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Updating finite element model using frequency domain decomposition method and bees algorithm

P. Alimouri[1], S. Moradi[2*], R. Chinipardaz[3]

[1] *PhD, Student, School of Mechanical Engineering, College of Engineering, University of Shahid Chamran Ahvaz*

[2] *Professor, School of Mechanical Engineering, College of Engineering, University of Shahid Chamran Ahvaz*

[3] *Professor, School of Statistics, College of Computer & Mathematics Science, University of Shahid Chamran Ahvaz*

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Abstract

The following study deals with the updating the finite element model of structures using the operational modal analysis. The updating process uses an evolutionary optimization algorithm, namely bees algorithm which applies instinctive behavior of honeybees for finding food sources. To determine the uncertain updated parameters such as geometry and material properties of the structure, local and global sensitivity analyses have been performed. The sum of the squared errors between the natural frequencies obtained from operational modal analysis and the finite element method is used to define the objective function. The experimental natural frequencies are determined by frequency domain decomposition technique which is considered as an efficient operational modal analysis method. To verify the accuracy of the proposed algorithm, it is implemented on a three-story structure to update its finite element model. Moreover, to study the efficiency of bees algorithm, its results are compared with those particle swarm optimization and Nelder and Mead methods. The results show that this algorithm leads more accurate results with faster convergence. In addition, modal assurance criterion is calculated for updated finite element model and frequency domain decomposition technique. Moreover, finding the best locations of acceleration and shaker mounting in order to accurate experiments are explained.

Keywords: Finite Element Model, Operational Modal Analysis, Frequency Domain Decomposition, Bees Algorithm, Sensitivity Analysis.

[*] Corresponding Author. Tel.: +98613330010
Email Address: moradis@scu.ac.ir

1. Introduction

Due to increase in demand for increased efficiency and reduced weight of structures in modern industries such as aviation and aerospace industries, developing an accurate understanding of dynamic and vibrational behavior of systems and providing a precise model to describe their behavior are necessary. Accurate dynamic model ensures accuracy of subsequent analyses done on the structure such as structural health monitoring, damage detection and many others [1].

During the last few decades, interest for modal analysis raised remarkably. One of the most important ways to identify dynamic characteristics of civil or mechanical structures such as dams, bridges, ships and airplanes is experimental modal analysis [2]. Nowadays modal analysis is divided in two important branches, empirical modal analysis (EMA) and operational modal analysis (OMA) [3]. During in EMA, both the applied force and response of structure are measured. Then, by combination of this information and identification methods, dynamic properties of each structure can be obtained. There are several methods to find modal parameters using EMA which can be found in [2]. However, providing and measuring input forces in real condition are approximately impossible, therefore for solving this drawback OMA was created. Response of structure due to any external force is the sole information used in operational modal analysis [3]. Operational modal analysis methods are categorized in frequency and time domains [4]. Peak picking, transmissibility and frequency domain decomposition are the most demanding methods in frequency domain, whereas stochastic subspace identification, linear autoregressive method and Ibrahim's time domain method are considered as the most important methods in time domain. Increasing demands of various industries for precise calculation of modal parameters have resulted in expanding operational modal analysis in the past two decades. James et al. [5] formulated natural excitation method based on applying cross-correlations between obtained responses from structures. This method is considered as the first method in operational modal analysis. In addition Ibrahim time domain [6], polyreference and eigensystem realization algorithms [7] are second generation of operational modal analysis in time domain. However, the most powerful method in time OMA is stochastic subspace identification method which was created by Van Overschee [8]. Order of system is considered as the most important factor in most of OMA modal analysis in time domain. Inappropriate order selection causes essential problem to identify accurate modal parameters [4]. Peak-picking is the first OMA method in frequency domain which is based on power spectral density (PSD) [9]. This method could not distinct close natural frequencies and this is the main drawback of this method.

To overcome this problem, frequency domain decomposition (FDD) was created [3]. This method is based on PSD and singular value decomposition transform (SVD). FDD method is capable to detect closed modes; however it is not able to estimate damping in structures. Enhanced frequency domain decomposition (EFDD) which is considered as the next generation of FDD method eliminated this problem and detects damping in structures [10, 11]. Pioldi et al [12] created a new version of FDD method as refined FDD method. The main characteristic of this new method is to calculate modal parameters of structures which have high damping. Then, they applied their method on a frame and obtain dynamic parameters with an acceptable accuracy.

Updating the finite element model is considered as an inverse approach which involves reducing the difference between finite element model and physical model. Therefore, the updating process involves an optimization gradient based optimization techniques have been used extensively in the finite element model updating. Collins et al. [13] have used inverse eigensensitivity method for updating the finite element model. However, failing to find the global optimum point of system is one of the fundamental problems of these methods. In addition, the existence of optimum points in system boundaries results in the reduction in efficiency of these methods. In order to solve these drawbacks, the intelligent optimization methods such as genetic algorithm, bee algorithm and particle swarm optimization algorithm were invented. These methods do not need to use gradient information and are not sensitive to initial guess. Consequently, considering these characteristics, these methods can be used in finite element model updating. Dunn et al. [14] applied genetic algorithm for updating the finite element model of a F/A-18 aircraft based on experimental data. Moradi et al. [15] updated a piping system in finite element model using bee algorithm and classical modal analysis data. Then, they compared their results with genetic and PSO algorithms. Malekzehtab et al. [16] applied genetic algorithm for finite element model updating and damage detection of a jacket offshore platform. Their objective function was defined based on the natural frequencies and mode shape of the offshore platform. Chouksey et al. [17] utilized experimental data for updating a rotating shaft with two journal bearing supports. They modeled journal bearing supports with linear and rotating springs and dampers.

