# Unravelling the anomalous gauge boson couplings in $Z W^{ \pm}$production at the LHC and the role of spin- 1 polarizations 

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Abstract: We study the anomalous triple gauge boson couplings (aTGC) in $Z W^{ \pm}$production in $3 l+E_{T}$ channel at the LHC for $\sqrt{s}=13 \mathrm{TeV}$. We use cross sections, azimuthal asymmetry, forward-backward asymmetry and polarization asymmetries of $Z$ and reconstructed $W$ to estimate simultaneous limits on the anomalous couplings for both effective vertex formalism as well as effective operator approach using Markov-Chain-Monte-Carlo (MCMC) method for luminosities $35.9 \mathrm{fb}^{-1}, 100 \mathrm{fb}^{-1}, 300 \mathrm{fb}^{-1}, 1000 \mathrm{fb}^{-1}$ and $3000 \mathrm{fb}^{-1}$. The trilepton invariant mass $\left(m_{3 l}\right)$ and the transverse momentum of $Z\left(p_{T}(Z)\right)$ are found to be sensitive to the aTGC for the cross sections as well as for the asymmetries. We observed that the asymmetries significantly improve the measurement of anomalous couplings at the high-luminosity LHC if a deviation from the Standard Model (SM) is observed.

Keywords: Beyond Standard Model, Effective Field Theories

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## 1 Introduction

After the discovery of Higgs [1, 2], the Large Hadron Collider (LHC) has been looking for new physics beyond the SM (BSM) needed to address many open questions such as neutrino oscillation, dark matter, baryogenesis, etc. with higher energies and higher luminosities. Unfortunately, no new physics has been found [3-7] except a few fluctuations (e.g., refs. [8-10]). One could expect that the new physics scale is too heavy to be directly explored by the LHC, and they may leave some footprints in the available energy range. They will modify the structure of the SM vertices or bring some new vertices, often through higher-dimensional operators with the SM fields. These new vertices and/or the extra contribution to the SM vertices are termed as anomalous in the sense that they are not present in the SM at leading order (LO). The electroweak sector will get affected by the anomalous bosonic self-couplings, which alter the paradigm of electroweak symmetry breaking (EWSB). To understand the EWSB mechanism, one needs precise measurements of the couplings in the bosonic sector of the SM. Here, we choose to focus on the charge sector by probing the $W W Z$ anomalous couplings in the $Z W^{ \pm}$production at the LHC. The $W W Z$ anomalous triple gauge boson couplings (aTGC) may be obtained by higher dimension effective operators made out of SM fields suppressed by a new physics scale $\Lambda$. The effective Lagrangian including the higher dimension effective operators $(\mathscr{O})$ to the SM Lagrangian $\left(\mathscr{L}_{\text {SM }}\right)$ is treated to be

$$
\begin{equation*}
\mathscr{L}_{\mathrm{eft}}=\mathscr{L}_{\mathrm{SM}}+\sum_{i} \frac{c_{i}^{\mathscr{O}(6)}}{\Lambda^{2}} \mathscr{O}_{i}^{(6)}+\sum_{i} \frac{c_{i}^{\mathscr{O}(8)}}{\Lambda^{4}} \mathscr{O}_{i}^{(8)}+\ldots, \tag{1.1}
\end{equation*}
$$

with $c_{i}^{\mathscr{O}(6,8)}$ being the couplings of the dimension- $(6,8)$ operators $\mathscr{O}_{i}^{(6,8)}$. The effective operators in the Hagiwara-Ishihara- Szalapski-Zeppenfeld (HISZ) basis up to dimension-6 contributing to $W W Z / \gamma$ couplings, in general, are [11, 12]

$$
\begin{align*}
\mathscr{O}_{W W W} & =\operatorname{Tr}\left[W_{\mu \nu} W^{\nu \rho} W_{\rho}^{\mu}\right] \\
\mathscr{O}_{W} & =\left(D_{\mu} \Phi\right)^{\dagger} W^{\mu \nu}\left(D_{\nu} \Phi\right) \\
\mathscr{O}_{B} & =\left(D_{\mu} \Phi\right)^{\dagger} B^{\mu \nu}\left(D_{\nu} \Phi\right) \\
\mathscr{O}_{\widetilde{W W W}} & =\operatorname{Tr}\left[\tilde{W}_{\mu \nu} W^{\nu \rho} W_{\rho}^{\mu}\right] \\
\mathscr{O}_{\widetilde{W}} & =\left(D_{\mu} \Phi\right)^{\dagger} \tilde{W}^{\mu \nu}\left(D_{\nu} \Phi\right) \tag{1.2}
\end{align*}
$$

Among these operators $\mathscr{O}_{W W W}, \mathscr{O}_{W}$ and $\mathscr{O}_{B}$ are $C P$-even, while $\mathscr{O}_{W W W}$ and $\mathscr{O}_{\widetilde{W}}$ are $C P$ odd. On the other hand, the $W W Z$ anomalous couplings may be parametrized in a model independent way with the most general Lorentz invariant form factors or vertex factors given by [13]

$$
\begin{align*}
\mathscr{L}_{W W Z}= & i g_{W W Z}\left[\left(1+\Delta g_{1}^{Z}\right)\left(W_{\mu \nu}^{+} W^{-\mu}-W^{+\mu} W_{\mu \nu}^{-}\right) Z^{\nu}+\frac{\lambda^{Z}}{m_{W}^{2}} W_{\mu}^{+\nu} W_{\nu}^{-\rho} Z_{\rho}^{\mu}\right. \\
& \left.+\frac{\widetilde{\lambda^{Z}}}{m_{W}^{2}} W_{\mu}^{+\nu} W_{\nu}^{-\rho} \widetilde{Z}_{\rho}^{\mu}+\left(1+\Delta \kappa^{Z}\right) W_{\mu}^{+} W_{\nu}^{-} Z^{\mu \nu}+\widetilde{\kappa^{Z}} W_{\mu}^{+} W_{\nu}^{-} \widetilde{Z}^{\mu \nu}\right] \tag{1.3}
\end{align*}
$$

where $W_{\mu \nu}^{ \pm}=\partial_{\mu} W_{\nu}^{ \pm}-\partial_{\nu} W_{\mu}^{ \pm}, Z_{\mu \nu}=\partial_{\mu} Z_{\nu}-\partial_{\nu} Z_{\mu}, \widetilde{Z}^{\mu \nu}=1 / 2 \epsilon^{\mu \nu \rho \sigma} Z_{\rho \sigma}$, and the overall coupling constants is given as $g_{W W Z}=-g \cos \theta_{W}, \theta_{W}$ being the weak mixing angle. The couplings $\Delta g_{1}^{Z}, \Delta \kappa^{Z}$ and $\lambda^{Z}$ of eq. (1.3) are $C P$-even, while $\widetilde{\kappa^{Z}}$ and $\widetilde{\lambda^{Z}}$ are $C P$-odd in nature. All the anomalous couplings vanish in the $S M$. In the $S U(2) \times U(1)$ gauge group, the coupling $\left(c_{i}^{\mathscr{L}}\right)$ of the Lagrangian in eq. (1.3) are related to the couplings $c_{i}^{\mathscr{O}}$ of the operators in eq. (1.2) as [11, 12, 14]

$$
\begin{align*}
\Delta g_{1}^{Z} & =c_{W} \frac{m_{Z}^{2}}{2 \Lambda^{2}} \\
\lambda^{Z} & =c_{W W W} \frac{3 g^{2} m_{W}^{2}}{2 \Lambda^{2}} \\
\widetilde{\lambda^{Z}} & =c_{\widetilde{W W W}} \frac{3 g^{2} m_{W}^{2}}{2 \Lambda^{2}} \\
\Delta \kappa^{Z} & =\left(c_{W}-c_{B} \tan ^{2} \theta_{W}\right) \frac{m_{W}^{2}}{2 \Lambda^{2}} \\
\widetilde{\kappa^{Z}} & =-c_{\widetilde{W}} \tan ^{2} \theta_{W} \frac{m_{W}^{2}}{2 \Lambda^{2}} \tag{1.4}
\end{align*}
$$

We label the anomalous couplings of three scenarios for later uses as follows: the couplings of the operators in eq. (1.2), the couplings of effective vertices in $\mathscr{L}_{W W V}$ in eq. (1.3) and the vertex couplings translated from the operators in eq. (1.4) are labelled as $c_{i}^{\mathscr{O}}, c_{i}^{\mathscr{L}}$, and $c_{i}^{\mathscr{L}_{g}}$, respectively.

| $c_{i}^{\varrho}$ | Limits $\left(\mathrm{TeV}^{-2}\right)$ | Remark |
| :--- | :--- | :--- |
| $\frac{c_{W W W}}{\Lambda^{2}}$ | $[-1.58,+1.59]$ | CMS $\sqrt{s}=13 \mathrm{TeV}, \mathscr{L}=35.9 \mathrm{fb}^{-1}, \mathrm{SU}(2) \times \mathrm{U}(1)[72]$ |
| $\frac{c_{W}}{\Lambda^{2}}$ | $[-2.00,+2.65]$ | CMS $[72]$ |
| $\frac{c_{B}}{\Lambda^{2}}$ | $[-8.78,+8.54]$ | CMS $[72]$ |
| $\frac{c_{W \widetilde{W W}} \Lambda^{2}}{\Lambda^{2}}$ | $[-11,+11]$ | ATLAS $\sqrt{s}=7(8) \mathrm{TeV}, \mathscr{L}=4.7(20.2) \mathrm{fb}^{-1}[61]$ |
| $\frac{c_{\widetilde{W}}^{\Lambda^{2}}}{}$ | $[-580,580]$ | ATLAS $[61]$ |
| $c_{i}^{\mathscr{L}_{g}}$ | Limits $\left(\times 10^{-2}\right)$ | Remark |
| $\lambda^{Z}$ | $[-0.65,+0.66]$ | CMS $[72]$ |
| $\Delta g_{1}^{Z}$ | $[-0.61,+0.74]$ | CMS $[72]$ |
| $\Delta \kappa^{Z}$ | $[-0.79,+0.82]$ | CMS $[72]$ |
| $\widetilde{\lambda^{Z}}$ | $[-4.7,+4.6]$ | ATLAS $[61]$ |
| $\widetilde{\kappa^{Z}}$ | $[-14,-1]$ | DELPHI $(\mathrm{LEP2}) \sqrt{s}=189-209 \mathrm{GeV}, \mathscr{L}=520 \mathrm{pb}^{-1}[55]$ |

Table 1. The list of tightest constraints observed on the effective operators and the effective vertices in $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge at $95 \%$ C.L. from experiments.

These anomalous gauge boson self-couplings may be obtained from some high scale new physics such as MSSM [15-17], extra dimension [18, 19], Georgi-Machacek model [20], etc. by integrating out the heavy degrees of freedom. Some of these couplings can also be obtained at loop level within the SM $[21,22]$. There have been a lot of studies to probe the anomalous $W W Z / \gamma$ couplings in the effective operator method as well as in the effective vertex factor approach subjected to $\mathrm{SU}(2) \times \mathrm{U}(1)$ invariance for various colliders: for $e^{+}$-$e^{-}$linear collider [13, 23-34], for the Large Hadron electron collider (LHeC) [35-37], $e-\gamma$ collider [38] and for the LHC [30, 31, 39-52]. Some $C P$-odd $W W V$ couplings have been studied in refs. [33, 48]. Direct measurement of these charged aTGC has been performed at the LEP [53-56], Tevatron [57, 58], LHC [59-74] and Tevatron-LHC [75]. The most stringent constraints on the operators $\left(c_{i}^{\mathscr{O}}\right)$ are obtained in ref. [72] for $C P$-even ones and in ref. [61] for $C P$-odd ones, and they are listed in table 1. These limits translated to the effective vertices $\left(c_{i}^{\mathscr{L}_{g}}\right)$ are also given in table 1.

