Uplink Decentralized Joint Bandwidth and Power Allocation for Energy-Efficient Operation in a Heterogeneous Wireless Medium

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***Abstract*—In this paper, energy efficient uplink communications are investigated for battery-constrained mobile terminals (MTs) with service quality requirements and multi-homing capabilities. A heterogeneous wireless medium is considered, where MTs communicate with base stations (BSs) and access points (APs) of different networks in overlapped coverage. Different from the existing works, we develop a quality of service (QoS)-based optimization framework for joint uplink bandwidth and power allocation to maximize energy efficiency for a set of MTs with multi-homing capabilities. The proposed framework is implemented in a decentralized architecture, through coordination among BSs/APs of different networks and MTs, which is a desirable feature when different networks are operated by different service providers. A suboptimal framework is presented with a reduced computational complexity as compared with the optimal framework. Simulation results demonstrate the improved performance of both the optimal and suboptimal frameworks over a state-of-the-art benchmark.**

***Index Terms*—Energy efficient communications, green communications, multi-homing resource allocation, heterogeneous wireless medium.**

I. Introduction

The past decade has witnessed significant advances in the design of mobile terminals (MTs) and the offered communication services for mobile users. Specifically, MTs are currently equipped with processing and display capabilities that enable them to support voice, video, and data calls. In addition, MTs are capable of establishing simultaneous communications with base stations (BSs) and access points (APs) of different networks, through multiple radio interfaces and using the multi-homing feature [2]. However, such an advancement results in high energy consumption of the MTs. It has been shown that there exists an exponential increase in the gap between the MT demand for energy and the offered battery capacity [3]. The operational time of an MT in between battery chargings is considered to be a significant factor in the user perceived quality-of-service (QoS) [4], [5]. Energy efficient (*green*) communication techniques have been proposed as a promising solution to regulate the MT energy usage while satisfying the user required QoS.

In literature, energy efficient techniques can be classified into two broad categories based on the call traffic load. On-off switching of the MT radio interface is adopted at a bursty/low call traffic load to achieve energy saving [6] - [13]. At a continuous/high call traffic load, energy efficient radio resource allocation is employed [14] - [20]. With overlapped coverage from different networks, the wireless medium has become a heterogeneous environment. Using its multiple radio interfaces and through the multi-homing capability, an MT is capable of establishing two modes of communications, namely single-network and multi-homing access [2], [31] - [33]. In a single-network access, the MT connects to the best wireless network at its location, while in multi-homing access, the MT connects to all wireless networks available at its location. Hence, energy efficient radio resource allocation is further classified into single-network [14] - [17] and multi-homing [18] - [20] techniques, based on the number of utilized radio interfaces at the MT. Multi-homing resource allocation can achieve higher energy efficiency compared to the single-network access, due to the potential disparity in: 1) wireless channels between the MT and different BSs/APs and 2) available radio resources at different BSs/APs. However, the multi-homing feature is challenged by the incurred additional energy cost to simultaneously activate multiple radio interfaces of the MT, along with the associated signaling overhead. Hence, efficient radio resource allocation is required for multi-homing MTs to address the aforementioned challenges while achieving the target benefits.

One limitation with the existing multi-homing energy efficient radio resource allocation mechanisms in a heterogeneous wireless medium is that these solutions focus only on optimal power allocation to different radio interfaces of the MT, given an allocated bandwidth. Hence, the main focus so far is on exploiting the diversity in fading channels and propagation losses between the MT and different BSs/APs in order to enhance the uplink energy efficiency. However, further improvement can be achieved by exploiting the disparity in available radio resources at the BSs/APs of different networks. This calls for a joint optimization framework for bandwidth and power

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This work was presented in part at IEEE ICC 2014 [1].

allocation to maximize uplink energy efficiency for a set of MTs with multi-homing capabilities. Furthermore, the existing resource aggregation schemes (e.g., carrier aggregation in long-term evolution (LTE)-advanced [21] - [23]) assume a scenario where all resources belong to the same service provider. Hence, centralized resource allocation schemes can be adopted. On the other hand, in a heterogeneous networking environment, the aggregated resources are operated by different service providers. Hence, novel decentralized mechanisms should be investigated to enable coordination among MTs and BSs/APs of different networks so as to satisfy the target QoS in an energy efficient manner.

In this paper, we propose a QoS-based optimization framework for joint uplink bandwidth and power allocation to maximize energy efficiency for MTs in a heterogeneous wireless medium. Specifically, we summarize the contributions of this work as follows:

- The energy efficient uplink communication problem is formulated to jointly allocate uplink transmission bandwidth and power to a set of MTs, with minimum required QoS and multi-homing capabilities, from a set of BSs/APs with overlapped coverage. In dealing with a multi-user system, we aim to maximize the performance of an MT that achieves the minimum energy efficiency. In addition, the heterogeneity of the wireless medium is captured in the problem formulation, in terms of different service areas, channel conditions, available radio resources at BSs/APs of different networks, and different maximum transmit power at the MTs.
- We show that the radio resource allocation problem is a max-min concave-convex fractional program [34]. Using a parametric approach, the problem is transformed into a convex optimization problem that can be solved efficiently through the Lagrangian decomposition approach.
- Based on the problem solution, we propose an optimal framework for joint bandwidth and power allocation. The framework is implemented in a decentralized architecture, through coordination among BSs/APs of different networks and MTs, which is desirable in case different networks are operated by different service providers.
- To reduce the associated computational complexity and signaling overhead, we propose a suboptimal radio resource allocation framework for joint bandwidth and power allocation.
- The performance of the proposed optimal and suboptimal frameworks is evaluated in comparison with a state-of-the-art benchmark. The benchmark relies only on optimal power allocation to maximize the uplink energy efficiency. Simulation results demonstrate the improved performance of the optimal and suboptimal frameworks in terms of the achieved total energy efficiency, minimum energy efficiency, and total throughput, QoS satisfaction, and reduced computational complexity and signaling overhead of the suboptimal framework.

The rest of the paper is organized as follows. The related work is reviewed in Section II. The system model is presented in Section III. In Section IV, the uplink energy efficient communication problem is formulated and the parametric approach and the Lagrangian decomposition technique are employed to find the optimal bandwidth and power allocation. The decentralized optimal framework is outlined in Section V. Signaling overhead and computational complexity studies are presented in Section VI and a suboptimal framework is developed. Also, a benchmark is presented for comparison. Simulation results and discussions are given in Section VI. Finally, conclusions and future work are given in Section VII. Table I summarizes the important mathematical symbols used in this paper, and Appendices present the proofs of Proposition 1 and Theorem 1.

II. Related Work

Uplink energy efficient radio resource allocation mechanisms can be classified into single-network and multi-homing access techniques, based on the number of utilized radio interfaces at the MT. Specifically, a single-network radio resource allocation mechanism activates a single radio interface of the MT at a time and connects to only one network at a time. On the other hand, a multi-homing resource allocation mechanism employs multiple radio interfaces of the MT and connects it to multiple networks simultaneously.

In the single-network access mechanisms, energy efficient uplink communication is investigated for a set of MTs within one network with specific wireless access technology, e.g., orthogonal frequency division multiple access (OFDMA). The carrier aggregation technique of an OFDMA network is employed in [14] to enable MTs to use multiple carriers to achieve high data rate communications in the uplink with an improved energy efficiency. In [15], a central scheduler is developed to optimize the uplink energy efficiency across an OFDMA network by allocating the system bandwidth among MTs. In [16], subcarrier assignment, modulation, and transmit power adjustment are investigated to optimize the sum of users' bit-per-joule in a multi-cell multi-user OFDMA network. The users' energy efficiency in a multi-cell uplink OFDMA network is maximized in [17] through noncooperative games for subcarrier allocation and transmit power control. One drawback of single-network access mechanisms is that they do not fully exploit the available resources in the heterogeneous wireless medium in terms of diverse wireless channel conditions between the MT and different BSs/APs and radio resource availability at different BSs/APs.