Moradi and Alimouri [18] used the differential quadrature method, experimental modal analysis and bees optimization method in order to obtain location, depth and size of cracks in structures. Torres et al. [19] used operational modal analysis to update FEM Metropolitan Cathedral of Santiago Chile. They used FDD and EFDD methods to obtain dynamic parameters of structure. Then, an objective function defined based on mode shapes and natural frequencies to update FE model of this structure. Ebrahimi et al. [20] updated finite model of a cutting harvest using FDD technique. Then, they could reduce the vibration of this machine by adding some masses to real model.

In this study, the modal parameters of the system are calculated empirically using frequency domain decomposition method. Then effective parameters in model are obtained by sensitivity analysis. Next, an objective function based on natural frequencies calculated by operational modal analysis and finite element model is defined to update finite element model, and finally, this objective function minimized by the bees algorithm. To evaluate the proposed algorithm, these steps are applied on a three-story structure and the results are compared with PSO and Neader-Mead results. Moreover, in order to ensure accurate results from operational modal analysis, the appropriate location of accelerometers and excitation by shaker are determined.

2. Theory

2.1. *FDD Method*

The FDD method is based on a conventional relation between input and output of a system [3]. The input is assumed be a stationary random process and the basic relation in FDD method is stated by Eq.(1).

$$G_{yy} = H(j\omega) G_{xx} H(j\omega)^H \qquad -\infty < \omega < \infty \tag{1}$$

In the above equation $G_{xx}(\omega)\in R^{p\times p}$ is defined as power spectral density function (PSD) and p is regarded as number of input channels. In this equation $G_{yy}(\omega) \in R^{q\times q}$ is output PSD matrix and q is number of output signals. $H(\omega)$ is the $(p \times q)$ frequency response function (FRF) matrix and overbar H denotes Hermitian operator. Therefore, the best method for showing FRF matrix is based on as [3, 11, 21]:

$$H(j\omega) = \sum_{k=1}^{k} \left(\frac{R_k}{j\omega - \lambda_k} + \frac{\bar{R}_k}{j\omega - \bar{\lambda}_k} \right) \qquad -\infty < \omega < \infty \tag{2}$$

where k is considered as the number of vibration modes, λ_k and $\bar{\lambda}_k$ are the poles of the FRF function, and R_k is the $(p \times q)$ residue matrix and it can be depict as [12]:

$$R_k = \phi_k \Gamma_k^T \tag{3}$$

where $\phi_k = [\ \phi_{k1}\ \phi_{k2} \dots \phi_{km}\ ]^T$ and $\Gamma_k = [\ \Gamma_{k1}\ \Gamma_{k2} \dots \Gamma_{kr}\ ]^T$ are the $k_{th}\in R^{p\times 1}$ mode shape vector and modal participation factor vector, respectively [21]. When all output measurements are equal by input references, $H(\omega)$ becomes a square matrix. Then, Eq.(1), and Eq.(2), can be rewritten as [21]:

$$G_{xx}(j\omega) = \sum_{k=1}^{n} \frac{A_k}{j\omega - \lambda_k} + \frac{A_k^H}{-j\omega - \bar{\lambda}_k} + \frac{\bar{A}_k}{j\omega - \lambda_k} + \frac{A_k^T}{-j\omega - \bar{\lambda}_k} \tag{4}$$

where A_k is the residue matrix of the PSD output. Since, for the PSD output, the residue matrix is an $(q \times q)$ matrix is obtained by [21]:

$$A_k = R_k G_{yy} \left(\sum_{s=1}^{n} \frac{R_k}{-\lambda_k - \lambda_s} + \frac{\bar{R}_k}{-\lambda_k - \bar{\lambda}_s} \right) \tag{5}$$

For light structural damping (small damping in most of civil structure ratios $\zeta \approx .1$), the pole can be expressed in an approximate form. Thus, in the close of the k^{th} natural frequency, only the $\bar{R}_k$ term exist, therefore the residue matrix can be derived from Eq.(3) as [11, 12]:

$$A_k = \left[\frac{R_k}{2(\zeta_k \omega_k - i\omega_k)} + \frac{\bar{R}_k}{2\zeta_k \omega_k} \right] G_{FF} \bar{R}_k^T \cong \frac{\bar{R}_k G_{FF} R_k^T}{2\zeta_k \omega_k} = d_k \bar{\phi}_k \phi_k^T \tag{6}$$

If the damping ratio is low the power spectral density matrix can be written as below:

$$G_{xx}(j\omega) = \sum_{k\in Sub}^{n} \frac{d_k \phi_k \phi_k^T}{j\omega - \lambda_k} + \frac{\bar{d}_k \phi_k \phi_k^T}{j\omega - \bar{\lambda}_k} \qquad -\infty < \omega < \infty \tag{7}$$

Eq.(7) is expressed for negative and positive natural frequency range; however in industries application only the positive part will be considered, therefore Eq.(7) can be converted to Eq.(8) [11, 12, 21]:

$$G_{xx}(j\omega) \cong \sum_{k\in Sub}^{n} \frac{d_k \phi_k \phi_k^T}{j\omega - \lambda_k} = \bar{\Phi} \left\{ diag \left[\mathrm{Re} \left(\frac{2d_k}{i\omega - \lambda_k} \right) \right] \right\} \Phi^T \tag{8}$$

Finally singular value decomposition (SVD) is used to decompose output power spectral density $G_{yy}(\omega)$. SVD transform decomposes $Gyy(\omega)$ into singular values and singular vectors. Then, by using these singular values and singular vectors natural frequencies and mode shapes of structures will be obtained, respectively. In Fig. 1 algorithm of FDD method is explained.

2.2. *Selection the best freedom degrees for operational modal test*

In order to transfer excitation energy to all degrees of freedom of a system, it should be noted that the excitation points should not be located near nodes of mode shapes of the system. By using optimum driving point (ODP) parameter given in Eq.(9), it can be determined that how near the degrees of freedom are to mode nodes of the system.