In this article, we intend to study the $W W Z$ anomalous couplings in $Z W^{ \pm}$production at the LHC at $\sqrt{s}=13 \mathrm{TeV}$ using the cross sections, forward-backward asymmetries, and polarizations asymmetries [53, 76-81] of $Z$ and $W^{ \pm}$in the $3 l+E_{T}$ channel. The polarizations of $Z$ and $W$ have been used recently for various BSM studies [82-88] along with studies of anomalous gauge boson couplings [53, 80, 89, 90]. The polarizations of $W^{ \pm} / Z$ have been estimated earlier in $Z W^{ \pm}$production [91-93] and also have been measured recently at the LHC [94] in the SM. We note that the $Z W^{ \pm}$processes also contain anomalous couplings other than aTGC, such as the anomalous $Z q \bar{q}$ couplings, and they affect the measurement of aTGC $[28,46,52]$. However, the main aim of this paper is to demonstrate the usefulness of polarization observables in probing possible new physics. For simplicity, we restrict our analysis to possible anomalous couplings only in the bosonic sector of the SM.



Figure 1. Sample of Born level Feynman diagrams for $Z W^{+}$production in the $e^{+} e^{-} \mu^{+} \nu_{\mu}$ channel at the LHC. The diagrams for $Z W^{-}$can be obtained by charge conjugation. The shaded blob represents the presence of anomalous $W W V$ couplings on top of SM.

| Process | Obtained at | $\sigma_{\mathrm{LO}}(\mathrm{fb})$ | $\sigma_{\text {NLO }}(\mathrm{fb})$ | $\sigma_{\text {NNLO }}(\mathrm{fb})$ |
| :--- | :--- | :--- | :--- | :--- |
| $p p \rightarrow e^{+} e^{-} \mu^{+} \nu_{\mu}$ | MATRIX | $22.08_{-6.2 \%}^{+5.2 \%}$ | $43.95_{-4.3 \%}^{+5.4 \%}$ | $48.55_{-2.0 \%}^{+2.2 \%}$ |
|  | mg5_aMC | $22.02_{-7.2 \%}^{+6.1 \%}$ | $43.63_{-6.6 \%}^{+6.6 \%}$ | - |
| $p p \rightarrow e^{+} e^{-} \mu^{-} \bar{\nu}_{\mu}$ | MATRIX | $14.45_{-6.7 \%}^{+5.6 \%}$ | $30.04_{-4.5 \%}^{+5.6 \%}$ | $33.39_{-2.1 \%}^{+2.3 \%}$ |
|  | mg5_aMC | $14.38_{-7.6 \%}^{+6.4 \%}$ | $29.85_{-6.8 \%}^{+6.8 \%}$ | - |
| $p p \rightarrow 3 l+E_{T}$ | MATRIX [97] | $148.4_{-6.4 \%}^{+5.4 \%}$ | $301.4_{-4.4 \%}^{+5.1 \%}$ | $334.3_{-2.1 \%}^{+2.3 \%}$ |
| $p p \rightarrow 3 l+E_{T}$ | CMS [98] | $258.0 \pm 8.1 \%$ | $(\text { stat })_{-7.7 \%}^{+7: 4 \%}$ | (syst) $\pm 3.1$ (lumi) |

Table 2. The theoretical estimates and experimental measurements of the cross sections of $Z W^{ \pm}$ production in the $e^{+} e^{-} \mu^{ \pm} \nu_{\mu} / \bar{\nu}_{\mu}$ channels at $\sqrt{s}=13 \mathrm{TeV}$ at the LHC for CMS fiducial phase-space. The uncertainties in the theoretical estimates are due to scale variation.

We will begin in section 2 by providing the estimates of the cross sections for CMS fiducial phase-space by MATRIX [95], MADGraph5_aMC@NLO [96] and investigate their sensitivities to the anomalous couplings. Section 3 is devoted to polarization asymmetries of $Z$ and $W$ and the reconstruction of longitudinal momenta of the missing neutrino. In section 4, we perform a simultaneous analysis using MCMC to obtain limits on the anomalous couplings along with a toy measurement of non-zero aTGC and conclude in section 5 .

## 2 Signal cross sections and their sensitivity to anomalous couplings

The processes of interest are the $Z W^{ \pm}$production in the $3 l+E_{T}$ channel at the LHC. The representative Feynman diagrams at Born level are displayed in figure 1 containing doubly-resonant processes (upper-row) as well as singly-resonant processes (lower-row). The presence of anomalous $W W Z$ couplings is shown by the shaded blob. While this may


Figure 2. The differential distributions of $m_{3 l}$ (top-row) and $p_{T}(Z)$ (bottom-row) in the $Z W^{+}$ (left-column) and $Z W^{-}$(right-column) production in the $e^{+} e^{-} \mu^{ \pm}+E_{T}$ channel at the LHC for $\sqrt{s}=13 \mathrm{TeV}$ at LO and NLO in QCD obtained using MATRIX [95, 97, 99-104] for CMS fiducial phase-space.
contain the $W W \gamma$ couplings due to the off-shell $\gamma$, this has been cut out by $Z$ selection cuts, described later. The leading order result ( $\sigma_{\mathrm{LO}}^{\mathrm{th}}=148.4 \mathrm{fb}$ estimated by MATRIX in ref. [97]) for the $3 l+E_{T}$ cross section at the LHC is way below the measured cross section at the LHC ( $\sigma_{\text {exp }}^{\text {CMS }}=258 \mathrm{fb}$ measured by CMS [98]). Higher-order corrections to the tree level result are thus necessary. The next-to-leading order (NLO) corrections in QCD appear in the vertices connected to the quarks (see, figure 1) with either QCD loops or QCD radiations from the quarks. The SM cross sections of $Z W^{ \pm}$production in the $e^{+} e^{-} \mu^{ \pm}$ channel obtained by MATRIX and MadGraph5_aMC@NLO v2.6.4 (mg5_aMC) for $\sqrt{s}=13 \mathrm{TeV}$ for the CMS fiducial phase-phase region are presented in the table 2. The CMS fiducial


Figure 3. The differential distributions of $m_{3 l}$ and $p_{T}(Z)$ in the $W^{+} Z$ production in the $e^{+} e^{-} \mu^{+} \nu_{\mu}$ channel at the LHC at $\sqrt{s}=13 \mathrm{TeV}$ and $\mathscr{L}=35.9 \mathrm{fb}^{-1}$ at NLO in QCD for SM and five benchmark anomalous couplings.
phase-phase region [98] is given by

$$
\begin{array}{ccc}
p_{T}\left(l_{Z, 1}\right)>20 \mathrm{GeV}, & p_{T}\left(l_{Z, 2}\right)>10 \mathrm{GeV}, & p_{T}\left(l_{W}\right)>20 \mathrm{GeV} \\
\left|\eta_{l}\right|<2.5, & 60 \mathrm{GeV}<m_{l_{Z}^{+} l_{Z}}<120 \mathrm{GeV}, & m_{l^{+} l^{-}}>4 \mathrm{GeV}
\end{array}
$$

We use the values of the SM input parameters the same as used in ref. [97] (default in MATRIX). A fixed renormalization $\left(\mu_{R}\right)$ and factorization $\left(\mu_{F}\right)$ scale of $\mu_{R}=\mu_{F}=\mu_{0}=$ $\frac{1}{2}\left(m_{Z}+m_{W}\right)$ is used, and the uncertainties are estimated by varying the $\mu_{R}$ and $\mu_{F}$ in the range of $0.5 \mu_{0} \leq \mu_{R}, \mu_{F} \leq 2 \mu_{0}$, with the constraint $0.5 \leq \mu_{R} / \mu_{F} \leq 2$ and shown in table 2. We use the NNPDF3.0 [105] sets of parton distribution functions (PDFs) with $\alpha_{s}\left(m_{Z}\right)=0.118$ for our calculations. The combined result for all leptonic channels given in ref. [97] and the measured cross section by CMS [98] are also presented in the same table. The uncertainties in the theoretical estimates are due to scale variation. The result obtained by MATRIX and mg5_aMC matches quite well at both LO and NLO level. The NLO corrections have increased the LO cross section by up to $100 \%$ and the next-to-next-toleading order (NNLO) cross section is further increased by $10 \%$ from the NLO value. The higher order corrections to the cross section vary with kinematical variable like $m_{3 l}$ and $p_{T}(Z)$, as shown in figure 2 obtained by MATRIX [95, 97, 99-104]. The lower panels display the respective bin-by-bin ratios to the NLO central predictions. The NLO to LO ratio does not appear to be constant over the range of $m_{3 l}$ and $p_{T}(Z)$. Thus a simple $k$-factor with LO events can not be used as a proxy for NLO events. We use results from mg5_aMC, including NLO QCD corrections, for our analysis in the rest of the paper.

The signals for the $e^{+} e^{-} \mu^{+}$and $e^{+} e^{-} \mu^{-}$are generated separately using mg5_aMC at NLO in QCD for SM as well as SM including aTGC. We use the FeynRules [106] to generate QCD NLO UFO model of the Lagrangian in eq. (1.3) for mg5_aMC. These signals are then used as a proxy for the $3 l+E_{T}$ final state up to a factor of four for the four channels. For these, the $p_{T}$ cut for $e^{ \pm}$and $\mu^{ \pm}$are kept at the same value, i.e., $p_{T}(l)>10 \mathrm{GeV}$. We use a


Figure 4. The sensitivities of cross sections to the five benchmark aTGC as a function of the lower cut on $m_{3 l}$ and $p_{T}(Z)$ in the $Z W^{ \pm}$production at the LHC at $\sqrt{s}=13 \mathrm{TeV}$ and $\mathscr{L}=35.9 \mathrm{fb}^{-1}$.
threshold for the trilepton invariant mass $\left(m_{3 l}\right)$ of 100 GeV to select the doubly resonant contribution of trilepton final state. Later we will see that a cut of $m_{3 l} \geq 100 \mathrm{GeV}$ is required to improve the sensitivities of the observables to the anomalous couplings. The event selection cuts for this analysis are thus,

$$
\begin{array}{cc}
p_{T}(l)>10 \mathrm{GeV}, & \left|\eta_{l}\right|<2.5, \\
60 \mathrm{GeV}<m_{l_{Z}^{+} l_{Z}^{-}}<120 \mathrm{GeV}, & m_{l^{+} l^{-}}>4 \mathrm{GeV}, \tag{2.2}
\end{array} m_{3 l}>100 \mathrm{GeV} .
$$

