Up and beyond: Building a mountain in the Netherlands

Up and Beyond - Building a Mountain in the Netherlands

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Abstract

We discuss the idea of building a 2 km high mountain in the Netherlands. In this paper, we give suggestions on three important areas for the completion of this project. Issues like location, structure and sustainability are investigated and discussed in detail.

KEYWORDS: building a mountain, high structure, the Netherlands

1 Introduction

The Netherlands does not have any tall mountains. Indeed, its name even derives from the fact that it is essentially flat. According to Thijs Zonneveld, a journalist and former professional cyclist, this is a serious shortcoming of his country. As a possible remedy, he proposed *building* a 2 kilometer high mountain in the Netherlands. The response was immense. Immediately, there was a lot of excitement at the prospect of building a mountain, but also a fair amount of skepticism about whether it can actually be done (see [11]). In this report we aim to address some of the obstacles and opportunities that may arise in the construction of such a mountain.

The idea of building a massive structure is not new. In the past, numerous plans have been proposed for extremely tall buildings and structures. However, what all these plans have in common is that they never left the drawing board. The Dutch, however, are renowned for their large-scale engineering works such as the dikes, polders, and the Delta Works. Still, it is not hard to see that building a mountain would dwarf these accomplishments by comparison. Consider

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that currently, at a height of 828 meters, the Burj Khalifa in Dubai is the tallest building in the world – truly a marvel of modern engineering. Imagine then the extremely special care and consideration, the vast amount of work and the incredible ingenuity that is required to achieve a structure that is more than double that height.

After Thijs Zonneveld proposed building a mountain, a group of companies joined forces in the organization 'Die Berg komt er!' [9]. The aim of this organization is to bring Zonneveld's vision into reality and build a mountain. Right now, they concentrate on studying the feasibility of building a 2 kilometer high mountain in the Netherlands. One of the companies involved, Bartels Consulting Engineers, brought this problem to the Study Group Mathematics with Industry (SWI) held in Eindhoven from 30 January to 3 February 2012 to aid in this investigation.

The main questions Bartels Consulting Engineers posed at the SWI were:

1. Where should the mountain be built?
2. What shape, size and structure should the mountain have?
3. Which materials can be used to build a mountain?
4. How will the mountain impact the environment, soil levels and (local) weather?
5. Can the mountain be made sustainable?
6. How could one set up the necessary infrastructure?
7. How can the mountain be used? (Both during construction and after completion.)

Our aim during the SWI was to answer these questions as best we could. In Section 2, we discuss possible locations for the mountain and in Section 3, we discuss the impact a mountain would have on the ground it is built on. In Section 4 we make some general remarks about possible ways of constructing the mountain, in Section 5 we take a more in-depth look at possible materials that may be used, and in Section 6 we discuss possibilities for making the mountain sustainable. In Section 7 we conclude with a summary of our results, we present our conclusions and make suggestions for further research.

One final remark: we will assume throughout this report that the man-made mountain will have a height of 2 kilometers and a width at the base of roughly 14 kilometers.

2 Location

At the start of the Study Group, Bartels Consulting Engineers handed us the following selection of eight possible locations to build the mountain, which are listed in the table below. The numbers in the table corresponding to the locations are positioned on the map of the Netherlands in Figure 1.

Number of location	Description	In sea/on land
1	near Bergen aan Zee	in sea
2	near The Hague	in sea
3	off the coast of Zeeland	in sea
4	off the coast of Texel	in sea
5	in the IJsselmeer, near the Afsluitdijk	in sea
6	in the IJsselmeer, close to Flevoland	in sea
7	in the Markermeer	in sea
8	in the province Flevoland	on land

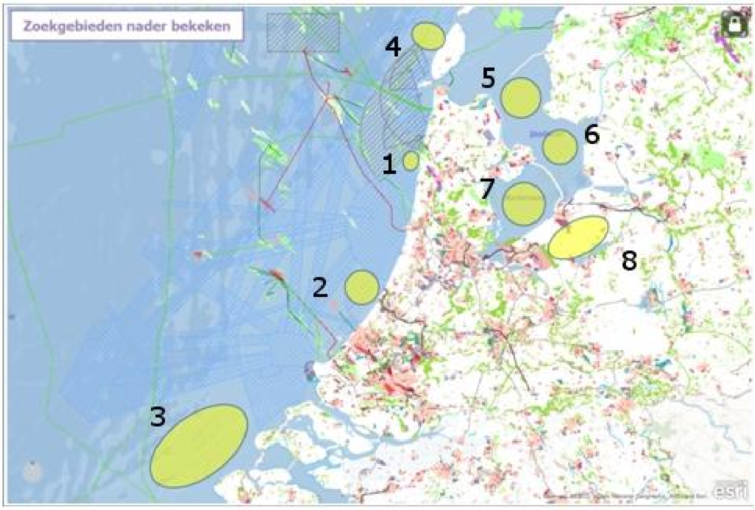

Figure 1: Eight possible locations to build the mountain. The numbers correspond to the numbers in the table of locations.

To select the most suitable location, we formulated several criteria:

- *Flight routes*. Schiphol is one of the largest airports in Europe. The air traffic should not be hindered in any way by the mountain. Hence, the mountain should not be placed on a flight route of Schiphol. These flight routes are depicted in Figure 2. Locations 2 and 7 are on busy flight routes, making them less preferable as a building location.