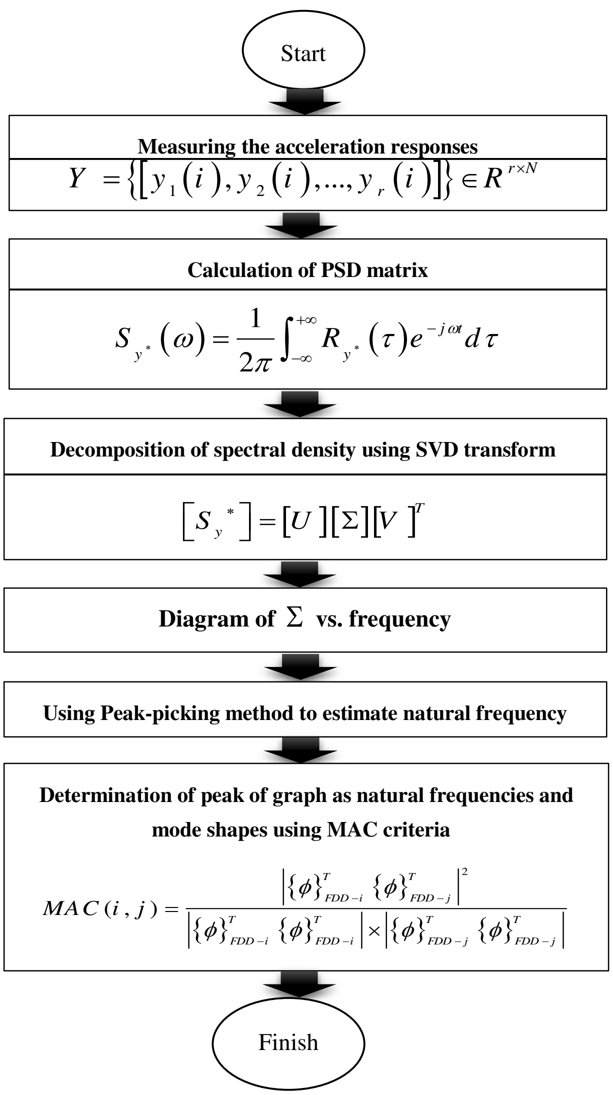

Figure 1: Flowchart of FDD method

$$\mathrm{ODP}(j) = \prod_{r1=1}^{m} \left\| \phi_{jr1}^2 \right\| \qquad (9)$$

where, j and m are degree of freedom and number of mode shapes of the system, respectively. The points of which ODP values are zero or close to zero are not suitable for stimulation of system, because they are placed in mode shape nodes or near them. By contrast, points with maximal ODP values are appropriate for excitation and mounting accelerometers. While exciting a structure by a shaker, the possibility of interference between shaker and structure establishes which should be minimized. Each shaker is composed of a system of mass, spring and damper and any interference between it and the structure causes changes in signal generated by the shaker. To reduce this effect, shaker should be connected to the structure at where average acceleration is of minimum value. The average acceleration is defined using average driving dof displacement (ADDOFA) as in Eq.(10).

$$ADDOFA(j) = \sum_{r1=1}^{m} \frac{\phi_{jr1}^2}{\omega_{r1}^2} \qquad (10)$$

The possibility of interference excitation in points with high ADDOFA is higher than in other points; therefore, the points with higher ODP/ADDOFA ratio are considered more appropriate points for excitation.

2.3. *Sensitivity Analysis*

Various mathematical models have been implemented for estimation of complex phenomena in different areas such as engineering, economy and physics. Identification of effective parameters in these mathematical models is the major challenge which users often deal with. Sensitivity analysis is one of the most important methods able to identify the parameters that have the greatest impact on the results. In other words, any small changes in sensitive parameters would result in significant changes in output.

Once at a time index (OAT) is considered as one of the most applicable criteria for determination of sensitive parameters. Eq. (11) states index (OAT) for identification of sensitive parameters. In this case, according to Eq. (11), a dimensionless index is considered in order to remove the effects of various units of parameters.

$$\mathrm{OAT} = \frac{\Delta Y}{\Delta X}\frac{X}{Y} \qquad (11)$$

where, X and Y are input and output parameters of the model, respectively. Moreover, factor X/Y is defined as a normalized coefficient to eliminate the effects of the units. The OAT index defined in Eq.(12)

measures the local sensitivity. Global sensitivity index (GSI) is defined by Eq. (12) in which overall sensitivity can be estimated.

$$\mathrm{GSI} = \frac{Y_{\max} - Y_{\min}}{Y_{\max}} \qquad (12)$$

here, Y_{min} and Y_{max} are minimum and maximum output of model using upper and lower limits bounds of the input parameters, respectively. According to Eqs. (11) and (12), parameters that play fundamental role in output can be identified. In this research, input parameters are physical properties of structure and natural frequencies are considered as outputs. By using sensitivity analysis, physical parameters which are suitable for optimization algorithm can be determined.

2.4. *Bees Algorithm*

Bees optimization algorithm is classified in evolutionary algorithms such as genetic algorithm, PSO algorithm and so on. There is an organized social behavior among bees which can be used for solving complex optimization problems. There are scout bees in each swarm whose main task is to find food sources for their hives. As scout bees find new food sources, they return to their hives and evaluate the different discovered gardens based on specific parameters. Then, scout bees by using toggle dance provide direction, distance and amount of nectar in these gardens for worker bees. Then, the worker bees fly to the detected locations. The number of worker bees sent has direct proportion to nectar amount available in the detected garden and reverse proportion to its distance. In other words, more worker bees are sent to gardens which have more nectars and short distance to hive. Therefore, this strategy enables bees swarm to obtain food sources in an efficient procedure. From N_t random solutions, N_{t1} solutions which have higher fitness values are considered as the best solutions. Then, N_{t2} solutions are selected as elite ones among the best solutions. In order to find better solution, the best solutions neighborhood is searched. Therefore, n_{t1} and n_{t2} denote the number of neighborhoods searched around the best and elite solutions, respectively (n_{t1}<n_{t2}). Then, the remaining solutions are chosen randomly in the search space to find other solutions Eq. (13) indicate formation of a new generation in bees algorithm.