We explore the effect of aTGC in the distributions of $m_{3 l}$ and $p_{T}(Z)$ in both $Z W^{+}$ and $Z W^{-}$production and show them in figure 3. The distribution of $m_{3 l}$ in the left-panel and $p_{T}(Z)$ in the right-panel in the $e^{+} e^{-} \mu^{+} \nu_{\mu}$ channel are shown for SM (filled/green) and five anomalous benchmark couplings ${ }^{1}$ of $\Delta g_{1}^{Z}=-0.02$ (solid/black), $\lambda^{Z}=+0.01$ (dashed/blue), $\Delta \kappa^{Z}=+0.2$ (dotted/red), $\widetilde{\lambda^{Z}}=+0.01$ (dash-dotted/orange) and $\widetilde{\kappa^{Z}}=$ +0.2 (dashed-dotdotted/magenta) with events normalised to an integrated luminosity of $\mathscr{L}=35.9 \mathrm{fb}^{-1}$. The higher $m_{3 l}$ and higher $p_{T}(Z)$ seem to have higher sensitivity to the

[^0]anomalous couplings which is due to higher momentum transfer at higher energies, for example see ref. [34]. We study the sensitivity of total cross section to the anomalous couplings by varying lower cut on $m_{3 l}$ and $p_{T}(Z)$ for the above mentioned five benchmark scenarios. The sensitivity of an observable $\mathscr{O}\left(c_{i}\right)$ to coupling $c_{i}$ is defined as
\[

$$
\begin{equation*}
\mathscr{S} \mathscr{O}\left(c_{i}\right)=\frac{\left|\mathscr{O}\left(c_{i}\right)-\mathscr{O}\left(c_{i}=0\right)\right|}{\delta \mathscr{O}}, \tag{2.3}
\end{equation*}
$$

\]

where $\delta \mathscr{O}$ is the estimated error in $\mathscr{O}$. For cross sections and asymmetries, the errors are

$$
\begin{equation*}
\delta \sigma=\sqrt{\frac{\sigma}{\mathscr{L}}+\left(\epsilon_{\sigma} \sigma\right)^{2}} \quad \text { and } \quad \delta A_{i}=\sqrt{\frac{1-A_{i}^{2}}{\mathscr{L} \times \sigma}+\epsilon_{A}^{2}}, \tag{2.4}
\end{equation*}
$$

where $\mathscr{L}$ is the integrated luminosity and $\epsilon_{\sigma}$ and $\epsilon_{A}$ are the systematic uncertainties for the cross section and the asymmetries, respectively. The sensitivities of the cross sections, ignoring the systematic uncertainty, for the five benchmark cases (as used in figure 3) are shown in figure 4 for $Z W^{+}$in the upper-row and for $Z W^{-}$in the lower-row as a function of lower cut of $m_{3 l}$ (left-column) and $p_{T}(Z)$ (right-column) for luminosity of $\mathscr{L}=35.9 \mathrm{fb}^{-1}$. It is clear that the sensitivities increase as the cut increases for both $m_{3 l}$ and $p_{T}(Z)$ for couplings $\Delta g_{1}^{Z}, \lambda^{Z}$ and $\widetilde{\lambda^{Z}}$, while they decrease just after $\sim 150 \mathrm{GeV}$ of cuts for the couplings $\Delta \kappa^{Z}$ and $\widetilde{\kappa^{Z}}$. This can also be seen in figure 3 where $\Delta \kappa^{Z}$ and $\widetilde{\kappa^{Z}}$ contribute more than other three couplings for $m_{3 l}<0.8 \mathrm{TeV}$ and $p_{T}(Z)<0.6 \mathrm{TeV}$. Taking hints from figure 4 , we identify four bins in $m_{3 l}-p_{T}(Z)$ plane to maximize the sensitivity of all the couplings. These four bins are given by,

$$
\begin{array}{lll}
\text { Bin } 11: & 400 \mathrm{GeV}<m_{3 l}<1500 \mathrm{GeV}, & 200 \mathrm{GeV}<p_{T}(Z)<1200 \mathrm{GeV}, \\
\text { Bin }_{12}: & 400 \mathrm{GeV}<m_{3 l}<1500 \mathrm{GeV}, & p_{T}(Z)>1200 \mathrm{GeV}, \\
\text { Bin }_{21}: & m_{3 l}>1500 \mathrm{GeV}, & 200 \mathrm{GeV}<p_{T}(Z)<1200 \mathrm{GeV}, \\
\text { Bin }_{22}: & m_{3 l}>1500 \mathrm{GeV}, & p_{T}(Z)>1200 \mathrm{GeV} .
\end{array}
$$

The sensitivities of the cross sections to the benchmark anomalous couplings are calculated in the said four bins for luminosity of $\mathscr{L}=35.9 \mathrm{fb}^{-1}$ and they are shown in table 3 in both $Z W^{+}$and $Z W^{-}$productions. As expected, we see that $\operatorname{Bin}_{22}$ has the higher sensitivity to couplings $\Delta g_{1}^{Z}, \lambda^{Z}$ and $\widetilde{\lambda^{Z}}$, while $B i n_{11}$ has higher, but comparable sensitivity to couplings $\Delta \kappa^{Z}$ and $\widetilde{\kappa^{Z}}$. The simultaneous cuts on both the variable have increased the sensitivity by a significant amount as compared to the individual cuts. For example, figure 4 shows that cross section in $Z W^{+}$has a maximum sensitivity of 15 and 22 on $\Delta g_{1}^{Z}=-0.02$ for individual $m_{3 l}$ and $p_{T}(Z)$ lower cuts, respectively. While imposing simultaneous lower cuts on both the variable, the same sensitivity increases to 44.5 (in $\operatorname{Bin}_{22}$ ).

At the LHC, the other contributions to the $3 l+E_{T}$ channel come from the production of $Z Z, Z \gamma, Z+j, t \bar{t}, W t, W W+j, t \bar{t}+V, t Z, V V V$ as has been studied by CMS [74, 98] and ATLAS $[94,107]$. The total non- $Z W$ contributions listed above is about $40 \%$ of the $Z W$ contributions [98]. We include these extra contributions to the cross sections while estimating limits on the anomalous couplings in section 4.

|  | $Z W^{+}$ |  |  |  |  | $Z W^{-}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| aTGC | Bin $_{11}$ | Bin $_{12}$ | Bin $_{21}$ | Bin $_{22}$ | Bin $_{11}$ | Bin $_{12}$ | Bin $_{21}$ | Bin $_{22}$ |  |
| $\Delta g_{1}^{Z}=-0.02$ | 1.17 | 1.14 | 7.52 | 44.5 | 0.32 | 2.10 | 3.95 | 23.19 |  |
| $\lambda^{Z}=0.01$ | 3.08 | 5.37 | 6.08 | 26.2 | 1.58 | 2.63 | 3.32 | 13.68 |  |
| $\Delta \kappa^{Z}=0.2$ | 8.52 | 0.50 | 3.28 | 4.87 | 5.01 | 0.15 | 1.64 | 2.40 |  |
| $\widetilde{\lambda^{Z}}=0.01$ | 3.20 | 5.56 | 6.18 | 27.2 | 1.70 | 2.69 | 3.37 | 13.83 |  |
| $\widetilde{\kappa^{Z}}=0.2$ | 6.50 | 0.60 | 3.15 | 4.89 | 3.86 | 0.22 | 1.65 | 2.36 |  |

Table 3. The sensitivities of the cross sections on the five benchmark aTGC in the four bins (see eq. (2.5)) of $m_{3 l}$ and $p_{T}(Z)$ in the $Z W^{ \pm}$productions at the LHC at $\sqrt{s}=13 \mathrm{TeV}$ and $\mathscr{L}=35 \mathrm{fb}^{-1}$.

## 3 Polarization observables of $Z$ and $W^{ \pm}$along with other angular asymmetries

Being a spin-1 particle, the $Z / W(V)$ offers eight additional observables related to their eight degrees of polarizations apart from their production cross sections. The angular distributions of the daughter particle reveal the polarizations of the mother particle $V$. The normalised decay angular distribution of the daughter fermion $f\left(l_{Z} / l_{W}\right)$ from the decay of $V$ is given by [78]

$$
\begin{aligned}
\frac{1}{\sigma} \frac{d \sigma}{d \Omega_{f}}= & \frac{3}{8 \pi}\left[\left(\frac{2}{3}-(1-3 \delta) \frac{T_{z z}}{\sqrt{6}}\right)+\alpha p_{z} \cos \theta_{f}+\sqrt{\frac{3}{2}}(1-3 \delta) T_{z z} \cos ^{2} \theta_{f}\right. \\
& +\left(\alpha p_{x}+2 \sqrt{\frac{2}{3}}(1-3 \delta) T_{x z} \cos \theta_{f}\right) \sin \theta_{f} \cos \phi_{f} \\
& +\left(\alpha p_{y}+2 \sqrt{\frac{2}{3}}(1-3 \delta) T_{y z} \cos \theta_{f}\right) \sin \theta_{f} \sin \phi_{f} \\
& +(1-3 \delta)\left(\frac{T_{x x}-T_{y y}}{\sqrt{6}}\right) \sin ^{2} \theta_{f} \cos \left(2 \phi_{f}\right) \\
& \left.+\sqrt{\frac{2}{3}}(1-3 \delta) T_{x y} \sin ^{2} \theta_{f} \sin \left(2 \phi_{f}\right)\right]
\end{aligned}
$$

Here $\theta_{f}, \phi_{f}$ are the polar and the azimuthal orientation of the fermion $f$, in the rest frame of the particle $(V)$ with its would be momentum along the $z$-direction. For massless final state fermions, we have $\delta=0$ and $\alpha=\left(R_{f}^{2}-L_{f}^{2}\right) /\left(R_{f}^{2}+L_{f}^{2}\right)$ for $Z$ with $Z f \bar{f}$ coupling to be $\gamma^{\mu}\left(L_{f} P_{L}+R_{f} P_{R}\right)$ and $\alpha=-1$ for $W^{ \pm}$. The quantities $p_{x}, p_{y}$, and $p_{z}$ are the three vector polarizations and $T_{x y}, T_{x z}, T_{y z}, T_{x x}-T_{y y}$, and $T_{z z}$ are the five independent tensor polarizations of the particle $V$. These polarizations $p_{i}$ and $T_{i j}$ are calculable from asymmetries constructed from the decay angular information of lepton using eq. (3.1). For example, the polarization parameters $p_{z}$ and $T_{x z}$ can be calculated from the asymmetries $A_{z}$ and $A_{x z}$, respectively, as

$$
\begin{aligned}
A_{z} & =\frac{1}{\sigma}\left[\int_{0}^{\frac{\pi}{2}} \frac{d \sigma}{d \theta_{f}} d \theta_{f}-\int_{\frac{\pi}{2}}^{\pi} \frac{d \sigma}{d \theta_{f}} d \theta_{f}\right] \equiv \frac{\sigma\left(\cos \theta_{f}>0\right)-\sigma\left(\cos \theta_{f}<0\right)}{\sigma\left(\cos \theta_{f}>0\right)+\sigma\left(\cos \theta_{f}<0\right)} \\
& =\frac{3 \alpha p_{z}}{4}
\end{aligned}
$$

$$
\begin{align*}
A_{x z}= & \frac{1}{\sigma}\left[\left(\int_{\theta=0}^{\frac{\pi}{2}} \int_{\phi=-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d \sigma}{d \Omega_{f}} d \Omega_{f}+\int_{\theta=\frac{\pi}{2}}^{\pi} \int_{\phi=\frac{\pi}{2}}^{\frac{3 \pi}{2}} \frac{d \sigma}{d \Omega_{f}} d \Omega_{f}\right)\right. \\
& \left.\quad-\left(\int_{\theta=0}^{\frac{\pi}{2}} \int_{\phi=\frac{\pi}{2}}^{\frac{3 \pi}{2}} \frac{d \sigma}{d \Omega_{f}} d \Omega_{f}+\int_{\theta=\frac{\pi}{2}}^{\pi} \int_{\phi=-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d \sigma}{d \Omega_{f}} d \Omega_{f}\right)\right] \\
\equiv & \frac{\sigma\left(\cos \theta_{f} \cos \phi_{f}>0\right)-\sigma\left(\cos \theta_{f} \cos \phi_{f}<0\right)}{\sigma\left(\cos \theta_{f} \cos \phi_{f}>0\right)+\sigma\left(\cos \theta_{f} \cos \phi_{f}<0\right)} \\
= & \frac{2}{\pi} \sqrt{\frac{2}{3}}(1-3 \delta) T_{x z} . \tag{3.2}
\end{align*}
$$

Similarly one can construct asymmetries corresponding to each of the other polarizations $p_{i}$ and $T_{i j}$, see ref. [80] for details.

