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UNSTEADY MHD BLASIUS AND SAKIADIS FLOWS WITH VARIABLE THERMAL CONDUCTIVITY IN THE PRESENCE OF THERMAL RADIATION AND VISCOUS DISSIPATION

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ABSTRACT

A theoretical analysis has been carried out to investigate the influence of unsteadiness on the laminar two-phase magnetohydrodynamic nanofluid flow filled with porous medium under the combined effects of Brownian motion and thermophoresis. Thermal variable conductivity, thermal radiation and viscous dissipation effects are also considered in this numerical study. The highly nonlinear partial differential equations are transformed into a set of coupled nonlinear ordinary differential equations through suitable similarity transformations. The resultant ordinary differential equations are then numerically solved using the spectral quasilinearization method. The effects of the pertinent physical parameters over the fluid velocity, temperature, concentration, skin friction, Nusselt and Sherwood numbers for both Blasius and Sakiadis flows are presented in graphical and tabular forms and discussed. It is found that the two types of flows behave differently when the physical parameters are varied.

Keywords: *Brownian motion, thermophoresis, spectral quasilinearization method.*

1. INTRODUCTION

Several researchers have been attracted to investigate electrically conducting fluid flows due to their day-to-day applications in numerous engineering areas, such as paper production, cooling of metallic sheets, plasma studies, petroleum industries, cooling of nuclear reactors and magnetohydrodynamic power generators. Blasius (1908) was the first to solve the problem of the fluid flow along a horizontal stationary semi-infinite plate in a moving with a constant velocity. This gave birth to what is now widely known as Blasius boundary layer flow. The fluid motion in a Blasius problem is produced by the free stream. Howarth (1938) later found out a numerical solution to the Blasius problem. The desired cooling effect can be achieved by controlling the rate of heat transfer through the application of the MHD flow in electrically conducting fluid. Further, Lighthill (1950) discussed the theory of heat transfer through a laminar boundary layer. Liu and Chang Liu and Chang (2008), later developed a numerical technique by using new transformation that transforms the governing equation into a nonlinear second-order- boundary value problem. Aminikhah and Kazemi (2016) employed the elastic B-spline approximations to construct to a numerical solution for solving the Blasius equation.

In 1961, Sakiadis (1961) improved the Blasius problem to consider the flow over a moving plate or sheet through an otherwise environment of a quiescent fluid. Through his pioneering work, solutions to this different class of boundary layer problems are significantly different from those of boundary layer flows over stationary surfaces. Subsequently, Tsou *et al.* (1967) managed to experimentally and analytically showed that such flows are physically realizable by confirming the data from Sakiadis. Since then, many researchers have obtained analytical/numerical solutions for different aspects of this class of boundary-layer problems. This class of boundary-layer problem is now known as Sakiadis boundary layer flow. Crane (1970) studied the flow over a stretching sheet which moves in its own plane at a velocity that varies linearly with the distance from the slit. Wang (2004) solved the classical Blasius equation using the Adomian decomposition method. Chamkha (2004) investigated the problem of unsteady, two dimensional, laminar boundary layer flow of a viscous, incompressible electrically conducting and heat fluid over a semi-infinite plate. Choi (1995) proposed the suspension of nanoparticles in a base fluid like water, Ethylene glycol and glycol. This gave birth to a type of fluid flow now know as nanofluids. It is known that conventional fluids such as water, are poor heat transfer fluids. Nanofluids have the potential to reduce thermal resistances and industrial groups such as electronics, medical, food and manufacturing are benefiting from these improved heat transfer. Nanotechnology has been an ongoing hot topic health as researchers claim that nanoparticles could present possible dangers in environment, Mnyusiwalla *et al.* (2003). Sharma *et al.* (2017) investigated the steady two dimensional laminar magnetohydrodynamic slip flow and heat transfer of a viscous incompressible and electrically conducting fluid past a flat exponentially non-conducting stretching porous sheet.

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Buongiorno (2006) explained how the increase in thermal conductivity of nanofluids and also developed a model that considered the particle Brownian motion and thermophoresis. Kuznetsov and Nield (2010) investigated the effects of Brownian motion and thermophoresis on the natural convective boundary layer flow of a nanofluid. Oyelakin *et al.* (2016) reported the combing Dufour and Soret effects on the heat and mass transfer in a Casson nanofluid flow over an unsteady stretching sheet with thermal radiation and heat generation. The study also investigated the effects of partial slip, convective thermal boundary condition, Brownian and thermophoresis diffusion coefficients. Ahmad and Ishak (2017) studied a Blasius and Sakiadis problem in nanofluids. The Buongiormo model was used by Anuar and Bachok (2016) to investigate thermophysical properties of nano-liquids due to Blasius and Sakiadis problem. Hady *et al.* (2013) investigated the density effects on the flow and heat transfer characteristics of the Blasius and Sakiadis flow in a nanofluid in the presence of thermal radiation under convective surface boundary condition. Sekhar *et al.* (2017) theoretically analysed the flow of heat transfer characteristics of Sakiadus and Blasius flow of an MHD Maxwell fluid with an exponential heat source/sink. Cortell (2005) discussed the problem in Blasius and Sakiadis by considering thermal radiation effects with convective surface boundary conditions. Mabood *et al.* (2016) theoretically investigated the effects of volume fraction of nanoparticles, suction/injection and convective heat and mass transfer parameter on MHD stagnation point flow of water based nanofluid (Cu and Ag). Narsu and Kumar (2018) investigated the effects of unsteady MHD chemically reacting radiated Blasius with variable conductivity and a Sakiadis flow under a convective surface boundary condition. Reddy and B. Mallikarjuna (2018) developed a mathematical model to analyze the double diffusive convective of Casson fluid over an inclined stretching sheet with Cattaneo-Christov Heat Flux model. Recently, Mjankwi *et al.* (2019) studied the unsteady MHD flow of a nanofluid with variable fluid properties over an inclined stretching sheet in the presence of thermal radiation and chemical reaction taking into account variable thermal conductivity and diffusion coefficient. Krishna *et al.* (2019) theoretically studied the effect of MHD field on the classical Blasius and Sakiadis flows of heat transfer characteristics with variable conditions and variable properties. **?** studied the effect of non-uniform slot suction or injection into a steady mixed convective MHD boundary layer flow over a vertical wedge embedded in a porous medium. Astuti *et al.* (2019) attempted to provide information and limitations of the use of nanofluids in designing a cooling system in the ingineering field.

Motivated by the above studies, the current study aims at investigating the unsteady laminar two phase (Blasius and Sakiadis) MHD nanofluid flow filled with porous medium under the combined effects of Brownian motion and thermophoresis as well as variable thermal conductivity. The innovation of this paper lies in the unification of more factors into the governing equations and an attempt to give a thorough analysis of how the flow properties are affected by these factors. Also the novelty of the current paper lies in the application of the recently developed numerical method to solve these highly nonlinear equations. In this research this governing equations which are highly nonlinear partial differential equations are transformed into nonlinear ordinary differential equations. The newly developed spectral quasi-linearization method will be applied to obtain numerical solutions to the current model. The effects or influences of nondimensional governing pertinent parameter on the dimensionless velocity, temperature and concentration profiles as well skin friction, Nusselt number and Sherwood number will be discussed with the aid of graphs and/or tables.