$$N_{new} = \underbrace{\left((N_{t1} - N_{t2})n_{t1} + (N_{t2})n_{t2}\right)}_{N_{t1}selected} + \left(N_{old} - N_{t1}\right) \qquad (13)$$

where, N_{new} and N_{old} are the new generation and the previous generation, respectively. Additionally, number of population in each generation is fixed. These steps continue whenever the convergence criteria happened. In this research, an objective function is defined for finite element model updating. This objective function is based on natural frequencies obtained from finite element method and operational modal analysis (Eq. (14)).

$$Error\ function\left(\{Z\}\right) = \sum_{j=1}^{s}\left(\Omega_{c,j} - \Omega_{uc,j}\right)^2 \qquad (14)$$

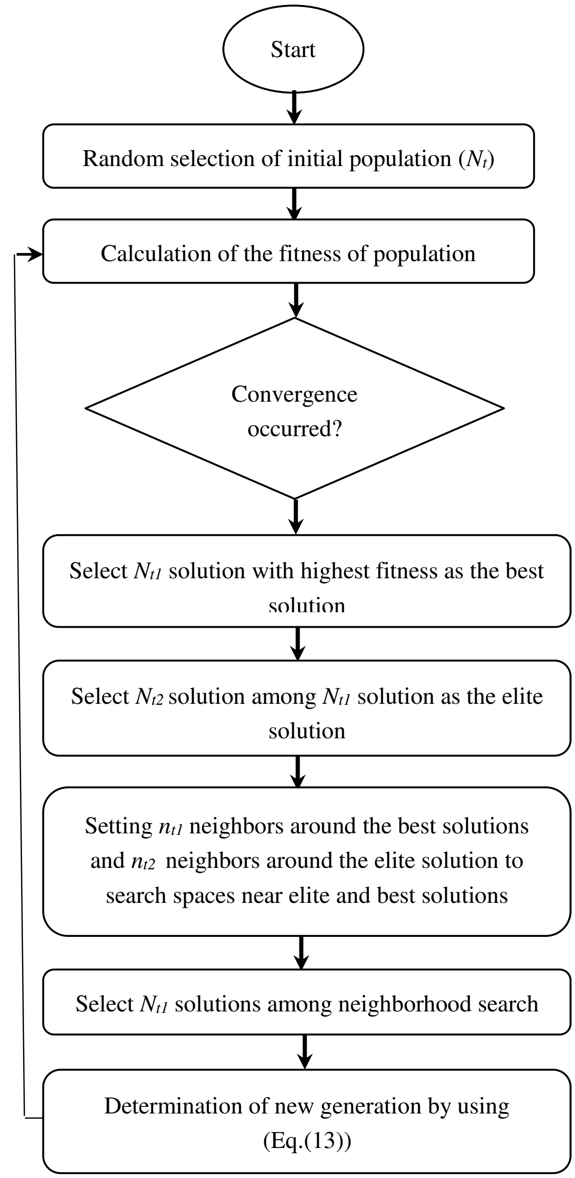

Figure 2: Flowchart of bees optimization algorithm

where s is the number of natural frequencies used in optimization problem, $\Omega_{c,j}$ denotes natural frequencies obtained from finite element method and $\Omega_{uc,j}$ denotes natural frequencies calculated using operational modal analysis. Moreover, Z vector includes design parameters which are identified by sensitivity analysis (section (2-3)). By minimization of objective function (Eq. (14)) using bees optimization algorithm, design parameters are obtained and a precise finite element model based on real structure is designed. Fig.2 shows the flowchart of bees algorithm.

3. Results and discussion

The main objective of this research is to optimize the finite element model of structures by using a combination of FDD method, sensitivity analysis and finite element method. The algorithm of the proposed method used is described in Fig. 3. To verify the algorithm, a three-story structure is built and the proposed algorithm is tasted on it. Results are depicted in the following steps.

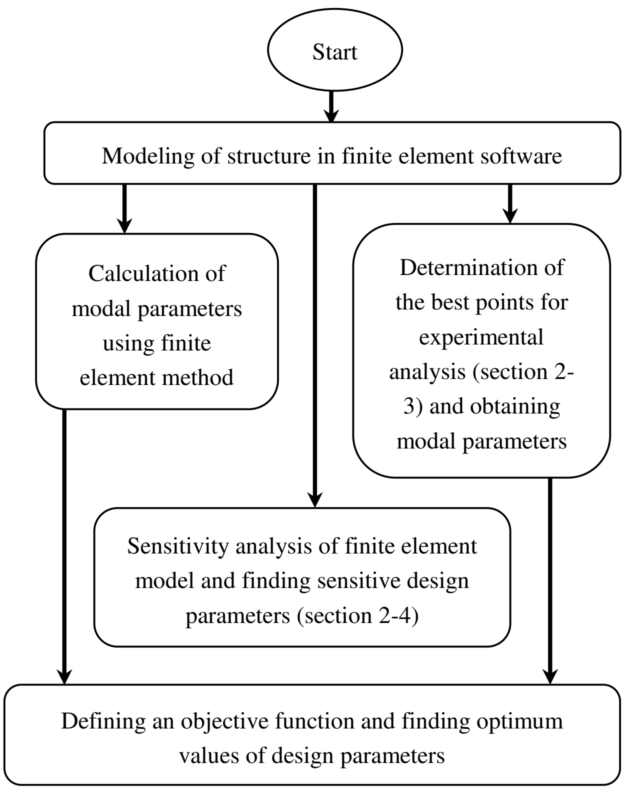

Figure 3: Finite model updating algorithm

3.1. *Finite element model*

Fig. 4 shows sketch of the three-story structure. This structure is built by steel and all connections are welded. Moreover, anchor bolts are used to connect the frame to the ground.

In Table 1, all dimensions of the three-story frame are presented.