The $Z$ and the $W^{ \pm}$bosons produced in the $Z W^{ \pm}$production are not forward-backward symmetric owing to only a $t$-channel diagram and not having an $u$-channel diagram (see figure 1). These provide an extra observable, the forward-backward asymmetry defined as

$$
\begin{equation*}
A_{\mathrm{fb}}^{V}=\frac{\sigma\left(\cos \theta_{V}>0\right)-\sigma\left(\cos \theta_{V}<0\right)}{\sigma\left(\cos \theta_{V}>0\right)+\sigma\left(\cos \theta_{V}<0\right)} \tag{3.3}
\end{equation*}
$$

$\theta_{V}$ is the production angle of the $V$ w.r.t. the colliding quark-direction. One more angular variable sensitive to aTGC is the angular separation of the lepton $l_{W}$ from $W^{ \pm}$and the $Z$ in the transverse plane, i.e,

$$
\begin{equation*}
\Delta \phi\left(l_{W}, Z\right)=\cos ^{-1}\left(\frac{\vec{p}_{T}\left(l_{W}\right) \cdot \vec{p}_{T}(Z)}{p_{T}\left(l_{W}\right) p_{T}(Z)}\right) \tag{3.4}
\end{equation*}
$$

One can construct an asymmetry based on the $\Delta \phi$ as,

$$
\begin{equation*}
A_{\Delta \phi}=\frac{\sigma\left(\cos \left(\Delta \phi\left(l_{W}, Z\right)\right)>0\right)-\sigma\left(\cos \left(\Delta \phi\left(l_{W}, Z\right)\right)<0\right)}{\sigma\left(\cos \left(\Delta \phi\left(l_{W}, Z\right)\right)>0\right)+\sigma\left(\cos \left(\Delta \phi\left(l_{W}, Z\right)\right)<0\right)} . \tag{3.5}
\end{equation*}
$$

The sensitivities of $A_{\Delta \phi}$ to the five benchmark aTGC are shown in figure 5 as a function of lower cuts on $p_{T}(Z)$ in both $Z W^{ \pm}$for luminosity of $\mathscr{L}=35.9 \mathrm{fb}^{-1}$. A choice of $p_{T}(Z)^{\text {low }}=$ 300 GeV appears to be an optimal choice for sensitivity for all the couplings. The $m_{3 l}$ cut, however, reduces the sensitivities to all the aTGC.

To construct the asymmetries, we need to set a reference frame and assign the leptons to the correct mother spin-1 particle. For the present process with missing neutrino, we face a set of challenges in constructing the asymmetries. These are discussed below.

Selecting $Z$ candidate leptons. The $Z$ boson momenta is required to be reconstructed to obtain all the asymmetries which require the right pairing of the $Z$ boson leptons $l_{Z}^{+}$and $l_{Z}^{-}$. Although the opposite flavour channels $e^{+} e^{-} \mu^{ \pm} / \mu^{+} \mu^{-} e^{ \pm}$are safe, the same flavour channels $e^{+} e^{-} e^{ \pm} / \mu^{+} \mu^{-} \mu^{ \pm}$suffer ambiguity to select the right $Z$ boson candidate leptons. The right paring of leptons for the $Z$ boson in the same flavoured channel is possible with $\geq 96.5 \%$ accuracy for $m_{3 l}>100 \mathrm{GeV}$ and $\geq 99.8 \%$ accuracy for $m_{3 l}>550 \mathrm{GeV}$ in both SM and benchmark aTGC by requiring a smaller value of $\left|m_{Z}-m_{l^{+} l^{-}}\right|$. This small miss pairing is neglected to use the $2 e \mu \nu_{\mu}$ channel as a proxy for a $3 l+E_{T}$ final state with good enough accuracy.


Figure 5. The sensitivity of the asymmetry $A_{\Delta \phi}$ on the five benchmark aTGC as a function of the lower cut on $p_{T}(Z)$ in the $Z W^{ \pm}$production at the LHC at $\sqrt{s}=13 \mathrm{TeV}$ and $\mathscr{L}=35.9 \mathrm{fb}^{-1}$. The legend labels are same as in figure 4.

The reconstruction of neutrino momentum. The other major issue is to obtain the asymmetries related to $W^{ \pm}$bosons, which require to reconstruct their momenta. As the neutrino from $W^{ \pm}$goes missing, reconstruction of $W^{ \pm}$boson momenta is possible with a two-fold ambiguity using the transverse missing energy $D_{T} / E_{T}$ and the on-shell $W$ mass $\left(m_{W}\right)$ constrain. The two solutions for the longitudinal momentum of the missing neutrino are given by

$$
\begin{equation*}
p_{z}(\nu)_{ \pm}=\frac{-\beta p_{z}\left(l_{W}\right) \pm E\left(l_{W}\right) \sqrt{D}}{p_{T}^{2}\left(l_{W}\right)} \tag{3.6}
\end{equation*}
$$

with

$$
\begin{equation*}
D=\beta^{2}-p_{T}^{2}(\nu) p_{T}^{2}\left(l_{W}\right), \quad \beta=m_{W}^{2}+p_{x}\left(l_{W}\right) p_{x}(\nu)+p_{y}\left(l_{W}\right) p_{y}(\nu) . \tag{3.7}
\end{equation*}
$$

Because the $W$ is not produced on-shell all the time, among the two solutions of neutrino longitudinal momenta, one of them will be closer to the true value, and another will be far from the true value. There are no suitable selector or discriminator to select the correct solution from the two solutions. Even if we substitute the Monte-Carlo truth $m_{W}$ to solve for $p_{z}(\nu)$, we don't have any discriminator to distinguish between the two solutions $p_{z}(\nu)_{ \pm}$. The smaller value of $\left|p_{z}(\nu)\right|$ corresponds to the correct solution only for $\approx 65 \%$ times on average in $Z W^{+}$and little lower in $Z W^{-}$production. One more discriminator, which is $\| \beta_{Z}\left|-\left|\beta_{W}\right|\right|$, the smaller value of this can choose the correct solution a little over the boundary, i.e., $\approx 55 \%$. We have tried machine-learning approaches (artificial neural network) to select the correct solutions, but the accuracy was not better than $65 \%$. In some cases, we have $D<0$ with the on-shell $W$. For these cases, either one can throw those events (which affects the distribution and statistics), or one can vary the $m_{W}$ from its central value to have $D>0$. Here, we follow the latter. So, as the best available option, we choose the smaller value of $\left|p_{z}(\nu)\right|$ to be the correct solution to reconstruct the $W$ boson momenta. At this point, it becomes important to explore the effect of reconstruction on asymmetries and their sensitivities to aTGC. To this end, we consider three scenarios:


Figure 6. The sensitivity of some polarization asymmetries of $W^{+}\left(Z W^{+}\right)$on some benchmark aTGC for three scenarios: with absolute truth (Abs. True) information of neutrino in solid/blue lines, with the close to true reconstructed solution of neutrino (Reco. True) in dotted/red lines and with the smaller $\left|p_{z}(\nu)\right|$ to be the true solution ( $\left.\operatorname{Small}\left|\mathbf{p}_{\mathbf{z}}(\nu)\right|\right)$ in dash-dotted/blue lines as a function of the lower cut on $p_{T}(Z)$ (top-left-panel) and $m_{3 l}$ (top-right-panel) at $\sqrt{s}=13 \mathrm{TeV}$ and $\mathscr{L}=100 \mathrm{fb}^{-1}$. The scatter plot of the total $\chi^{2}$ for about 100 aTGC points using all the asymmetries of $W^{ \pm}$for Reco. True in $x$-axis with Small $\left|\mathbf{p}_{\mathbf{z}}(\nu)\right|$ in $y$-axis is shown in the bottom-panel.

Abs. True: the first thing is to use the Monte-Carlo truth events and estimate the asymmetries in the lab frame. The observables in this scenario are directly related to the dynamics up to a rotation of frame [76, 108, 109].

Reco. True: using the pole mass of $W$ in eq. (3.7) and choosing the solution closer to the Monte-Carlo true value is the best that one can do in reconstruction. The goal of any reconstruction algorithm would be to become as close to this scenario as possible.

Small $\left|\boldsymbol{p}_{\boldsymbol{z}}(\boldsymbol{\nu})\right|$ : this choice is the best available realistic algorithm which we will be using for the analysis.

The values of reconstructed asymmetries and hence polarizations get shifted from Abs. True case. In the case of Reco. True, the shifts are roughly constant, while in the case of Small $\left|\mathbf{p}_{\mathbf{z}}(\nu)\right|$, the shifts are not constant over varying lower cuts on $m_{3 l}$ and $p_{T}(Z)$ due to the $35 \%$ wrong choice. It is, thus, expected that the reconstructed sensitivities
to aTGC remain the same in Reco. True and change in the Small $\left|\mathbf{p}_{\mathbf{z}}(\nu)\right|$ case when compared to the Abs. True case. In the Small $\left|\mathbf{p}_{\mathbf{z}}(\nu)\right|$ reconstruction case, sensitivities of some asymmetries to aTGC are less than that of the Abs. True case, while they are higher for some other asymmetries. This is illustrated in figure 6 (top-row) comparing the sensitivity of some polarization asymmetries of $W^{+}$, e.g., $A_{y}$ to $\kappa^{Z}=+0.2$ in cross ( $\times$ ) points, $A_{z}$ to $\Delta g_{1}^{Z}=-0.02$ in square ( $\left(\right.$ ) points, and $A_{z z}$ to $\Delta \kappa^{Z}=+0.2$ in circular ( $\odot$ ) points for the three scenarios of Abs. True (solid/blue line), Reco. True (dotted/red) and Small $\left|\mathbf{p}_{\mathbf{z}}(\nu)\right|$ (dash-dotted/blue) for varying lower cuts on $p_{T}(Z)$ and $m_{3 l}$ in $Z W^{+}$ production with a luminosity of $\mathscr{L}=100 \mathrm{fb}^{-1}$. The sensitivities are roughly the same for Abs. True and Reco. True reconstruction in all the asymmetries for both $p_{T}(Z)$ and $m_{3 l}$ cuts. In the Small $\left|\mathbf{p}_{\mathbf{z}}(\nu)\right|$ reconstruction case, sensitivity is smaller for $A_{z z}$, higher for $A_{y}$, and it depends on cut for $A_{z}$ when compared to the Abs. True case. When all the $W$ asymmetries are combined, the total $\chi^{2}$ is higher in the $\operatorname{Small}\left|\mathbf{p}_{\mathbf{z}}(\nu)\right|$ case compared to the Reco. True case for about 100 chosen benchmark points, see figure 6 (bottom-panel). Here, a total $\chi^{2}$ of all the asymmetries of $W\left(A_{i}^{W}\right)$ for a set of benchmark points $\left(\left\{c_{i}\right\}\right)$ is given by

$$
\begin{equation*}
\chi^{2}\left(A_{i}^{W}\right)\left(\left\{c_{i}\right\}\right)=\sum_{j}^{N=9}\left(\mathscr{S} A_{j}^{W}\left(\left\{c_{i}\right\}\right)\right)^{2} . \tag{3.8}
\end{equation*}
$$

The said increment of $\chi^{2}$ is observed in both $W^{+} Z\left(\odot /\right.$ blue) and $W^{-} Z(\odot / \mathrm{red})$ production processes. So even if we are not able to reconstruct the $W$ and hence its polarization observables correctly, realistic effects end up enhancing the overall sensitivity of the observables to the aTGC.