In multi-homing access mechanisms, the MT multiple radio interfaces are utilized to enhance energy efficiency. Cooperation among MTs in transmitting their data packets to the BS using their multiple radio interfaces is investigated in [18]. Specifically, using the short-range radio interfaces (e.g., via the bluetooth technology), MTs exchange their data packets among each other and then forward the data packets using the cellular interface to the BS. In [19], MTs relay the source message to the destination, using their multiple radio interfaces, in a decode-and-forward fashion. In [20], uplink energy efficiency is enhanced for an MT through multi-homing communications with all available BSs/APs. However, the mechanisms proposed in [18] - [20] mainly focus on exploiting

TABLE I
SUMMARY OF IMPORTANT SYMBOLS

Symbol	Definition
B_{nsm}	Bandwidth allocated from network n BS/AP s to MT m
B_{ns}	Total bandwidth available at network n BS/AP s
d_{nsm}	Distance between MT m and network n BS/AP s
h_{nsm}	Channel power gain between MT m and network n BS/AP s
L	Lagrangian function
$\mathcal{M}$	Set of MTs in the geographical region
$\mathcal{M}_{ns}$	Subset of MTs in the coverage area of network n BS/AP s
$\mathcal{N}$	Set of available networks in the geographical region
N_0	One-sided noise power spectral density
P_{nsm}	Transmission power allocated by MT m to the radio interface communicating with network n BS/AP s
P_m	Total power consumption by MT m
P_m^{T}	Total available power at MT m
P_{ns}^{T}	Maximum transmission power of MT m radio interface communicating with network n BS/AP s
Q_{nsm}^{F}	Fixed circuit power consumption of MT m radio interface communicating with network n BS/AP s
Q_{nsm}^{D}	Dynamic circuit power consumption of MT m radio interface communicating with network n BS/AP s
R_{nsm}	Achieved data rate by MT m on its radio interface communicating with network n BS/AP s
R_m	Total achieved data rate by MT m
$R_m^{\min}$	Minimum required data rate by MT m
$\mathcal{S}_n$	Set of BSs/APs of network n covering the geographical region
η_m	Achieved energy efficiency by MT m
λ	Non-negative parameter used in transforming the concave-convex fractional program into a convex optimization program
ϕ_m	Lagrangian multiplier for the required data rate constraint
μ_m	Lagrangian multiplier for the minimum energy efficiency constraint
ν_m	Lagrangian multiplier for the total power consumption constraint
β_{ns}	Lagrangian multiplier for the total bandwidth allocation constraint
ω_{nsm}	Lagrangian multiplier for the maximum transmission power constraint
α	Path loss exponent
ρ	Power amplifier efficiency
Ω_{nsm}	Average channel power gain between MT m and network n BS/AP s

the diversity in wireless channel conditions among the MT and different BSs/APs and neglect the diversity in available resources (e.g., bandwidth) at the BSs/APs, which can further enhance the energy efficiency.

In this paper, we present an optimization framework to jointly allocate uplink transmission bandwidth and power for a set of MTs with multi-homing capabilities in a heterogeneous wireless medium. In literature, joint bandwidth and power allocation has been investigated for OFDMA networks through sub-carrier assignment and power control, e.g., [16] and [17]. However, the related works are limited to single-network access. Hence, the associated mechanisms cannot be directly applied to the heterogeneous wireless medium due to the following: 1) In a heterogeneous wireless medium, the coverage area is partitioned into a set of service areas, each is uniquely covered by a subset of networks, different from the OFDMA single-network access, which is described by a single service area and involves no coverage overlap among different networks; 2) In OFDMA single-network access, the MT is served by one BS while in a heterogeneous wireless medium, the MT can be served by BSs/APs at different distances (hence, the MT suffers from different path losses), which affects the radio resource allocation decision; 3) In OFDMA single-network access, no coordination is required among BSs of different networks for resource allocation, different from the heterogeneous wireless networks scenario. In this paper, we formulate the problem to capture the heterogeneity of the wireless medium, in terms of different service areas, channel conditions, available radio resources at BSs/APs of different networks, and different maximum power levels at the MTs.

In LTE-advanced networks, joint carrier component selection and power control has been studied in the context of carrier aggregation. Although carrier aggregation is similar in concept to multi-homing resource aggregation, multi-homing supports simultaneous use of different radio access technologies, unlike carrier aggregation. Furthermore, in LTE-advanced, all contiguous or dis-contiguous) carrier components are operated by the same service provider [21] - [23]. As a result, in LTE-advanced networks, central resource allocation can be adopted through a central resource manager. However, in a heterogeneous wireless medium with multiple operators, decentralized coordination among MTs and BSs/APs of different networks is required.

Moreover, aiming at optimal joint bandwidth and power allocation, we deal with a multi-user system to account for the MTs' competition over the shared bandwidth, different from [20] that studies only optimal power allocation and hence deals with a single-user system. In this regard, we aim to maximize the performance of the MT that has the minimum achieved energy efficiency. To further reduce the associated signaling overhead and computational complexity, a suboptimal framework is presented.

III. SYSTEM MODEL

A geographical region is considered where a set, $\mathcal{N} = \{1, 2, \ldots, N\}$, of wireless networks is available, as shown in Figure 1. Different networks are operated in separate frequency bands by different service providers, and as a result no interference exists among these networks. Specifically, the set, $\mathcal{N}$, contains cellular networks with heterogeneous cell sizes

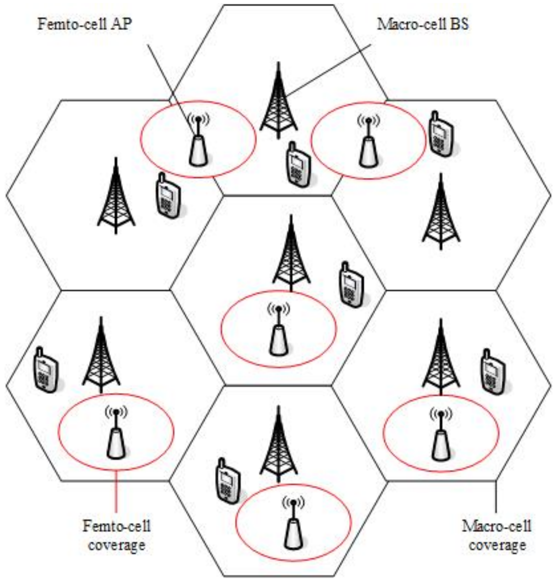


Fig. 1. The network coverage areas [2]

(e.g., macro, pico, and femto-cells) and overlapped coverage areas. Each network, $n \in \mathcal{N}$, has a set $\mathcal{S}_n = \{1, 2, \ldots, S_n\}$ of BSs/APs in the geographical region. Interference management schemes (e.g., frequency reuse [24] - [27]) are implemented for interference mitigation among BSs/APs within the same network. Due to the overlapped coverage from BSs/APs of different networks, the geographical region is partitioned into a set of service areas. A unique subset of BSs/APs covers each service area. The total bandwidth available at network n BS/AP s is denoted by B_{ns}. A cooperative networking scenario is considered where different networks in $\mathcal{N}$ cooperate in radio resource allocation through signaling exchange over a backbone [2].

A set of MTs $\mathcal{M} = \{1, 2, \ldots, M\}$ performs uplink multi-homing video transmission in the geographical region[1]. Let $\mathcal{M}_{ns} \subseteq \mathcal{M}$ denote the subset of MTs which lie in the coverage area of network n BS/AP s. Using the multiple radio interfaces and through the multi-homing capability, each MT can communicate with multiple BSs/APs simultaneously. The bandwidth allocated on the uplink from network n BS/AP s to MT m is denoted by B_{nsm}, where $B_{nsm} = 0$ for $m \notin \mathcal{M}_{ns}$. Let P_{nsm} represent the transmission power allocated by MT m to its radio interface communicating with network n BS/AP s. Denote ρ as the power amplifier efficiency. Hence, the MT transmission power consumption on each radio interface is given by P_{nsm}/ρ [29]. The MT circuit power consumption for each radio interface has two components. The first component is a fixed circuit power consumption for each MT radio interface and is given by Q^{F}_{nsm}, which captures the power consumption of the radio frequency (RF) chain, i.e., digital-to-analog converter, RF filter, local oscillator, and mixer. The second component is a dynamic part that refers to the digital circuit power consumption and scales with the allocated transmission bandwidth (as bandwidth increases, more computations and base band processing are required). The dynamic component is expressed as [30]

$$Q^{\text{D}}_{nsm} = Q^{\text{ref}}_{\text{D}} + \sigma_{nsm} \frac{B_{nsm}}{B_{\text{ref}}} \tag{1}$$

where $Q^{\text{ref}}_{\text{D}}$ denotes the reference digital circuit power consumption for a reference bandwidth B_{ref} and σ_{nsm} is a proportionality constant. For $m \notin \mathcal{M}_{ns}$, $P_{nsm} = Q^{\text{F}}_{nsm} = Q^{\text{D}}_{nsm} = 0$. Denote $Q^{\text{F}}_{nsm} + Q^{\text{ref}}_{\text{D}}$ by Q_{nsm} and $\sigma_{nsm}/B_{\text{ref}}$ by ζ_{nsm}. Hence, the MT total power consumption for each radio interface is given by

$$P^{\text{T}}_{nsm} = \frac{P_{nsm}}{\rho_{nsm}} + Q_{nsm} + \zeta_{nsm} B_{nsm}. \tag{2}$$

Due to technology limitation, each MT radio interface has a maximum transmission power of P^{T}_{ns}. The maximum power constraint at MT m is given by P^{T}_{m}. The MT target service quality can be obtained using a minimum data rate of $R^{\min}_{m}$ for MT m.