In Fig. 5, finite element model of three-story structure in ANSYS software is displayed.

Solid 186 Element is used for building this model. Additionally, in this model, welds are taken into account as a change in Young's modulus in connections.

26781 elements were used to build the model. Applying the boundary conditions the natural frequencies of the frame were calculated by the eigenvalue analysis in the finite element software (ANSYS) and the results are listed in Table 2.

3.2. *Appropriate points for shaker and accelerometer installation*

Acquiring modal parameters needs an accurate planning for conducting experiments. Appropriate location of the accelerometers and shakers can lead to more accurate modal parameters estimation. According to section (2-2), by using ODP and ODP/ADDOFA, the best points of accelerometers measurement and structure stimulation by shaker can be detected. Fig. 6 and Table 3 display and tabulated these points in *y* direction, respectively.

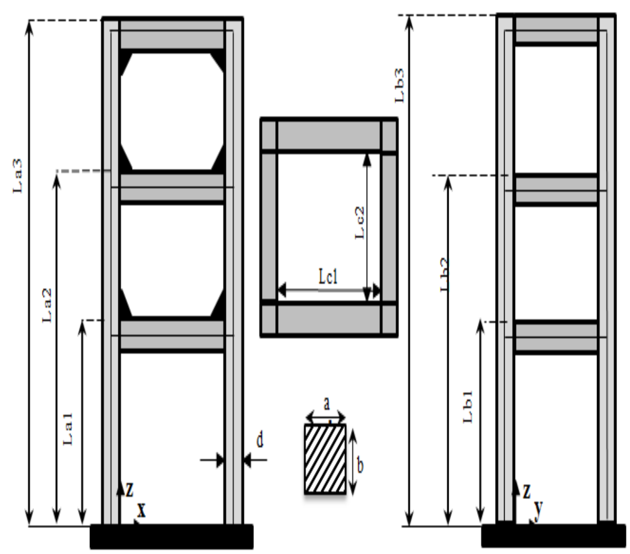

Figure 4: Plan of three views of the three-story structure

Table 1: Dimensions of the three-story frame

No	parameter	symbol	Length (m)
1		La1	0.600
2	Length of horizontal members in the y-z plane	La2	1.100
3		La3	1.600
4		Lb1	0.600
5	Height of horizontal members in the x-z plane	Lb2	1.100
6		Lb3	1.600
7	Length of horizontal members in the x-z plane	Lc1	0.500
8		Lc2	0.400
9	Cross section	a	0.016
10		b	0.016

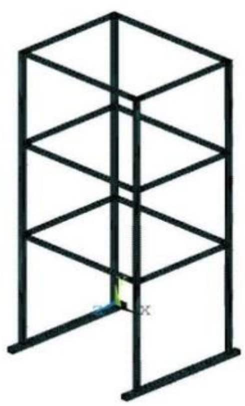

Figure 5: Finite element model of the three-story structure

Table 2. Natural frequencies obtained by FEM (rad/sec)

Freq	1st	2nd	3rd	4th	5th	6th
FEM	73.05	84.70	92.86	234.56	276.03	326.48

Fig. 7 shows the best places for accelerometers and shaker installation in *x* direction and Table 4 presents their corresponding coordinates

3.3. *Sensitivity analysis*

In order to determine the effective parameters for finite element model updating, sensitivity analysis was applied on the three-story frame according to section (2-3). Table 5 presents lower and upper limits of design parameters for sensitivity analysis (see section (2-3)).

Figs. 8 and 9 present local and global sensitivities for the three-story structure respectively.

As is clear from Fig. 8, maximum local sensitivity belongs to those design parameters related to length of members and physical characteristics of structure such as Young's modulus and density, however the minimum values are related to Young's modulus of weld connections and the cross-section of members. Moreover, in global sensitivity analysis, approximately the same parameters considered in local sensitivity analysis have direct impact on the natural frequencies obtained.

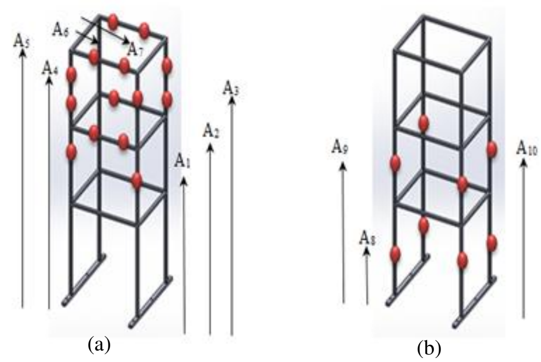

Figure 6: a- Best location of accelerometer installation in y direction b- Best location of shaker installation in y direction

Table 3: Coordinates of the best locations for accelerometers and shaker installation in y direction

Point	Length (cm)	point	Length (cm)
A_1	95	A_6	12
A_2	110	A_7	46
A_3	122	A_8	44
A_4	144	A_9	86
A_5	160	A_{10}	105

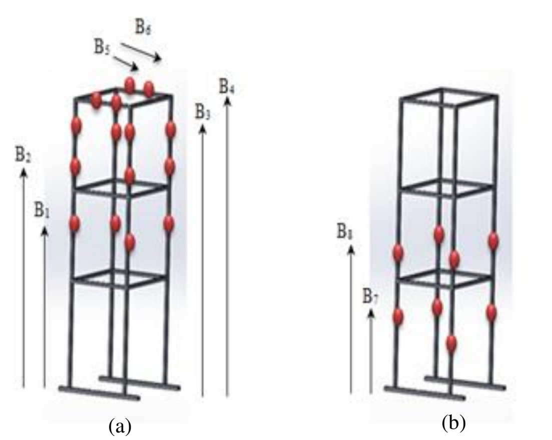

Figure 7: a-The Best location of accelerometer installation in *x* direction b-The Best location of shaker installation in *x* direction

