Updated constraints on the minimal supergravity model

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Abstract

Recently, refinements have been made on both the theoretical and experimental determinations of *i.*) the mass of the lightest Higgs scalar, *ii.*) the relic density of cold dark matter in the universe, *iii.*) the branching fraction for the radiative $b \to s\gamma$ decay, *iv.*) the muon anomalous magnetic moment, and *v.*) the flavor violating decay $B_s \to \mu^+\mu^-$. In this work, we present constraints from each of these quantities on the minimal supergravity model as embedded in the updated version of the computer program ISAJET v7.64. Improvements and updates since our published work are especially emphasized. The combination of constraints points to certain favored regions of model parameter space where collider and non-accelerator SUSY searches may be more focused.

1 Introduction

Particle physics models including supersymmetry solve a host of problems occurring in non-supersymmetric theories, and predict a variety of new matter states— the sparticles— at or around the TeV scale[1]. The so-called *minimal* supergravity (mSUGRA) model (sometimes also referred to as the CMSSM) has traditionally been the most popular choice for phenomenological SUSY analyses. In mSUGRA, it is assumed that the Minimal Supersymmetric Standard Model (MSSM) is valid from the weak scale all the way up to the GUT scale $M_{GUT} \simeq 2\times10^{16}$ GeV, where the gauge couplings g_1 and g_2 unify. In many of the early SUGRA models[2], a simple choice of Kähler metric G_i^j and gauge kinetic function f_{AB} led to *universal* soft SUSY breaking scalar masses (m_0), gaugino masses ($m_{1/2}$) and A-terms (A_0) at M_{GUT}. This assumption of universality in the scalar sector leads to the phenomenologically required suppression of flavor violating processes that are supersymmetric in origin. In the mSUGRA model, we thus assume universal scalar

*Talk given by C. Balázs at SUSY02.

masses, gaugino masses (as a consequence of assuming grand unification) and A-terms. We will also require that electroweak symmetry is broken radiatively (REWSB), allowing us to fix the magnitude, but not the sign, of the superpotential Higgs mass term μ so as to obtain the correct value of M_Z. Finally, we trade the bilinear soft supersymmetry breaking (SSB) parameter B for $\tan\beta$ (the ratio of Higgs field vacuum expectation values). Thus, the parameter set

$$m_0,\ m_{1/2},\ A_0,\ \tan\beta,\ \text{ and }\ sign(\mu) \tag{1}$$

completely determines the spectrum of supersymmetric matter and physical Higgs fields.

In our calculations, we use ISAJET v7.64 [3] since this version includes a number of improvements in calculating the SUSY particle mass spectrum compared to v7.58 used in Ref.[4]. Once the SUSY and Higgs masses and mixings are known, then a host of observables may be calculated, and compared against experimental measurements. The most important of these include:

- lower limits on sparticle and Higgs boson masses from new particle searches at LEP2,
- the relic density of neutralinos originating from the Big Bang,
- the branching fraction of the flavor changing decay $b \to s\gamma$,
- the value of muon anomalous magnetic moment $a_\mu = (g-2)_\mu/2$ and
- the lower bound on the rate for the rare decay $B_s \to \mu^+\mu^-$.

Our goal is to delineate the mSUGRA parameter space region consistent with all these constraints. In our analysis, we incorporate a new calculation of the neutralino relic density that has recently become available[5]. We also present improved $b \to s\gamma$ branching fraction predictions in accord with the current ISAJET release. We discuss constraints imposed by the measurement of the muon anomalous magnetic moment. Finally, we delineate the region of mSUGRA parameter space excluded by the CDF lower limit[6] on the branching fraction of $B_s \to \mu^+\mu^-$. This constraint is important for very large $\tan\beta$'s[7].

Within the mSUGRA framework, the parameters m_0 and $m_{1/2}$ are the most important for fixing the scale of sparticle masses. The m_0-$m_{1/2}$ plane (for fixed values of other parameters) is convenient for a simultaneous display of these constraints, and hence, of parameter regions in accord with all experimental data.

