

Unscented SLAM for Large-Scale Outdoor Environments

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Abstract—This paper presents an experimentally validated alternative to the classical extended Kalman filter approach to the solution of the probabilistic state-space Simultaneous Localization and Mapping (SLAM) problem. Several authors have recently reported the divergence of this classical approach due to the linearization of the inherent non-linear nature of the SLAM problem. Hence, the approach described in this work aims to avoid the analytical linearization based on Taylor-series expansion of both the model and measurement equations by using the unscented filter. An innovation-based consistency checking validates the feasibility and applicability of the unscented SLAM approach to a real large-scale outdoor exploration mission.

Index Terms—SLAM, Unscented filtering, consistency, normalized innovation squared test

I. INTRODUCTION

The simultaneous localization and mapping (SLAM) is the problem to determine the localization of an autonomous vehicle on an unknown environment and, concurrently, to learn the most significant features of the environment to safely and robustly complete the commanded mission. Different approaches to the solution of this problem have been reported for indoor [3], outdoor [7], underwater [15] and air-borne [13] applications.

From a theoretical view-point the SLAM problem can be interpreted as a non-linear filtering problem which involves a pair of discrete-time stochastic processes, $\{\mathbf{x}_k, 0 \leq k < \infty\}$ named *state process*, and $\{\mathbf{z}_k, 0 \leq k < \infty\}$, named *measurement process* with a given joint probability density function (pdf). The solution consists of the recursive computation of the conditional pdf (the posterior pdf) of \mathbf{x}_k given the observations in the past $\{\mathbf{z}_l, 0 \leq l \leq k\}$ and the previous knowledge about \mathbf{x}_k (the prior pdf). Unfortunately, the optimal solution to this problem cannot be obtained in a finite time [14].

The most popular feasible approximation to SLAM, dates back to the seminal work of [16] where a discrete-time state-space framework, named the *stochastic map* was originally presented. Assuming linearization of the state and measurement models and Gaussianity of the underlying pdf's, the

approximate solution to the non-linear filtering problem was obtained by the Extended Kalman Filter (EKF) [2]. Recently, different improvements of the classical algorithm have been reported in the literature [7], [17], [18]. Due to the inherent non-linearity of the SLAM problem, consistency checking of the state estimation is of paramount importance. As reported in [6] the solution of the EKF-based SLAM is consistent in the linear-Gaussian case, however for the general non-linear problem it is well known that linearization can lead to filter divergence [2], as has been confirmed in carefully designed experiments [9], [4] which isolate the effects of linearization errors in the EKF-based approach.

In [10] and references therein, Julier and Uhlmann proposed the *unscented filter* as an alternative to the EKF for recursive state estimation, increasing the accuracy of the state estimation with a comparable computational cost in low-dimensional systems. At time step k , the unscented filter estimates the first two moments of the underlying pdf (i.e. mean and covariance) by a linear weighted regression of evaluations of the true non-linear models at the so-called *sigma-points*. This method resembles the most ambitious particle filtering [19] approach although in this case, small-scale deterministic sampling is used as opposed to the large-scale random sampling used in particle filtering. As proved in the literature, an $\mathcal{O}(n)$, with n the dimension of the state vector, number of sigma-points precisely and accurately approximates the estimated moments up to second order in the Gaussian case. From a practical view-point, the increased accuracy in the computation of the mean and covariance of the state pdf suggests a reduction in the ambiguity of data association, hence, low-complexity validation gate approaches could reliably and robustly be utilized. The first implementation of the unscented filter in SLAM was reported in [5] where preliminary work on a small-scale indoor environment was considered. More recently, [1] describes the application of the *unscented transformation* to the vehicle states by assuming linearity both in the prediction of the map features and the update of the complete state vector.

This paper focus on the SLAM problem from the perspective of the unscented filter, demonstrating its feasibility and on-line consistency for a large-scale outdoor mission.

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A controlled simulation experiment, which isolates the effects of linearization errors, describes the improvement in the consistency of the unscented filter over the EKF-based approach. The rest of the paper is structured as follows: Section II presents the discrete-time state-space approach to SLAM, where a general recursive algorithm for the approximate solution is sketched. The concept of consistency of a state estimator is given in section III. Then, the unscented SLAM approach is described in sections IV and V. Finally, experimental results validate the proposed approach for large-scale outdoor environment.