Reference $\boldsymbol{z}$-axis for polarizations. The other challenge to obtain the polarization of $V$ is that one needs a reference axis ( $z$-axis) to get the momentum direction of $V$, which is not possible at the LHC as it is a symmetric collider. Thus, for the asymmetries related to $Z$ boson, we consider the direction of total visible longitudinal momenta as an unambiguous choice for positive $z$-axis. For the case of $W$, the direction of the reconstructed boost is used as a proxy for the positive $z$-axis. The latter choice is inspired by the fact that in $q^{\prime} \bar{q}$ fusion the quark is supposed to have larger momentum than the anti-quark at the LHC, thus the above proxy could stand statistically for the direction of the quark direction.

List of observables. The set of observables used in this analysis are,
$\sigma_{i}$ : the cross sections in four bins (4),
$A_{\mathrm{pol}}^{Z}$ : eight polarization asymmetries of $Z(8)$,
$A_{\mathrm{fb}}^{Z}$ : forward-backward asymmetry of $Z(1)$,
$A_{\Delta \phi}$ : azimuthal asymmetry (1),
$A_{\mathrm{pol}}^{W}$ : eight polarization asymmetries of reconstructed $W$ (8),
$A_{\mathrm{fb}}^{W}$ : forward-backward asymmetry of reconstructed $W(1),{ }^{2}$

[^1]which make a total of $N(\mathscr{O})=(4+8+1+1+8+1) \times 2=46$ observables including both processes. All the asymmetry from $Z$ side and all the asymmetries from $W$ side are termed as $A_{i}^{Z}$ and $A_{i}^{W}$, respectively, for the latter uses. The total $\chi^{2}$ for all observables would be the quadratic sum of sensitivities (eq. (2.3)) given by
\[

$$
\begin{equation*}
\chi_{\mathrm{tot}}^{2}\left(c_{i}\right)=\sum_{j}^{N=46}\left(\mathscr{S} \mathscr{O}_{j}\left(c_{i}\right)\right)^{2} . \tag{3.9}
\end{equation*}
$$

\]

We use these set of observables in some chosen kinematical region to obtain limits on aTGC in the next section.

## 4 Measurement of the anomalous couplings

We study the sensitivity of all the $(N(\mathscr{O})=46)$ observables for varying lower cuts on $m_{3 l}$ and $p_{T}(Z)$ separately as well as simultaneously (grid scan in step of 50 GeV in each direction) for the chosen benchmark anomalous couplings. The maximum sensitivities are observed for simultaneous lower cuts on $m_{3 l}$ and $p_{T}(Z)$ given in table 4 for all the asymmetries in both $Z W^{ \pm}$processes. Some of these cuts can be realised from figure $5 \& 6$. The SM values of the asymmetries of $Z$ and $W$ and their corresponding polarizations for the selection cuts (sel.cut in eq. (2.2)) and for the optimized cuts (opt.cut in table 4) are listed in table 6 in appendix B for completeness. We use the cross sections in the four bins and all the asymmetries with the optimized cuts to obtain limits on the anomalous couplings for both effective vertices and effective operators. We use the semi-analytical expressions for the observables fitted with the simulated data from mg5_aMC. The details of the fitting procedures are described in appendix A . The uncertainty on the cross sections and asymmetries are taken as $\epsilon_{\sigma}=20 \%$ and $\epsilon_{A}=2 \%$, respectively consistent with the analysis by CMS [98] and ATLAS [94]. We note that these uncertainties are not considered in the previous sections for qualitative analysis and optimization of cuts.

The sensitivities of all the observables to the aTGC are studied by varying oneparameter, two-parameter, and all-parameter at a time in the optimized cut region. We look at the $\chi^{2}=4$ contours in the $\Delta \kappa^{Z}-\kappa^{Z}$ plane for a luminosity of $\mathscr{L}=100 \mathrm{fb}^{-1}$ for various combinations of asymmetries and cross sections and show them in figure 7. We observe that the $Z$-asymmetries $\left(A_{i}^{Z}\right)$ are weaker than the $W$-asymmetries $\left(A_{i}^{W}\right) ; A_{i}^{W}$ provides very symmetric limits, while $A_{i}^{Z}$ has a sense of directionality. The $A_{\Delta \phi}$ is better than both $A_{i}^{Z}$ and $A_{i}^{W}$ in most of the directions in $\Delta \kappa^{Z}-\widetilde{\kappa^{Z}}$ plane. After combining $A_{i}^{Z}, A_{i}^{W}$ and $A_{\Delta \phi}$, we get a tighter contours; but the shape is dictated by $A_{\Delta \phi}$. We see (figure 7 right-panel) that the cross sections have higher sensitivity compared to the asymmetries to the aTGC. The cross sections dominate constraining the couplings, while the contribution from the asymmetries remain sub-dominant at best. Although the directional constraints provided by the asymmetries get washed away when combined with the cross sections, they are expected to remain prominent to extract non-zero couplings should a deviation from the SM be observed. This possibility is discussed in subsection 4.2.

| $\mathscr{O}$ | $Z$ in $Z W^{+}$ | $Z$ in $Z W^{-}$ | $W^{ \pm}$in $Z W^{ \pm}$ |
| :---: | :---: | :---: | :---: |
| $A_{x}$ | $(200,100)$ | $(100,150)$ | $(250,0)$ |
| $A_{y}$ | $(150,100)$ | $(100,100)$ | $"$ |
| $A_{z}$ | $(550,50)$ | $(100,250)$ | $"$ |
| $A_{x y}$ | $(150,100)$ | $(150,100)$ | $"$ |
| $A_{x z}$ | $(150,0)$ | $(200,50)$ | $"$ |
| $A_{y z}$ | $(100,50)$ | $(100,0)$ | $"$ |
| $A_{x^{2}-y^{2}}$ | $(400,150)$ | $(300,100)$ | $"$ |
| $A_{z z}$ | $(550,0)$ | $(300,400)$ | $"$ |
| $A_{\mathrm{fb}}$ | $(300,0)$ | $(550,0)$ | $"$ |
| $Z W^{+}$ |  |  |  |
| $A_{\Delta \phi}$ | $(100,300)$ |  | $Z W^{-}$ |

Table 4. The list of optimized lower cuts (opt.cut) in GeV on $\left(m_{3 l}, p_{T}(Z)\right)$ for various asymmetries to maximize their sensitivity to the anomalous couplings.

$$
\begin{array}{rrr}
A_{i}^{Z} & - & A_{\Delta \phi}-\cdots \\
A_{i}^{W} & -\cdots & A_{\Delta \phi}+A_{i}^{Z}
\end{array}
$$

$$
A_{\Delta \phi}+A_{i}^{Z}+A_{i}^{W}-\quad \sigma_{i}+A_{\Delta \phi}+A_{i}^{Z}
$$

$$
\sigma_{i} \cdots \cdots \quad \sigma_{i}+A_{\Delta \phi}+A_{i}^{Z}+A_{i}^{W}-
$$



Figure 7. The $\chi^{2}=4$ contours are shown in the $\Delta \kappa^{Z}-\widetilde{\kappa^{Z}}$ plane with different asymmetries and their combinations in the left-panel, various combinations of the cross sections and asymmetries in the right-panel for $\sqrt{s}=13 \mathrm{TeV}$ and $\mathscr{L}=100 \mathrm{fb}^{-1}$. The contour for $A_{\Delta \phi}+A_{i}^{Z}+A_{i}^{W}$ (thicksolid/black line ) is repeated in both panel for comparison.

### 4.1 Limits on the couplings

We extract simultaneous limits on all the anomalous couplings using all the observables using MCMC method. We perform this analysis in two ways: $(i)$ vary effective vertex factors couplings $\left(c_{i}^{\mathscr{L}}\right)$ and (ii) vary effective operators couplings $\left(c_{i}^{\mathscr{O}}\right)$ and translate them in to effective vertex factors couplings $\left(c_{i}^{\mathscr{L}_{g}}\right)$ using eq. (1.4). The $95 \%$ BCI (Bayesian confidence interval) obtained on aTGC are listed in table 5 for five choices of integrated luminosi-