The channel power gain between MT m and network n BS/AP s is denoted by h_{nsm}, which captures both the wireless channel Rayleigh fading and the path loss[2]. Let d_{nsm} denote the distance between MT m and network n BS/AP s. The associated path loss is given by $d^{-\alpha}_{nsm}$, where α is the path loss exponent. Let κ_{nsm} be a Rayleigh random variable associated with the link between MT m and network n BS/AP s. The channel power gain between MT m and network n BS/AP s is given by

$$h_{nsm} = \kappa_{nsm} d^{-\alpha}_{nsm}. \tag{3}$$

The one-sided noise power spectral density is denoted by N_0.

IV. Problem Formulation

In this section, the joint bandwidth and power allocation problem is formulated to maximize energy efficiency for a set of MTs with QoS requirements given by $R^{\min}_{m}$. In the next section, we present an optimal decentralized energy efficient radio resource allocation framework, based on the problem solution.

According to Shannon formula, the data rate achieved by MT m using the radio interface communicating with network n BS/AP s is given by[3]

$$R_{nsm} = B_{nsm} \log_2\left(1 + \frac{P_{nsm} h_{nsm}}{N_0 B_{nsm}}\right), \qquad \forall n, s, m. \tag{4}$$

The total achieved data rate by MT m is $R_m = \sum_n \sum_s R_{nsm}$, which should satisfy the required QoS, i.e.,

$$R_m \geq R^{\min}_{m}, \qquad \forall m. \tag{5}$$

The total allocated bandwidth by network n BS/AP s should not be larger than the total available bandwidth, i.e.,

$$\sum_{m \in \mathcal{M}_{ns}} B_{nsm} \leq B_{ns}, \qquad \forall n, s. \tag{6}$$

[1]The video packets streamed among the multiple radio interfaces in transmission can be achieved using a packet scheduling algorithm as in [28].

[2]The power consumed in the channel state information (CSI) reporting is negligible as compared with the power consumed in the actual data transmission.

[3]The data rate formula in (4) can be modified to account for the spectral efficiency of different technology standards.

Given the technical limitation on the maximum transmission power for each radio interface, we have

$$P_{nsm} \leq P_{ns}^{\mathrm{T}}, \quad \forall n, s, m. \tag{7}$$

The MT total power consumption includes both data transmission and circuit power consumption for all active radio interfaces, i.e., for MT m, $P_m = \sum_n \sum_s P_{nsm}^{\mathrm{T}}$. The total power consumption for MT m, P_m, should satisfy the MT maximum power constraint, i.e.,

$$P_m \leq P_m^{\mathrm{T}}, \quad \forall m \in \mathcal{M}. \tag{8}$$

Define the energy efficiency of MT m, η_m, as a ratio of the total achieved data rate to the total power consumption, i.e., $\eta_m = R_m/P_m$. The objective is to maximize the minimum achieved energy efficiency η_m for $m \in \mathcal{M}$. This is obtained through joint bandwidth and power allocation from all networks in $\mathcal{N}$ to all MTs in $\mathcal{M}$, while satisfying the required minimum transmission rates and the total bandwidth and power constraints. Hence, the problem is formulated as

$$\begin{aligned} \max_{B_{nsm}, P_{nsm}} \quad & \{\min_{m \in \mathcal{M}} \eta_m\} \\ s.t. \quad & (5) - (8), \\ & B_{nsm}, P_{nsm} \geq 0, \quad \forall n, s, m. \end{aligned} \tag{9}$$

Problem (9) is classified as a *max-min fractional program* [34].

Proposition 1. Problem (9) is a concave-convex fractional program.

Proof. See Appendix A.

From Proposition 1, (9) can be transformed into a convex optimization problem, for a given parameter λ, using a parametric approach [34]. The optimal value of λ, which results in the optimal bandwidth and power allocation for (9), can be obtained through an iterative algorithm.

A. The Parametric Approach

For a non-negative parameter $\lambda = \min_{m \in \mathcal{M}} \eta_m$, (9) can be transformed into

$$\begin{aligned} F(\lambda) = \max_{B_{nsm}, P_{nsm}} \quad & \{\min_{m \in \mathcal{M}} \{R_m - \lambda P_m\}\} \\ s.t. \quad & (5) - (8), \\ & B_{nsm}, P_{nsm} \geq 0, \quad \forall n, s, m. \end{aligned} \tag{10}$$

The optimal solution of (9) can be determined by finding a root of equation $F(\lambda) = 0$, which can be obtained using a Dinkelbach-type algorithm, as given in Algorithm 1 [35].

Algorithm 1 Dinkelbach-type Procedure

Initialization: $\{B_{nsm}(1), P_{nsm}(1)\} > 0\ \forall n, s, m$, $\lambda(1) = \min_{m \in \mathcal{M}} \eta_m$, $i = 1$;
while $F(\lambda(i)) \neq 0$ **do**
  Solve (10) for optimal $\{B_{nsm}(i), P_{nsm}(i)\}$;
  $\lambda(i+1) = \min_{m \in \mathcal{M}} \eta_m(i)$;
  $i \longleftarrow i + 1$;
end while
Output: B_{nsm}, $P_{nsm}\ \forall n, s, m$.

Algorithm 1 converges to the optimal solution of (9) in a finite number of iterations [35].

In the following, we focus on solving (10), as it constitutes an important step in Algorithm 1. Letting $\theta = \min_{m \in \mathcal{M}} \{R_m - \lambda P_m\}$, (10) can be re-written as

$$\begin{aligned} \max_{B_{nsm}, P_{nsm}} \quad & \theta \\ s.t. \quad & \theta \leq R_m - \lambda P_m, \quad \forall m \\ & (5) - (8), \\ & B_{nsm}, P_{nsm} \geq 0, \quad \forall n, s, m. \end{aligned} \tag{11}$$

Since (11) has a linear objective function and convex constraints, it is a convex optimization problem [36]. The Lagrangian function for (11) can be expressed as

$$L = \theta(1 - \sum_{m \in \mathcal{M}} \mu_m) + \sum_{m \in \mathcal{M}} L_m + \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}_n} L_{ns}, \tag{12}$$

where L_m and L_{ns} are given by

$$\begin{aligned} L_m = \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}_n} \{ & (\mu_m + \phi_m) R_{nsm} - (\lambda \mu_m + \nu_m) \\ & \cdot (\frac{P_{nsm}}{\rho} + Q_{nsm} + \zeta_{nsm} B_{nsm}) + \omega_{nsm}(P_{ns}^{\mathrm{T}} - P_{nsm}) \\ & - \phi_m R_{m,\min} + \nu_m P_m^{\mathrm{T}} \}, \end{aligned} \tag{13}$$

$$L_{ns} = \beta_{ns}\{B_{ns} - \sum_{m \in \mathcal{M}_{ns}} B_{nsm}\}, \tag{14}$$

where μ_m is a Lagrangian multiplier for the first constraint in (11), and ϕ_m, β_{ns}, ω_{nsm}, and ν_m are Lagrangian multipliers for constraints (5) - (8), respectively.

In the following, we maximize L to find the optimal bandwidth and power allocation for a given value of λ.