II. PROBABILISTIC STATE-SPACE GAUSSIAN SLAM

In the probabilistic state-space to SLAM, the vehicle R and a set of environment features $\mathcal{F} = \{F_1, \dots, F_n\}$ is represented by a stochastic state vector \mathbf{x}^W with estimated mean $\hat{\mathbf{x}}^W$ and estimated error covariance \mathbf{P}^W :

$$\hat{\mathbf{x}}^W = \begin{bmatrix} \hat{\mathbf{x}}_R^W \\ \hat{\mathbf{x}}_{\mathcal{F}}^W \end{bmatrix}; \mathbf{P}^W = \begin{bmatrix} \mathbf{P}_{RR}^W & \mathbf{P}_{R\mathcal{F}}^W \\ \mathbf{P}_{\mathcal{F}R}^W & \mathbf{P}_{\mathcal{F}\mathcal{F}}^W \end{bmatrix} \quad (1)$$

where $\hat{\mathbf{x}}_R^W$ is the estimated location of the vehicle with respect to (wrt) a base reference frame W , $\hat{\mathbf{x}}_{\mathcal{F}}^W$ is the estimated location of the features also wrt W , \mathbf{P}_{RR}^W is the estimated error covariance of the location of R , $\mathbf{P}_{\mathcal{F}\mathcal{F}}^W$ is the estimated error covariance of the location of the features, and finally, $\mathbf{P}_{R\mathcal{F}}^W$ represents the cross-covariance between the different elements of the state vector. Additionally, it is generally assumed that the underlying probability density function is Gaussian, hence, at time step k , $\mathbf{x}_k^W \sim \mathcal{N}(\hat{\mathbf{x}}_k^W, \mathbf{P}_k^W)$.

When the vehicle moves from position at step $k-1$ to position at step k , the stochastic state vector changes according to the non-linear motion equation:

$$\mathbf{x}_k^W = \mathbf{f}_k(\mathbf{x}_{k-1}^W, \mathbf{x}_{R_k}^{R_{k-1}}) \quad (2)$$

where the uncertain relative motion $\mathbf{x}_{R_k}^{R_{k-1}}$ is estimated by odometry and assumed to be white Gaussian.

From a Bayesian view-point, suppose that the stochastic map $\mathbf{x}_{k-1}^W \sim \mathcal{N}(\hat{\mathbf{x}}_{k-1}^W, \mathbf{P}_{k-1}^W)$ is available at time $k-1$, then, the predicted stochastic map at time k result from:

$$\begin{aligned} \hat{\mathbf{x}}_{k|k-1}^W &= E[\mathbf{f}_k(\mathbf{x}_{k-1}^W, \mathbf{x}_{R_k}^{R_{k-1}})] \\ \mathbf{P}_{k|k-1} &= E[(\mathbf{x}_k^W - \hat{\mathbf{x}}_{k|k-1}^W)(\mathbf{x}_k^W - \hat{\mathbf{x}}_{k|k-1}^W)^T] \end{aligned} \quad (3)$$

On-board sensors provide, at time k , the observation \mathbf{z}_k related to the state vector \mathbf{x}_k^W by the non-linear measurement equation:

$$\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k^W, \mathbf{x}_{\mathcal{E}_k}^{R_k}) \quad (4)$$

where $\mathbf{x}_{\mathcal{E}_k}^{R_k}$ represents the set of white Gaussian gathered observations. This new information about the state vector, can be incorporated into the state by using the update equations of a linear (in the measurements) estimator:

$$\begin{aligned} \hat{\mathbf{x}}_k^W &= \hat{\mathbf{x}}_{k|k-1}^W + \mathbf{P}_{\mathbf{z}\mathbf{z}} \mathbf{P}_{\mathbf{z}\mathbf{x}}^{-1} (\mathbf{z}_k - \hat{\mathbf{z}}_k) \\ \mathbf{P}_k^W &= \mathbf{P}_{k|k-1}^W - \mathbf{P}_{\mathbf{z}\mathbf{x}} \mathbf{P}_{\mathbf{z}\mathbf{z}}^{-1} \mathbf{P}_{\mathbf{z}\mathbf{x}} \end{aligned} \quad (5)$$