2 Constraints and calculations in the mSUGRA model

Constraints from LEP2 searches

Based on negative searches for superpartners at LEP2, we require

- $m_{\widetilde{W}_1} > 103.5$ GeV and $m_{\tilde{e}_{L,R}} > 99$ GeV provided $m_{\tilde{\ell}} - m_{\widetilde{Z}_1} > 10$ GeV,

which is the most stringent of the slepton mass limits. The LEP2 experiments also set a limit on the SM Higgs boson mass: $m_{H_{SM}} > 114.1$ GeV[8]. In our mSUGRA parameter space scans, the lightest SUSY Higgs boson h is almost always SM-like. The exception occurs when the value of m_A becomes low at very large values of $\tan\beta$. For clarity, we show contours where

- $m_h > 114.1$ GeV,

and will direct the reader's attention to any regions where this bound might fail.

Neutralino relic density

Measurements of galactic rotation curves, binding of galactic clusters, and the large scale structure of the universe all point to the need for significant amounts of cold dark matter (CDM) in the universe. In addition, recent measurements of the power structure of the cosmic microwave background, and measurements of distant supernovae, point to a cold dark matter density[9]

- $0.1 < \Omega_{CDM}h^2 < 0.3$.

The lightest neutralino of mSUGRA is an excellent candidate for relic CDM particles in the universe. The upper limit above represents a true constraint, while the corresponding lower limit is flexible, since there may be additional sources of CDM such as axions, or states associated with the hidden sector and/or extra dimensions.

To estimate the relic density of neutralinos in the mSUGRA model, we use the recent calculation in Ref. [5]. In that work, all relevant neutralino annihilation and co-annihilation reactions are evaluated at tree level using the CompHEP[10] program. The annihilation cross section times velocity is relativistically thermally averaged[11], which is important for obtaining the correct neutralino relic density in the vicinity of annihilations through s-channel resonances.

The $b \to s\gamma$ branching fraction

The branching fraction $BF(b \to s\gamma)$ has recently been measured by the BELLE[12], CLEO[13] and ALEPH[14] collaborations. Combining statistical and systematic errors in quadrature, these measurements give $(3.36 \pm 0.67) \times 10^{-4}$ (BELLE), $(3.21 \pm 0.51) \times 10^{-4}$ (CLEO) and $(3.11 \pm 1.07) \times 10^{-4}$ (ALEPH). A weighted averaging of these results yields $BF(b \to s\gamma) = (3.25 \pm 0.37) \times 10^{-4}$. The 95% CL range corresponds to $\pm 2\sigma$ away from the mean. To this we should add uncertainty in the theoretical evaluation, which within the SM dominantly comes from the scale uncertainty, and is about 10%. Together, these imply the bounds,

- $2.16 \times 10^{-4} < BF(b \to s\gamma) < 4.34 \times 10^{-4}$.

In our study, we show contours of $BF(b \to s\gamma)$ of 2, 3, 4 and 5×10^{-4}.

The calculation of $BF(b \to s\gamma)$ used here is based upon the program of Ref. [15]. In our calculations, we also implement the running b-quark mass including SUSY threshold corrections as calculated in ISAJET; these effects can be important at large values of the parameter $\tan\beta$[16]. Our value of the SM $b \to s\gamma$ branching fraction yields 3.4×10^{-4}, with a scale uncertainty of 10%.

Muon anomalous magnetic moment

The muon anomalous magnetic moment $a_\mu = (g-2)_\mu/2$ has been recently measured to high precision by the E821 experiment[17]: $a_\mu = 11659204(7)(5) \times 10^{-10}$. The most challenging parts of the SM calculation are the hadronic light-by-light[18] and vacuum polarization (HVP)[19] contributions and their uncertainties. Presently these results are in dispute. In the case of the HVP the use of tau decay data can reduce the error, but the interpretation of these data is somewhat controversial[20]. Thus, the deviation of the measurement from the SM depends on which prediction is taken into account. According to the recent analysis by Hagiwara et al.[19]:

- $11.5 < \delta a_\mu \times 10^{10} < 60.7$.