B. Power Allocation to the MT Different Radio Interfaces

In this subsection, the optimal allocated power at the MT different radio interfaces is derived, given the bandwidth allocation $B_{nsm}\ \forall n, s, m$, ϕ_m, and $\mu_m\ \forall m$. Using the Karush-Kuhn-Tucker (KKT) conditions [36], we have

$$\frac{\partial L_m}{\partial P_{nsm}} = (\mu_m + \phi_m) \frac{\partial R_{nsm}}{\partial P_{nsm}} - \frac{1}{\rho}(\lambda \mu_m + \nu_m) - \omega_{nsm} = 0. \tag{15}$$

From (4) and (15), we have

$$P_{nsm}^* = B_{nsm}[\frac{\mu_m + \phi_m}{\ln(2)\{\frac{1}{\rho}(\lambda \mu_m + \nu_m^*) + \omega_{nsm}^*\}} - \frac{N_0}{h_{nsm}}]^+, \quad \forall n, s, m \tag{16}$$

where $[\cdot]^+$ is a projection on the positive quadrature to account for $P_{nsm} \geq 0$. The optimal values of ω_{nsm}^* and ν_m^* can be obtained by solving the dual problem using a gradient descent method [36]. Hence, we have

$$\omega_{nsm}(i+1) = [\omega_{nsm}(i) - \varepsilon_1(P_{ns}^{\mathrm{T}} - P_{nsm}(i))]^+, \tag{17}$$

$$\begin{aligned} \nu_m(i+1) = [\nu_m(i) - \varepsilon_2(P_m^{\mathrm{T}} - \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}_n} \{ & \frac{P_{nsm(i)}}{\rho} \\ & + Q_{nsm} + \zeta_{nsm} B_{nsm}\})]^+ \end{aligned} \tag{18}$$

where ε_1 and ε_2 are sufficiently small step sizes. From (17) and (18), the optimal values of ω_{nsm} and ν_m ensure that the optimal transmission power allocated for each radio interface satisfies the maximum transmission power constraint and the total power allocated by the MT satisfies its maximum total power constraint. An iterative algorithm can be used to update ω_{nsm} and ν_m until the optimal P_{nsm} value $\forall n, s$ is found, as shown in Algorithm 2, where ϵ is a small tolerance value.

Algorithm 2 Power Allocation to Each Radio Interface for Every MT m

Input: B_{nsm} $\forall n, s$, μ_m, ϕ_m, and λ;
Initialization: $\omega_{nsm}(1) \geq 0$ and $\nu_m(1) \geq 0$, $i = 1$, $J = 1$;
while $J = 1$ **do**
  for $n \in \mathcal{N}$ **do**
    for $s \in \mathcal{S}_n$ **do**
      $P_{nsm}(i) = B_{nsm}[\frac{\mu_m+\phi_m}{\ln(2)\{\frac{1}{\rho}(\lambda\mu_m+\nu_m(i))+\omega_{nsm}(i)\}} - \frac{N_0}{h_{nsm}}]^+$;
      $\omega_{nsm}(i+1) = [\omega_{nsm}(i) - \varepsilon_1(P^{\mathrm{T}}_{ns} - P_{nsm}(i))]^+$;
    end for
  end for
  $\nu_m(i+1) = [\nu_m(i) - \varepsilon_2(P^{\mathrm{T}}_m - \sum_{n\in\mathcal{N}}\sum_{s\in\mathcal{S}_n}\{\frac{P_{nsm}(i)}{\rho} + Q_{nsm} + \zeta_{nsm}B_{nsm}\})]^+$;
  if $|P_{nsm}(i) - P_{nsm}(i-1)| \leq \epsilon$ **then**
    $J = 0$;
  else
    $i \longleftarrow i + 1$;
  end if
end while
Output: P^*_{nsm} $\forall n, s$.

C. Bandwidth Allocation at Every Network BS/AP

In this subsection, the optimal allocated bandwidth at different BSs/APs to each MT is derived, given the allocated power P^*_{nsm} and ν^*_m, calculated in the previous subsection, ϕ_m, and μ_m $\forall m$.

Applying the KKT conditions, we have

$$\frac{\partial L}{\partial B_{nsm}} = (\mu_m + \phi_m)\frac{\partial R_{nsm}}{\partial B_{nsm}} - \zeta_{nsm}(\lambda\mu_m + \nu^*_m) - \beta_{ns} = 0. \tag{19}$$

Hence, we have

$$(\frac{\partial R_{nsm}}{\partial B_{nsm}})^{-1} = \frac{\mu_m + \phi_m}{\zeta_{nsm}(\lambda\mu_m + \nu^*_m) + \beta_{ns}}, \quad \forall n, s. \tag{20}$$

From (4) and (20), the bandwidth allocation solution can be found, using the Newton's method, as the positive real root of

$$\log_2(1 + \frac{P^*_{nsm}h_{nsm}}{N_0 B^*_{nsm}}) - \frac{P^*_{nsm}h_{nsm}}{\ln(2)(N_0 B^*_{nsm} + P^*_{nsm}h_{nsm})} = \frac{\zeta_{nsm}(\lambda\mu_m + \nu^*_m) + \beta^*_{ns}}{\mu_m + \phi_m}. \tag{21}$$

In addition, the optimal value of β^*_{ns} can be obtained by solving the dual problem using a gradient descent method, i.e.,

$$\beta_{ns}(i+1) = [\beta_{ns}(i) - \varepsilon_3(B_{ns} - \sum_{m\in\mathcal{M}_{ns}} B_{nsm}(i))]^+, \quad \forall n, s \tag{22}$$

where ε_3 is a sufficiently small step size. The Lagrangian multiplier β_{ns} is calculated at each BS/AP to guarantee that the total allocated bandwidth by each BS/AP satisfies its total available bandwidth. An iterative algorithm can be used to update β_{ns} until the optimal B_{nsm} value $\forall n, s, m$ is found for a given value of P_{nsm}, ν_m, ϕ_m, and μ_m $\forall m$, as shown in Algorithm 3.

Algorithm 3 Bandwidth Allocation at Each Network BS/AP to Each MT

Input: P^*_{nsm} $\forall n, s, m$, ν^*_m, ϕ_m, and μ_m;
Initialization: $\beta_{ns}(1) \geq 0$, $i = 1$, $J = 1$;
while $J = 1$ **do**
  Find $B^*_{nsm}(i)$ as the positive real root of (21);
  $\beta_{ns}(i+1) = [\beta_{ns}(i) - \varepsilon_3(B_{ns} - \sum_{m\in\mathcal{M}_{ns}} B_{nsm}(i))]^+$;
  if $|B_{nsm}(i) - B_{nsm}(i-1)| \leq \epsilon$ **then**
    $J = 0$;
  else
    $i \longleftarrow i + 1$;
  end if
end while
Output: B^*_{nsm} $\forall n, s$.

V. Optimal Uplink Energy Efficient Radio Resource Allocation Framework with QoS Guarantee

In this section, the optimal values of ϕ_m and μ_m $\forall m$, which result in the optimal solution of (11) at a given λ value, are derived. Also, a summary of the joint bandwidth and power allocation framework is presented.

A. Finding the Optimal μ_m for a Given ϕ_m

Applying the KKT conditions, we have

$$\frac{\partial L}{\partial \theta} = 0. \tag{23}$$

From (12), we have

$$\sum_{m\in\mathcal{M}} \mu_m = 1. \tag{24}$$

To account for (24), we modify the power and bandwidth allocation expressions in (16) and (21), as follows. Using (24), we re-write (16) as

$$P^*_{nsm} = B_{nsm}[\frac{\frac{\mu_m}{\sum_m \mu_m} + \phi_m}{\ln(2)\{\frac{1}{\rho}(\lambda\frac{\mu_m}{\sum_m \mu_m} + \nu^*_m) + \omega^*_{nsm}\}} - \frac{N_0}{h_{nsm}}]^+, \quad \forall n, s, m. \tag{25}$$

Hence, we have

$$P^*_{nsm} = B_{nsm}[\frac{\mu_m + \widetilde{\phi}_m}{\ln(2)\{\frac{1}{\rho}(\lambda\mu_m + \widetilde{\nu}^*_m) + \widetilde{\omega}^*_{nsm}\}} - \frac{N_0}{h_{nsm}}]^+, \quad \forall n, s, m \tag{26}$$

where $\widetilde{\phi}_m = \phi_m \sum_m \mu_m$, $\widetilde{\nu}_m = \nu_m \sum_m \mu_m$, and $\widetilde{\omega}_{nsm} = \omega_{nsm} \sum_m \mu_m$. Similarly, from (21) and (24), we have

$$\log_2(1 + \frac{P^*_{nsm}h_{nsm}}{N_0 B^*_{nsm}}) - \frac{P^*_{nsm}h_{nsm}}{\ln(2)(N_0 B^*_{nsm} + P^*_{nsm}h_{nsm})} = \frac{\zeta_{nsm}(\lambda\mu_m + \widetilde{\nu}^*_m) + \widetilde{\beta}^*_{ns}}{\mu_m + \widetilde{\phi}_m} \tag{27}$$

where $\widetilde{\beta}_{ns} = \beta_{ns}\sum_m \mu_m$.The optimal μ_m values can be obtained by solving the dual problem using a gradient descent method, and we have

$$\mu_m(i+1) = [\mu_m(i) - \varepsilon_4(R_m(i) - \lambda P_m(i) - \theta)]^+, \quad \forall m \quad (28)$$

where ε_4 is a sufficiently small step size. An iterative algorithm can be used to update μ_m to find the optimal P_{nsm} and B_{nsm} values $\forall n, s, m$, for a given value of ϕ_m $\forall m$ and λ. Define $f_m = \{R_m - \lambda P_m\}$. Theorem 1 gives the termination condition for the update of μ_m.

