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Universal properties of the two-dimensional Kuramoto-Sivashinsky equation

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Jayaprakash, Hayot, and Pandit Reply: We disagree completely with the Comment that L'vov and Procaccia [1] make on our Letter [2], so we explain the coarse-graining procedure and the renormalization-group (RG) arguments we use in the light of their Comment.

Our coarse-graining procedure is like a prefacing transformation which maps the Kuramoto-Sivashinsky (KS) equation onto the Kardar-Parisi-Zhang (KPZ) equation. This procedure yields effective values for the diffusion coefficient and the strengths of the nonlinear and noise terms in the KPZ equation. These effective values can, in general, depend on the cutoff Λ that we use in our coarse-graining procedure, but they do not, *indicating that our coarse-graining procedure is meaningful* (see Ref. [2] for details). Contrary to the assertion of L'vov and Procaccia [1], the Λ independence of the effective diffusion constant $\nu_\Lambda(k)$ *does not imply* that the dynamical exponent $z=2$. It merely implies that the diffusion term (in the effective KPZ equation) is of the form $\nu_B k^2$ on large enough length scales, with $\nu_B > 0$ the Λ -independent value of $\nu_\Lambda(k)$. Our coarse-graining procedure also yields the nonlinear and noise terms of the KPZ equation. The nontrivial value of z (≈ 1.6 in two dimensions) follows from the long-distance and long-time behavior of this KPZ equation [3]. We remind the reader that the same coarse-graining procedure [4] yields a Λ -independent value for $\nu_\Lambda(k)$ even in one dimension, where $z = \frac{3}{2}$ for the KPZ equation, and the identity of the long-distance and long-time behaviors of the KS and KPZ equations are well established by a number of authors [4] including L'vov *et al.* [5]. Thus the Λ independence of $\nu_\Lambda(k)$ does not imply that z is 2 contrary to their claim.

L'vov and Procaccia are not accurate in their discussion of our criticism of their earlier work [6,7]. We now address their comments for the cases $d=3$ and $d=2$ separately.

In $d=3$ we maintain that the nonlocal, strong-coupling solution of L'vov and Procaccia [7] exhibits the same infrared behavior as free-field theory (which describes a smooth interface) in that both two-point correlation and response functions are the same for both theories. The issue is the nature of the asymptotic behavior and not the way in which this behavior is reached. We agree that this behavior is reached after "renormalization" of the νk^2 term by small- k fluctuations arising from the nonlinear term; however, this does not alter the nature of the fixed point.

In $d=2$ we numerically coarse grain the KS equation to derive an effective KPZ equation; we do not just "allege" that this is the effective equation, we establish it. Our point of view is as follows: The small- k fluctuations renormalize the bare (negative) ν to a positive value, thus yielding a KPZ equation. On these length scales the equation is near the free-field fixed point and the nonlinear term is marginally relevant. Well-known results for the KPZ equation imply therefore that, under RG iterations, the system flows away from the free-field fixed point, first displaying logarithmic corrections and eventually flowing to the strong-coupling fixed point. We believe that the numerical results of Ref. [6] saw this region of logarithmic corrections. The crossover from free-field to strong-coupling behaviors is exponentially slow because of the marginality of the nonlinear term. Again the crossover does not connect two strong-coupling fixed points as stated by L'vov and Procaccia [1].

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