| $c_{i}^{\mathscr{L}}\left(10^{-3}\right)$ | $35.9 \mathrm{fb}^{-1}$ | $100 \mathrm{fb}^{-1}$ | $300 \mathrm{fb}^{-1}$ | $1000 \mathrm{fb}^{-1}$ | $3000 \mathrm{fb}^{-1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta g_{1}^{Z}$ | $\begin{array}{r} +2.15 \\ { }_{-4.20} \end{array}$ | $\begin{array}{r} +1.50 \\ +3.47 \end{array}$ | $\begin{aligned} & +0.963 \\ & -2.92 \end{aligned}$ | $\begin{aligned} & +0.565 \\ & -2.48 \end{aligned}$ | $\begin{aligned} & +0.318 \\ & { }_{-2.17} \end{aligned}$ |
| $\lambda^{Z}$ | $\begin{aligned} & +2.11 \\ & -2.24 \end{aligned}$ | $\begin{array}{r} +1.66 \\ -1.78 \\ \hline \end{array}$ | $\begin{array}{r} +1.30 \\ -1.42 \\ \hline \end{array}$ | $\begin{array}{r} +1.01 \\ -1.14 \end{array}$ | $\begin{aligned} & +0.811 \\ & -0.931 \end{aligned}$ |
| $\Delta \kappa^{Z}$ | $\begin{aligned} & +83.5 \\ & { }_{-83.0} \end{aligned}$ | $\begin{aligned} & +66.6 \\ & { }_{-64.1} \end{aligned}$ | $\begin{array}{r} +52.8 \\ +47.9 \\ \hline \end{array}$ | $\begin{array}{r} +42.1 \\ -34.2 \end{array}$ | $\begin{array}{r} +36.0 \\ -27.2 \end{array}$ |
| $\widetilde{\lambda^{Z}}$ | $\begin{array}{r} +2.19 \\ -2.19 \end{array}$ | $\begin{array}{r} +1.72 \\ -1.74 \end{array}$ | $\begin{aligned} & +1.36 \\ & { }_{-1.38} \end{aligned}$ | $\begin{array}{r} +1.09 \\ -1.09 \end{array}$ | $\begin{aligned} & +0.883 \\ & { }_{-0.884}^{+} \end{aligned}$ |
| $\widetilde{\kappa^{Z}}$ | $\begin{aligned} & +86.2 \\ & -88.4 \end{aligned}$ | $\begin{array}{r} +67.5 \\ { }_{-70.4} \end{array}$ | $\begin{array}{r} +51.8 \\ { }_{-54.9} \end{array}$ | $\begin{array}{r} +40.1 \\ -43.2 \end{array}$ | $\begin{array}{r} +33.9 \\ { }_{-36.7} \end{array}$ |
| $c_{i}^{\mathscr{O}}\left(\mathrm{TeV}^{-2}\right)$ |  |  |  |  |  |
| $\frac{c_{W W W}}{\Lambda^{2}}$ | $\begin{aligned} & +0.540 \\ & { }_{-0.565}^{+} \end{aligned}$ | $\begin{aligned} & +0.426 \\ & { }_{-0.445}^{+} \end{aligned}$ | $\begin{aligned} & +0.327 \\ & -0.365 \end{aligned}$ | $\begin{aligned} & +0.257 \\ & { }_{-0.258} \end{aligned}$ | $\begin{aligned} & +0.200 \\ & { }_{-0.238} \end{aligned}$ |
| $\frac{c_{W}}{\Lambda^{2}}$ | $\begin{aligned} & +0.504 \\ & { }_{-0.747} \end{aligned}$ | $\begin{aligned} & +0.397 \\ & { }_{-0.683}^{+} \end{aligned}$ | +0.274 +0.624 | $\begin{aligned} & +0.196 \\ & { }_{-0.390} \end{aligned}$ | $\begin{aligned} & +0.138 \\ & { }_{-0.381}^{+0} \end{aligned}$ |
| $\frac{c_{B}}{\Lambda^{2}}$ | +67.8 +67.1 | +60.1 +59.2 | +47.6 +52.6 | $\begin{array}{r} +30.9 \\ -33.3 \end{array}$ | $\begin{aligned} & +27.0 \\ & -30.1 \end{aligned}$ |
| $\frac{c_{\widetilde{W W W}}}{\Lambda^{2}}$ | $\begin{aligned} & +0.516 \\ & -0.514 \end{aligned}$ | $\begin{aligned} & +0.415 \\ & -0.430 \end{aligned}$ | $\begin{aligned} & +0.339 \\ & -0.342 \end{aligned}$ | $\begin{aligned} & +0.252 \\ & -0.244 \end{aligned}$ | $\begin{aligned} & +0.209 \\ & -0.216 \end{aligned}$ |
| $\frac{c_{\widetilde{W}}}{\Lambda^{2}}$ | ${ }_{-68.5}^{+69.2}$ | $\begin{array}{r} +61.2 \\ -60.4 \end{array}$ | $\begin{array}{r} +52.7 \\ +52.0 \\ \hline-8 \end{array}$ | $\begin{array}{r} +34.2 \\ { }_{-32.7} \end{array}$ | $\begin{array}{r} +31.0 \\ { }_{-30.0} \end{array}$ |
| $c_{i}^{\mathscr{L}_{g}}\left(10^{-3}\right)$ |  |  |  |  |  |
| $\Delta g_{1}^{Z}$ | $\begin{array}{r} +2.10 \\ -3.10 \\ \hline \end{array}$ | $\begin{aligned} & +1.65 \\ & -2.84 \\ & \hline \end{aligned}$ | $\begin{aligned} & +1.14 \\ & { }_{-2.59} \end{aligned}$ | $\begin{aligned} & +0.814 \\ & -1.62 \end{aligned}$ | $\begin{aligned} & +0.576 \\ & -1.58 \end{aligned}$ |
| $\lambda^{Z}$ | $\begin{aligned} & +2.21 \\ & -2.31 \end{aligned}$ | $\begin{array}{r} +1.74 \\ -1.82 \\ \hline \end{array}$ | $\begin{array}{r} +1.34 \\ -1.49 \end{array}$ | $\begin{aligned} & +1.05 \\ & -1.06 \\ & \hline \end{aligned}$ | $\begin{aligned} & +0.822 \\ & -0.975 \end{aligned}$ |
| $\Delta \kappa^{Z}$ | $\begin{array}{r} +62.1 \\ -63.4 \\ \hline \end{array}$ | $\begin{array}{r} +54.6 \\ -56.4 \end{array}$ | $\begin{array}{r} +48.3 \\ -44.8 \\ \hline \end{array}$ | $\begin{array}{r} +30.6 \\ -29.1 \\ \hline \end{array}$ | $\begin{array}{r} +27.6 \\ -25.6 \\ \hline \end{array}$ |
| $\widetilde{\lambda^{Z}}$ | $\begin{aligned} & +2.11 \\ & -2.10 \end{aligned}$ | $\begin{aligned} & +1.70 \\ & -1.76 \end{aligned}$ | $\begin{aligned} & +1.39 \\ & -1.40 \end{aligned}$ | $\begin{aligned} & +1.03 \\ & -1.00 \end{aligned}$ | $\begin{aligned} & +0.857 \\ & { }_{-0.882}^{+} \end{aligned}$ |
| $\widetilde{\kappa^{Z}}$ | ${ }_{-64.5}^{+63.8}$ | $\begin{aligned} & +56.3 \\ & { }_{-57.1} \end{aligned}$ | $\begin{array}{r} +48.4 \\ { }_{-49.1} \end{array}$ | $\begin{array}{r} +30.5 \\ { }_{-31.9} \end{array}$ | $\begin{aligned} & +28.0 \\ & { }_{-28.9} \end{aligned}$ |

Table 5. The list of simultaneous limits from MCMC at $95 \%$ BCI on the effective vertex couplings $c_{i}^{\mathscr{L}}$ and the effective operator couplings $c_{i}^{\mathscr{O}}$ along with translated limits on effective vertices $c_{i}^{\mathscr{L}_{g}}$ for various luminosities with the notation ligher limit lower limit $\equiv[$ lower limit, higher limit $]$.
ties: $\mathscr{L}=35.9 \mathrm{fb}^{-1}, \mathscr{L}=100 \mathrm{fb}^{-1}, \mathscr{L}=300 \mathrm{fb}^{-1}, \mathscr{L}=1000 \mathrm{fb}^{-1}$ and $\mathscr{L}=3000 \mathrm{fb}^{-1}$. The correlation among the parameters are studied (using GetDist [110, 111]) and they are shown in figure 8 along with $1 D$ projections for effective vertex factors. The limits on the couplings get tighter as the luminosity is increased, as it should be. The shape of the contours are very circular in all two-parameter projections as the cross sections dominate in constraining the aTGC. The same conclusions are drawn when effective operators are varied as independent parameters. The limits on $c_{i}^{\mathscr{L}_{g}}$ are tighter compared to the limits on $c_{i}^{\mathscr{L}}$ (see table 5); the comparison between them are shown in the two-parameter marginalised plane in figure 9 in $\Delta g_{1}^{Z}-\kappa^{Z}, \lambda^{Z}-\widetilde{\lambda^{Z}}$ and $\kappa^{Z}-\widetilde{\kappa^{Z}}$ planes as representative for luminosity $\mathscr{L}=100 \mathrm{fb}^{-1}$ (outer contours) and $\mathscr{L}=1000 \mathrm{fb}^{-1}$ (inner contours). The limits and the contours are roughly the same in $\lambda^{Z}-\widetilde{\lambda^{Z}}$ plane. The contours are more symmetric around the SM for $c_{i}^{\mathscr{L}_{g}}$ compared to $c_{i}^{\mathscr{L}}$, e.g., see $\Delta g_{1}^{Z}-\kappa^{Z}$ plane. The limits obtained here for luminosity $35.9 \mathrm{fb}^{-1}$ are better than the experimentally observed limits at the LHC given in table 1 except on $c_{B}$ and hence on $\Delta \kappa^{Z}$. This is because the LHC analysis [72] uses $W W$


Figure 8. All the marginalised $1 D$ projections and $2 D$ projections at $95 \% \mathrm{BCI}$ from MCMC in triangular array for the effective vertices $\left(c_{i}^{\mathscr{L}}\right)$ for various luminosities at $\sqrt{s}=13 \mathrm{TeV}$ using all the observables.
production on top of $Z W$ production, whereas we only use $Z W$ production process. But our limits on the couplings are better when compared with the $Z W$ production process alone at the LHC [74]. In figure 10, we present the comparison of limits obtained by the CMS analyses with $Z W+W W$ [72] process and $Z W$ [74] with our estimate with two parameter $95 \% \mathrm{BCI}$ contours in the $c_{W W W} / \Lambda^{2}-c_{W} / \Lambda^{2}$ plane (left-panel) and $c_{W} / \Lambda^{2}-c_{B} / \Lambda^{2}$ plane (right-panel). The contour in the plane $c_{W W W} / \Lambda^{2}-c_{W} / \Lambda^{2}$ in our estimate (We expect) (solid/green line) is tighter compared to both CMS $Z W+W W$ (dashed/black line) and CMS $Z W$ analyses (dotted/blue line). This is because we use binned cross sections in the analysis. The limit on the couplings $c_{B} / \Lambda^{2}$ (right-panel) on the other hand is tighter, yet comparable, with CMS $Z W$ and weaker than the CMS $Z W+W W$ analysis because the $Z W$ process itself is less sensitive to $c_{W}$.


Figure 9. The marginalised $2 D$ projections at $95 \%$ BCI from MCMC in the $\Delta g_{1}^{Z}-\Delta \kappa^{Z}, \lambda^{Z}{ }_{-}$ $\widetilde{\lambda^{Z}}$, and $\Delta \kappa^{Z}-\widetilde{\kappa^{Z}}$ planes are shown in solid/red when the effective vertex factors $\left(c_{i}^{\mathscr{L}}\right)$ are treated independent, while shown in dashed/green when the operators are treated independent $\left(c_{i}^{\mathscr{L}_{g}}\right)$ for luminosities $\mathscr{L}=1000 \mathrm{fb}^{-1}$ (two inner contours) and $\mathscr{L}=100 \mathrm{fb}^{-1}$ (two outer contours) at $\sqrt{s}=$ 13 TeV using all the observables.


Figure 10. The two parameter $95 \%$ C.L. contours in the $c_{W W W} / \Lambda^{2}-c_{W} / \Lambda^{2}$ plane (left-panel) and $c_{W} / \Lambda^{2}-c_{B} / \Lambda^{2}$ plane (right-panel) for our estimate (We expect) in solid/green lines, for CMS $Z W+W W$ in dashed/black lines and for CMS $Z W$ in dotted/blue lines at $\sqrt{s}=13 \mathrm{TeV}$ and $\mathscr{L}=35.9 \mathrm{fb}^{-1}$ using all the observables.