- *Shipping routes*. The seaport of Rotterdam is one of the largest in the world and it is crucial for the Dutch economy. Therefore, it is unwise to hinder

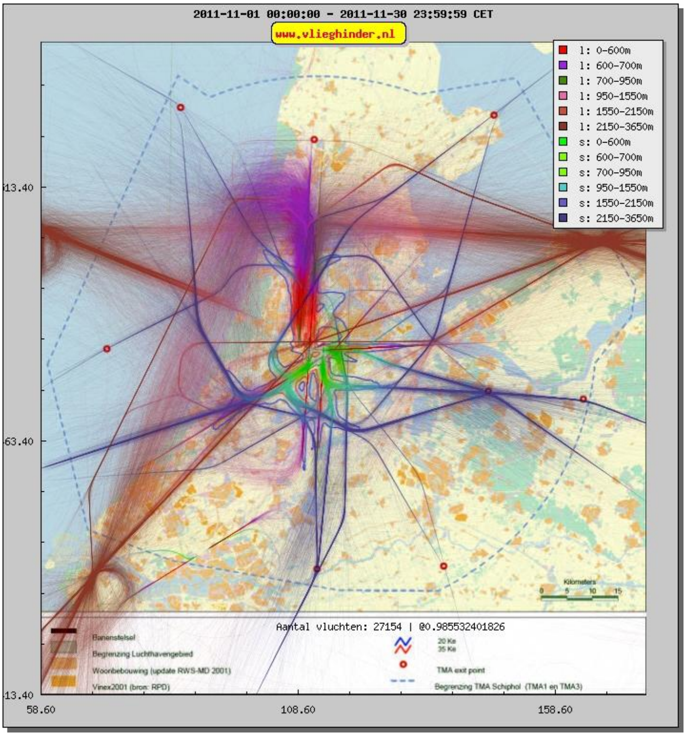

Figure 2: Routes of flights from and to Schiphol.

ship traffic to and from this seaport. Moreover, ships coming from the IJssel crossing the IJsselmeer should not be hindered either. This effectively rules out locations 3 and 6.

- *Sea currents*. The sea currents are quite strong near the North Sea coast. Building a mountain there would have a severe influence on the flow of these currents. This would cause changes in the location and shape of sand banks and would cause coastal erosion. Also, the currents will put stress on the building foundations. It would need to be verified through modeling and simulation, but it is likely that building a mountain at location 2 would have just such an amplifying effect on the local currents. This does, however, raise the question whether the power of these currents could be harnessed for energy production, for instance, by large turbines. Again, (computer) modeling would likely provide some insights.

- *Protected environment/nature*. Sustainable development and renewable energy will be a key issue in this project. Naturally, the construction and placement should have a minimal impact on the existing ecosystem, flora and fauna. Proximity of protected environments is therefore an important limiting aspect. Location 4 is close to the island of Texel, which has a rich and rather unique ecosystem, and for this reason we recommend against building the mountain there.

- *Accessibility*. Construction resources (e.g. people, material, machines) need to efficiently reach the construction site. This is hard to assess for a given location as it depends on many different factors, and needs to be looked at in more detail in future studies.
- *Impact on society*. Existing societal structures (e.g. cities, infrastructure) need to experience as little interference as possible from the project. Location 5, in particular, does not meet this criterium, as it is near important infrastructure (i.e. ‘de Afsluitdijk’).

Under these criteria, only locations 1 and 8 do not raise any immediate objections: one is in sea and the other is on land, see Figure 3. As we will explain below in Section 3, we prefer location 1, which is in the sea near Bergen aan Zee because we estimate that the soil will respond rather extremely to the pressure that the mountain would exert, and this would have less severe consequences if this happened off-shore.

Figure 3: The two locations that we believe are most favorable. One location is in the North Sea, about 15 km offshore near Bergen aan Zee. The other location is on land, situated in the province Flevoland.

3 Soil mechanics

In this section we focus on the effect of the mountain on soil, by estimating how far the mountain will sink into the ground. We also estimate how much soil will be displaced, and whether this will cause hills or depressions to form nearby. Since these effects would likely need to be prevented, we propose some methods to do so.

3.1 Model

Let us first look at a model for soil mechanics that is popular in the field of high-rise buildings (see, for example, [19]). Figure 4 shows a schematic picture of the soil and the load exerted on it by the building. The lines in the ground illustrate the curves along which the soil would slide if the load of the building is too high.

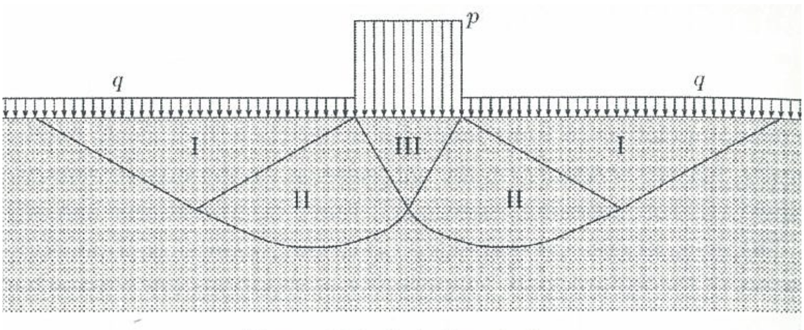

Figure 4: Popular model for the failing of soil caused by skyscrapers, [19].

The model states that if the load is higher than a certain value, the soil will slide. More specifically, zone III will move downwards, sliding along both zones II. As a result, zones II will slide sideways along the curved line at the bottom. This causes zones I to slide sideways and to rotate a bit. The result is that the ground will rise in the shape of a funnel.