Theorem 1. At the optimal point, for each m with $\phi_m \geq 0$, either $f_m = \theta^*$ or $f_m > \theta^*$ and $\mu_m^* = 0$.

Proof. See Appendix B.

From Theorem 1, at optimality, MTs with $\mu_m^* > 0$ have equal f_m value, which is denoted as θ^* (i.e., optimal value of θ for the given ϕ_m $\forall m$ values). Other MTs with $\mu_m^* = 0$ must have f_m values greater than θ^*.

B. Finding the Optimal ϕ_m

The optimal ϕ_m^* values can be obtained by solving the dual problem using a gradient descent method, i.e.,

$$\phi_m(i+1) = [\phi_m(i) - \varepsilon_5(R_m(i) - R_m^{\min})]^+, \quad \forall m \quad (29)$$

where ε_5 is a sufficiently small step size.

At optimality, the radio resource allocation process terminates when all MTs have $R_m \geq R_m^{\min}$.

Algorithm 4 gives the optimal solution of (11) for a given value of λ. In Algorithm 4, we iterate over P_{nsm} and B_{nsm} until convergence to find the optimal joint bandwidth and power allocation solution that maximizes the minimum energy efficiency in the region and satisfies the required QoS by all MTs.

Algorithm 4 Joint Bandwidth and Power Allocation for a Given λ

Input: λ;
Initialization: $\phi_m \geq 0$ and $\mu_m \geq 0$ $\forall m$, P_{nsm} and B_{nsm} $\forall n, s, m$, $i = 1$, $K = 1$;
while $K = 1$ **do**
  Every MT broadcasts to all serving BSs/APs its $\phi_m(i)$ value;
  Initialization: $J = 1$;
  while $J = 1$ **do**
    Every MT broadcasts to all serving BSs/APs its η_m, $f_m(i)$, and $P_{nsm}(i)$ values;
    Every BS/AP determines $\theta_{ns}(i) = \min_{m \in \mathcal{M}_{ns}} f_m(i)$;
    All BSs/APs exchange information regarding $\theta_{ns}(i)$ and determine $\theta(i) = \min_{n,s} \theta_{ns}(i)$;
    if for every m, $f_m(i) = \theta(i)$ or $f_m(i) > \theta(i)$ for m with $\mu_m(i) = 0$ **then**
      $J = 0$;
    else
      All BSs/APs update $\mu_m(i)$ for all MTs according to (28);
      All BSs/APs exchange their information to find $\sum_m \mu_m(i)$ and broadcast this value to all MTs;
      All BSs/APs allocate bandwidth to all MTs using Algorithm 3 (by replacing $\beta_{ns}(i)$, $\nu_m(i)$, and $\phi_m(i)$ by $\widetilde{\beta}_{ns}(i)$, $\widetilde{\nu}_m(i)$, and $\widetilde{\phi}_m(i)$, respectively);
      All MTs allocate power to their radio interfaces using Algorithm 2 (by replacing $\phi_m(i)$, $\nu_m(i)$, and $\omega_{nsm}(i)$ by $\widetilde{\phi}_m(i)$, $\widetilde{\nu}_m(i)$, and $\widetilde{\omega}_{nsm}(i)$, respectively);
    end if
  end while
  if For every m, $|R_m(i) - R_m(i-1)| \leq \epsilon$ **then**
    $K = 0$;
  else
    All MTs update $\phi_m(i)$ value using (29);
    $i \longleftarrow i + 1$;
  end if
end while
Output: B_{nsm}^*, P_{nsm}^*, $\forall n, s, m$.

C. Summary of the Proposed Optimal Framework

Using Algorithms 1 - 4, the decentralized uplink energy efficient radio resource allocation framework is summarized in the following 10 steps:

Step 1. The BSs/APs start with initial bandwidth allocation to all MTs in service and initialize a μ_m value for every MT. The MTs start with initial power allocation to their different radio interfaces. Every MT calculates its initial η_m value and broadcasts it along with an initial ϕ_m value to the serving BSs/APs.

Step 2. The BSs/APs exchange their information to find the value $\lambda = \min_{m \in \mathcal{M}} \eta_m$, as shown in Algorithm 1.

Step 3. The BSs/APs check a termination condition (as shown in Algorithm 1): $F(\lambda) = 0$? If the condition is true, the framework is terminated; otherwise, go to step 4.

Step 4. Every MT broadcasts to its serving BSs/APs its f_m and P_{nsm} values. All BSs/APs exchange their information regarding their minimum f_m and find $\theta = \min_m f_m$, as shown in Algorithm 4.

Step 5. The BSs/APs check a termination condition (as shown in Algorithm 4): $f_m = \theta$ $\forall m$ with $\mu_m > 0$ and $f_m > \theta$ $\forall m$ with $\mu_m = 0$? If the condition is true, go to step 9; otherwise, all BSs/APs update the μ_m values, as shown in Algorithm 4. Also, all BSs/APs exchange their information to find $\sum_m \mu_m$ and broadcast this value to all MTs.

Step 6. All BSs/APs allocate their radio resources (e.g., bandwidth) to all MTs in service using Algorithm 3.

Step 7. All MTs perform power allocation to their radio interfaces using Algorithm 2.

Step 8. Go to step 4.

Step 9. Every MT checks its total achieved data rate R_m. If R_m did not converge, the MTs update their ϕ_m value and broadcasts it to all serving BSs/APs, as shown in Algorithm 4. Go to step 4.

Step 10. If R_m converges, every MT transmits its η_m value to all serving BSs/APs. Go to step 2.

VI. Suboptimal Uplink Energy Efficient Radio Radio Resource Allocation Framework

In this section, we discuss the signaling overhead required by the optimal framework. Furthermore, we perform a complexity analysis study. A suboptimal framework is proposed to reduce the associated signaling overhead and computational complexity. Finally, a benchmark is presented for comparison purposes.

A. Signaling Overhead and Computational Complexity

The optimal radio resource allocation framework requires that the serving BSs/APs obtain information regarding the CSI, h_{nsm}, in order to perform the resource allocation. Let the time be partitioned into time slots of equal duration. Assume that the CSI remains fixed during each time slot and changes from one time slot to another. Hence, the CSI information should be updated (reported) every time slot. In addition, information regarding variables update (λ and θ exchanged on the backbone among BSs/APs and λ, ϕ_m and μ_m exchanged among MTs and BSs/APs over the air interface).

Let I_D denote the number of iterations required for the convergence of the Dinkelbach-type procedure given in Algorithm 1. The computational complexity of the optimal radio resource allocation algorithm is determined by the complexity of solving the dual problem. The complexity of the gradient method is polynomial in the number of dual variables [37]. Hence, the computational complexity is given by $O(I_D M^2 \sum_n S_n)$. The optimal framework has an online computational complexity that is quadratic in the number of MT M. In a system with a large M, the online computational complexity will be high, which could make it infeasible for the algorithm to run within every time slot of fixed CSI.

To further reduce the associated signaling overhead and computational complexity, in the following we present a suboptimal framework.

B. Suboptimal Framework

In the optimal framework, every time the CSI changes, the radio resource allocation has to be updated. This incurs high signaling overhead over both the backbone connecting the BSs/APs and the air interfaces. In order to reduce the associated signaling overhead and computational complexity, a two-step suboptimal framework is proposed. The first step is executed only once during an initialization phase and is to set the values of the variables λ, ϕ_m, and μ_m. The variables are calculated based on the average channel gain, $\Omega_{nsm} = \mathbb{E}\{h_{nsm}\}$, where $\mathbb{E}\{\cdot\}$ denotes the expectation. The second step updates the radio resource allocation given the time slot CSI. The two steps are explained in details next.