A different assessment of the theoretical uncertainties[19] using the procedure described in ref.[4] gives,

- $-16.7 < \delta a_\mu \times 10^{10} < 49.1$.

In view of the theoretical uncertainty, we only present contours of δa_μ, as calculated using the program developed in [21], and leave it to the reader to decide the extent of the parameter region allowed by the data.

$B_s \to \mu^+\mu^-$ **decay**

The branching fraction of B_s to a pair of muons has been experimentally bounded by CDF[6]:

- $BF(B_s \to \mu^+\mu^-) < 2.6 \times 10^{-6}$.

A potentially important contribution to this decay is mediated by the neutral states in the Higgs sector of supersymmetric models. While this branching fraction is very small within the SM ($BF_{SM}(B_s \to \mu^+\mu^-) \simeq 3.4 \times 10^{-9}$), the amplitude for the Higgs-mediated decay of B_s grows as $\tan^3\beta$ within the SUSY framework, and hence can completely dominate the SM contribution if $\tan\beta$ is large. In our analysis we use the results from the last paper in Ref.[7] to delineate the region of mSUGRA parameters excluded by the CDF upper limit on its branching fraction.

3 Results

To generate numerical results, in this work we use ISAJET v7.64 that includes several improvements over v7.58 which was used in Ref.[4]. These changes lead to important differences in the figures when compared with Ref.[4]. Notably, the boundary of the region excluded by the lack of REWSB moved to higher m_0 values and the allowed relic density region along this boundary changed, especially for the lower $\tan\beta$ values. Furthermore, for high $\tan\beta$ and $\mu < 0$ the diagonal corridors allowed by the relic density are considerably shifted and narrowed. Finally, the area allowed by relic density near the boundary of the stau LSP region shrank at low $\tan\beta$'s.

Our first results are plotted in Fig.1. Here, we show the m_0 *vs.* $m_{1/2}$ plane for $A_0 = 0$, $\tan\beta = 10$ and both signs of μ. The red shaded regions are excluded either due to a lack of REWSB (right-hand side), or a stau LSP (left-hand side). The magenta region is excluded by searches for charginos and sleptons at LEP2. The region below the red contour is excluded by LEP2 Higgs searches, since here $m_h < 114.1$ GeV. In addition, we show regions of neutralino relic density with green contours marking $\Omega_{\widetilde{Z}_1}h^2 = 0.1$ (dotted), 0.3 (dashed) and 1.0 (solid). The region right to the solid green contour has $\Omega_{\widetilde{Z}_1}h^2 > 1$, and would thus be excluded since the age of the universe would be less than 10 billion years. There is no constraint arising from $B_s \to \mu^+\mu^-$ decay at $\tan\beta = 10$.

For $\mu < 0$ the magenta contours denote values of $BF(b \to s\gamma) = 4$ and 5×10^{-4} and the blue contours denote values of $\delta a_\mu = -30, -10, -5, -2$ and -1×10^{-10}, moving