These are exactly the effects that we want to avoid during and after the construction phase. However, it is not clear whether the above model can still be applied in our case. The expected area of the base of the mountain is several orders of magnitude greater than the area of the base of a large building. Furthermore, the load of the mountain on the soil (being orders of magnitude greater than the load caused by skyscrapers) causes zones II to be several kilometers deep. The soil is too heterogeneous at these depths (also containing rock type areas) for these failure curves to make sense. A more realistic model is thus needed to comprehensively understand the effects of the load of a mountain.

The shape of the mountain has not yet been decided upon, so we allow ourselves to make major simplifications in modeling the mountain and the soil. A schematic picture is given by Figure 5. We model the mountain as a solid cone. The assumptions on the size, weight and other relevant quantities are given in Table 1. For the expected weight of the mountain, we assume that it will be constructed mainly from concrete. The value of the resulting weight is estimated in Section 4. Note that these estimates only serve the purpose of supplying rough estimates on the weight and base area of the mountain.

Soil consists of layers, and our model needs to take this into account. We only use common knowledge about these layers, which can be found for example in

Quantity	Value	Description
A	$1.5 \cdot 10^8$ m^2	base area of the mountain
d	$1.4 \cdot 10^4$ m	diameter of the mountain
E	$2.0 \cdot 10^7$ N m^{-2}	static stress-strain modulus for the soil
g	9.8 m s^{-2}	Earth gravitational acceleration
h	$2 \cdot 10^3$ m	height of the mountain
ℓ	$5.0 \cdot 10^2$ m	thickness of the soft soil layer
m	$6.9 \cdot 10^{12}$ kg	mass of the mountain

Table 1: Quantities and their units.

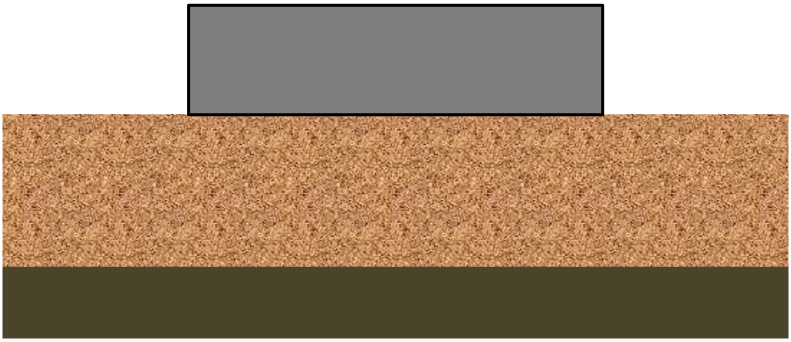

Figure 5: Schematic picture of our model of the mountain and the two soil layers. The first layer (light brown) is the soft layer. The bottom layer (dark brown) is modeled as a hard layer (not deformable), and consists of a second and third layer as mentioned below. The thickness of the layers as depicted here does not correspond to the actual ratio between them.

[10].

The first layer from the top, formed during the Quaternary period (this period started 2.6 million years ago and is still ongoing) is very thin in comparison to the other layers below, so we will neglect this layer in our model.

The second layer, formed during the Neogene period (between 23 and 2.6 million years ago) consists mainly of sublayers of either clay or sand. Its thickness depends heavily on the location. In Flevoland, it is about 500 meters thick (see [7]).

The third layer, formed during the Paleocene period (between 66 and 23 million years ago) also consists mainly of clay and sand, but because of the pressure induced by the second layer, it is more compressed. In Flevoland, this layer is also about 500 meters thick (see [7]).

The fourth layer, formed during the Cretaceous period (between 146 and 66 million years ago) consists mainly of limestone.

Based on these data, we propose a model for the soil in Flevoland with some major simplifications. We propose a two-layer system, with a soft (mobile) layer to model the Neogene layer, and a rigid (immobile) layer to model the Paleocene and Cretaceous layer. Depending on the phenomena we want to study, we model the soft layer either by an incompressible viscous fluid or by an elastic medium with a linear stress-strain relation given by $\sigma = E\epsilon$ (see Table 1), where σ denotes the stress and ϵ the strain. The value of E that we take is based upon the values of this modulus for soft clay and loose sand (see [12], Table 2-7 on page 99).

Since the values of E for the Paleocene and the Cretaceous layer are at least an order of magnitude greater, we model these layers as being rigid. Hence our model of the soil consists of two layers; the soft layer on top, and a rigid layer below (see Figure 5).

3.2 Results

First we calculate how far the mountain will sink into the soil in our model. This distance is indicated by $\Delta\ell$. We will model the part of the soft soil layer under the mountain as an elastic medium for which we have the linear stress-strain relation

$$\sigma = E\epsilon. \tag{1}$$

The stress on the soil is given by the force that the mountain exerts on the soil divided by the area: $\sigma = F/A$ (for the sake of simplicity, we assume here that the force is equally distributed over the area). F is the gravitational force of the mountain, which is given by the mass of the mountain times the gravitational acceleration: $F = mg$. The strain is given by the ratio between the thickness of the soil and how far it is compressed: $\epsilon = \Delta\ell/\ell$. Substitution of these quantities into (1) yields

$$\frac{mg}{A} = E\frac{\Delta\ell}{\ell}.$$

Hence, we obtain

$$\Delta\ell = \frac{mg\ell}{EA} \approx 11 \text{ m}, \tag{2}$$

where we have used the values in Table 1.