2. MATHEMATICAL FORMULATION

We consider the unsteady laminar two-phase (Blasius and Sakiadis) flow filled with porous media under the combined effects of Brownian motion and thermophoresis as well as thermal radiation. Variable thermal conductivity is also considered in this study. The cartesian coordinate has the origin at the leading edge with the $x-$axis extending along the sheet in the flow direction and the $y-$axis is measured perpendicular to the flow. The unsteady fluid flow, heat and mass transfer start at time(t) =0 and the sheet is being stretched with velocity $U_w(x,t)$ along the $x-$axis while the origin is kept fixed.

The temperature and the concentration fields of the sheet are respectively, $T_w(x,t)$ and $C_w(x,t)$ and are summed to be linear functions of x. The thermo-physical properties of the sheet and the ambient fluid are assumed to be constant except density variations and thermal conductivity which we assume to be linearly varying with temperature. Under these assumptions together with the Boussinesq and boundary layer approximations, the governing equations for the model under consideration are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}\pm g\beta(T-T_\infty)\pm g\beta_c(C-C_\infty) -\frac{\sigma B_0^2}{\rho}u-\frac{\varphi}{K_o}u, \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(J(T)\frac{\partial T}{\partial y}\right)-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} +\frac{\tau}{\rho c_p}\left(D_B\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right)+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2, \tag{3}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-K_n(C-C_\infty). \tag{4}$$

The dimensional boundary conditions are:

1. Blasius problem:

$$u=0,\ v=0 \quad \text{at } y=0 \tag{5}$$
$$u=U \quad \text{as } y\to\infty \tag{6}$$

2. Sakiadis problem:

$$u=U, v=0 \quad \text{at } y=0, \tag{7}$$
$$u=0 \quad \text{as } y\to\infty \tag{8}$$

$$T=T_w,\ \ C=C_w \ \text{ at } y=0, \quad T=T_\infty,\ C=C_\infty,\ \text{as } y\to\infty. \tag{9}$$

The thermal conductivity is assumed to linearly vary with temperature, (Narsu and Kumar, 2018) as:

$$J(T)=J_\infty\left(1+\epsilon\left(\frac{T-T_\infty}{T_w-T_\infty}\right)\right), \tag{10}$$

ϵ is a small parameter known as the variable conductivity parameter, and J_∞ is the thermal conductivity of the fluid far away from the sheet. The radiative heat flux following Hossain *et al.* (1999) and Raptis (1998), can be expressed(Roseland approximation) as

$$q_r=-\frac{4\sigma^*}{3\kappa^*}\frac{\partial T^4}{\partial y}. \tag{11}$$

Here, σ^* and κ^* are respectively, the Stephan-Boltzmann constant and the mean absorption coefficient. We assume that the temperature different within flow is such that the term T^4 can be expressed as a linear function of temperature. Hence, expanding T^4 in Taylor series about T_∞ and neglecting higher-order terms, we obtain

$$T^4\approx 4T_\infty^3 T-T_\infty^4. \tag{12}$$

The stretching velocity is assumed as $U_w(x,t)=ax/(1-ct)$, where a and c are constants (with $a\geq 0$ and $c\geq 0$, where $ct\leq 1$) and both have dimension $(time)^{-1}$ and a as the initial stretching rate $a/(1-ct)$ and it is increasing with time. We also assume the surface temperature, $T_w(x,t)$ and concentration, $C_w(x,t)$ of the stretching to vary with the

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distance x and an inverse square law for its decrease with time in the following form:

$$T_w(x,t) = T_\infty + \frac{bx}{(1-ct)^2}, \quad C_w(x,t) = C_\infty + \frac{bx}{(1-ct)^2}, \tag{13}$$

We choose these particular forms of $U_w(x,t)$, $T_w(x,t)$ and $C_w(x,t)$ to obtain similarity transformation. We introduce the stream function ψ which is defined in the usual way, a similarity variable η and the following similarity transformations.

$$\eta = \left(\frac{a}{\nu(1-ct)}\right)^{\frac{1}{2}} y, \quad \psi = \left(\frac{\nu a}{(1-ct)}\right)^{\frac{1}{2}} x f(\eta)$$
$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \quad \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty}, \tag{14}$$

where, $\psi(x,y,t)$ is a stream function defined as $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$. The engineering important characteristics of the flow are the skin friction coefficient, the local Nusselt and Sherwood numbers, defined respectively, as:

$$C_f = \frac{\tau_w}{\rho U_w^2/2}, \quad Nu_x = \frac{x q_w}{k_\infty}(T_w - T_\infty), \quad Sh_x = \frac{x j_x}{k_\infty(C_w - C_\infty)}, \tag{15}$$

where $\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$, $q_w = -k_\infty\left(\frac{\partial T}{\partial y} + q_r\right)$, $j_3 = -k_\infty\left(\frac{\partial C}{\partial y}\right)_{y=0}$. Upon substituting the similarity variables into the governing equations we obtain:

$$\frac{d^3f}{d\eta^3} + f\frac{d^2f}{d\eta^2} - \left(\frac{df}{d\eta}\right)^2 - A\left(\frac{df}{d\eta} + \frac{\eta}{2}\frac{d^2f}{d\eta^2}\right) \pm \lambda_T\theta \pm \lambda_C\phi$$
$$-(M+K)\frac{df}{d\eta} = 0, \tag{16}$$

$$\frac{1}{Pr}\left(\frac{d^2\theta}{d\eta^2} + \varepsilon\theta\frac{d^2\theta}{d\eta^2} + \varepsilon\left(\frac{d\theta}{d\eta}\right)^2 + N_r\frac{d^2\theta}{d\eta^2}\right) - A\left(2\theta + \frac{\eta}{2}\frac{d\theta}{d\eta}\right)$$
$$-\begin{vmatrix} \frac{df}{d\eta} & \frac{d\theta}{d\eta} \\ f & \theta \end{vmatrix} + N_b\frac{d\theta}{d\eta}\frac{d\phi}{d\eta} + N_t\left(\frac{d\theta}{d\eta}\right)^2 + E_c\left(\frac{d^2f}{d\eta^2}\right)^2 = 0, \tag{17}$$

$$\frac{d^2\phi}{d\eta^2} - LeA\left(2\phi + \frac{\eta}{2}\frac{d\phi}{d\eta}\right) + Le\begin{vmatrix} \frac{d\phi}{d\eta} & \frac{df}{d\eta} \\ \phi & f \end{vmatrix} + \frac{N_t}{N_b}\frac{d^2\theta}{d\eta^2}$$
$$-LeKr\phi = 0. \tag{18}$$