Table 4: Coordinates of the best locations for accelerometers and shaker installation in *x* direction

point	Length (cm)	point	Length (cm)
B_1	91	B_5	13
B_2	120	B_6	33
B_3	148	B_7	50
B_4	160	B_8	72

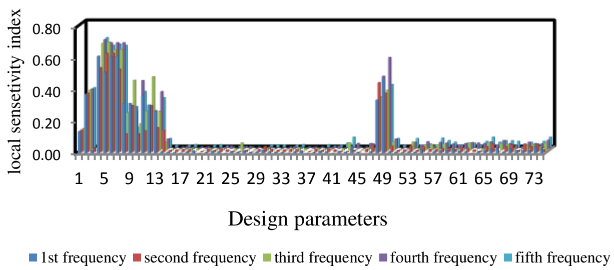

Figure 8: Results from local sensitivity analysis

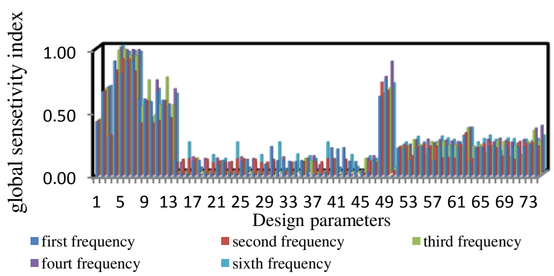

Figure 9: Results from global sensitivity analysis

Table 5. Upper and lower bounds of design parameters

no	Parameters	Symbol (unit)	Lower limit	Upper limit
1	Density	ρ (kg/m^3)	7600	8000
2	Young's modulus	E (Gpa)	180	220
3	Poisson's ratio	ν ([])	0.2	0.4
4		Da1 (m)	1.55	1.65
5 6 7	Length of vertical members	Da2 (m) Da3 (m) Da4 (m)	1.55 1.55 1.55	1.65 1.65 1.65
8 9 10	Height of horizontal members in x-z plane	Dd1 (m) Dd2 (m) Dd3 (m)	0.58 1.07 1.55	0.62 1.14 1.65
11 12 13	Height of horizontal members in y-z plane	De1 (m) De2 (m) De3 (m)	0.58 1.07 1.55	0.62 1.14 1.65
14-29	Width of cross section	a1….a 16 (m)	0.015 5	0.0165
31-45	Length of cross section	b1….b 16 (m)	0.015 5	0.0165
46	Length of horizontal bars in x-z plane	Dc (m)	0.48	0.52
47	Length of horizontal bars in y-z plane	Dd (m)	0.38	0.42
58-71	Weld Young's modulus	E1…E 24 (Gpa)	180	220

3.4. *Operational modal analysis*

Operational modal analysis is regarded as a subset of modal analysis that only depends on output responses. In this research, the FDD method is applied to identify dynamic parameters of the three-story structure. Random inputs is the main assumption of FDD method, thus for random stimulation of the three-story structure, an electro-dynamic shaker is applied. This electro-dynamic shaker can produce random, burst random, Pseudo random, Sweep random and Periodic random signals. Then, by using accelerometers attached on the structure, the output signals are captured and send to time recorder software. Fig. 10 presents all of equipment used in operational modal analysis of the structure.

In Fig. 11 the time response captured by the accelerometers under random excitation is displayed. Moreover, in this experiment, sampling rate 16328 was considered and 5 accelerometers gathered acceleration of structure simultaneously. Accelerometers are piezo-Tronic type (A120/V) and

TIRA is the brand name of shaker which is used in this experiment.

Natural frequencies obtained from various kinds of signals using FDD method are presented in Table 6. Additionally, the last row of Table 6 presents natural frequencies calculated by classic modal analysis.

Figure10: Equipment used in experimental modal analysis

Fig. 12 shows relative error percentage of natural frequencies of the structure obtained from operational modal analysis and classic modal analysis (hammer test). According to this figure, the natural frequencies calculated by pusedo random signals are more consistent with those obtained from classic modal analysis and consequently, only this signal will be used in the following sections of this research to obtain the updated model. Frequency diagram of FDD method for pusedo random excitation signal in *x* direction is depicted in Fig. 13. The natural frequencies of the three-story structure obtained from FDD and finite element methods plus relative error these are tabulated in Table 7.

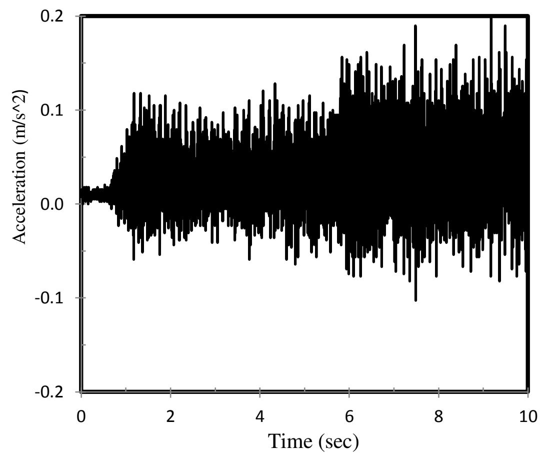

Figure 11: Response of the three-story structure under random excitation

3.5. *Bees updating results*

According to Table 7, the relative error of natural frequencies obtained by FDD and FEM methods is high and it suggests that there might be a defect in finite element modeling. This defect may be due to introducing physical properties such as density, Young' module and so on to the software or can be related to geometric properties of structure such as length, thickness and size of members. For solving this problem, bees algorithm is applied in order to minimize this error. Three different sets of design parameters are defined for bees algorithm. The first set includes parameters obtained from sensitivity analysis, second set includes parameters not important for sensitivity analysis and material properties and the third set consists of all design parameters. In this research, these sets are the parameters used in Table 8.