To predict how the soil around the mountain will react, we model the soft soil layer as an incompressible fluid, while using the result given by (2). A schematic view of this situation is given by Figure 6. If the mountain sinks a distance of $\Delta\ell = 11$ m, then this means that the following volume of soil needs to be displaced around the mountain:

$$V = \Delta\ell A \approx 1.7 \cdot 10^9 \text{ m}^3.$$

If this volume of soil would be distributed over a ring-shaped area around the mountain up to 3 kilometers away from the mountain, this would mean that on average, in this area the ground would rise 11 meters upwards.

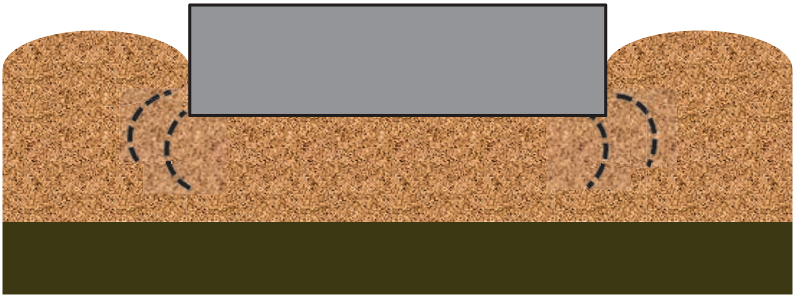

Figure 6: Schematic picture of the situation after the mountain has sunk into the soil. The shape of the surface of the soil in the surroundings is artificial.

3.3 Discussion

Since our model is a huge simplification of reality, our estimate that the mountain will sink 11 meters into the soil may be far from accurate. It is important to enhance the model to get a more accurate estimate. The estimates can be improved significantly by running computer simulations. This would allow one to take into account a number of aspects that our estimates ignore, such as the fact that the load is likely not uniformly distributed, and that the soil is in reality a complex and highly heterogeneous medium.

However, suppose that our estimate of 11 meters is of the right order of magnitude (or too small), then major problems can be expected while building the mountain. Even if one can come up with a solution that keeps the structure from collapsing while the mountain sinks into the soil, the problem remains that a huge amount of soil will be displaced to the surrounding area. If the mountain is going to be built on land, this could cause serious problems. If the mountain is going to be built in sea, it is still necessary to investigate whether this excess of soil can cause problems, but it seems less likely.

It may be better to prevent the mountain from sinking into the soil. A naive approach would be to use a foundation with long concrete pillars (approx. 100 m); see Figure 7. This has several advantages: first, the soil gets compressed by the driving force of the pillars, thus becoming more resistant to the load. Second, these concrete pillars will experience a shear force by the surrounding soil, which will carry a significant part of the weight of the mountain.

The downside of this approach is that one needs to cover at least 30% of the base area of the mountain by pillars to prevent the concrete from failing under the bulk pressure induced by the weight of the mountain.

A more innovative approach would make the mountain less dense than the soil that it rests on (see Figure 7). Since the soil behaves like a liquid on the length scale of the mountain, this idea is based on the same principle that makes a boat float on water. However, this is likely not possible with currently available materials and construction methods, and so would call for a major innovation.

Figure 7: Two different types of foundation: (concrete) pillars or a very light structure.

To conclude, our model indicates that one cannot neglect the depth that the mountain will sink into the soil. This will not only make it more difficult to construct the mountain, but will also raise the ground surrounding the mountain. Therefore, we advise to investigate how one can prevent the mountain from sinking into the soil. We propose to look either at a deep foundation with pillars, or to make the mountain much less dense than the soil it rests on.

4 Structure

It should be noted first off that there are no physical objections to building a mountain. However, the practicality of such an endeavor can be called into question.

Due to the broad nature of the project it is difficult to make precise statements about the sort of structure that this construction should conform to. We do know that it should have the appropriate shape and height, and it should be stable enough to serve as a platform for the construction of other buildings and facilitate activities that involve a large number of people. Based on this, some considerations can readily be made.

Comparing our man-made mountain with those in nature might seem like an obvious starting point; however, such comparison does not yield any useful insights. When it comes to natural mountains there is no intention to them, nor, for that matter, a design that one could copy effectively.

An obvious approach to building a mountain would be to simply pile on sand and rocks until a mountain is created. A number of artificial archipelagos have been created this way off the coast of Dubai. For instance, consider the 'The World' archipelago. This is a group of islands that have been shaped to resemble a map of the earth when viewed from above. It has a surface area of roughly 5.6 km^2, and an average elevation of 13 m when measured from the sea floor. Approximately 0.3 km^3 of sand was deposited over the course of 5 years by an approximate work force of $30,000$ men and women. At the start of construction the cost was estimated at 14 billion dollars. A report by Fugro NPA Satellite

Mapping suggested that the islands were both eroding and sinking; Nakheel, the company in charge of development of 'The World', denied those claims (see [6]).

It would seem then that scaling this construction method to the size of a mountain, where approximately 100 km^3 of sand would have to be deposited, is not feasible for reasons of cost and stability.

Let us then consider the scenario in which the mountain is merely a structure that from the outside looks like a mountain, so that a comparison with a high-rise building is more appropriate.