Where the following definitions:

$$A = \frac{c}{a}, \quad N_t = \frac{\tau D_T(T_w - T_\infty)}{\nu T_\infty}, \quad N_b = \frac{\tau D_B(C_w - C_\infty)}{\nu},$$
$$\lambda_T = \frac{g\beta_T x(T_w - T_\infty)}{U_w^2}, \lambda_C = \frac{g\beta_C x(C_w - C_\infty)}{U_w^2}, \quad N_r = \frac{16\sigma^* T_\infty^3}{3\kappa^* J_\infty},$$
$$E_c = \frac{U_w^2}{c_p(T_w - T_\infty)}, \quad M = \frac{\sigma B_0^2(1-ct)}{a\rho},$$
$$K = \frac{\varphi(1-ct)}{aK_o}, \quad Pr = \frac{\rho\nu c_p}{J_\infty}, \quad Le = \frac{\nu}{D_B}.$$

The dimensionless boundary conditions (5) - (9) becomes:

1. Blasius problem:

$$f(0) = 0, \; f'(0) = 0, \; \theta(0) = 1, \; \phi(0) = 1 \quad \text{at } \eta = 0 \tag{19}$$
$$f'(\infty) = 1, \; \theta(\infty) = 0, \; \phi(\infty) = 0 \quad \text{as } \eta \to \infty \tag{20}$$

2. Sakiadis problem:

$$f(0) = 0, \; f'(0) = 1, \; \theta(0) = 1, \; \phi(0) = 1 \quad \text{at } \eta = 0, \tag{21}$$
$$f'(\infty) = 0, \; \theta(\infty) = 0, \; \phi(\infty) = 0 \quad \text{as } \eta \to \infty \tag{22}$$

3. NUMERICAL SOLUTION OF THE PROBLEM

In this work we use the spectral quasi-linearization method (SQLM) to solve the system of non-linear ordinary differential equations (16) - (18) subject to boundary conditions (19) - (22). The SQLM introduced by Motsa *et al.* Motsa (2013) is a combination of a quasi-linearization method (QLM) and Chebyshev spectral collocation method. The QLM, originally proposed by Bellman and Kalaba (1965), is generally a Newton-Raphson based method for solving functional equations. The iterative scheme obtained from the QLM is then solved using the Chebyshev pseudospectral method. For further reading on the derivation of the quasi-linearization scheme we refer the interested reader to Motsa *et al.* (2014). Applying the SQLM on the system of non-linear ordinary differential equations (16) - (18) gives an iterative sequence of linear equations:

$$a_{0,s}\frac{d^3f_{s+1}}{d\eta^3} + a_{1,s}\frac{d^2f_{s+1}}{d\eta^2} + a_{2,s}\frac{df_{s+1}}{d\eta} + a_{3,s}f_{s+1} \pm a_{4,s}\theta_{s+1}$$
$$\pm a_{5,s}\phi_{s+1} = R_{1,s}, \tag{23}$$
$$b_{0,s}\frac{d^2\theta_{s+1}}{d\eta^2} + b_{1,s}\frac{d\theta_{s+1}}{d\eta} + b_{2,s}\theta_{s+1} + b_{3,s}\frac{d^2f_{s+1}}{d\eta^2} + b_{4,s}\frac{df_{s+1}}{d\eta}$$
$$+b_{5,s}f_{s+1} + b_{6,s}\frac{d\phi_{s+1}}{d\eta} = R_{2,s}, \tag{24}$$
$$c_{0,s}\frac{d^2\phi_{s+1}}{d\eta^2} + c_{1,s}\frac{d\phi_{s+1}}{d\eta} + c_{2,s}\phi_{s+1} + c_{3,s}\frac{df_{s+1}}{d\eta} + c_{4,s}f_{s+1}$$
$$+c_{5,s}\frac{d^2\theta_{s+1}}{d\eta^2} = R_{3,s}, \tag{25}$$

subject to the same boundary conditions (19) - (21) except that they are now evaluated at the current iteration level denoted by the subscript $s+1$. The variable coefficients have the following definitions:

$$a_{0,s} = 1, \quad a_{1,s} = f_s - \frac{A\eta}{2}, \quad a_{2,s} = -2\frac{df_s}{d\eta} - A - M - K,$$
$$a_{3,s} = \frac{d^2f_s}{d\eta^2}, a_{4,s} = \lambda_T,$$
$$a_{5,s} = \lambda_C, \quad b_{0,s} = \frac{1}{Pr} + \frac{\epsilon\theta_s}{Pr} + \frac{Nr}{Pr},$$
$$b_{1,s} = \frac{2\epsilon}{Pr}\frac{d\theta_s}{d\eta} - \frac{A\eta}{2} + f_s + N_b\frac{d\phi_s}{d\eta} + 2N_t\frac{d\theta_s}{d\eta},$$
$$b_{2,s} = \frac{\epsilon}{Pr}\frac{d^2\theta_s}{d\eta^2} - 2A - \frac{df_s}{d\eta},$$
$$b_{3,s} = 2E_c\frac{d^2f_s}{d\eta^2}, \quad b_{4,s} = -\theta_s, \quad b_{5,s} = \frac{d\theta_s}{d\eta}, \quad b_{6,s} = N_b\frac{d\theta_s}{d\eta},$$
$$c_{0,s} = 1, \quad c_{1,s} = Lef_s - Le\frac{A\eta}{2}, \quad c_{2,s} = -2LeA - Le\frac{df_s}{d\eta} - LeK_r,$$
$$c_{3,s} = -Le\phi_s,$$
$$c_{4,s} = Le\frac{d\phi_s}{d\eta}, \quad c_{5,s} = Le\frac{N_t}{N_b}, \quad R_{1,s} = f_s\frac{d^2f_s}{d\eta^2} - \left(\frac{df}{d\eta}\right)^2,$$
$$R_{2,s} = \frac{\epsilon\theta_s}{Pr}\frac{d^2\theta_s}{d\eta^2} + \frac{\epsilon}{Pr}\left(\frac{d\theta_s}{d\eta}\right)^2 -$$
$$\theta_s\frac{df_s}{d\eta} + f_s\frac{d\theta_s}{d\eta} + N_b\frac{d\theta_s}{d\eta}\frac{d\phi_s}{d\eta} + N_t\left(\frac{d\theta_s}{d\eta}\right)^2 + E_c\left(\frac{d^2f_s}{d\eta^2}\right)^2,$$
$$R_{3,s} = Lef_s\frac{d\phi_s}{d\eta} - Le\phi_s\frac{df_s}{d\eta}.$$

To apply the spectral collocation method, it is convenient to migrate from the physical domain $[0,\infty)$ to the computational domain $[-1,1]$ using a linear transformation $x(\eta) = \frac{2}{L_\infty}\eta - 1$. L_∞ is a sufficiently large number used to replace the semi-infinite interval $[0,\infty)$ with the closed

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interval $[0, L_\infty]$.