with:

$$\begin{aligned} \mathbf{P}_{\mathbf{z}\mathbf{z}} &= E[(\mathbf{z}_k - \hat{\mathbf{z}}_k)(\mathbf{z}_k - \hat{\mathbf{z}}_k)^T] \\ \mathbf{P}_{\mathbf{z}\mathbf{x}} &= E[(\mathbf{x}_k^W - \hat{\mathbf{x}}_{k|k-1}^W)(\mathbf{z}_k - \hat{\mathbf{z}}_k)^T] \end{aligned} \quad (6)$$

The classical EKF-based SLAM approach computes the estimated covariance matrices of equations (3) and (6) by first-order *analytical* linearization of the motion and measurement models respectively. As shown in the literature [9], [4] this approach frequently leads to filter divergence after only a few update steps. This paper shows the improvement achieved by using *statistical* linearization techniques [10] instead.

III. SLAM CONSISTENCY

Let $\hat{\mathbf{x}}_k^W$ and \mathbf{P}_k^W be the first two moments of the SLAM state estimated at time k . The state estimator is called *consistent* [2] if its state estimation error $\mathbf{x}_k^W - \hat{\mathbf{x}}_k^W$ is *unbiased*, i.e. $E[\mathbf{x}_k^W - \hat{\mathbf{x}}_k^W] = \mathbf{0}$ and the actual Mean Square Error matches the filter calculated covariances:

$$E[(\mathbf{x}_k^W - \hat{\mathbf{x}}_k^W)(\mathbf{x}_k^W - \hat{\mathbf{x}}_k^W)^T] = \mathbf{P}_k^W \quad (7)$$

When the ground true solution for the state variables is available, a statistical test for filter consistency can be carried out on the Normalized Estimation Error Squared (NEES):

$$(\mathbf{x}_k^W - \hat{\mathbf{x}}_k^W)^T (\mathbf{P}_k^W)^{-1} (\mathbf{x}_k^W - \hat{\mathbf{x}}_k^W) \leq \chi_{r,1-\alpha}^2 \quad (8)$$

where $\chi_{r,1-\alpha}^2$ is a threshold obtained from the χ^2 distribution with $r = \dim(\mathbf{x}_k^W)$ degrees of freedom, and α the desired significance level (usually 0.05).

Unfortunately, for most real-time applications, the ground true solution for the state variables would not be available. However, a statistical test for real-time consistency could still be carried out, in this case, on the Normalized Innovation Squared (NIS):

$$\nu_k^T \mathbf{S}_k^{-1} \nu_k \leq \chi_{r,1-\alpha}^2 \quad (9)$$

where $\nu_k = \mathbf{z}_k - \hat{\mathbf{z}}_k$ is called the *innovation* of the filter, and now $r = \dim(\nu_k)$.

In practice, one of the most critical factors that jeopardize the consistency of any SLAM algorithm are the incorrect data associations between observations and map features due to optimistic estimation of location uncertainty.

IV. UNSCENTED FILTERING

The core of unscented filtering is the so-called *Unscented Transformation* (UT) [11]. Small-scale deterministic sampling from the prior pdf provides a minimal set of samples, named *sigma points*, which capture the moments of the underlying density function. The moments of the posterior pdf are then obtained by means of weighted linear regression of evaluations of the non-linear function at the selected samples points. Figure 1 compares the propagation of a pdf through a nonlinear function obtained by analytical linearization and by the unscented transformation (i.e. statistical linearization).