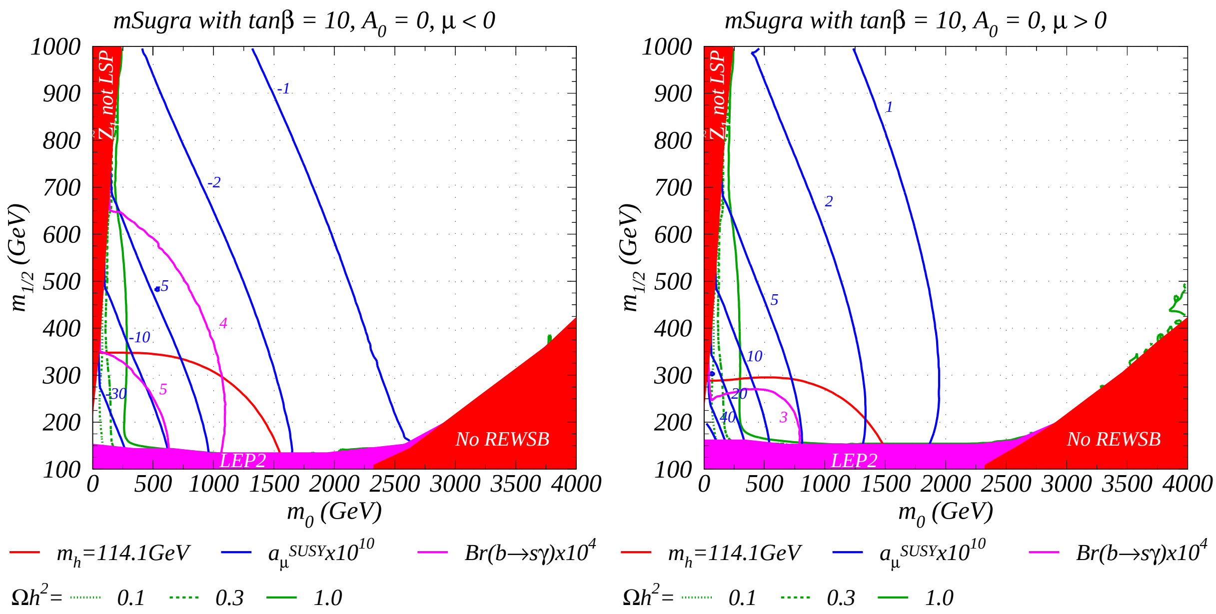

Figure 1: Plot of constraints for the mSUGRA model in the m_0 *vs.* $m_{1/2}$ plane for $\tan\beta = 10$ and $A_0 = 0$. We plot contours of the CDM relic density, $m_h = 114.1$ GeV, the muon anomalous magnetic moment a_μ ($\times 10^{10}$) and contours of $b \to s\gamma$ branching fraction ($\times 10^4$).

from lower left to upper right. An intriguing feature of the plot is that the region with the allowed relic density in the lower left part, where neutralinos mainly annihilate via t-channel slepton exchange to lepton-anti-lepton pairs is essentially excluded by the m_h, $b \to s\gamma$ and δa_μ constraints. That leaves two allowed regions with a preferred relic density: one that runs near the stau LSP region, where $\tilde\tau_1 - \widetilde{Z}_1$ co-annihilation effects reduce an otherwise large relic density (as pointed out by Ellis *et al.*[22]). This region has a highly fine-tuned relic density, since a slight change in m_0 leads to either too light or too heavy of a $\tilde\tau_1$ mass to give $0.1 < \Omega h^2 < 0.3$[23, 5]. The other runs parallel to the REWSB excluded region for $m_{1/2} > 400$ GeV in the "focus point" SUSY region. It occurs when the $\widetilde{Z}_1$ has a sufficiently large higgsino component that annihilation into WW, ZZ and Zh pairs reduces the relic density[24, 5].

For $\mu > 0$ almost the entire plane shown is in accord with the measured branching fraction of $b \to s\gamma$. The blue contours denote values of $\delta a_\mu = 60$, 40, 20, 10, 5, 2 and 1×10^{-10}. Constraints from δa_μ as well as from $B_s \to \mu^+\mu^-$ are not relevant for this case. In this case the slepton annihilation region of relic density has a small surviving region just beyond the Higgs mass contour. For the most part, to attain a preferred value of neutralino relic density, one must again live in the stau co-annihilation region. A final possibility is to be in the slepton annihilation region, but then the value of m_h should be slightly beyond the LEP2 limit; in this case, a Higgs boson signal may be detected in Run 2 of the Fermilab Tevatron[25].