$$\begin{bmatrix} \mathbf{A_{11}} & \mathbf{A_{12}} & \mathbf{A_{13}} \\ \mathbf{A_{21}} & \mathbf{A_{22}} & \mathbf{A_{23}} \\ \mathbf{A_{31}} & \mathbf{A_{32}} & \mathbf{A_{33}} \end{bmatrix} \begin{bmatrix} \mathbf{F}_{s+1} \\ \mathbf{\Theta}_{s+1} \\ \mathbf{\Phi}_{s+1} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{1,s} \\ \mathbf{R}_{2,s} \\ \mathbf{R}_{2,s} \end{bmatrix} \tag{26}$$

where

$$\mathbf{A}_{11} = \mathbf{a}_{0,s}\mathbf{D}^3 + \mathbf{a}_{1,s}\mathbf{D}^2 + \mathbf{a}_{2,s}\mathbf{D} + \mathbf{a}_{3,s}, \quad \mathbf{A}_{12} = \mathbf{a}_{4,s}, \quad \mathbf{A}_{13} = \mathbf{a}_{5,s},$$
$$\mathbf{A}_{21} = \mathbf{b}_{3,s}\mathbf{D}^2 + \mathbf{b}_{4,s}\mathbf{D} + \mathbf{b}_{5,s}, \ \mathbf{A}_{22} = \mathbf{b}_{0,s}\mathbf{D}^2 + \mathbf{b}_{1,s}\mathbf{D} + \mathbf{b}_{2,s},$$
$$\mathbf{A}_{23} = \mathbf{b}_{6,s}\mathbf{D}, \ \mathbf{A}_{31} = \mathbf{c}_{3,s}\mathbf{D} + \mathbf{c}_{4,s}, \ \mathbf{A}_{32} = \mathbf{c}_{5,s}\mathbf{D}^2,$$
$$\mathbf{A}_{33} = \mathbf{c}_{0,s}\mathbf{D}^2 + \mathbf{c}_{1,s}\mathbf{D} + \mathbf{c}_{2,s},$$
$$\mathbf{F}_{s+1} = [f_{s+1}(\eta_0), f_{s+1}(\eta_1), \cdots, f_{s+1}(\eta_{N-1}, f_{s+1}(\eta_N)]^T,$$
$$\mathbf{\Theta}_{s+1} = [\theta_{s+1}(\eta_0), \theta_{s+1}(\eta_1), \cdots, \theta_{s+1}(\eta_{N-1}, \ \theta_{s+1}(\eta_N)]^T,$$
$$\mathbf{\Phi}_{s+1} = [\phi_{s+1}(\eta_0), \phi_{s+1}(\eta_1), \cdots, \phi_{s+1}(\eta_{N-1}, \phi_{s+1}(\eta_N)]^T.$$

4. RESULTS AND DISCUSSION

The differential equations under study were solved using SQLM. The convergence of the SQLM solution was confirmed by the use of solution based errors. These errors are obtained by computing the differences between approximate solutions at the previous and current iteration levels s and $s+1$, respectively. The error norms are defined as follows:

$$Error_f = max_{0\le i\le N}||F_{s+1,i} - F_{s,i}||_\infty,$$
$$Error_\theta = max_{0\le i\le N}||\theta_{s+1,i} - \theta_{s,i}||_\infty,$$
$$Error_\phi = max_{0\le i\le N}||\phi_{s+1,i} - \phi_{s,i}||_\infty.$$

Figure 1 shows that the error in each of the functions f, θ and ϕ decreases with increasing number of iterations. The solutions converge after the 6th, 7th and 8th iterations, respectively.

For all the numerical results in this work, the default parameter values considered are: $\lambda_T = 3$, $\lambda_C = 1$, $Nb = 0.2$, $\epsilon = 0.2$, $A = 0.2$, $M = 0.5$, $Le = 0.3$.

Figure 2 displays the effects of the unsteadiness (A) on the velocity profiles of both the Blasius and Sakiadis flows. Though the flows behave differently, the velocity profiles are reduced when the unsteadiness parameter increases. We observe that stretching of sheet can be used as a flow stabilizer as the boundary layer thicknesses are reduced with increasing values of the unsteadiness parameter. Figures 3 and 4 show how the temperature and concentration profiles are respectively, influenced by the unsteadiness parameter. We observe that both boundary layer thickness are reduced with increasing values of the unsteadiness parameter A. In both figures we notice that the Blasius flow has thinker boundary layers flow than the Sakiadis flow. As the unsteadiness parameter comes as the sheet is stretching, increasing its values implies the increase of the surface area over which the fluid flows. This explains why the fluid properties like velocity, temperature concentration are reduced as depicted in Figures 2, 3 and 4.

Figure 5 depicts the effect of varying the values of the variable thermal conductivity parameter. The fluid velocity is greatly enhanced by increasing the thermal conductivity of the fluid in both Blasius and Sakiadis flows. As can be seen on Figures 6 and 7 an increase in the thermal conductivity of the flow system enhances the rate of heat transfer thus causing the thickening of both the thermal and solutal boundary layers. Increasing the values of the thermal conductivity implies the increase of more temperature distribution within the flow system this then physically results in the thickening of the thermal boundary layer. Physically, heat generation within the fluid, lightens the fluid particles thus increasing the fluid velocity in both cases. However, though it has little effect on the concentration, the heating of the fluid implies acceleration processes like chemical reaction which in turn reduces the fluid concentration within the flow system. The influence of the Brownian motion parameter Nb on the fluid velocity profiles on both types of flow is depicted on Figure