In general, tall buildings are constructed because of their ability to provide large usable spaces while taking up a small area at the ground level. This is quite an attractive feature for buildings to have in large cities, where space for construction is scarce. It is important to understand however that the motivation for building exceedingly tall high-rise buildings, such as the current record holder, Burj Khalifa in Dubai, is not economical but mainly comes from the prestige that goes with owning (and demonstrating the ability to construct) such a tall structure. It is commonly believed that construction costs grow exponentially with the height of the building. This, put together with the extra structural precautions required to make the building withstand extreme winds and earthquakes, not to mention the vast stresses and strains generated by the weight of the building itself, calls into question whether it is sensible or cost-effective to build so tall a building. Their main commonality with a man-made mountain would therefore be that both these constructions are built mostly for their impressive height, rather than for their practicality. We thus have to ask ourselves whether the stated goals of having a structure that can both function as a mountain on the outside and provide practical and cost-effective spaces on the interior are at all compatible.

There are two major differences between an artificial mountain and a high-rise building. First, with an artificial mountain it is more important to have a functional exterior than a functional interior, while with high-rise buildings it is the other way around. Whereas the facade of ordinary buildings typically will not contribute significantly to the total weight, with an artificial mountain one would expect its facade – e.g. the soil and secondary buildings on top of it – to have considerable weight. The second difference is that the width-to-height aspect ratio in high-rise buildings is roughly 3:7 while for a mountain these values would most likely be inverted. Also, high-rise buildings in general have vertical facades, whereas we want the facade of our artificial mountain to be slanted, in order to use it.

The general rule with high-rise buildings, when it comes to structure, is that about 30% of the volume of the building is made up of structural elements such as walls and pillars. Although it is conceivable that a clever method of construction, such as a (geodesic) dome, could reduce this number in case of an artificial mountain, it seems unlikely that any dramatic improvements can be made. While the lower height-to-width aspect ratio would facilitate a more widely and evenly distributed pressure at the base of the building, the central section of the base, where the mountain is tallest, would have to withstand large pressures and would therefore need to be of a higher density and be made of materials that could withstand extreme compressive forces. Furthermore, the surface of the structure would have to be heavily reinforced to facilitate secondary structures, such as other buildings, roads, or simply soil and flora, natural structures like rivers and lakes, and ice formations – factors which are usually not relevant in traditional buildings. In short, it may even be optimistic to assume that only 30% of the volume of the mountain would correspond to structural elements. This number is important since it can be used to estimate the minimal volume of materials needed in the construction based on an approximation of the volume of the mountain.

Finally, it had been suggested to us that the mountain could be constructed in stages. Here, for instance, one would start with a small hill and gradually expand it over time, so that during the construction the mountain is already functional in some way. The following question then arises: How does the expansion relate to the amount of materials added? That is, at which stage of the construction would one have a structure capable of satisfying at least some of the functional requirements? To tackle this, we can express the height of the mountain as a function of the volume: let h be the height of the mountain, V the volume of the mountain and ρ_m the ratio of material to air inside the mountain. Assume that the mountain has a conical shape with a fixed slope s, then

$$h = \left(\frac{3}{\pi}s^2\rho_m V\right)^{\frac{1}{3}}. \tag{3}$$

Hence, setting $\rho_m = 30\%$ and $s = 2/7$, starting with 1 km^3 of material would result in a hill of roughly 450 meters tall. Adding another 1 km^3 (spreading it uniformly over the surface) would increase the height to 571 meters. Repeating this process, the gain in height per added unit of volume decreases steeply. Indeed, increasing the volume of the mountain from 99 km^3 to 100 km^3 would result in a height gain of only 8 meters (cf. Figure 8). Looking at it from another perspective, one could say that if one wanted to double the height, one would have to use 8 times the material already used. Also note that the slow-down occurs at the end of the construction in this scenario. Different ways of adding the material (e.g. building the mountain in layers) could move this phenomenon to a different stage in the construction. The fact remains, however, that this slow-down has to occur at some stage of the construction. This intrinsic slow-down is thus another serious issue to contend with, if the mountain is to be a (financial) success.

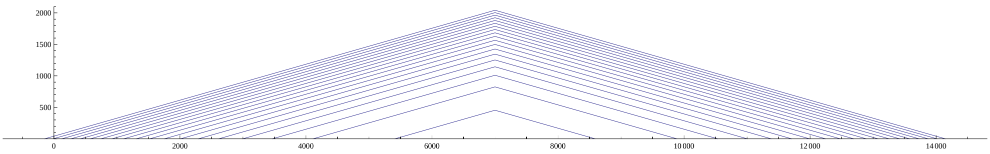

Figure 8: Schematic of the cross section of a conical mountain for 5 km^3 increments with a first layer of 1 km^3.

5 Materials

One of the largest limiting factors when building a mountain would be the amount of materials needed, the production costs of this material, its availability and the environmental impact of mining and/or manufacturing materials in such large quantities.

In this section we again approximate the total volume of the mountain by assuming its shape resembles a cone. For the sake of comparison we consider a cone with a diameter at the base of 14 km and a height of 2 km, resulting in a volume of approximately 100 km^3. Under the optimistic assumption that only 30% of this volume would be comprised of actual material we reach a total volume of 30 km^3 of material.

In Table 2 we consider for a few common construction materials with some rough estimates for the total amount of material needed, how many yearly world productions that represents, the total prices, and how many times the total yearly emissions of carbon dioxide of the Netherlands (= 176 mega tonnes of CO_2 per year [1]) the production of these quantities of materials would represent.

In all rows the values refer to 30 km^3 worth of material.