Fig. 14 depicts convergence of bees algorithm based on different sets of design parameters. As it is clear from Fig. 14, faster and more precise convergence is obtained using sets 1 and 3 than set 2. However, this convergence occurred with 14 design parameters by set 1 and 71 design parameters in set 3. Additionally, run time of the optimization process in set 1 is lower than that in set 3; this shows that optimization using set 1 could achieve optimal response in minimum time. Moreover, since all of design parameters in set 1 were obtained by sensitivity analysis, it is clear that sensitivity analysis is capable of identifying effective parameters in optimization process.

Table 6. Experimental natural frequencies calculated by FDD and classical methods

method	Input signal	Frequency no.					
		1st	2nd	3rd	4th	5th	6th
FDD (method)	Random	61.120	73.710	91.130	205.180	238.360	293.260
	Burst random	60.190	74.430	93.380	207.670	241.090	297.780
	Pseudo random	61.870	71.890	89.680	206.170	237.830	293.010
	Sweep random	65.130	78.410	97.130	212.450	241.390	296.670
	Periodic random	54.130	77.120	98.230	216.890	249.870	301.230
Classical modal	hammer	62.328	71.341	89.380	206.240	238.010	292.210

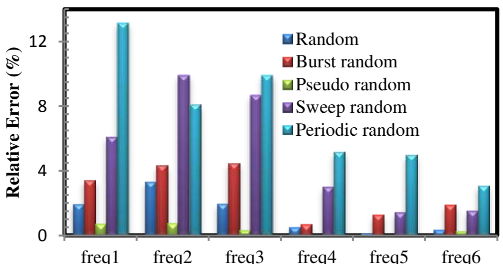

Figure12: Relative error percentage of natural frequencies of FDD method and classic modal analysis

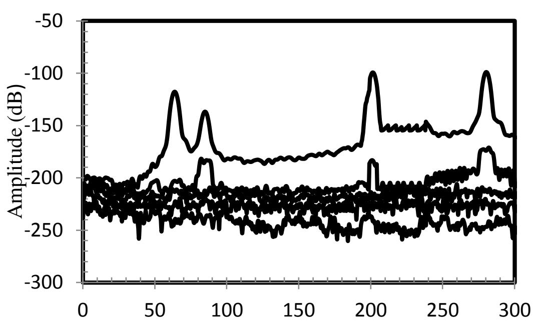

Figure13: Diagram of decomposition of power spectral density using SVD decomposition in x direction

Table 7: Natural frequencies obtained from FDD and FEM method-(rad/sec)

Frequency no.	FDD method	FEM method	Relative error (%)
1	61.870	73.050	15.304
2	71.890	84.700	15.124
3	89.680	92.864	3.429
4	206.170	234.562	12.104
5	237.830	276.033	13.840
6	293.010	326.483	10.252

Table 8: Design parameter sets used in FEM updating

Design parameters	Set 1	Set 2	Set 3
ρ	√	√	√
E	√	√	√
Da1	√	×	√
Da2	√	×	√
Da3	√	×	√
Da4	√	×	√
Dd1	√	×	√
Dd2	√	×	√
Dd3	√	×	√
De1	√	×	√
De2	√	×	√
De3	√	×	√
$a_1 \ldots a_{16}$	×	√	√
$b_1 \ldots b_{16}$	×	√	√
Dc	√	×	√
Dd	√	×	√
E1…E24	×	√	√
ν	×	√	√

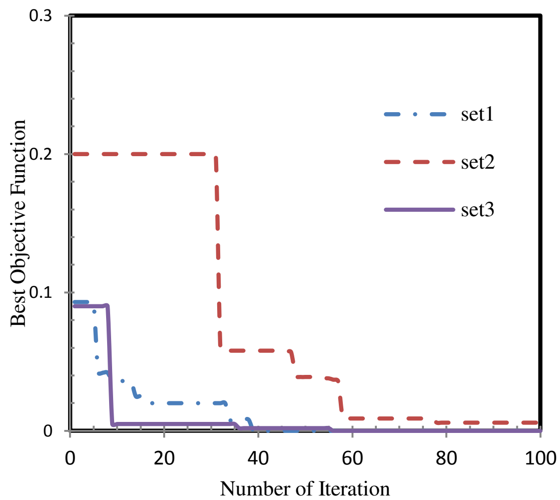

Figure14: Convergence of bees algorithm

Table 9 presents control parameters utilized in bees optimization algorithm. These values have been obtained by empirical studies. Table 10 lists the natural frequencies obtained from updated finite element model done by bees optimization algorithm. It can be deduced from Tables 7 and 10 that the relative error between natural frequencies calculated by FEM method and the experimental modal analysis decreased remarkably after model updating. Total relative error before model updating was %60.075 that reduced to %1.785 after model updating.

In order to verify performance of bees optimization algorithm in finite model updating, the finite element model of three-story structure is updated by PSO and Nelder-Mead optimization algorithm; then results of these algorithms are compared with each other. PSO method is an evolutionary optimization method designed by Kennedy and Eberhart [22]. This method is inspired by migration behavior of birds or fish schooling. PSO algorithm is based on particles movements in which each particle can be considered as a possible optimal solution. Particles in PSO method follow a very simple behavior: trying for neighboring particles success and their success. Finding optimal solution area in the search space with a large number of dimensions is the result of mass movement behavior.

Nelder-Mead algorithm is a direct method for finding minimal value of an unconstrained n-dimensional objective function presented by Nelder and Mead [23]. This method is based on the comparison of the function value in the n + 1 vertices of a simplex and replacing the worse vertex in terms of the objective function with a new point. Moreover, Nelder-Mead method works only by using the function values and does not need any function derivatives; thus it is classified as a direct method.

Fig. 15 portrays convergence diagram of bees algorithm, PSO and Nelder-Mead method for finite model updating of three-story structure using first set of design parameters.