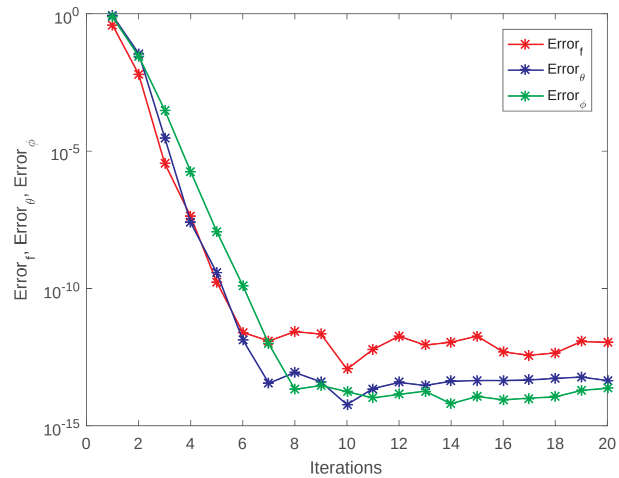

Fig. 1 Infinity error norms against iterations

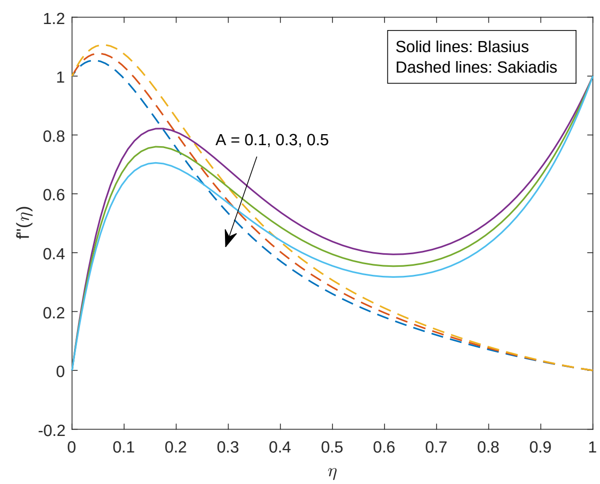

Fig. 2 The influence of A on the velocity profiles

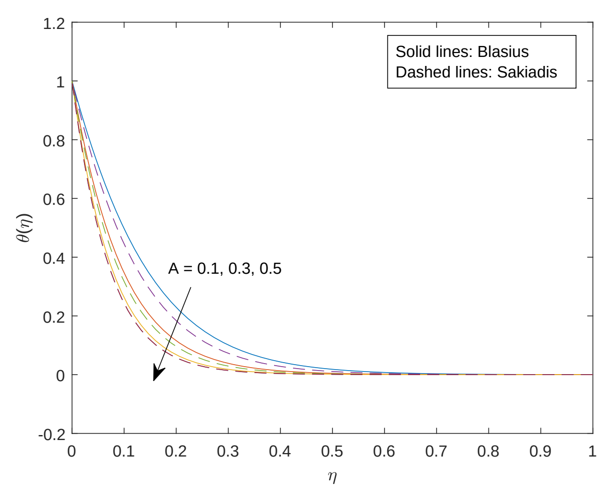

Fig. 3 The influence of A on the temperature profiles

8. The velocity profiles are reduced as the Brownian motion parameter increases with increasing values of the Brownian motion parameter thus

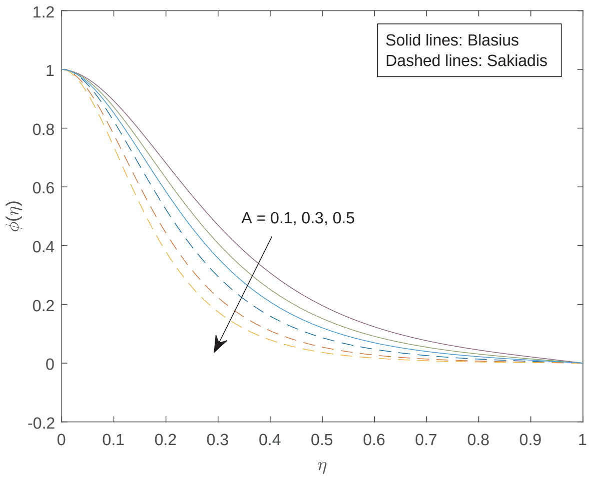

Fig. 4 The influence of A on the concentration profiles

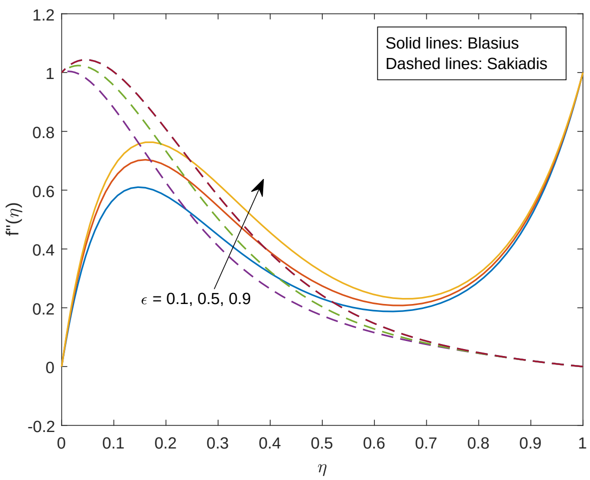

Fig. 5 The influence of ϵ on the velocity profiles

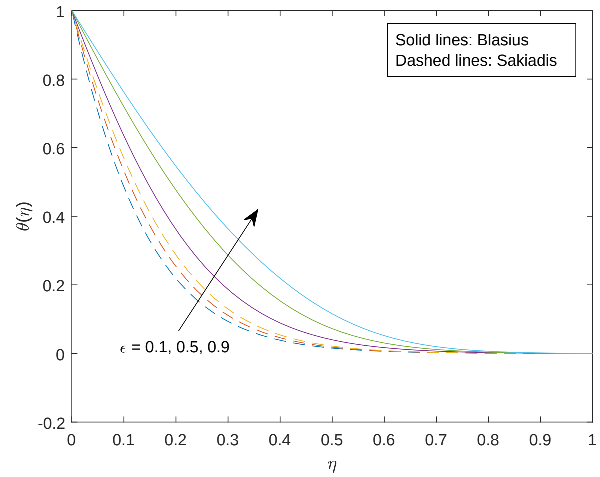

Fig. 6 The influence of ϵ on the temperature profiles

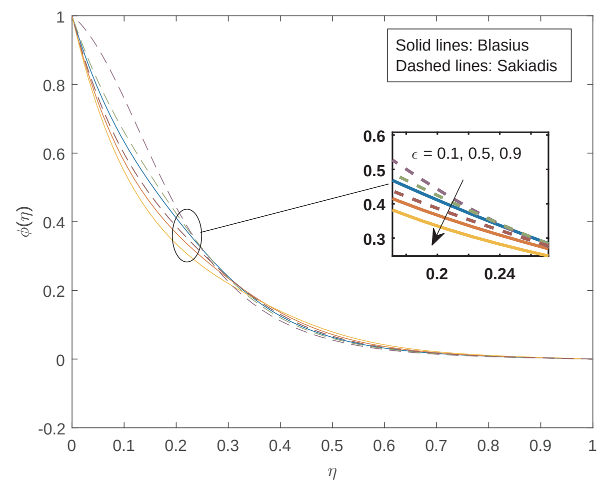

Fig. 7 The influence of ϵ on the concentration profiles

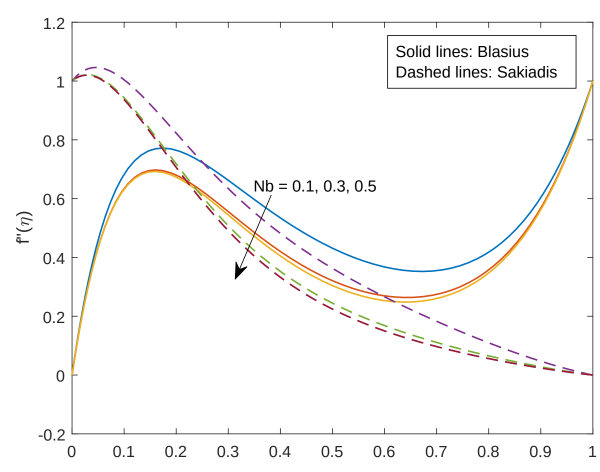

Fig. 8 The influence of Nb on the velocity profiles

increases the fluid temperature, Figure 9 . We clearly observe in Figure 10 that increasing the values of the Brownian motion parameter causes a sharp increase of the concentration profiles near the surface wall. The influence is significantly clear on the Blasius flow. However, away from the wall the concentration profiles decrease with increasing values of the Brownian motion. Physically, Brownian motion is the random motion of particles suspended in a fluid. Therefore increasing the Brownian motion parameter results in the slowing down of the overall fluid flow. This can be likened to traffic congestion. However, the collision of particles with the fast moving molecules within the fluid results in the heat generation thus increasing the temperature distribution within the system.