Materials	Required (10^{13} kg)	World Prod. (Years)	Total Price (10^{12} Euro)	NL CO_2 Emissions (Years)
Rock	7.50	–	1.95	–
Sand	7.95	–	2.10	–
Concrete	6.90	3.8	3.90	357
Plastic	2.91	29.1	29.10	1005
Steel	24.00	30.0	135.00	641
Glass	7.80	–	270.00	422

Table 2: Estimates on the required material, see [2], [5], [8], [4].

The table is quite clear: a man-made mountain cannot be built using traditional construction materials. The CO_2 emissions in particular stand out. An obvious requirement of any building materials used would therefore be that, besides availability, they could be produced in a more-or-less CO_2 neutral way. This may very well be the biggest hurdle when building a mountain. Note that even if our estimated material density of 30% is off by, say, three orders of magnitude, that is, one could build it with a material density of 0.03% instead, then this would still result in the equivalent of 350 years worth of Dutch CO_2 production, in the most favorable case.

To these estimates one would have to add costs, both financial and environmental, for transporting these materials. Not to mention salaries for the workforce which would likely have to consist of tens of thousands of people, the material required, the costs of purchasing the land where the mountain would be built and many other costs which would add significantly to the already exorbitant numbers seen above.

Another obstacle is simply the amount of time required to finish the project; if we take the fastest material to produce from Table 2 it would take almost 40 years just to produce the material, provided that one could buy 10% of all the material made in the world during that period.

Taking all the above in consideration, it is clear that the current materials and techniques are simply not sufficient for this project. Innovative ideas are needed to see the mountain become reality. One step forward that comes to mind is to explore new construction techniques based on cellular or foam-like structures. Geodesic domes are a prime example of this. They use a minimal amount of material to cover a large area. The drawback is that they are expensive, difficult to build, and they cannot withstand the large pressures that for instance a classically constructed building can handle. As an illustration of how geodesic domes could be used to construct a mountain, consider the artist's impression of a multi-stage building consisting of dome-like elements, shown in Figure 9. Note that in this impression, the mountain 'grows' outward from a certain initial core structure.

6 Sustainability

In this section, we investigate how natural resources can be used to produce renewable energy at the site of the mountain, during and after construction, and how the mountain can be designed in such a way that the most is gained from the available resources. We will focus on means of generating energy that exploit the mountain's height, since there would be no point in generating energy on a mountain if it could be generated more efficiently elsewhere.

The aim in building a mountain would be to construct a so-called Zero-Energy Building. This is a popular term to describe a building with a zero net energy consumption and zero carbon emissions per year. Hence, the total amount of energy produced on-site should at least compensate the total amount of energy used in the building, but also, it should compensate for the energy spent during the construction phase. Considering that the estimates on the CO_2 emissions in

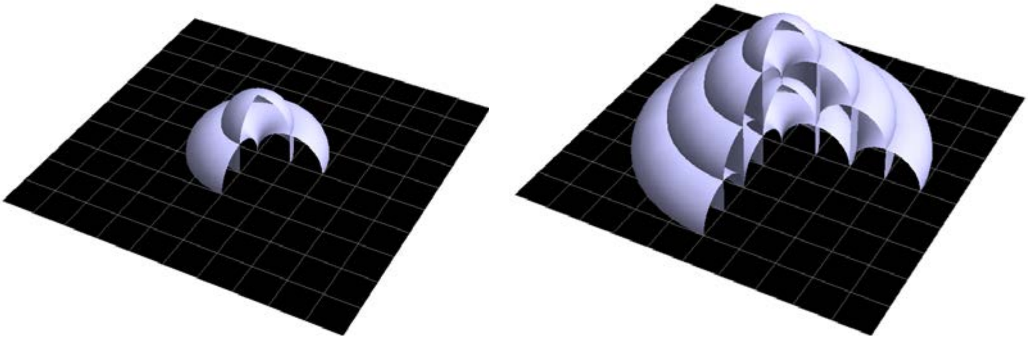

Figure 9: An artist's impression on building a mountain in stages.

the previous section were enormous, compensation for these emissions poses another serious problem.

We propose to use wind, sunlight and water as possible sources of energy.

6.1 Wind energy

Windmills have historically played a major role in the Netherlands by providing an alternative to water driven mills. More recently, wind power is being used as a renewable energy source. Today, around 2500 wind turbines are operational in the Netherlands, with a total wind energy production of 4000 GWh/year.

In the last decades, much research has been performed concerning wind energy production and the efficiency of wind turbines. This research varies from development of new types of wind turbines to determining the optimal spacing between turbines in a field of wind turbines, a so-called wind farm.

Commercial wind turbines, usually vertical, have a height varying from 80 to 150 meter producing around 2 – 5 MW. Although many features of the turbine play a role in determining the capacity of the turbine, the major difference in capacity is due to the blade length.

The capacity of a wind turbine can be determined as follows (W: wind power)

$$W = \frac{1}{2} A_{\text{swept}} \rho_{\text{air}} V^3. \tag{4}$$

Here, A_{swept} is a number that is mainly determined by the blade length, ρ_{air} is the density of air and V is the wind speed. This formula shows that the amount of energy produced is very much influenced by the wind speed. Optimization studies have shown that the ideal spacing between different turbines in a wind farm is about 12-20 times the blade length, since then the wind speed arriving at a turbine is not affected by the surrounding turbines. Thus, in order to achieve the most efficient wind energy production level, this has to be taken into account when placing wind turbines in a wind farm.

Since our aim is to build a 2 km high mountain, it is worthwhile to look at how high altitudes affect the wind speed and influence the energy production, when using wind turbines. As can be seen from equation (4), the power produced is proportional to the cube of the wind speed, i.e. $W \propto V^3$.