We next turn to our results for $\tan\beta = 30$ shown in Fig.2. The gray region in the bottom left corner of the plot is excluded because $m^2_{\tilde\tau_1} < 0$. In this case, the allowed region of the relic density in the lower-left has expanded considerably owing to enhanced

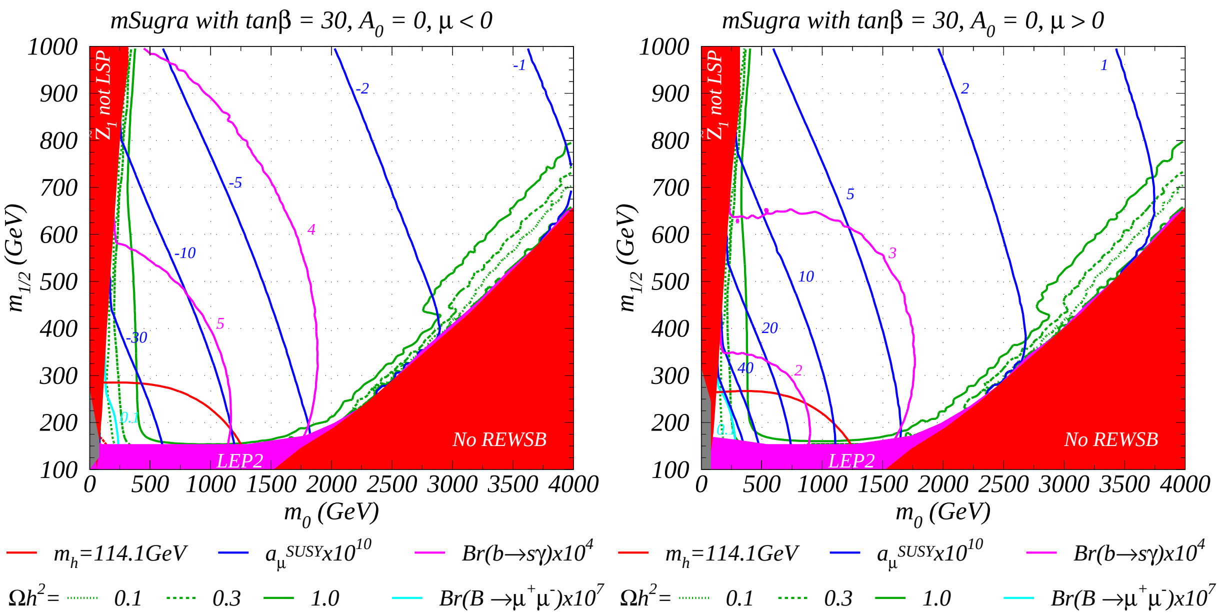

Figure 2: Same as Fig. 1, but for $\tan\beta = 30$. The light blue contour labeled 0.1 denotes where $B(B_s \to \mu^+\mu^-) = 0.1 \times 10^{-7}$. In subsequent figures these branching fractions contours are all labeled in units of 10^{-7}.

neutralino annihilation to $b\bar{b}$ and $\tau\bar{\tau}$ at large $\tan\beta$. Both lighter values of $m_{\tilde{\tau}_1}$ and $m_{\tilde{b}_1}$ and also large τ and b Yukawa couplings at large $\tan\beta$ enhance these t-channel annihilation rates through virtual staus and sbottoms. Unfortunately, for $\mu < 0$ the region excluded by $BF(b \to s\gamma)$ and by δa_μ also expands, and most of the cosmologically preferred region is again ruled out. As before, we are left with the corridors of stau co-annihilation and an enlarged focus point scenario[24, 5] as the only surviving regions.

For $\mu > 0$ the magenta contours of $BF(b \to s\gamma)$ correspond to 2 and 3×10^{-4}. Thus, the lower left region is excluded since it leads to too *low* a value of $BF(b \to s\gamma)$. The δa_μ contours begin from lower left with 60×10^{-10}, then proceed to 40, 20, 10, 5, 2 and 1×10^{-10}. A fraction of the slepton annihilation region of relic density is excluded also by too large a value of δa_μ. Of course, a reasonable relic density may also be achieved in the stau co-annihilation and focus point regions of parameter space.