Figure 11 depicts the influence of the Eckert number on the velocity profiles. We notice that in both the Blasius and Sakiadis flows, there are sharp increases of the velocity profiles as the Eckert number increases. Though the Eckert number does not directly influence the momentum equation but through the energy equation it has significant influence on the velocity profile. Physically, improving the dissipation causes the interaction among the particles less therefore this causes the velocity profiles to be depressed. We also remark that increasing the values of the Eckert number generates heat energy in the fluid due to frictional heating. This in urn enhances the temperature at any given point as can be clearly see in Figure 12.

The influence of the thermophoresis parameter on the velocity profiles is depicted on Figure 13. The velocity profiles are increasing as the values of the thermophoresis parameter Nt, increase. We also observe

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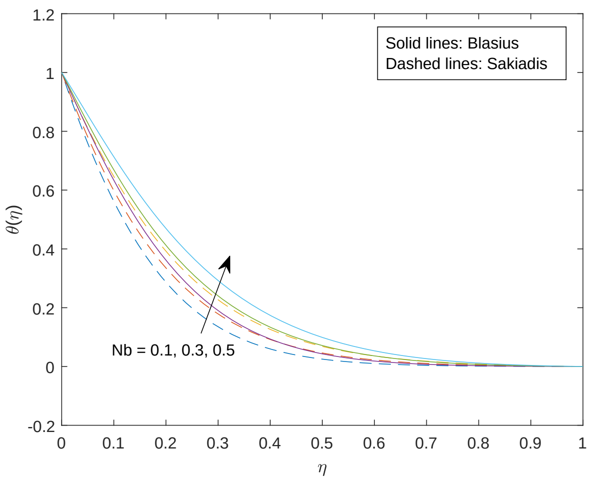

Fig. 9 The influence of Nb on the temperature profiles

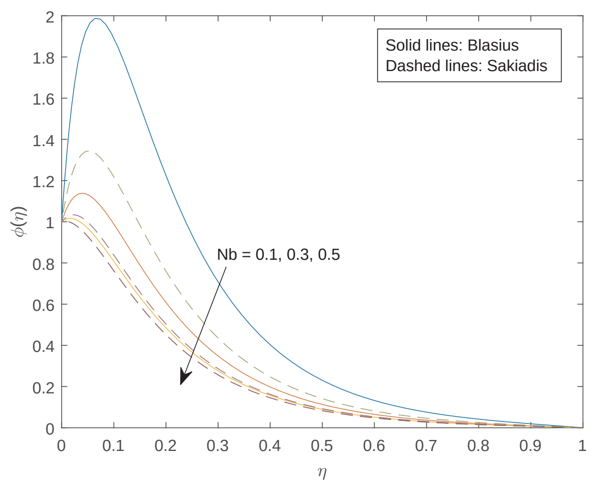

Fig. 10 The influence of Nb on the concentration profiles

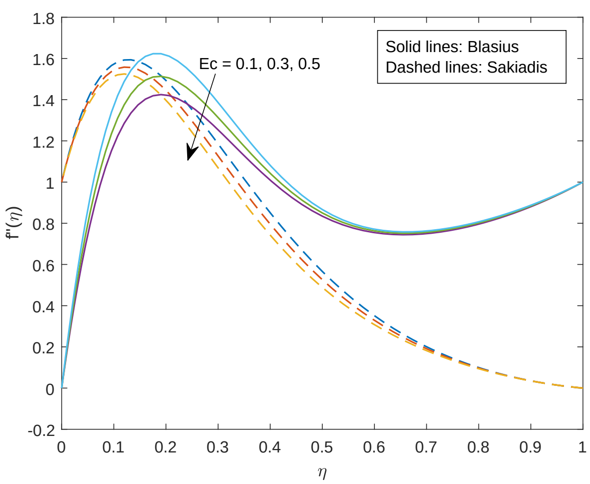

Fig. 11 The influence of Ec on the velocity profiles

from Figures 14 and 15, that the temperature and solutal boundary layers are significantly increased by increasing values of the thermophoresis

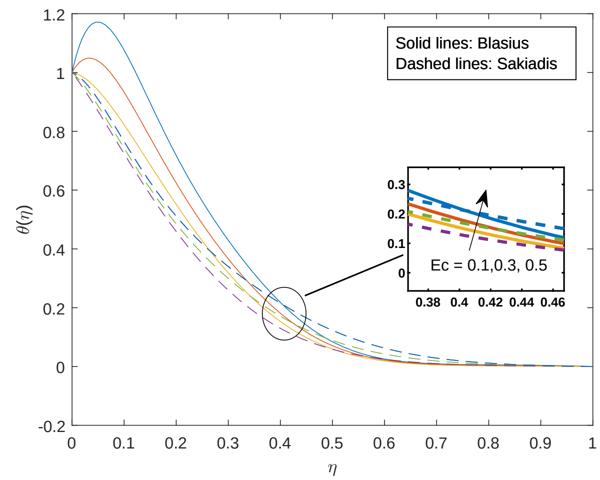

Fig. 12 The influence of Ec on the temperature profiles

parameter. Physically, the force induced by thermophoresis has the tendency to diffuse the particles from hotter to cooler areas thus increasing the temperature and concentration distribution within the fluid flow.

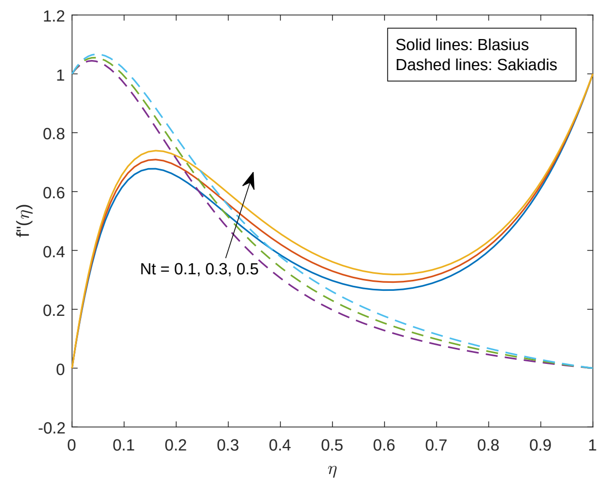

Fig. 13 The influence of Nt on the velocity profiles

Figures 16 and 17 display the influence of the thermal radiation parameter on the velocity and temperature profiles in both two types of flows. It is clearly observed that the thermal radiation parameter significantly enhances both the velocity and temperature profiles. This is because increasing the values of the thermal radiation parameter results in the generation of heat energy in the flow.

Figures 18 - 20 respectively, depict the influence of the porous parameter K on the fluid velocity, temperature and concentration. It is clearly seen on Figure 18 that the dimensionless velocity decreases for both cases of Blasius and Sakiadis characteristics. Physically, it must be noted that as K increases, the porous medium becomes tighter, therefore the resistance against the flow increases. Thus this reduction in fluid velocity makes more heat and concentration of the fluid to increase as displayed in Figures 19 and 20.

