the nonlocal Lagrangian in order to evaluate the possible effect of the nonlocal field in the CP violating kaon decay. The observable CP violating parameter is

$$\eta = \frac{\langle f | L^{NL} | K_2 \rangle}{\langle f | L^{NL} | K_1 \rangle} \tag{9}$$

where $|K_1\rangle$ and $|K_2\rangle$ are short and long-lived CP conserving kaon eigenstates and $|f\rangle$ is the final two pion state respectively. The Lagrangian L^{NL} in Eq. (9)

is an approximate nonlocal Lagrangian. Its lowest order term consists of the first-order derivative of the field involved in the interaction

$$L^{NL} = g \int d^4x \, \phi(x) \left(1 - a_\mu \partial^\mu\right) K(x) \tag{10}$$

where ϕ and K are pion and kaon field respectively. One notes that the nonlocal expression of the CP violating parameter (9) differs from that of the local expression in so far as the CP violating structure in the state function is transformed into the Lagrangian.[3] The explicit computation of the CP violating parameter (9) with nonlocal Lagrangian (10) leads to

$$\eta_{00} = \eta_{+-} \cong ia_0 \frac{m_K c}{\hbar} \tag{11}$$

What is most remarkable in this is that we found a formula in which the universal length a_0 is directly linked with observable quantities. Since a_0 and $\lambda\!\!\!^{-}_K = \hbar/m_K c$ are real numbers, the relation (11) becomes

$$|\eta| \sin \phi = a_0/\lambda\!\!\!^{-}_K \tag{12}$$

If we use the "world average value" of the old observed data[11]

$$|\eta_{+-}| \simeq |\eta_{00}| \simeq 2 \times 10^{-3} \tag{13}$$

and

$$\phi_{+-} \simeq \phi_{00} \simeq 43^\circ \quad .$$

The magnitude of the universal length becomes

$$a_0 = 0.52 \times 10^{-16} \text{ cm} \tag{14}$$

However, using the averaged value of the most recent observation on the CP violating parameters by several groups[12] as reported by Nygren[13] at the 1973 Berkeley meeting

$$|\eta_{+-}| \simeq |\eta_{00}| \simeq 2.3 \times 10^{-3} \tag{15}$$

and

$$\phi_{+-} \simeq \phi_{00} \simeq 50^{o}$$

the magnitude of the universal length becomes

$$a_0 = 0.69 \times 10^{-16} \text{ cm} \tag{16}$$

The question of what makes $a_0 = 0.69 \times 10^{-16}$ cm the universal length is, among others, discussed in detail elsewhere,[10] and can be answered indirectly as we compare this with a characteristic length obtained from different sources. From the general belief[14, 15] that a universal length plays an essential role in the weak interaction at small distances and from the universality of Fermi's weak interaction constant and its dimensionality, one may define a universal length

$$g = a_0^2 \hbar c \tag{17}$$

The observed lifetime of the weak decay particle and the V-A theory provide the magnitude of the constant g (including radiative corrections)

$$g = 14.3 \times 10^{-50} \text{ erg cm}^3 \tag{18}$$

Thus the universal length can be determined from Eq. (17)

$$a_0 = 0.67 \times 10^{-16} \text{ cm} \tag{19}$$

The agreement between the two values of Eq. (16) and Eq. (19) which were obtained from different sources is remarkable. It is interesting to see that the suggested test of the violation of the Lorentz invariance in the form of a lifetime shift can indeed be observed at $a_0 = 0.69 \times 10^{-16}$ cm. Would this suggest that there will be a new domain of physics[16] with the length scale of the order of 10^{-16} cm?

After completion of the manuscript, Dr. T. Pavlopoulos has pointed out the possible link between the characteristic length $G^{1/2} \simeq 0.6 \times 10^{-16}$ cm introduced in connection with a model of a composite structure of lepton by Greenberg and Yodh[17] and the universal length evaluated by the present author, 0.69×10^{-16} cm. More detailed discussion on this subject will be dealt with elsewhere.

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14. Heisenberg was the first one to point out that Fermi's weak decay theory could possibly be applied to the particle production (in connection with the possible mechanism of cosmic ray shower). If particles are to be produced in this formulation, the wavelength of the colliding particles must be smaller than a characteristic length (at that time, the length was speculated to be of the order of 10^{-13} cm, the classical electron radius). In other words, in this region, weak interaction can become strong interaction. See W. Heisenberg, Z. Physik 101, 533 (1936) and also W. Heisenberg, Ann. Physik 32, 20 (1938).

15. It has been shown that a possible modification of the space-time metrics in the domain of the order of 10^{-17} cm may lead to a violation of parity invariance. See V. G. Kadyshevskii, Soviet Physics Doklady 6, 36 (1961).