Next, we turn to Fig.3 where we examine the mSUGRA parameter plane for very large values of $\tan\beta = 45$ and $\mu < 0$. The gray and red regions are as in previous figures. The blue region is excluded because $m_A^2 < 0$, denoting again a lack of appropriate REWSB. The inner and outer red dashed lines are contours of $m_A = 100$ and $m_A = 200$ GeV, respectively. The former is roughly the lower bound on m_A from LEP experiments. In between these contours, h is not quite SM-like, and the mass bound from LEP may be somewhat lower than $m_h = 114.1$ GeV shown by the solid red contour, but outside the 200 GeV contour this bound should be valid. Much of the lower-left region is excluded by too high a value of $BF(b \to s\gamma)$ and too low a value of δa_μ. In addition, in this plane, the experimental limit on $B_s \to \mu^+\mu^-$ enters the lower-left, where values exceeding 26×10^{-7} are obtained. It seems that in the upper region which is favored by the $b \to s\gamma$ constraint, detection of $B_s \to \mu^+\mu^-$ at the Tevatron will be quite challenging.

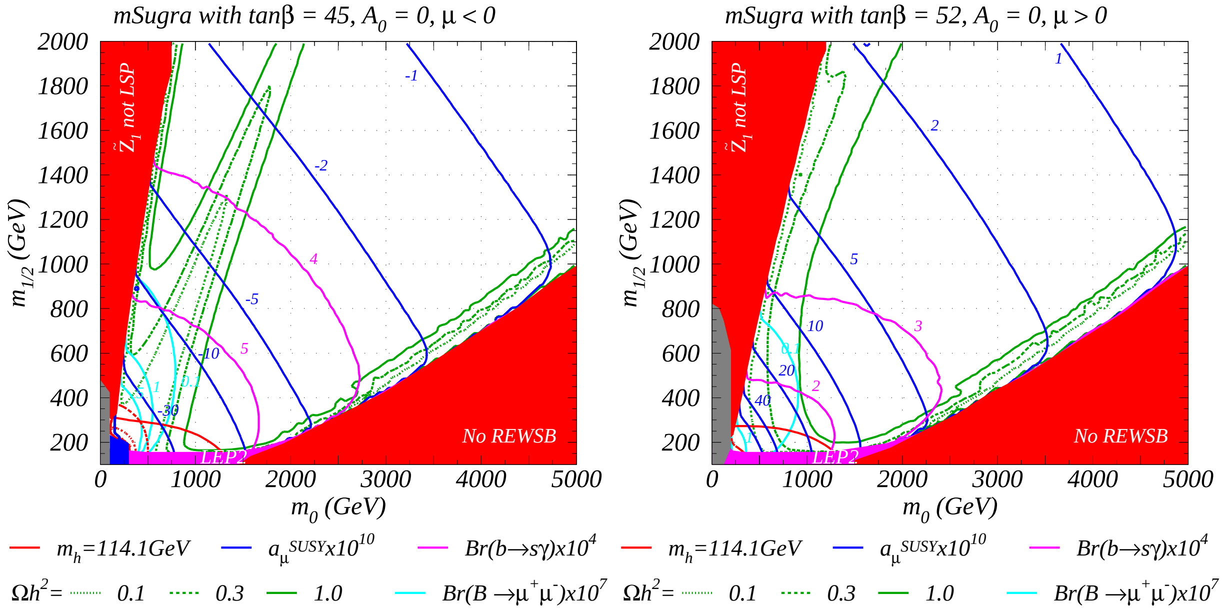

Figure 3: Same as Fig. 1, but for $\tan\beta = 45$, $\mu < 0$ and for $\tan\beta = 52$, $\mu > 0$. The inner and outer red dashed lines are contours of $m_A = 100$ and $m_A = 200$ GeV, respectively.