Table 1 displays the influence of the unsteadiness parameter, magnetic parameter, thermophoresis, Eckert number as well as the Lewis number on the skin friction, heat transfer and mass transfer causes the skin friction as well as the Nusselt number to be increased which the Sherwood number is reduced in the case of the Sakiadis flow. However,

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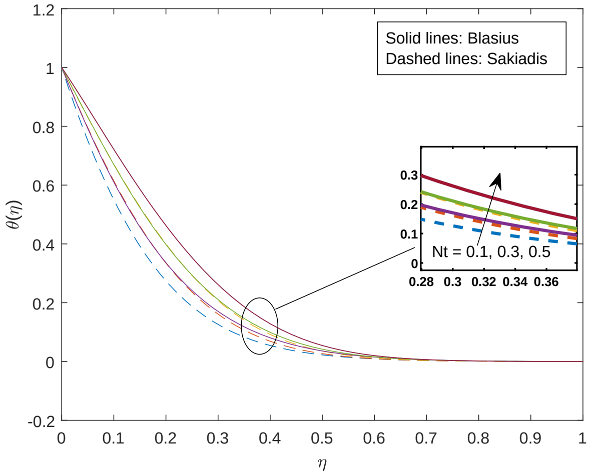

Fig. 14 The influence of Nt on the temperature profiles

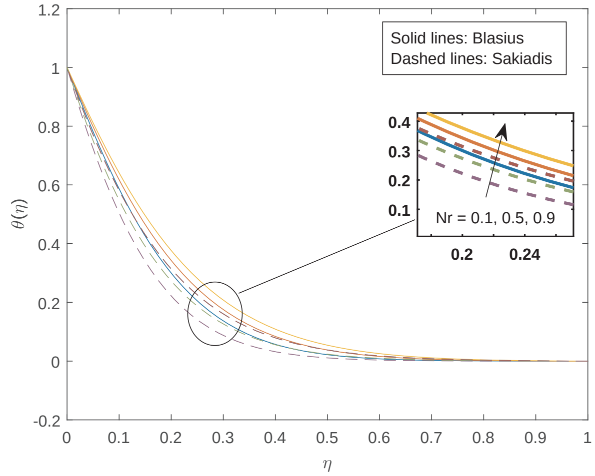

Fig. 17 The influence of Nr on the temperature profiles

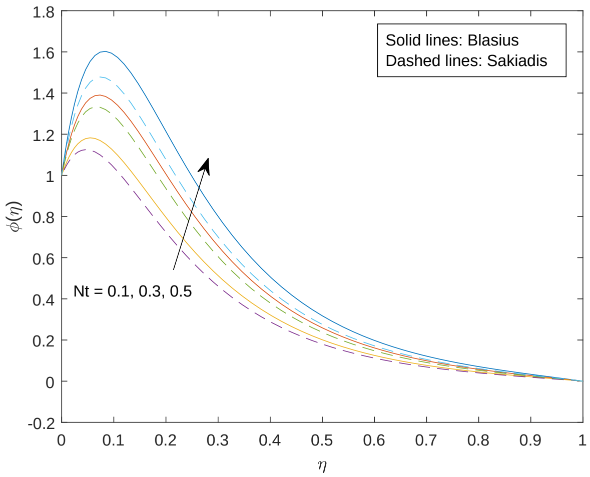

Fig. 15 The influence of Nt on the concentration profiles

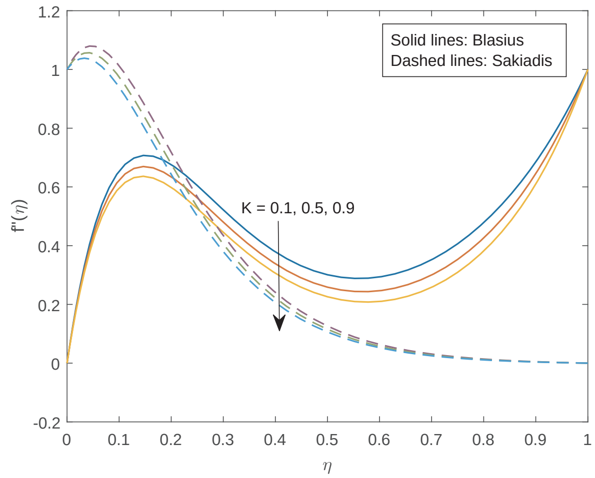

Fig. 18 The influence of K on the velocity profiles

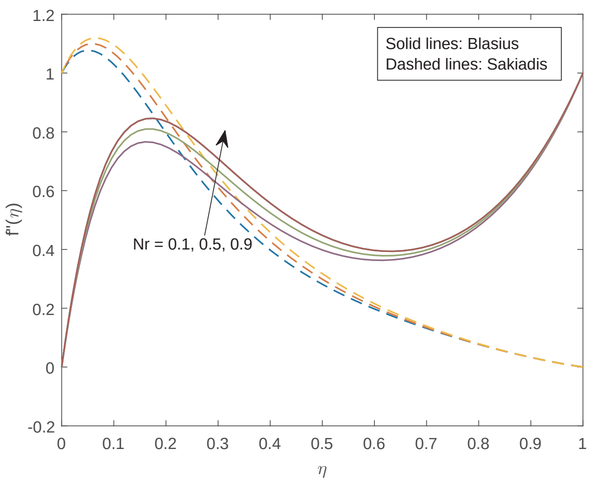

Fig. 16 The influence of Nr on the velocity profiles

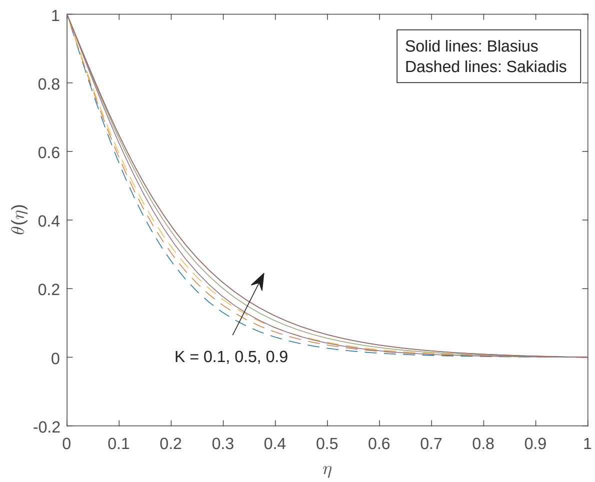

Fig. 19 The influence of K on the temperature profiles

an opposite trend happens in the case of the Blasius flow Table 2. The magnetic field, Lewis number and the Brownian motion parameters enchance the skin friction coefficient while it is reduced by respectively, increases of the values of the Eckert number and thermophoresis param-