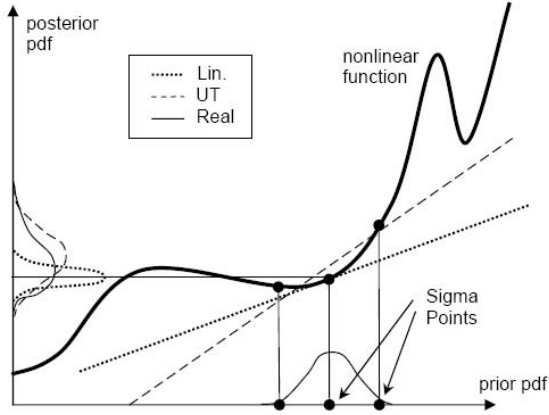


Fig. 1. Propagation of a pdf through a nonlinear function. The first order Taylor expansion (dotted) only use the mean point to compute the linear approximation, while the UT (dashed) approach the function with a linear regression of several sigma points. The actual distribution is the solid one. (Adapted from [20])

Recently, the *Scaled Unscented Transform* (SUT) has been proposed [12], [20] which allows the sigma-points to be scaled to an arbitrary dimension. For an N -dimensional state vector, a symmetric set of $2N + 1$ sigma-points is given by:

$$\begin{aligned} \mathcal{X}^{(0)} &= \hat{\mathbf{x}} \\ \mathcal{X}^{(j)} &= \hat{\mathbf{x}} + \left(\sqrt{(N + \lambda) \mathbf{P}} \right)_j, \quad j = 1, \dots, N \\ \mathcal{X}^{(j)} &= \hat{\mathbf{x}} - \left(\sqrt{(N + \lambda) \mathbf{P}} \right)_j, \quad j = N + 1, \dots, 2N \end{aligned} \quad (10)$$

where $\hat{\mathbf{x}}$ and \mathbf{P} are, respectively, the mean and covariance of the sampled pdf. This set of sigma-points captures the moments of underlying pdf up to the third-order for the Gaussian-case. For efficient computation of the matrix square root, a Cholesky decomposition $\mathbf{P} = \mathbf{S}\mathbf{S}^T$ is used. The parameter λ controls the scaling of the sigma-points.

Let \mathbf{x}_{k-1} be the state vector at time $k - 1$, and let $\{\mathcal{X}_{k-1}^{(0)}, \dots, \mathcal{X}_{k-1}^{(2N)}\}$ be the set of sigma-points computed from its pdf. Also, let the state evolution be characterized by the non-linear function:

$$\mathbf{x}_k = \mathbf{f}_k(\mathbf{x}_{k-1}) \quad (11)$$

Hence, the set of sigma-points are transformed by $2N + 1$ evaluations of the non-linear function at the sigma-points:

$$\mathcal{X}_k^{(j)} = \mathbf{f}_k(\mathcal{X}_{k-1}^{(j)}), \quad j = 0, \dots, 2N \quad (12)$$

Then, the first two moments of the density function of \mathbf{x}_k are computed by a weighted linear regression of the transformed sigma-points:

$$\hat{\mathbf{x}}_k = \sum_{j=0}^{2N} \omega_m^{(j)} \mathcal{X}_k^{(j)} \quad (13)$$

$$\mathbf{P}_k = \sum_{j=0}^{2N} \omega_c^{(j)} (\mathcal{X}_k^{(j)} - \hat{\mathbf{x}}_k)(\mathcal{X}_k^{(j)} - \hat{\mathbf{x}}_k)^T \quad (14)$$

where the weights are given by:

$$\begin{aligned} \omega_c^{(0)} &= \frac{\lambda}{N + \lambda} + (1 - \alpha^2 + \beta) \\ \omega_m^{(0)} &= \frac{\lambda}{N + \lambda} \\ \omega_m^{(j)} &= \omega_c^{(j)} = \frac{1}{2(N + \lambda)}, \quad j = 1, \dots, 2N \end{aligned} \quad (15)$$

where $\lambda = \alpha^2(N + \kappa) - N$. Note that the weights for the computation of the mean (m) and the covariance (c) are different in the 0-th component to compensate for scaling. For a Gaussian prior $N + \kappa = 3$ [12] and therefore, the numerical behavior of the SUT is the same as the *Central Difference Filter* with a step $h = \sqrt{3}$ and the *Gauss-Hermite Filter* with 3 points [8]. For a complete explanation of the parameters of the SUT refer to [20].