### 4.2 The role of asymmetries in parameter extraction

The asymmetries are sub-dominant in constraining the couplings much like seen in ref. [90] for $p p \rightarrow Z Z$ case. But the asymmetries have a sense of directionality in the parameter space. To see this, we perform a toy analysis to extract non-zero anomalous couplings with pseudo data generated for the set of anomalous couplings of

$$
\begin{equation*}
\text { aTGC-Bench : }\left\{\Delta g_{1}^{Z}, \lambda^{Z}, \Delta \kappa^{Z}, \widetilde{\lambda^{Z}}, \widetilde{\kappa^{Z}}\right\}=\{0.6,0.6,0.8,0.4,-10\} \times 10^{-2} \tag{4.1}
\end{equation*}
$$

$$
\ldots . . \sigma_{i} \quad--\sigma_{i}+A_{\Delta \phi}+A_{i}^{Z} \quad-\sigma_{i}+A_{\Delta \phi}+A_{i}^{Z}+A_{i}^{W} \quad \mid \mathscr{L}=300 \mathrm{fb}^{-1}
$$



Figure 11. The marginalised $2 D$ projections at $95 \% \mathrm{BCI}$ on $\Delta g_{1}^{Z}-\Delta \kappa^{Z}, \Delta g_{1}^{Z}-\widetilde{\kappa^{Z}}$, and $\Delta \kappa^{Z}{ }_{-}$ $\widetilde{\kappa^{Z}}$ planes from MCMC with observables $\sigma_{i}$ (dotted/red line), $\sigma_{i}+A_{\Delta \phi}+A_{i}^{Z}$ (dashed/blue line) and $\sigma_{i}+A_{\Delta \phi}+A_{i}^{Z}+A_{i}^{W}$ (solid/green line) for aTGC-Bench couplings $\left\{\Delta g_{1}^{Z}, \lambda^{Z}, \Delta \kappa^{Z}, \widetilde{\lambda^{Z}}, \widetilde{\kappa^{Z}}\right\}=$ $\{0.6,0.6,0.8,0.4,-10\} \times 10^{-2}$ at $\sqrt{s}=13 \mathrm{TeV}$ and $\mathscr{L}=300 \mathrm{fb}^{-1}$.


Figure 12. The marginalised $2 D$ projections at $95 \% \mathrm{BCI}$ on $\Delta g_{1}^{Z}-\Delta \kappa^{Z}, \Delta g_{1}^{Z}-\widetilde{\kappa^{Z}}$, and $\Delta \kappa^{Z}-\widetilde{\kappa^{Z}}$ planes from MCMC with observables $\sigma_{i}$ in top-row and $\sigma_{i}+A_{\Delta \phi}+A_{i}^{Z}+A_{i}^{W}$ in bottom-row for integrated luminosities $35.9 \mathrm{fb}^{-1}$ (outermost contours), $100 \mathrm{fb}^{-1}, 300 \mathrm{fb}^{-1}, 1000 \mathrm{fb}^{-1}$, and $3000 \mathrm{fb}^{-1}$ (innermost contours) for aTGC-Bench couplings $\left\{\Delta g_{1}^{Z}, \lambda^{Z}, \Delta \kappa^{Z}, \widetilde{\lambda^{Z}}, \widetilde{\kappa^{Z}}\right\}=\{0.6,0.6,0.8,0.4,-10\} \times$ $10^{-2}$ at $\sqrt{s}=13 \mathrm{TeV}$.
using MCMC method. These couplings are chosen to be within the current limits, see table 1. In figure 11, we show the posterior marginalised $2 D$ projections at $95 \% \mathrm{BCI}$ on $\Delta g_{1}^{Z}-\Delta \kappa^{Z}, \Delta g_{1}^{Z}-\widetilde{\kappa^{Z}}$, and $\Delta \kappa^{Z}-\widetilde{\kappa^{Z}}$ planes. We draw the contours using $\sigma_{i}$ only (dotted/red line), using $\sigma_{i}$ along with $A_{\Delta \phi}+A_{i}^{Z}$ (dashed/blue line) and all observables $\sigma_{i}+A_{\Delta \phi}+A_{i}^{Z}+$ $A_{i}^{W}$ (solid/green line) for integrated luminosity of $\mathscr{L}=300 \mathrm{fb}^{-1}$. The dot ( $\bullet$ ) points in the $2 D$ contours represent the SM point, while the star-marks $(\star)$ represent the couplings from aTGC-Bench. As the asymmetries $A_{\Delta \phi}$ and asymmetries of $Z\left(A_{i}^{Z}\right)$ are added on top of the cross sections, the measurement gets better and it improves further when the asymmetries of $W\left(A_{i}^{W}\right)$ are added. The cross sections are blind to the orientation of aTGC-Bench couplings and sensitive only to the magnitude of deviation from the SM. The asymmetries, however, give direction to the measurement, e.g., in $\Delta \kappa^{Z}-\widetilde{\kappa^{Z}}$ plane, $\sigma_{i}+A_{\Delta \phi}+A_{i}^{Z}$ give tight and directional constraints. The above three planes are shown again in figure 12 for varying luminosities of $35.9 \mathrm{fb}^{-1}$ (outermost contours), $100 \mathrm{fb}^{-1}, 300 \mathrm{fb}^{-1}, 1000 \mathrm{fb}^{-1}$, and $3000 \mathrm{fb}^{-1}$ (innermost contours) for observables $\sigma_{i}$ in top-row and $\sigma_{i}+A_{\Delta \phi}+A_{i}^{Z}$ in bottomrow. For higher luminosities, the $\sigma_{i}$ alone (top-row) do not yield improved limits nor do they give any hint towards the direction of aTGC-Bench. But the inclusion of asymmetries $\sigma_{i}+A_{i}$ (bottom-row) give increasingly accurate determination of the aTGC-Bench points with increasing luminosity. Thus this toy analysis indicate that the asymmetries would help in the measurement of anomalous couplings at high-luminosity provided an excess of events is observed at the LHC, and we interpret the deviation in terms of aTGC.

We note that the $3 l+E_{T}$ excess in the lower $p_{T}(Z)$ region at the LHC [74] interpreted by two extra scalers [112] may be fitted by aTGC, which is beyond the scope of this present work.

## 5 Conclusion

To conclude, we studied the $W W Z$ anomalous couplings in the $Z W^{ \pm}$production at the LHC and examined the role of polarization asymmetries together with $\Delta \phi\left(l_{W}, Z\right)$ asymmetry and forward-backward asymmetry on the estimation of limits on the anomalous couplings. We reconstructed the missing neutrino momentum by choosing the small $\left|p_{z}(\nu)\right|$ from the two-fold solutions and estimated the $W$ polarization asymmetries, while the $Z$ polarization asymmetries are kept free from any reconstruction ambiguity. We generated events at NLO in QCD in mg5_aMC for about 100 sets of anomalous couplings and used them for the numerical fitting of semi-analytic expressions of all the observables as a function of the couplings. We estimated simultaneous limits on the anomalous couplings using MCMC method for both effective vertex formalism and effective operator approach for luminosities $35.9 \mathrm{fb}^{-1}$, $100 \mathrm{fb}^{-1}, 300 \mathrm{fb}^{-1}, 1000 \mathrm{fb}^{-1}$ and $3000 \mathrm{fb}^{-1}$. The limits obtained for $\mathscr{L}=35.9 \mathrm{fb}^{-1}$ are tighter than the limits available at the LHC (see table $1 \& 5$ ) except on $c_{W}$ (and $\Delta \kappa^{Z}$ ). The asymmetries are helpful in extracting the values of anomalous couplings if a deviation from the SM is observed at the LHC. We performed a toy analysis of parameter extraction with some benchmark aTGC couplings and observed that the inclusion of asymmetries to the cross sections improves the parameter extraction significantly at high-luminosity.

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## A Fitting procedure in obtaining observables as a function of couplings

The SM +aTGC events are generated for about 100 set of couplings

$$
\left\{c_{i}\right\}=\left\{\Delta g_{1}^{Z}, \lambda^{Z}, \Delta \kappa^{Z}, \widetilde{\lambda^{Z}}, \widetilde{\kappa^{Z}}\right\}
$$

in both processes. The values of all the observables are obtained for the set couplings in the optimized cuts (table 4), and then those are used for numerical fitting to obtain the semi-analytical expression of all the observables as a function of the couplings. For the cross sections the following $C P$-even expression is used to fit the data:

$$
\begin{equation*}
\sigma\left(\left\{c_{i}\right\}\right)=\sigma_{\mathrm{SM}}+\sum_{i=1}^{3} c_{i} \times \sigma_{i}+\sum_{i=1}^{5}\left(c_{i}\right)^{2} \times \sigma_{i i}+\frac{1}{2} \sum_{i=1}^{3} \sum_{j(\neq i)=1}^{3} c_{i} c_{j} \times \sigma_{i j}+c_{4} c_{5} \times \sigma_{45} \tag{A.1}
\end{equation*}
$$

For asymmetries, the numerator and the denominator are fitted separately and then used as

$$
\begin{equation*}
A_{j}\left(\left\{c_{i}\right\}\right)=\frac{\Delta \sigma_{A_{j}}\left(\left\{c_{i}\right\}\right)}{\sigma_{A_{j}}\left(\left\{c_{i}\right\}\right)} \tag{A.2}
\end{equation*}
$$

The numerator ( $\Delta \sigma_{A}$ ) of $C P$-odd asymmetries are fitted with the $C P$-odd expression

$$
\begin{equation*}
\Delta \sigma_{A}\left(\left\{c_{i}\right\}\right)=\sum_{i=4}^{5} c_{i} \times \sigma_{i}+\sum_{i=1}^{3}\left(c_{i} c_{4} \times \sigma_{i 4}+c_{i} c_{5} \times \sigma_{i 5}\right) \tag{A.3}
\end{equation*}
$$

The denominator $\left(\sigma_{A_{j}}\right)$ of all the asymmetries and the numerator $\left(\Delta \sigma_{A}\right)$ of $C P$-even asymmetries are fitted with the $C P$-even expression given in eq. (A.1).

We use the MCMC method to fit the coefficients of the cross sections with positivity demand, i.e., $\sigma\left(\left\{c_{i}\right\}\right) \geq 0$. We use $80 \%$ data to fit the coefficients of the cross sections, and then the fitted expressions are validated against the rest $20 \%$ of the data and found to be matching within $2 \sigma \mathrm{MC}$ error. We generated $10^{7}$ events to keep the MC error as small as possible, even in the tightest optimized cuts. For example, the $A_{z z}$ in $Z W^{+}$has the tightest cut on $m_{3 l}$ (see table 4) and yet have very small ( $0.2 \%$ ) MC error (see table 6). In figure 13 fitted values of observables are compared against the simulated data for the cross section in two diagonal bins (top-panel) and the polarization asymmetries $A_{z}$ and $A_{x z}$ (bottom-panel) in $Z W^{+}$production in $e^{+} e^{-} \mu^{+} \nu_{\mu}$ channel as representative. The fitted values seem to agree with the simulated data used within the MC error.


Figure 13. The simulated data (in $x$-axis) vs. fitted values (in $y$-axis) for the cross section in the two diagonal bins (top-panel) and the polarization asymmetries $A_{z}$ and $A_{x z}$ (bottom-panel) in in $Z W^{+}$production in $e^{+} e^{-} \mu^{+} \nu_{\mu}$ channel at the LHC at $\sqrt{s}=13 \mathrm{TeV}$.