As can be seen from Fig. 15, bees algorithm converges faster than other optimization algorithms and its objective function has the lowest value among all investigated methods. The first set of design parameters optimized using various kinds of optimization methods are tabulated in Table 11. To evaluate the efficiency of optimization algorithms, two criteria are defined, practical reliability index and normalized price value. Practical reliability index is defined as the probability of the solution to reach a practical optimum. Additionally, a practical optimum is stated as an optimal solution within 0.1% of the final optimum response. In this essay, the best value of the objective function is calculated after 200 iterations; thus this objective function was used to define practical reliability. Furthermore, normalized price value is defined as the number of objective functions calculated to the practical reliability ratio.

Table 12 indicates values of practical reliability and normalized price for the three proposed optimization algorithms in this essay. According to Table 12, BA algorithm achieved the optimal solution with lower cost than other methods. To compare the mode shapes obtained from FDD and FEM methods, the mode assurance criterion (MAC) is defined. The relation of MAC criteria is stated in Eq.(15).

$$MAC(i,j)=\frac{\left|\{\phi\}_{FDD}^{T}\{\phi\}_{FEM}^{T}\right|^{2}}{\left|\{\phi\}_{FDD}^{T}\{\phi\}_{FDD}^{T}\right|\times\left|\{\phi\}_{FEM}^{T}\{\phi\}_{FEM}^{T}\right|} \qquad (15)$$

The MAC values for the mode shapes of FDD and FEM is shown in Fig. 16. As can be seen from this figure, the mode shapes calculated by these methods are compatible with each other and the value of MAC criteria is more than 0.8 for the same mode shapes.

4. Conclusions

In this study, finite model of a three-story structure was updated using BA optimization algorithm and FDD method. Several design parameters were selected for updating this model some of which were chosen by local and global sensitivity analyses. An objective function based on summation of the squared errors between natural frequencies obtained from finite element model and experiment model was defined. As the inputs to structure were random, only operational modal analysis is able to identify modal parameters of structure with an appropriate accuracy; therefor, FDD method as one of the strong parametric methods in operational modal analysis was performed on structure and could obtain modal parameters of real structure. In order to update finite element model, BA optimization method minimized the defined objective function and finally the best set of designed parameters was obtained. Results show that the combined errors between the first six natural frequencies were reduced from %60.075 to %1.785 after updating. For verification of the BA optimization algorithm, the proposed objective function was minimized by PSO and Nelder-Mead methods. Results showed that BA algorithm was faster than these methods and BA algorithm successfully brought finite element model closer to the real model. Moreover, the MAC values for the same modes are more than 0.8 indicated that mode shapes obtained by FDD and FEM method are compatible with each other.

Table 9: Control parameters of bees algorithm

Control parameter	values
Number of objective functions	40000
Neighborhood radius	0.01
N_{t1}	20
N_{t2}	12
n_{t1}	10
n_{t2}	15
N_t	100
Number of iterations	100

Table 10: Comparison of the natural frequencies from the updated finite element models and corresponding empirical values

Mode	Natural frequencies (rad/sec)		Relative error (%)
	Experimental model	Updated finite element model	
1	61.870	61.851	0.031
2	71.890	71.513	0.523
3	89.680	89.647	0.037
4	206.170	205.098	0.523
5	237.830	237.814	0.007
6	293.010	291.086	0.661

Table 11: Design parameters optimized using different optimization algorithms

Design parameters	BA	Nelder-Mead	PSO
ρ (kg/m^3)	7712	7643	7643
E (Gpa)	198	201	201
Da1	1.61	1.63	1.63
Da2	1.61	1.63	1.63
Da3	1.60	1.64	1.64
Da4	1.60	1.65	1.65
Dd1	0.58	0.60	0.60
Dd2	1.09	1.12	1.12
Dd3	1.63	1.64	1.64
De1	0.62	0.58	0.58
De2	1.08	1.10	1.10
De3	1.65	1.63	1.63
Dc	0.51	0.49	0.49
Dd	0.40	0.40	0.40

Table12: Practical reliabilities and normalized prices obtained from BA, PSO and Nelder-Mead methods

Method	practical reliability	normalized price
BA	0.9	44444.4
PSO	0.6	66666.7
Nelder-Mead	0.4	100000

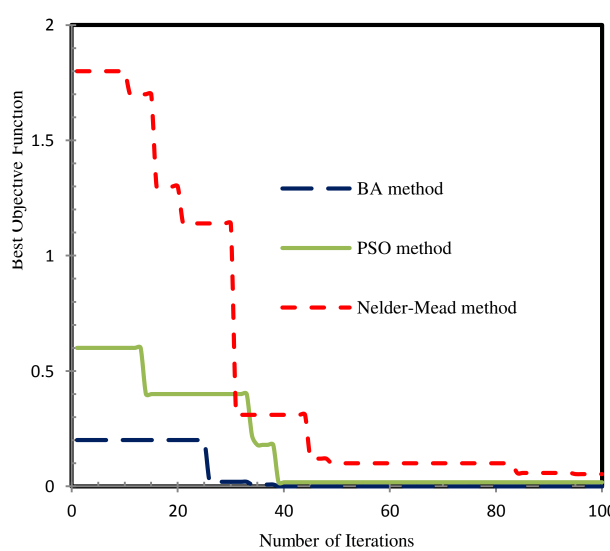

Figure 15: Convergence of various kinds of optimization method with set 1

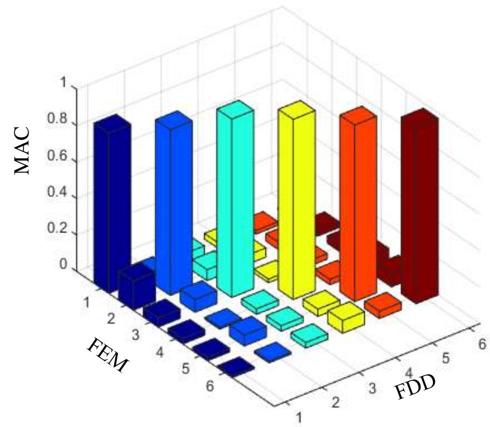

Figure 16: MAC criteria between FDD and FEM mode shapes

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