V. UNSCENTED SLAM

Let $\mathbf{x}_{k-1}^W \sim \mathcal{N}(\hat{\mathbf{x}}_{k-1}^W, \mathbf{P}_{k-1}^W)$ be the stochastic map available at time $k - 1$. Let $\mathbf{x}_{R_k}^{R_{k-1}} \sim \mathcal{N}(\hat{\mathbf{x}}_{R_k}^{R_{k-1}}, \mathbf{P}_{R_k}^{R_{k-1}})$ be the vehicle motion from time step $k - 1$ to time step k as estimated by odometry, and finally let $\mathcal{E} = \{E_1, \dots, E_m\}$ be the set of observations gathered by on-board sensors at time k , with a joint-Gaussian distribution with covariance matrix $\mathbf{P}_{\mathcal{E}_k}^{R_k}$. Assuming independence between the prior state estimation, the displacement and the set of available measurements, we define an augmented stochastic state vector with mean:

$$\hat{\mathbf{x}}_{a,k-1}^W = \begin{bmatrix} \hat{\mathbf{x}}_{R_{k-1}}^W \\ \hat{\mathbf{x}}_{\mathcal{F}_{k-1}}^W \\ \hat{\mathbf{x}}_{R_k}^{R_{k-1}} \\ \hat{\mathbf{x}}_{\mathcal{E}_k}^{R_k} \end{bmatrix} \quad (16)$$

and a block-diagonal covariance matrix:

$$\mathbf{P}_{a,k-1}^W = \begin{bmatrix} \mathbf{P}_{R_{k-1}}^W & \mathbf{P}_{R_{k-1}}^W & \mathbf{0} & \mathbf{0} \\ \mathbf{P}_{\mathcal{F}_{k-1}}^W & \mathbf{P}_{\mathcal{F}_{k-1}}^W & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{P}_{\mathcal{E}_k}^{R_k} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{P}_{R_k}^{R_{k-1}} \end{bmatrix} \quad (17)$$

By using the deterministic sampling algorithm described in section (IV) we obtain a small-size set of sigma-points which accurately represents the first two moments of the previous distribution: $\{\mathcal{X}_{a,k-1}^{(0)}, \dots, \mathcal{X}_{a,k-1}^{(2N)}\}$.

The set of sigma-points at time $k - 1$ is propagated forward in time through the non-linear state equation:

$$\mathbf{x}_{a,k|k-1}^W = \mathbf{f}_k(\mathbf{x}_{a,k-1}^W) \quad (18)$$

with

$$\begin{aligned}
\mathbf{x}_{R_k}^W &= \mathbf{x}_{R_{k-1}}^W \oplus \mathbf{x}_{R_k}^{R_{k-1}} \\
\mathbf{x}_{F_k}^W &= \mathbf{x}_{F_{k-1}}^W \\
\mathbf{x}_{E_{1,k}}^W &= \mathbf{x}_{R_{k-1}}^W \oplus \mathbf{x}_{R_k}^{R_{k-1}} \oplus \mathbf{x}_{E_{1,k}}^{R_k} \\
&\vdots \\
\mathbf{x}_{E_{m,k}}^W &= \mathbf{x}_{R_{k-1}}^W \oplus \mathbf{x}_{R_k}^{R_{k-1}} \oplus \mathbf{x}_{E_{m,k}}^{R_k}
\end{aligned} \tag{19}$$

where \oplus represents the composition of location vectors [3]. The predicted mean at time k of the augmented state vector and its estimated error covariance matrix are then computed from a linear weighted regression of the transformed sigma-points $\{\mathcal{X}_{a,k|k-1}^{(0)}, \dots, \mathcal{X}_{a,k|k-1}^{(2N)}\}$:

$$\hat{\mathbf{x}}_{a,k|k-1}^W = \sum_{j=0}^{2N} \omega_m^{(j)} \mathcal{X}_{a,k|k-1}^{(j)} \tag{20}$$

and,

$$\mathbf{P}_{a,k|k-1}^W = \sum_{j=0}^{2N} \omega_c^{(j)} (\mathcal{X}_{a,k|k-1}^{(j)} - \hat{\mathbf{x}}_{a,k|k-1}^W) (\mathcal{X}_{a,k|k-1}^{(j)} - \hat{\mathbf{x}}_{a,k|k-1}^W)^T \tag{21}$$