The effect of altitude on wind speed can be estimated as follows (see [15]):

$$V_x = V_y \left(\frac{h_x}{h_y}\right)^{\alpha}, \quad \alpha \equiv \frac{1}{7} \tag{5}$$

Here V_y is the wind speed at a given height h_y, take for example 10 meters, and V_x is the wind speed at altitude h_x. Formula (5) implies that at 2 km above sea level the wind speed is approximately twice the speed at sea level (10 meters). By using relation (4) we can see that this results in eight times more energy per turbine at 2 km altitude than at sea level.

As promising as it may seem, achieving this large gain may as of yet not be possible. Turbines constructed with today's technology are not built to cope with very high wind speeds. Nevertheless, recent developments raise hope for producing much more stable and strong wind turbines, see [13].

Another important aspect to consider is the stage-by-stage construction of the mountain. This provides the possibility to produce wind energy already after the construction of the first stage. Furthermore, since wind energy is as yet not often produced at high altitude, this way of construction will provide opportunity to develop and implement the cutting-edge technology in this field.

6.1.1 Tunnels in the mountain

One of the methods for wind energy production worth considering is wind tunneling. Making a long, relatively narrow tunnel through a building or mountain will cause air to be sucked in at a high velocity, similar to a chimney. This method is not widely used in traditional high-rise buildings due to stability problems, as having a high wind speed at the top introduces a large horizontal strain. However, stability in this sense is not an issue for the mountain, because of the height-to-width ratio is very small. Thus, tunneling appears to be a perfect way to use the construction to generate power from wind. Studies have shown that wind power can be increased by approximately 5-6 times with the tunneling effect, see [18]. Taking into account both the effects of altitude and wind tunneling, it seems that one could generate 30 to 40 times more power from a turbine in a wind tunnel at the top of the mountain than than one could generate from a traditional wind mill at sea level of the same size as the diameter of the tunnel.

6.2 Solar energy

A popular source of renewable energy is solar energy. Nowadays this is mostly collected using photovoltaic solar panels. Of course, an enormous area such as the surface of a mountain could easily serve as a subsoil for solar panels. Also, if the mountain would be built in layers, the panels could be installed on the first layer and reused in the next. This way there would be gain already in an early stage of construction. However, it should be noted that solar panels do not seem to be significantly more efficient at high altitude than at sea level. Still, they might be implemented in places that would otherwise go unused.

Heat storage could generate energy as well, and solar radiation could contribute to the heating. On sunny days, heat could be collected inside or outside the structure. We now discuss some possibilities to use heat in more detail.

6.2.1 Solar chimneys

A number of new techniques in renewable energy have been developed over the years, one of them is a so-called ‘solar chimney’, an installation which combines three simple techniques. It consists of three essential elements: a glass roof, a chimney and wind turbines. Basically, this construction works as follows. Solar radiation heats up air below the glass roof with open sides. Attached to this roof is a high chimney. Air at large altitudes is cooler, and the difference in temperature of the air below the glass roof and at the top of the chimney causes the hot air to rise, creating a draft within the chimney. This principle of air acceleration causes high wind speeds which can generate energy using wind turbines (see Figure 10).

To have an effective solar chimney, a large area at ground level should be covered by a transparent roof, so as to catch as much heat from the sun as possible. A single 1000 meter tall solar chimney can provide energy for $30,000$ Dutch households, see [16] (see also [3], [14] for further literature on solar chimneys).

Using solar chimneys has several advantages compared to other energy sources. For instance, since it uses both direct and diffuse radiation it is more suited to the Dutch weather conditions, whereas traditional solar power plants can only use direct radiation, which means they only work on sunny days. Also, because the construction of a solar chimney is relatively simple and there are few moving parts, the structure is very reliable (and therefore it requires little maintenance). Moreover, the power plant needs no cooling water, which is commonly used in solar power plants today. The greatest advantage, however, is that all the necessary technologies are already widely available and relatively cheap.

In 1982, a prototype solar chimney was built in Manzanares, Spain. It is a 195 m high chimney with a diameter of 10 m, with a collector that is 244 m in diameter. It achieved a yield of 50 kW. Designs for chimneys with a yield of 100 MW exist. Until now these chimneys have not often been used. There are two reasons for this. First, the efficiency of a solar chimney is fairly low compared to

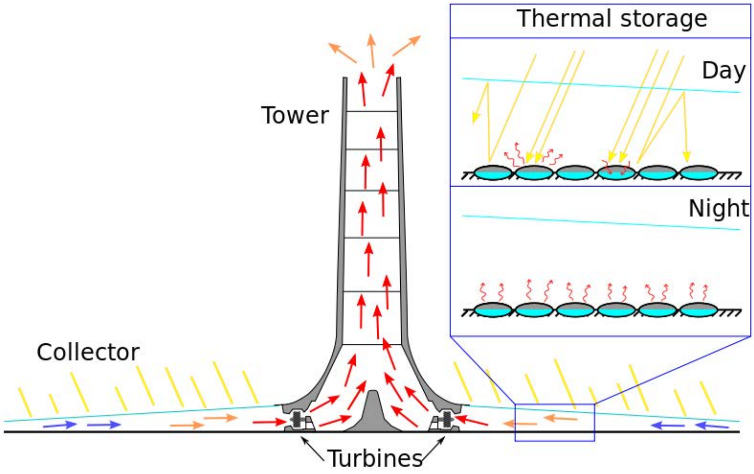

Figure 10: Principle of the solar chimney: a glass roof collector, chimney tube and wind turbines. The enlargement shows the use of water filled tubes.

that of traditional power plants. Second, their size often raises objections from the people who would have to live near them. Considering that we are trying to build a mountain that is much larger still, the latter issue seems to be minor by comparison.