In this figure, the relic density regions are qualitatively different from the lower $\tan\beta$ plots. A long diagonal strip running from lower-left to upper-right occurs because in this region, neutralinos annihilate very efficiently through s-channel A and H Higgs graphs, where the total Higgs widths are very large due to the large b and τ Yukawa couplings for the high value of $\tan\beta$ in this plot. Adjacent to this region allowed regions where neutralino annihilation is still dominated by the s-channel Higgs graphs, but in this case the annihilation is somewhat off-resonance. The A and H widths are so large that even if $|2m_{\widetilde{Z}_1} - m_{A(H)}|$ is relatively large, efficient annihilation can still take place. (An improvement of the Higgs widths is adopted for these plots compared to Ref.[4].)

For the case of $\mu > 0$, we show the mSUGRA parameter space plane for $\tan\beta = 52$. In this plane, the relic density annihilation corridor occurs near the boundary of the excluded $\tilde{\tau}_1$ LSP region. The width of the A and H Higgs scalars is very wide, so efficient s-channel annihilation through the Higgs poles can occur throughout much of the allowed parameter space. But the annihilation is not overly efficient due to the large breadth of the Higgs resonances. In much of the region with $m_{1/2} < 400$ GeV, the value of $BF(b \to s\gamma)$ is below 2×10^{-4}, so that some of the lower allowed relic density region where annihilation occurs through t-channel stau exchange is excluded. In contrast, the value of δa_μ is in the range of $10 - 40 \times 10^{-10}$, which is in accord with the E821 measurement. The value of m_h is almost always above 114.1 GeV, and the $BF(B_s \to \mu^+\mu^-)$ is always below 10^{-7}, and could (if at all) be detected with several years of main injector operation.

In conclusion, we have presented updated constraints on the mSUGRA model from *i.)* the LEP2 constraints on sparticle and Higgs boson masses, *ii.)* the neutralino relic density $\Omega_{\widetilde{Z}_1} h^2$, *iii.)* the branching fraction $BF(b \to s\gamma)$, *iv.)* the muon anomalous magnetic moment a_μ and *v.)* the leptonic decay $B_s \to \mu^+\mu^-$. Putting all five constraints together, we find favored regions of parameter space which may be categorized by the mechanism for annihilating relic neutralinos in the early universe:

- **1.** annihilation through t-channel slepton exchange (low m_0 and $m_{1/2}$),
- **2.** the stau co-annihilation region (very low m_0 but large $m_{1/2}$),
- **3.** the focus point region (large m_0 but low to intermediate $m_{1/2}$) and
- **4.** the flanks of the neutralino s-channel annihilation via A and H corridor at large $\tan\beta$ when Γ_A and Γ_H are very large.

To summarize, we find the five constraints considered in this work to be highly restrictive. Together, they rule out large regions of parameter space of the mSUGRA model, including much of the region where t-channel slepton annihilation of neutralinos occurs in the early universe. The surviving regions **1.-4.** have distinct characteristics of their SUSY spectrum, and should lead to distinctive SUSY signatures at colliders.

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References

[1] For recent reviews, see *e.g.* S. Martin, in *Perspectives on Supersymmetry*, edited by G. Kane (World Scientific), hep-ph/9709356; M. Drees, hep-ph/9611409; J. Bagger, hep-ph/9604232; X. Tata, *Proc. IX J. Swieca Summer School,* J. Barata, A. Malbousson, and S. Novaes, Eds., hep-ph/9706307; S. Dawson, Proc. TASI 97, J. Bagger, Ed., hep-ph/9712464.

[2] A. Chamseddine, R. Arnowitt, and P. Nath, *Phys. Rev. Lett.* **49** (1982) 970; R. Barbieri, S. Ferrara, and C. Savoy, *Phys. Lett.* **B 119** (1982) 343; L. J. Hall, J. Lykken, and S. Weinberg, *Phys. Rev.* **D 27** (1983) 2359; for a review, see H. P. Nilles, *Phys. Rept.* **110** (1984) 1.

[3] H. Baer, F. Paige, S. Protopopescu, and X. Tata, hep-ph/0001086.

[4] H. Baer, C. Balázs, A. Belyaev, J. K. Mizukoshi, X. Tata, and Y. Wang, JHEP **0207**, 050 (2002).