Data association provides the observation \mathbf{z}_k statistically compatible and related to the augmented state vector by a non-linear function \mathbf{h}_k :

$$\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_{a,k|k-1}^W) \tag{22}$$

Hence, the update of the estimated mean and estimated error covariance at time k follows from:

$$\hat{\mathbf{x}}_{a,k}^W = \hat{\mathbf{x}}_{a,k|k-1}^W + \mathbf{P}_{\mathbf{x}\nu} \mathbf{S}_k^{-1} (\mathbf{z}_k - \hat{\mathbf{z}}_k) \tag{23}$$

$$\mathbf{P}_{a,k}^W = \mathbf{P}_{a,k|k-1}^W - \mathbf{P}_{\mathbf{x}\nu} \mathbf{S}_k^{-1} \mathbf{P}_{\nu\mathbf{x}} \tag{24}$$

where

$$\hat{\mathbf{z}}_k = \sum_{j=0}^{2N} \omega_m^{(j)} \mathcal{Z}_k^{(j)} \tag{25}$$

with

$$\mathcal{Z}_k^{(j)} = \mathbf{h}_k(\mathcal{X}_{a,k|k-1}^{(j)}) \tag{26}$$

and

$$\mathbf{S}_k = \sum_{j=0}^{2N} \omega_c^{(j)} (\mathcal{Z}_k^{(j)} - \hat{\mathbf{z}}_k) (\mathcal{Z}_k^{(j)} - \hat{\mathbf{z}}_k)^T \tag{27}$$

$$\mathbf{P}_{\mathbf{x}\nu} = \sum_{j=0}^{2N} \omega_c^{(j)} (\mathcal{X}_{a,k|k-1}^{(j)} - \hat{\mathbf{x}}_{a,k|k-1}^W) (\mathcal{Z}_k^{(j)} - \hat{\mathbf{z}}_k)^T \tag{28}$$

As clearly stated in the previous equations, the expectancies of the stochastic vectors involved in the Bayesian solution to the non-linear SLAM problem, given by equations (3)-(7) are approximated by weighted evaluations of the model and measurement function at the deterministically selected sigma-points.

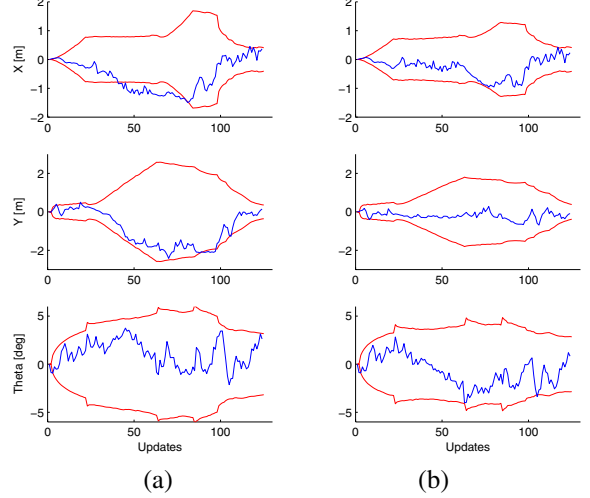


Fig. 2. Estimated errors and 2σ bounds for the (x, y, θ) components of the vehicle estimated location: (a) EKF-based SLAM and (b) Unscented SLAM.

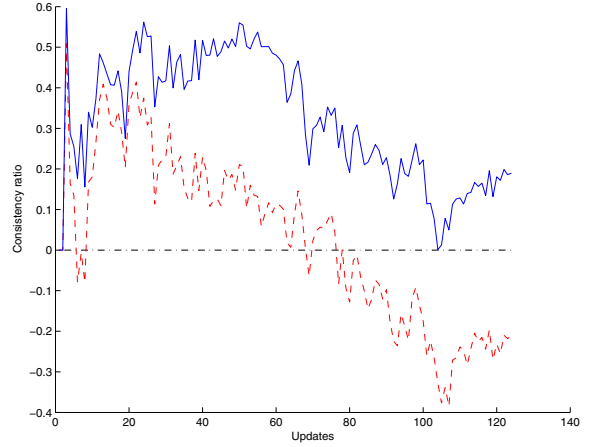


Fig. 3. Consistency ratio for EKF-based SLAM (dashed) and Unscented SLAM (solid) for the 120 m simulated loop trajectory.