16. Beside Eq. (12) which provides the relation between the universal length and CP violation, there has been discussion on the possible role of the universal length (characteristic length) in the problem of the rising proton total cross section. As an explanation of the rising total cross section the authors suggested broken scaling which in turn could possibly be a consequence of the nonpoint particle properties of the parton within which the characteristic length (universal length) plays a role.
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Professor L. B. Redei informed me that preliminary calculations on the Michel parameter using the modified V-A Lagrangian are being done by Dr. M. Sekine (Tokyo Tech. University). A presence of a characteristic length of the order of 10^{-17} cm seems to be necessary in these calculations, too. Private communication.

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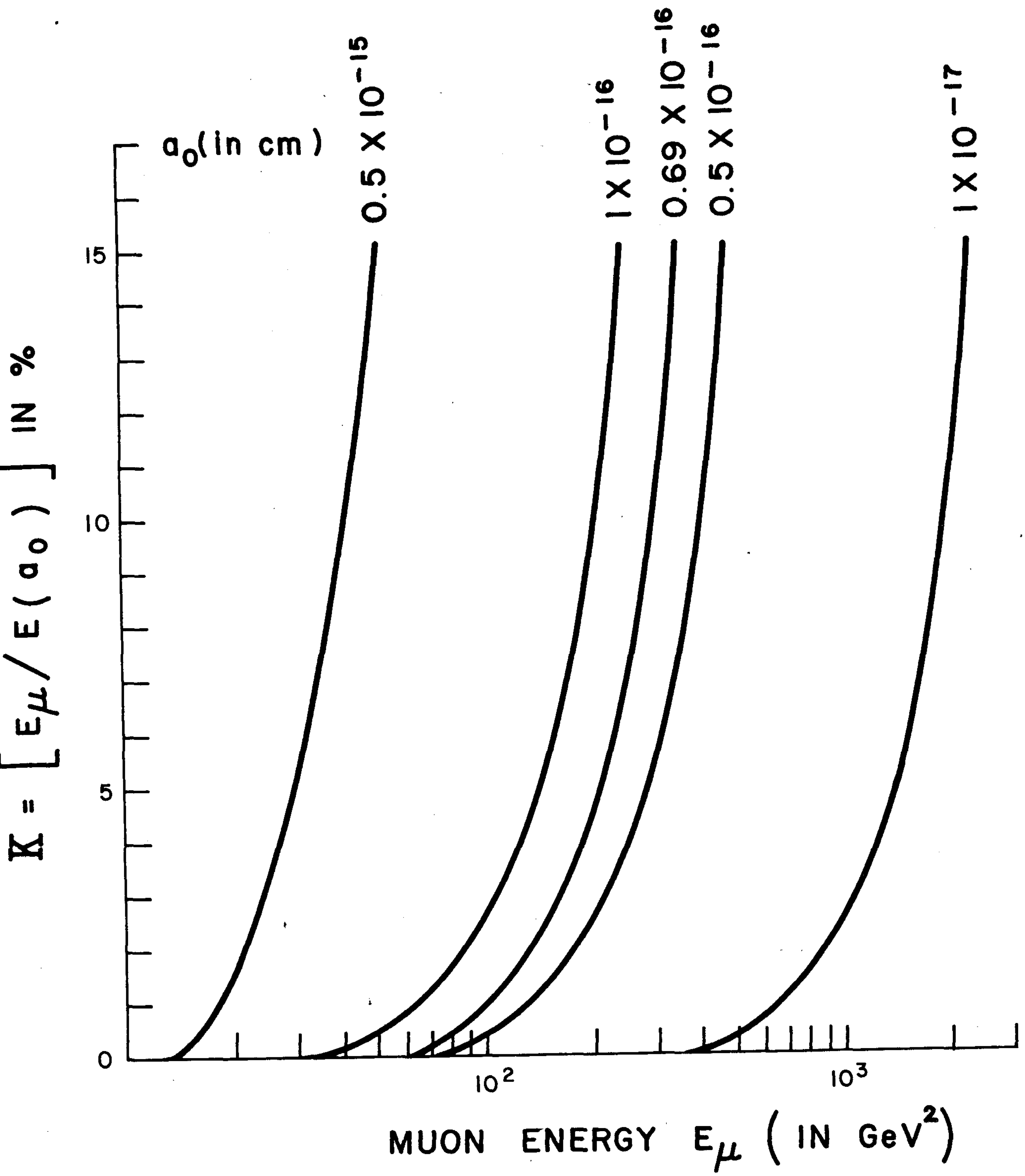
FIGURE CAPTION

The measure of the possible violation of the Lorentz invariance κ as a function of particle energy for several fixed parameters a_0, where

$$\kappa = \frac{E_\mu^2}{\left[3.23\,\lambda\!\!\!^{-}_\mu \frac{m_\mu c^2}{a_0}\ \mathrm{GeV}\right]^2} .$$



a_0(in cm)
0.5×10^{-15}
1×10^{-16}
0.69×10^{-16}
0.5×10^{-16}
1×10^{-17}
$K = \left[E_\mu / E(a_0) \right]^2$ IN %
0
5
10
15
10^2
10^3
MUON ENERGY E_μ (IN GeV^2)