[5] H. Baer, C. Balázs and A. Belyaev, *J. High Energy Phys.* **0203** (2002) 042.

[6] F. Abe *et al.*, (CDF Collaboration), *Phys. Rev.* **D 57** (1998) 3811.

[7] K. Babu and C. Kolda, *Phys. Rev. Lett.* **84** (2000) 228; A. Dedes, H. Dreiner, and U. Nierste, *Phys. Rev. Lett.* **87** (2001) 251804; R. Arnowitt, B. Dutta, T. Kamon, and M. Tanaka, hep-ph/0203069; J. K. Mizukoshi, X. Tata, and Y. Wang, hep-ph/0208078.

[8] LEP Higgs Working Group Collaboration, hep-ex/0107030.

[9] See *e.g.* W. L. Freedman, *Phys. Rev.* **333** (2000) 13.

[10] CompHEP v.33.23, by A. Pukhov *et al.*, hep-ph/9908288.

[11] P. Gondolo and G. Gelmini, *Nucl. Phys.* **B 360** (1991) 145; J. Edsjö and P. Gondolo, *Phys. Rev.* **D 56** (1997) 1879.

[12] K. Abe *et al.* (Belle Collaboration), *Phys. Lett.* **B 511** (2001) 151.

[13] D. Cronin-Hennessy *et al.* (Cleo Collaboration), *Phys. Rev. Lett.* **87** (2001) 251808.

[14] R. Barate *et al.* (Aleph Collaboration), *Phys. Lett.* **B 429** (1998) 169.

[15] H. Baer and M. Brhlik, *Phys. Rev.* **D 55** (1997) 3201; H. Baer, M. Brhlik, D. Castaño, and X. Tata, *Phys. Rev.* **D 58** (1998) 015007.

[16] G. Degrassi, P. Gambino, and G. Giudice, *J. High Energy Phys.* **0012** (2000) 009. M. Carena, D. Garcia, U. Nierste, and C. Wagner, *Phys. Lett.* **B 499** (2001) 141.

[17] G. W. Bennett *et al.* [Muon g-2 Collaboration], Phys. Rev. Lett. **89**, 101804 (2002) [Erratum-ibid. **89**, 129903 (2002)].

[18] M. Knecht and A. Nyffeler, *Phys. Rev.* **D 65** (2002) 073034; M. Knecht, A. Nyffeler, M. Perrottet, and E. De Rafael, *Phys. Rev. Lett.* **88** (2002) 071802; M. Hayakawa and T. Kinoshita, hep-ph/0112102; I. Blokland, A. Czarnecki, and K. Melnikov, *Phys. Rev. Lett.* **88** (2002) 071803; A. Nyffeler, hep-ph/0209329.

[19] K. Melnikov, *Int. J. Mod. Phys.* **A 16** (2001) 4591 (His updated analysis of the SM value of δa_μ was presented at the High Energy Physics Seminar, University of Hawaii, March 2002); F. Jegerlehner, hep-ph/0104304; K. Hagiwara, A. D. Martin, D. Nomura, and T. Teubner, hep-ph/0209187.

[20] M. Davier, S. Eidelman, A. Hocker, and Z. Zhang, hep-ph/0208177.

[21] H. Baer, C. Balázs, J. Ferrandis, and X. Tata, *Phys. Rev.* **D 64** (2001) 035004.

[22] J. Ellis, T. Falk, and K. Olive, *Phys. Lett.* **B 444** (1998) 367; J. Ellis, T. Falk, K. Olive and M. Srednicki, *Astropart. Phys.* **13** (2000) 181.

[23] J. Ellis and K. Olive, *Phys. Lett.* **B 514** (2002) 114.

[24] J. Feng, K. Matchev, and F. Wilczek, *Phys. Lett.* **B 482** (2000) 388 and *Phys. Rev.* **D 63** (2001) 045024.

[25] M. Carena *et al.*, hep-ph/0010338.