VI. EXPERIMENTAL RESULTS

This section presents both simulation and real experimental results to demonstrate the applicability of the unscented filter to the SLAM problem.

A. Simulation Results

A first experiment has been designed to isolate the effects of linearization errors on the consistency of the SLAM solution. A vehicle travels along a rectangular-shaped trajectory of 40×20 meters, i.e. a 120 m loop trajectory, moving 1 m per step. The map of the navigation area is composed of 2-D point features, with a feature density of about 0.2 feature/m. The vehicle is equipped with a range-bearing sensor with a maximum range of 15 meters and a 180 degrees frontal field-of-view. Gaussian-distributed synthetic errors were generated

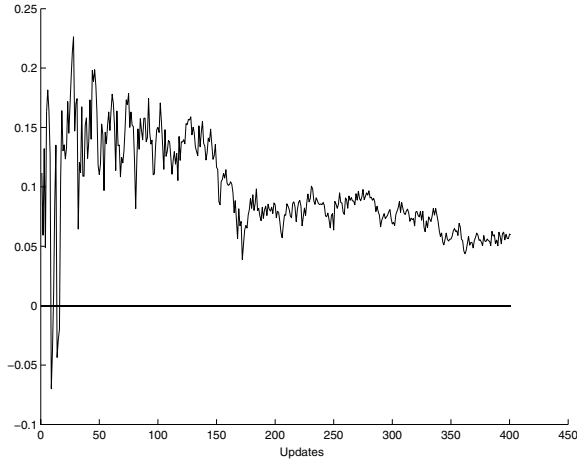


Fig. 4. Consistency ratio for Unscented SLAM in a 400-m long loop. At the end, the estimator remains consistent, so the loop could be close with simple data association strategies.

for both the sensor measurements and for the odometry model of the vehicle. Additionally, known data association is considered.

The evolution of the errors in the components of the vehicle estimated location are plotted in figure 2. In general, the unscented filter provides lower estimated errors than the EKF-based approach with slightly more tight uncertainty bounds, which would result in lower ambiguity of data association in a real application. Figure 3 plots the consistency ratio, $1 - \text{NEES} / \chi_{r,1-\alpha}^2$, for both the EKF-based SLAM and the Unscented SLAM. Clearly, as previously reported in the literature, the EKF-based approach becomes inconsistent after only a few update steps. Nevertheless, the Unscented filter remains consistent (up to 5% statistical error) during the complete vehicle trajectory. Furthermore, figure 4 illustrates that even after a 400-m loop trajectory, the Unscented SLAM remains consistent.

B. Large-Scale Outdoor Navigation

Experimentation with a well-known benchmark dataset [7] has been done to validate the real-time consistency of the estimated solution of our implementation of Unscented SLAM. The vehicle is equipped with a 2D laser rangefinder, a GPS sensor and an odometry unit (forward velocity and steering angle). During the experiment, the vehicle moves in a 3.5 km path along the Victoria Park, in Sydney, Australia. Then, the laser is used to detect and measure the position of trees, which are used as natural landmarks. The poor quality of the odometry and the presence of many spurious make the SLAM problem very interesting with this dataset.

Figure 5 describes the Unscented SLAM solution, with the complete vehicle trajectory and the detected environmental features with the estimated location uncertainty. Our approach keeps a high level of accuracy during all the path

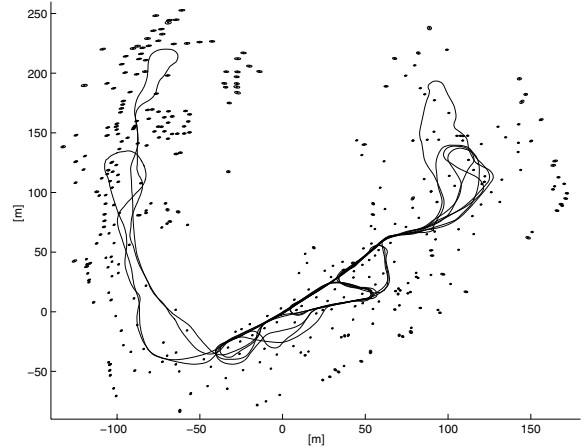


Fig. 5. Outdoor SLAM: Final 2D-point feature based map and vehicle estimated trajectory (Observe that the given trajectory has not been corrected backwards in time).