## B Standard Model values of the asymmetries and polarizations

In table 6, we show the SM estimates (with $1 \sigma$ MC error) of the polarization asymmetries of $Z$ and $W$ and their corresponding polarizations along with the other asymmetries for our selection cuts (sel.cut) given in eq. (2.2) and optimized cuts (opt.cut) given table 4. A number of events of $N \simeq 9.9 \times 10^{6}$ satisfy our selection cuts, which give the same error $\left(\delta A_{i}=1 / \sqrt{N}\right)$ for all the asymmetries, and hence they are given in the top row. As the optimized cuts for $W$ are the same for all the asymmetries, the errors for them are also given in the top row. For the optimized cuts of $Z$ observables, however, the number of events varies, and hence the MC errors are given to each asymmetries. The $C P$-odd polarizations $p_{y}, T_{x y}, T_{y z}$, and their corresponding asymmetries are consistent with zero in the SM within MC error.

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|  | $Z W^{+}$ |  |  |  | $Z W^{-}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $Z$ |  | $W^{+}$ |  | Z |  | $W^{-}$ |  |
| $O$ <br> $\delta A_{i}$ | $\begin{aligned} & \text { sel.cut } \\ & \pm 0.0003 \end{aligned}$ | opt.cut | $\begin{aligned} & \text { sel.cut } \\ & \pm 0.0003 \end{aligned}$ | $\begin{aligned} & \text { opt.cut } \\ & \pm 0.0007 \end{aligned}$ | $\begin{aligned} & \text { sel.cut } \\ & \pm 0.0003 \end{aligned}$ | opt.cut | $\begin{aligned} & \text { sel.cut } \\ & \pm 0.0003 \end{aligned}$ | $\begin{aligned} & \text { opt.cut } \\ & \pm 0.0007 \end{aligned}$ |
| $\begin{aligned} & A_{x} \\ & p_{x} \end{aligned}$ | $\begin{aligned} & -0.0196 \\ & +0.1192 \pm 0.0018 \end{aligned}$ | $\begin{aligned} & -0.0150 \pm 0.0008 \\ & +0.0912 \pm 0.0049 \end{aligned}$ | $\begin{aligned} & -0.2303 \\ & +0.3071 \pm 0.0004 \end{aligned}$ | $\begin{aligned} & -0.0550 \\ & 0.0733 \pm 0.0009 \end{aligned}$ | $\begin{aligned} & +0.0074 \\ & -0.0450 \pm 0.0018 \end{aligned}$ | $\begin{aligned} -0.0046 & \pm 0.0010 \\ +0.0280 & \pm 0.0061 \end{aligned}$ | $\begin{aligned} & -0.0826 \\ & +0.110 \pm 0.00041 \end{aligned}$ | $\begin{aligned} & -0.0001 \\ & +0.00013 \pm 0.0009 \end{aligned}$ |
| $\begin{aligned} & A_{y} \\ & p_{y} \end{aligned}$ | $\begin{aligned} & +0.0003 \\ & -0.0018 \pm 0.0018 \end{aligned}$ | $\begin{aligned} & +0.0004 \pm 0.0007 \\ & -0.0024 \pm 0.0146 \end{aligned}$ | $\begin{aligned} & -0.0007 \\ & +0.0009 \pm 0.0004 \end{aligned}$ | $\begin{aligned} & -0.0005 \\ & +0.0006 \pm 0.0009 \end{aligned}$ | $\begin{aligned} & -0.0013 \\ & +0.0079 \pm 0.0018 \end{aligned}$ | $\begin{aligned} & -0.0021 \pm 0.0007 \\ & +0.0127 \pm 0.0042 \end{aligned}$ | $\begin{aligned} & 0.0 \\ & 0.0 \pm 0.0004 \end{aligned}$ | $\begin{aligned} & +0.0007 \\ & -0.0009 \pm 0.0009 \end{aligned}$ |
| $\begin{aligned} & A_{z} \\ & p_{z} \end{aligned}$ | $\begin{aligned} & -0.0040 \\ & +0.0243 \pm 0.0018 \end{aligned}$ | $\begin{aligned} & +0.0502 \pm 0.0025 \\ & -0.3051 \pm 0.0152 \end{aligned}$ | $\begin{aligned} & +0.1337 \\ & -0.1783 \pm 0.0004 \end{aligned}$ | $\begin{aligned} & +0.6615 \\ & -0.8820 \pm 0.0009 \end{aligned}$ | $\begin{aligned} & +0.0316 \\ & -0.1921 \pm 0.0018 \end{aligned}$ | $\begin{aligned} & +0.0482 \pm 0.0019 \\ & -0.2930 \pm 0.0115 \end{aligned}$ | $\begin{aligned} & +0.1954 \\ & -0.2605 \pm 0.0004 \end{aligned}$ | $\begin{aligned} & +0.7381 \\ & -0.9841 \pm 0.0009 \end{aligned}$ |
| $\begin{aligned} & A_{x y} \\ & T_{x y} \end{aligned}$ | $\begin{aligned} & -0.0017 \\ & -0.0033 \pm 0.0006 \end{aligned}$ | $\begin{aligned} & +0.0005 \pm 0.0007 \\ & +0.00096 \pm 0.0013 \end{aligned}$ | $\begin{aligned} & -0.0011 \\ & -0.0021 \pm 0.0006 \end{aligned}$ | $\begin{aligned} & -0.0006 \\ & -0.0012 \pm 0.0013 \end{aligned}$ | $\begin{aligned} & +0.0008 \\ & +0.0015 \pm 0.0006 \end{aligned}$ | $\begin{aligned} & +0.0014 \pm 0.0007 \\ & +0.0027 \pm 0.0013 \end{aligned}$ | $\begin{aligned} & +0.0013 \\ & +0.0025 \pm 0.0006 \end{aligned}$ | $\begin{aligned} & -0.0003 \\ & -0.0006 \pm 0.0013 \end{aligned}$ |
| $\begin{aligned} & A_{x z} \\ & T_{x z} \end{aligned}$ | $\begin{aligned} & +0.0196 \\ & +0.0377 \pm 0.0006 \end{aligned}$ | $\begin{aligned} & +0.0914 \pm 0.0004 \\ & +0.1758 \pm 0.0008 \end{aligned}$ | $\begin{aligned} & +0.0048 \\ & +0.0092 \pm 0.0006 \end{aligned}$ | $\begin{aligned} & -0.0063 \\ & -0.0121 \pm 0.0013 \end{aligned}$ | $\begin{aligned} & +0.0961 \\ & +0.1849 \pm 0.0006 \end{aligned}$ | $\begin{aligned} & +0.0547 \pm 0.0006 \\ & +0.1052 \pm 0.0011 \end{aligned}$ | $\begin{aligned} & +0.0010 \\ & +0.0019 \pm 0.0006 \end{aligned}$ | $\begin{aligned} & -0.0136 \\ & -0.0262 \pm 0.0013 \end{aligned}$ |
| $\begin{aligned} & A_{y z} \\ & T_{y z} \end{aligned}$ | $\begin{aligned} & +0.0002 \\ & +0.0004 \pm 0.0006 \end{aligned}$ | $\begin{array}{r} -0.0001 \pm 0.0004 \\ -0.0002 \pm 0.0008 \end{array}$ | $\begin{aligned} & +0.0003 \\ & +0.0006 \pm 0.0006 \end{aligned}$ | $\begin{aligned} & -0.0005 \\ & -0.0009 \pm 0.0013 \end{aligned}$ | $\begin{aligned} & -0.0017 \\ & -0.0033 \pm 0.0006 \end{aligned}$ | $\begin{aligned} & -0.0016 \pm 0.0003 \\ & -0.0031 \pm 0.0006 \end{aligned}$ | $\begin{aligned} & +0.0001 \\ & +0.0002 \pm 0.0006 \end{aligned}$ | $\begin{aligned} & -0.0001 \\ & -0.0002 \pm 0.0013 \end{aligned}$ |
| $\begin{aligned} & A_{x^{2}-y^{2}} \\ & T_{x x}-T_{y y} \\ & \hline \end{aligned}$ | $\begin{aligned} & -0.0878 \\ & -0.3378 \pm 0.0011 \end{aligned}$ | $\begin{array}{\|l\|} \hline-0.0925 \pm 0.0019 \\ -0.3559 \pm 0.0073 \\ \hline \end{array}$ | $\begin{aligned} & \hline-0.0266 \\ & -0.1023 \pm 0.0011 \\ & \hline \end{aligned}$ | $\begin{aligned} & -0.1326 \\ & -0.5102 \pm 0.0027 \end{aligned}$ | $\begin{aligned} & -0.0935 \\ & -0.3597 \pm 0.0011 \\ & \hline \end{aligned}$ | $\begin{aligned} & -0.0899 \pm 0.0012 \\ & -0.3459 \pm 0.0046 \end{aligned}$ | $\begin{aligned} & -0.0923 \\ & -0.3551 \pm 0.0011 \end{aligned}$ | $\begin{aligned} & -0.1588 \\ & -0.6110 \pm 0.0027 \end{aligned}$ |
| $\begin{aligned} & A_{z z} \\ & T_{z z} \end{aligned}$ | $\begin{aligned} & -0.0137 \\ & -0.0298 \pm 0.0006 \end{aligned}$ | $\begin{aligned} & +0.0982 \pm 0.0024 \\ & +0.2138 \pm .0052 \end{aligned}$ | $\begin{aligned} & +0.0519 \\ & +0.1130 \pm 0.0006 \end{aligned}$ | $\begin{aligned} & +0.1406 \\ & +0.3061 \pm 0.0015 \end{aligned}$ | $\begin{aligned} & +0.0030 \\ & +0.0065 \pm 0.0006 \end{aligned}$ | $\begin{aligned} & +0.0863 \pm 0.0048 \\ & +0.1879 \pm 0.0104 \end{aligned}$ | $\begin{aligned} & +0.1046 \\ & +0.2277 \pm 0.0006 \end{aligned}$ | $\begin{aligned} & +0.2547 \\ & +0.5546 \pm 0.0015 \end{aligned}$ |
| $A_{\text {fb }}$ | +0.6829 | $+0.4475 \pm 0.0009$ | +0.4699 | $+0.2627$ | $+0.6696$ | $+0.2791 \pm 0.0025$ | $+0.2060$ | $+0.3174$ |
|  | sel.cut |  | opt.cut |  | sel.cut |  | opt.cut |  |
| $A_{\Delta \phi}$ | $-0.3756 \pm 0.0003$ |  | $-0.4151 \pm 0.0022$ |  | $-0.3880 \pm 0.0003$ |  | $-0.4208 \pm 0.0025$ |  |

Table 6. The SM values with $1 \sigma$ MC error of the polarization asymmetries of $Z$ and $W$ and their corresponding polarizations along with the other asymmetries in $Z W^{ \pm}$production in the $e^{+} e^{-} \mu^{ \pm}+E_{T}$ channel are shown for event selection cuts (sel.cut) given in eq. (2.2) and optimized cuts (opt.cut) given table 4.

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[^0]:    ${ }^{1}$ For each of these benchmark couplings, only one of the couplings is set to non-zero value such that it leads to $\sim 1 \sigma$ deviation in the total cross section. More benchmark scenarios ( $\sim 100$ ) with more than one parameters set to non-zero values at a time are also considered in later sections.

[^1]:    ${ }^{2}$ We note that the forward-backward asymmetry of $Z$ and $W$ are ideally the same in the CM frame. However, since we measure the $Z$ and $W \cos \theta$ w.r.t. different quantity, i.e., visible $p_{z}$ for $Z$ and reconstructed boost for $W$, they are practically different and we use them as two independent observables.