1) Initialization Phase: In this step, we aim to find the values of λ, ϕ_m, and μ_m to be used in the second step. Denote $\bar{\lambda}$, $\bar{\phi}_m$, and $\bar{\mu}_m$ as the values calculated based on the average channel gain Ω_{nsm}. In the following, we find $\bar{\lambda}$, $\bar{\phi}_m$, and $\bar{\mu}_m$ that maximize the minimum average energy efficiency while satisfying the average QoS constraints. The average achieved data rate for MT m communicating with BS/AP s of network n is given by [38]

$$\begin{aligned}\mathbb{E}\{R_{nsm}\} &= \int_0^{+\infty} B_{nsm}\log_2(1+\frac{P_{nsm}h_{nsm}}{N_0 B_{nsm}}) \\ &\quad \cdot \frac{1}{\Omega_{nsm}}\exp(-\frac{h_{nsm}}{\Omega_{nsm}})dh_{nsm} \\ &= \frac{B_{nsm}}{\ln(2)}\exp(\frac{N_0 B_{nsm}}{\Omega_{nsm}P_{nsm}})E_1(\frac{N_0 B_{nsm}}{\Omega_{nsm}P_{nsm}}) \quad (30)\end{aligned}$$

where $E_1(x) = \int_0^{+\infty}\exp(-x)x^{-1}dx$ is the exponential integral. From Lemma 2.1 in [38], we have

$$\mathbb{E}\{R_{nsm}\} \geq \frac{B_{nsm}}{2}\log_2(1+\frac{2\Omega_{nsm}P_{nsm}}{N_0 B_{nsm}}). \quad (31)$$

Since the radio channels fade independently, and using the bound given in (31), the average η_m is given by

$$\mathbb{E}\{\eta_m\} = \frac{\sum_n\sum_s \frac{B_{nsm}}{2}\log_2(1+\frac{2\Omega_{nsm}P_{nsm}}{N_0 B_{nsm}})}{\sum_n\sum_s\{\frac{P_{nsm}}{\rho_{nsm}} + Q_{nsm} + \zeta_{nsm}B_{nsm}\}} \quad (32)$$

and the average QoS constraint is given by

$$\sum_{n\in\mathcal{N}}\sum_{s\in\mathcal{S}_n}\frac{B_{nsm}}{2}\log_2(1+\frac{2\Omega_{nsm}P_{nsm}}{N_0 B_{nsm}}) \geq R_m^{\min}. \quad (33)$$

Hence, we aim to solve

$$\begin{aligned}&\max_{B_{nsm},P_{nsm}} && \{\min_{m\in\mathcal{M}}\mathbb{E}\{\eta_m\}\} \\ &s.t. && (6)-(8),(33), \\ & && B_{nsm},P_{nsm}\geq 0, \quad \forall n,s,m.\end{aligned} \quad (34)$$

The optimization (34) can be solved in a way similar to (9). Hence, using a similar analysis as in Section IV.C., the optimal power allocation of (34) is given by solving

$$P^*_{nsm} = \frac{B_{nsm}}{2}[\frac{\mu_m + \widetilde{\phi}_m}{\ln(2)\{\frac{1}{\rho}(\lambda\mu_m + \widetilde{\nu}^*_m) + \widetilde{\omega}^*_{nsm}\}} - \frac{N_0}{\Omega_{nsm}}]^+, \quad \forall n,s,m. \quad (35)$$

Also, using an analysis similar to that in Section IV.B., the optimal bandwidth allocation of (34) is given by

$$\log_2(1+\frac{P^*_{nsm}\Omega_{nsm}}{N_0\frac{B^*_{nsm}}{2}}) - \frac{P^*_{nsm}\Omega_{nsm}}{\ln(2)(N_0\frac{B^*_{nsm}}{2} + P^*_{nsm}\Omega_{nsm})} = \frac{\zeta_{nsm}(\lambda\mu_m + \widetilde{\nu}^*_m) + \widetilde{\beta}^*_{ns}}{\mu_m + \widetilde{\phi}_m}. \quad (36)$$

Therefore, $\bar{\lambda}$, $\bar{\phi}_m$, and $\bar{\mu}_m$ are found by solving Algorithms 1 - 4 while replacing the optimal power and bandwidth allocations in Algorithm 2 and Algorithm 3 by (35) and (36), respectively. The values of $\bar{\lambda}$, $\bar{\phi}_m$, and $\bar{\mu}_m$ are then exchanged among MTs and BSs/APs to be used in the next step.

2) Resource Allocation Update Phase: This phase takes place when the CSI changes every time slot. In this step, the power and bandwidth allocations are updated given the channel gain h_{nsm} in the current time slot using Algorithm 2 and Algorithm 3 while replacing λ, ϕ_m, and μ_m by $\bar{\lambda}$, $\bar{\phi}_m$, and $\bar{\mu}_m$, respectively, as calculated in the initialization phase.

Although the suboptimal framework relies on the average CSI in the initialization phase, this is used only to reduce the amount of signaling required to update the dual variables. In the resource allocation update phase of the suboptimal framework, the CSI information should be updated (reported) every time slot.

As compared with the optimal framework described in Section V, the suboptimal framework has a reduced computational complexity. Only Algorithm 3 is executed at each BS/AP and Algorithm 2 is executed at each MT for resource allocation update. Almost no signaling exchange takes place during the resource allocation updates, except for the allocated B_{nsm} values that are provided to each MT and the CSI information that is updated once at each time slot. While the initialization phase of the suboptimal framework incurs the same computational complexity $O(I_D M^2 \sum_n S_n)$, this is only executed once during the call setup. The resource allocation update phase that takes place every time slot has a computational complexity of $O(M \sum_n S_n)$, different from the optimal framework. Hence, the resource allocation update, which is executed within every time slot of fixed CSI, has an online computational complexity that is linear in M, which is more feasible.

C. Benchmark

The most relevant state-of-the-art research (e.g., [20]) investigates power allocation in a heterogeneous wireless medium for energy efficiency. Hence, given some bandwidth allocation from different networks, every MT independently allocates transmission power to its radio interfaces in order to maximize its own energy efficiency. That is, every MT solves

$$\begin{aligned} \max_{P_{nsm}\geq 0} \quad & \eta_m \\ s.t. \quad & R_m \geq R_m^{\min} \\ & P_{nsm} \leq P_{ns}^{\mathrm{T}}, \quad \forall n,s \\ & P_m \leq P_m^{\mathrm{T}}. \end{aligned} \tag{37}$$

Similar to (9), (37) is a concave-convex fractional program. Hence, a parametric approach can be used to transform (37) into a convex optimization problem. As a result, we have

$$\begin{aligned} G(\lambda) = \max_{P_{nsm}\geq 0} \quad & \{R_m - \lambda_m P_m\} \\ s.t. \quad & R_m \geq R_m^{\min} \\ & P_{nsm} \leq P_{ns}^{\mathrm{T}}, \quad \forall n,s \\ & P_m \leq P_m^{\mathrm{T}}. \end{aligned} \tag{38}$$

A Dinkelbach-type algorithm, similar to Algorithm 1, can be used to find the optimal solution of (38). Algorithm 5 gives the power allocation for the benchmark.