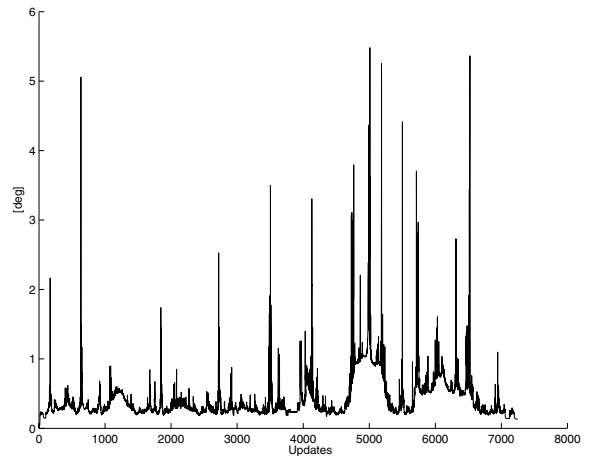


Fig. 6. 1σ bound for the estimated heading error of the vehicle along the trajectory as computed by the Unscented filter. The maximum standard deviation in the complete trajectory (around 3.5km) is lower than 0.5deg

enough to do the data association using only a simple nearest neighbor algorithm. For example, the heading of the robot is bounded by a standard deviation of 0.5 deg during all the trajectory, except in a reduced number of steps. Furthermore, the heading deviation is lower than 0.25 deg most of time (figure 6).

The evolution of the consistency ratio $1 - \text{NIS} / \chi_{r,1-\alpha}^2$ along the complete trajectory is presented in figure 7. The number of inconsistent updates, from the NIS view-point, is roughly 7%, sufficiently close to the theoretical 5%. Furthermore, some of these inconsistent steps are produced by spurious observations. So, we can conclude that the Unscented SLAM is statistically consistent in a 3.5 km outdoor trajectory.

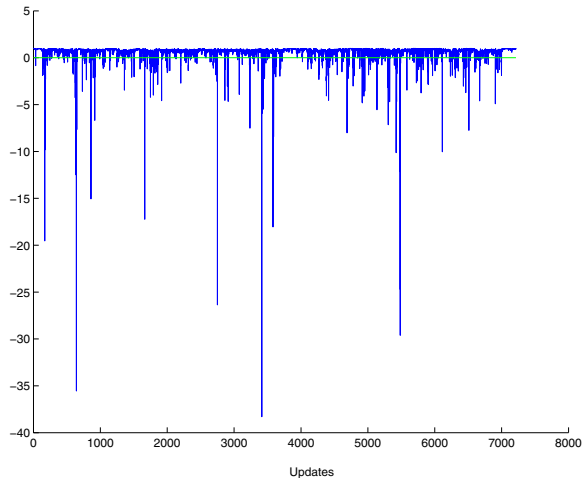


Fig. 7. Consistency ratio of complete evolution of the state vector of the Unscented SLAM. Only the 7% of the poses are inconsistent which is similar to the significant level (5%)

VII. CONCLUSIONS

In this work we have presented the Unscented SLAM, which represents a feasible framework to the solution of the simultaneous localization and mapping problem. Accuracy of state estimation has been increased (lower errors, tighter uncertainty bounds) over the classical EKF-based approach.

The paper has emphasized the improvement in the consistency of the sequential algorithm achieved by avoiding analytical linearization of the model equations. A normalized innovation-based consistency checking has been presented to support the applicability of the approach to a large-scale outdoor environment mission.

The promising results obtained in this work suggest that the Unscented SLAM approach could push forward, in both mission duration and size of the mapped environment, the consistency horizon of the SLAM problem.

Future work further elaborates on the Bayesian viewpoint to the SLAM problem for highly non-linear state and measurement model. In our opinion, closely related application domains, e.g. geometric vision, would readily benefit from the latest results obtained within the SLAM research community.

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