6.2.2 The 'Tower of MegaPower'

A similar idea to a solar chimney that may be investigated further is that of the so-called 'Tower of MegaPower' (ToMP). The idea was proposed in [17] where a more detailed description and a first study can be found.

A ToMP is a tall tunnel-shaped device that is placed upright above a body of water and that generates energy through the following process: at ground-level, water evaporates and rises upward through the tunnel. Due to the colder air at high altitudes, the vapor will condensate and fall back down. By placing a turbine in the condensed water's path, energy can be generated. In other words, a ToMP imitates the rain cycle in a sustained way.

Like a solar chimney, a ToMP seems very well suited for integration into the mountain.

6.3 Hydroelectric energy

A simple way of generating energy that is easily integrated in the project is hydroelectricity. One can construct several lakes on the mountain that collect rain-water. These lakes can be used to store and produce energy, but could also provide venues for recreation. The lakes can be constructed at different altitudes. This way, a surplus of generated energy can be stored by pumping up water from one lake to another, higher up the mountain. This energy can then later be reclaimed by using a hydroelectric power installation. This way, the system functions a lot like a battery. Apart from using the water to store and generate energy, this water can also be used in various ways to supply the needs of the mountain's facilities.

7 Summary, conclusions and recommendations

We have taken on the challenge posed by Bartels Consulting Engineers during the Study Group Mathematics with Industry in Eindhoven, the Netherlands, and investigated both the possibilities of and the difficulties in constructing a 2 kilometer tall artificial mountain in the Netherlands.

Where to build a mountain. The Netherlands is a relatively small but densely populated country. The mountain would have to be built in an uninhabited zone. This clearly creates a severe restriction. We considered eight possible locations suggested by the organization of 'Die Berg komt er!', and came to the conclusion that it is best to either build it on land in the province of Flevoland, or to build it in the North Sea near (the appropriately named) Bergen aan Zee. We chose these locations on the basis of several criteria imposed by the presence of settlements, industry, infrastructure and nature.

Another important factor to take into account when choosing a location is the effect such a massive structure would have on the underlying soil, and specifically on the areas around it.

Of the two locations that seem the most convenient, we believe that the mountain would best be placed in sea rather than on land, considering the effects the building of a mountain would have on the surrounding soil, as the soft Dutch soil would shift significantly under the load of a mountain. In fact, we estimated that without a proper foundation the mountain could sink as much as 11 meters into the ground, and that this displacement would result in raising the ground around the mountain on average by 11 meters as far as 3 kilometers away.

Furthermore, we conclude that to minimize the effects of the mountain on the surrounding soil, it would be best to come up with a method that would prevent the mountain from sinking at all. The traditional way would require many and long concrete pillars to be driven into the soil; alternatively, one could lower the average density of the mountain, for instance by building it on a cushion made of a very light material, so that the mountain floats on the soil, like a ship in the water.

We recommend that extensive measurements of the profile of the soil layers is performed, down to much greater depths than is standard practice. This data can be used in a computer model to assess the ramifications on the surrounding soil (and possibly, sea currents) and the structural requirements in much greater detail than we were able to, in a couple of days.

How to build a mountain. Building a 2 kilometer high mountain would be an endeavor of unprecedented scale in human history. Comparison to existing man-made archipelagos tell us that a solid mountain made of sand and rocks would cost too much, erode too easily, and involve too much work to be completed in a feasible amount of time. This leads us to conclude that instead one would need to apply more refined construction techniques that allow for rapid progress and that require significantly less material than a solid mountain would.

In Section 4 we presented a table that states for a number of common building materials the total price, a comparison of the necessary amount with the current world production, and a comparison with the total annual emission of CO_2 in the Netherlands. These estimates give us a strong indication that not enough of these materials is available (in the world) to finish the project on a time scale of decades. Furthermore, even if availability was not a problem, the production of these materials would be far too damaging for the (global) environment to be justifiable. Thus, we have to conclude that with the materials and techniques that are currently common, a 2 kilometer high mountain cannot be built.

Hence, the problem of producing massive quantities of cheap building materials without producing large amounts of CO_2 or other pollutants must first be resolved before a mountain can be built. This is certainly the most important and difficult problem that comes with building a mountain, but also the one where the reward is the greatest.

How to make the mountain sustainable. The mountain can be designed in such a way that electrical energy can be generated in several ways. Some modern ideas like a 'Tower of MegaPower', wind tunnels through the mountain and solar chimneys seem good candidates, since the mountain is very susceptible to the integration of such devices. Indeed, the integration of devices that rely on the difference in temperature at the ground and at greater altitude, or on the higher wind speeds that come with greater altitude, seem to offer the greatest rewards.

Summary. With the current techniques, materials and knowledge it is not possible to build a 2 kilometer high mountain. It seems that vast leaps in thinking about structural design and material use have to be made: the mountain needs to be as light as possible, as cheap as possible, as 'clean' as possible, and it needs to be built in a relatively short time as well. This opens up grand new challenges in material development, design and logistics.

As for the sustainability of the project, there are certainly many ways in which the mountain can be used to generate clean energy.

The building of a mountain in the Netherlands turns out to be both challenging and inspiring; it certainly invites one to go up and beyond.

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