Algorithm 5 Benchmark: Power Allocation at MT m

Input: $B_{nsm}\ \forall n, s$;
Initialization: $P_{nsm}(1) > 0\ \forall n, s,\ \lambda_m(1) = \eta_m,\ i_1 = 1$;
while $G(\lambda(i_1)) \neq 0$ **do**
  Initialization: $\phi_m(1) \geq 0,\ K = 1$;
  while $K = 1$ **do**
    Initialization: $\nu_m(1) \geq 0,\ J = 1,\ i_2 = 1$;
    while $J = 1$ **do**
      for $n \in \mathcal{N}$ **do**
        for $s \in \mathcal{S}_n$ **do**
          $P_{nsm}(i_2) = B_{nsm}[\frac{1+\phi_m(i_2)}{\ln(2)\{\frac{1}{\rho}(\lambda(i_1)+\nu_m(i_2))+\omega_{nsm}(i_2)\}} - \frac{N_0}{h_{nsm}}]^+$;
          $\omega_{nsm}(i_2+1) = [\omega_{nsm}(i_2) - \varepsilon_1(P_{ns}^{\mathrm{T}} - P_{nsm}(i_2))]^+$;
        end for
      end for
      $\nu_m(i_2+1) = [\nu_m(i_2) - \varepsilon_2(P_m^{\mathrm{T}} - \sum_{n\in\mathcal{N}}\sum_{s\in\mathcal{S}_n}\{\frac{P_{nsm}(i_2)}{\rho} + Q_{nsm} + \zeta_{nsm}B_{nsm}\})]^+$;
      if $|P_{nsm}(i_2) - P_{nsm}(i_2-1)| \leq \epsilon$ **then**
        $J = 0$;
      else
        $i_2 \longleftarrow i_2 + 1$;
      end if
    end while
    if $|R_m(i_2) - R_m(i_2-1)| \leq \epsilon$ **then**
      $K = 0$;
    else
      $\phi_m(i_2+1) = [\phi_m(i_2) - \varepsilon_3(R_m(i_2) - R_m^{\min})]^+$;
    end if
  end while
  $\lambda_m(i_1+1) = \eta_m(i_2)$;
  $i_1 \longleftarrow i_1 + 1$;
end while
Output: $P_{nsm}\ \forall n, s$.

The benchmark has a computational complexity of $O(I_D \sum_n S_n)$. The benchmark computational complexity is not a function of M since it requires no coordination for resource allocation among MTs. Instead, each MT independently allocates its transmission power to maximize its own energy efficiency. However, this comes at the cost of reduced energy efficiency and achieved throughput as compared with the suboptimal framework, as will be shown later.

VII. Simulation Results and Discussions

This section presents simulation results for the proposed framework for energy efficient uplink joint radio resource allocation. The simulation setting is shown in Figure 2. We consider a geographical region that is covered by a micro BS (indexed as 1) and two femto-cell APs (indexed as 2 and 3, respectively)[4]. The micro BS has a coverage area of 1.5 Km, while each femto AP has a coverage area of 20 m. Due to overlapped coverage among the BS and the two APs, three service areas can be distinguished. In the first and second areas, MTs can get service from both the micro BS and one femto AP. In the third service area, MTs can get service only from the micro BS. The simulation parameters are given in Table II, adopted from [29], [30], [39], [40], and [41]. In the simulation, the optimal framework requires a total of 5 iterations for the Dinkelback-type algorithm (Algorithm 1) to

[4] Adding more cells will only impact the associated computational complexity and signaling overhead, which has been quantified through complexity analysis using the O-notation.

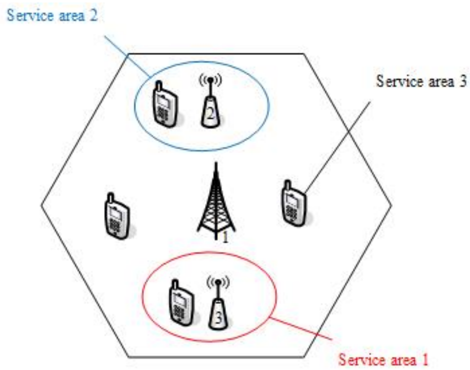


Fig. 2. The simulation setting. Three service areas can be distinguished, namely, service areas 1, 2, and 3.

TABLE II
SIMULATION PARAMETERS

Parameter	Value
B_1	10 MHz
$B_{2,3}$	5 MHz
N_0	-174 dBm/Hz
P_{ns}^{T}	501.2 mW
Q_{nsm}	100 mW
$R_m^{\min}$	Uniformly distributed in $[0, 50]$ Mbps
α	4
ρ	0.35
ζ	20×10^{-9} W/Hz

converge. Hence, in the following results, we set a maximum limit of 12 iterations in the optimal framework for Algorithm 1, while no maximum limit is imposed for the benchmark .

Two simulation cases are considered. In the first case, each service area has 5 MTs, and we show the performance of the optimal and suboptimal frameworks (using Algorithms 1 - 4 and the two phases in Section VI.B, respectively) as compared with the benchmark (using Algorithm 5). In the second case, each service area has 10 MTs. In this case, we show the results of the suboptimal framework as compared with the benchmark, due to computational complexity. In each of the following results, we vary the total power consumption at MTs, $P_m^{\mathrm{T}} = [0.5, 3]$ on the x-axis. The total available power is used in both data transmission and circuit power consumption. Over this range ($P_m^{\mathrm{T}} = [0.5, 3]$), we aim to investigate the performance of the proposed optimal and suboptimal frameworks compared with the benchmark in two situations. The first situation ($P_m^{\mathrm{T}} = [0.5, 1.5]$) represents comparable transmission and circuit power consumption values (due to low total available power). The second situation ($P_m^{\mathrm{T}} = (1.5, 3]$) represents large available transmission power compared with circuit power consumption (due to high total available power). Simulation results are averaged over 100 runs.

Figures 3a and 3b show the minimum and average achieved energy efficiency versus P_m^{T}, respectively. Given the simulation settings, energy efficiency is improved with P_m^{T}, as the MTs can enhance the achieved throughput at a slight increase in power consumption. With low total available power, lower energy efficiency is achieved due to the comparable values of transmission power consumption (which is translated into a useful term, i.e., throughput) and circuit power consumption (which does not contribute into the achieved throughput). With more total available power, more power can be consumed for data transmission which is translated into higher throughput and enhanced efficiency. As shown in the figures, the proposed optimal and suboptimal frameworks outperform the benchmark. This is mainly due to two reasons. Firstly, the proposed frameworks jointly optimize bandwidth, among MTs, and power allocation, at each MT, to maximize energy efficiency, unlike the benchmark, which optimizes only power allocation. Hence, in the new frameworks, bandwidth and power allocations are performed according to channel conditions at different radio interfaces of different MTs and the available energy at each MT. This results in the improved performance in Figures 3a and 3b. Secondly, the proposed frameworks aim to maximize the minimum energy efficiency in the geographical region, unlike the benchmark where every MT aims to maximize its own energy efficiency independent of other MTs. This results in the improved performance of the proposed frameworks in Figure 3a. The optimal framework has improved performance over the suboptimal framework due to the fact that the optimal framework calculates its dual variables at every time slot using the actual CSI, whereas the suboptimal framework is based on the average CSI. However, overall the suboptimal framework has an improved performance over the benchmark with a reduced signaling overhead and computational complexity, as will be shown in Figure 6. Furthermore, as the number of MTs increases in the system, lower energy efficiency is achieved. This is mainly due to the increased competition on the radio resources at the BS and APs, which lead to reduced bandwidth allocation per user, and hence a lower energy efficiency is achieved.

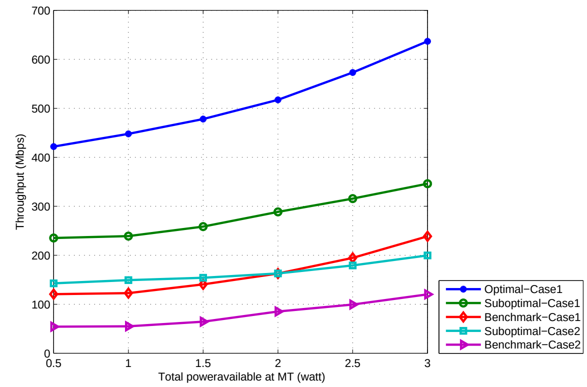
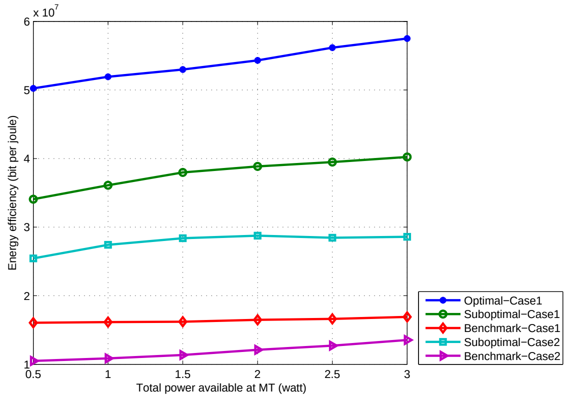
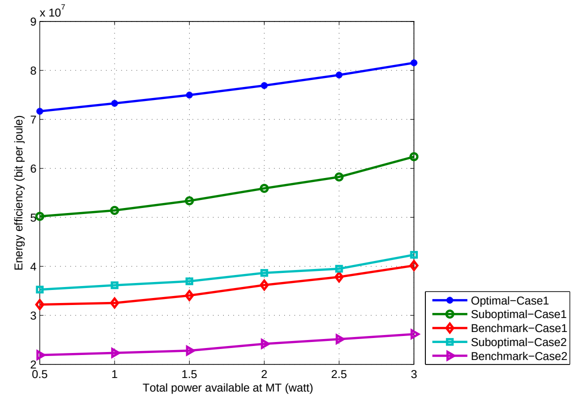


Fig. 4. The average achieved throughput versus the total power available at any MT.