DOI: 10.5098/hmt.14.18

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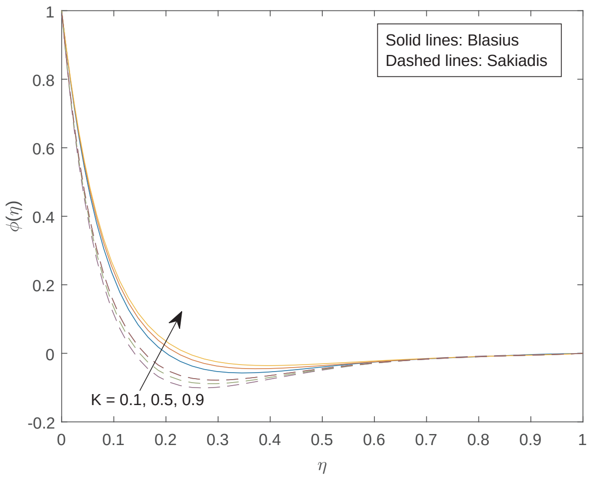

Fig. 20 The influence of K on the concentration profiles

eter in the Sakiadis case. However, the opposite trend of the influences of these parameter is observed in the Blasius flow case. We clearly observe in both flows that the Sherwood number is increased by increasing the values of the magnetic parameter as well as the Lewis number.

Table 1 The values of $f''(0,\xi)$, $-\theta'(0,\xi)$ and $-\phi'(0,\xi)$ for different values of the parameters A, M, ϵ, N_b, E_c and K for Sakiadis flow.

A	M	Nt	Nb	E_c	Le	$-f''(0,\xi)$	$-\theta'(0,\xi)$	$-\phi'(0,\xi)$
0.1						0.9131	0.6282	0.2242
0.3						1.0029	0.6805	0.2167
0.5						1.0872	0.7293	0.2111
	0.5					0.9587	0.6805	0.2167
	1.0					1.1526	0.6628	0.2190
	1.5					1.3270	0.6477	0.2211
		0.1				0.9783	0.6883	0.3683
		0.3				0.9587	0.6805	0.2167
		0.5				0.9399	0.6727	0.0712
			0.1			0.8375	0.7359	0.8424
			0.3			0.9402	0.6995	0.4667
			0.5			0.9587	0.6805	0.2167
				0.1		0.9584	0.6680	0.2239
				0.3		0.9576	0.6429	0.2384
				0.5		0.9569	0.6179	0.2528
					0.1	0.9587	0.6805	0.2167
					0.3	0.9982	0.6563	0.5587
					0.5	1.0241	0.6410	0.8118

5. CONCLUSION

The current problem investigates the influence of significant parameters on fluid velocity, temperature and concentration as the skin friction, the Nusselt and Sherwood numbers. The governing partial differential equations are first reduced into a system of nonlinear ordinary differential equations. The resultant ordinary differential equations are then numerically by an efficient spectral quasi-linearization method. This method was found to be very accurate with tolerance levels of between 10^{12} and 10^{-14}. Based on the current investigation, the following main observations are made:

1. The hydrodynamic boundary layer decreases with increasing values of the stretching parameter, the magnetic parameter, the Brownian motion parameter, and Eckert number, Lewis number and porosity parameter.

2. The fluid velocity increases as the values of variable thermal conductivity parameter, the thermal radiation parameter as well the thermophoresis parameter.

3. The fluid temperature is an increasing function of the magnetic, porosity, thermal conductivity, thermal radiation, Brownian motion parameters as well as the Eckert number.

4. The stretching parameter, the thermal conductivity, the Brownian motion as well as the Lewis number were found to reduce the fluid concentration.

5. The concentration was observed to be increasing as the values of the magnetic, thermophoresis and porosity parameters.

6. The magnetic field parameter, the Lewis number and the Brownian motion enhance the skin friction coefficient whilst it is reduced by respectively, increasing the values of the Eckert number and thermophoresis in the Sakiadis case. However, the opposite trend of the influences of these parameters is observed in the Blasius flow case.

Table 2 The values of $f''(0,\xi)$, $-\theta'(0,\xi)$ and $-\phi'(0,\xi)$ for different values of the parameters A, M, ϵ, N_b, E_c and K for Blasius flow.

A	M	Nt	Nb	E_c	Le	$-f''(0,\xi)$	$-\theta'(0,\xi)$	$-\phi'(0,\xi)$
0.1						0.5378	0.3974	0.2875
0.3						0.5083	0.5213	0.2613
0.5						0.4829	0.6280	0.2452
	0.5					0.6381	0.4905	0.2701
	1.0					0.5716	0.4742	0.2715
	1.5					0.5225	0.4619	0.2729
		0.1				0.5123	0.4695	0.3455
		0.3				0.5225	0.4619	0.2729
		0.5				0.5321	0.4544	0.2058
			0.1			0.6444	0.5076	0.7300
			0.3			0.5482	0.4706	0.6315
			0.5			0.5321	0.4544	0.2059
				0.1		0.5226	0.4588	0.2745
				0.3		0.5223	0.4525	0.2778
				0.5		0.5233	0.4462	0.2812
					0.1	0.5225	0.4619	0.2729
					0.3	0.4979	0.4463	0.5000
					0.5	0.4814	0.4364	0.6675

We observed in the current study that the most dominating parameters on the fluid properties are unsteadiness parameter, the thermal conductivity parameter, the Brownian motion parameter and the porosity parameter. Lastly, we recommend that the since both the Blasius and the Sakiadis flows are similarly affected or influenced by all the parameters analysed in the current problem, there's really no need to investigate them simultaneously.

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NOMENCLATURE

A	unsteadiness parameter
a	non negative constant(s^{-1})
b	non negative constant K/m
c	non negative constant(s^{-1})
C	fluid concentration mass (kg/kmol)
C_∞	ambient fluid concentration (kg/kmol)
c_p	specific heat (J/kg · K)
D_B	Brownian diffusion constant (m^2/s)
D_m	mass diffusivity m^2/s
D_T	thermophoretic diffusion coefficient (m^2/s)
Ec	Eckert number
g	acceleration due to gravity m/s^2
K	porous medium parameter m^2
$J(T)$	variable thermal conductivity (W/m · K)
J_∞	thermal conductivity far away from the sheet (W/m · K)
Le	Lewis number
M	magnetic parameter
N_b	Brownian motion parameter
N_r	thermal radiation parameter
N_t	thermophoresis parameter
Pr	Prandtl number
q_w	heat flux W/m^2
t	time (s)
T	fluid temperature (K)
T_∞	ambient fluid temperature (K)
u	velocity in the x-direction (m/s)
$U_w(x,t)$	velocity of the stretched sheet
v	velocity component in the y-direction
x	coordinate (m)
y	coordinate (m)
Greek Symbols	
β	coefficient of thermal viscosity
ε	a small parameter known as variable conductivity parameter
ρ	fluid density (kg/m^3)
σ^*	Stefan-Boltzmann constant (W/m^2)
κ^*	mean absorption constant
ν	kinematic viscosity
λ_T	thermal Grashof number
λ_C	concentration Grashof number
$\psi(x,y,t)$	a stream function
Superscripts	
w	at the wall surface
∞	ambient environment

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