Figure 4 shows the average achieved throughput versus P_m^{T}. Given the simulation settings, an improved throughput can be achieved, since more available power can be used to enhance the data rate for each MT. In addition, the proposed frameworks can achieve a higher total throughput than the benchmark. This is mainly because the proposed frameworks allocate bandwidth among MTs, and power at each MT, based on the channel conditions at MTs, different from the



(a) Minimum achieved energy efficiency



(b) Average achieved energy efficiency

Fig. 3. The achieved energy efficiency versus the total power available at any MT.

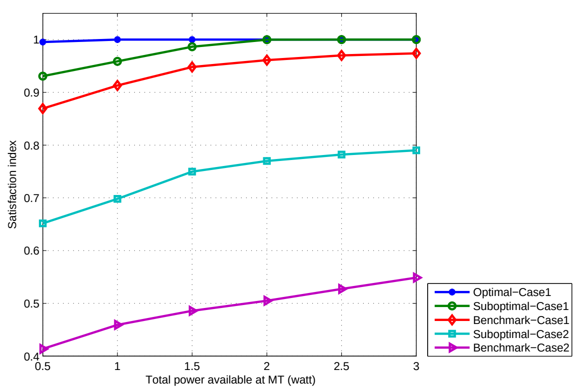


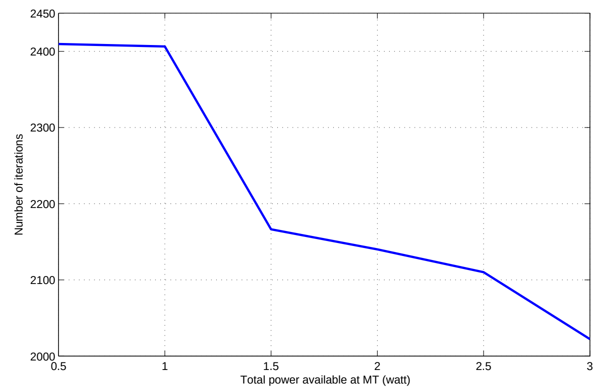
Fig. 5. The average achieved satisfaction index versus the total power available at any MT.

benchmark that allocates bandwidth to MTs independent of their channel conditions. Again, the suboptimal framework achieves a lower throughput than the optimal framework as its dual variables are based on the average CSI. Furthermore, a reduced throughput is achieved with an increased number of MTs due to the increased competition on the available bandwidth at the BS and APs.

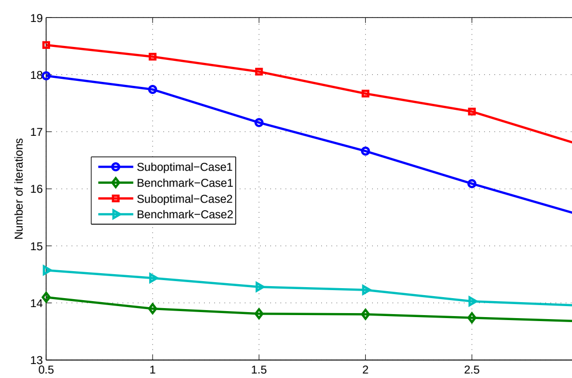
Figure 5 shows the average satisfaction index of MTs versus P_m^{T}. The satisfaction index captures the ability of the radio resource allocation frameworks to satisfy the QoS requirements of the MTs. Specifically, the satisfaction index is defined as [37]

$$SI = \mathbb{E}\{\mathbb{1}_{R_m \geq R_m^{\min}} + \mathbb{1}_{R_m < R_m^{\min}} \frac{R_m}{R_m^{\min}}\} \tag{39}$$

where $\mathbb{1}_a = 1$ if a is satisfied, and 0 otherwise. As shown in Figure 5, the optimal framework always achieves a satisfaction index of 1. Overall, the suboptimal framework has an improved satisfaction index over the benchmark. This is mainly due to the improved achieved throughput of the suboptimal framework as compared with the benchmark, as shown in Figure 4. While the suboptimal framework and benchmark satisfy the minimum required data rates of the MTs, the



(a) Optimal framework-case 1



(b) Suboptimal framework and benchamrk

Fig. 6. The number of iterations versus the total power available at any MT.

suboptimal framework achieves much higher throughput than the benchmark due to the CSI-based bandwidth allocation, which leads to higher satisfaction index.

Figure 6 shows the computational complexity of the radio resource allocation frameworks, in terms of total number of iterations per user per time slot, versus P_m^{T}. The number of iterations includes the required iterations for all dual variables to converge. As shown in Figure 6a, the number of iterations for the optimal framework (in case 1) is above 2000. On the other hand, as shown in Figure 6b, the suboptimal framework and the benchmark has a close number of iterations (around 18 for the suboptimal framework and around 14 for the benchmark). It should be noted that for the suboptimal framework and the benchmark, all such iterations are executed at the MTs (and BS/APs for the suboptimal framework), and hence do not require information exchange over the air interface. Hence, while the suboptimal framework has performance close to the benchmark in terms of computational complexity and signaling overhead, it has higher energy efficiency and throughput than the bechmark and much reduced computational complexity in comparison with the optimal framework. Furthermore, as the number of MTs increase, the number of iterations is increased due to the competition among MTs for the available bandwidth.

VIII. Conclusions and Future Work

In this paper, we propose a joint bandwidth and power allocation framework to maximize energy efficiency in a heterogeneous wireless medium. MTs are equipped with multiple radio interfaces, support the multi-homing capability, and have minimum required data rates. The proposed framework jointly allocates bandwidth among MTs from different BSs/APs, and transmission power to the radio interfaces of each MT, so as to maximize the minimum energy efficiency in the heterogeneous network. A desirable feature of the proposed framework is that it can be implemented in a decentralized manner among BSs/APs of different networks and MTs. A suboptimal framework is also presented to reduce the associated signaling overhead and computational complexity. Simulation results demonstrate the improved performance of the optimal and suboptimal frameworks over a state-of-the-art benchmark in terms of the minimum and total achieved energy efficiency, and the total achieved throughput and the reduced computational complexity and signaling overhead of the suboptimal framework.

In our future work, to assess the percentage improvement in the achieved energy efficiency, we will compare the proposed frameworks with general multi-homing algorithms [2] that aim to satisfy average/peak data rates in absence of energy consumption consideration. Moreover, in a multi-user system model, fairness issues should be addressed. A future extension of this work is to consider a max-min fair (lexicographic max-min) radio resource allocation. Furthermore, for practical implementation, a discrete set of data rates should be supported by each radio interface. We will investigate a mixed integer non-linear program (MINLP) that maximizes energy efficiency while supporting a discrete set of data rates for each radio interface. In addition to joint bandwidth and power allocation for each user, sub-channel selection within each BS will also be investigated for frequency selective fading channels.

Appendix A
Proof of Proposition 1

In order to prove that (9) is a concave-convex fractional program, we prove the concavity of R_m. Let $c_1 = \frac{1}{\ln(2)}$ and $c_2 = \frac{h_{nsm}}{N_0}$. We first prove the concavity of R_{nsm} in the decision variables B_{nsm} and P_{nsm}. The Hessian matrix of R_{nsm} can be expressed as

$$H = \frac{1}{(B_{nsm}+c_2P_{nsm})^2}\begin{bmatrix} -\frac{c_1c_2^2P_{nsm}^2}{B_{nsm}} & c_1c_2^2P_{nsm} \\ c_1c_2^2P_{nsm} & -c_1c_2^2B_{nsm} \end{bmatrix}.$$

As both H_{11} and H_{22} are negative and the second principal minor of H is 0, H is negative semidefinite [36]. Thus, R_{nsm} is concave in both B_{nsm} and P_{nsm}. As R_m is a sum of concave functions, R_m is also concave [36]. Since the numerator of η_m, i.e., R_m, is concave, the denominator is convex, and the constraints constitute a convex set in B_{nsm} and P_{nsm}, (9) is concave-convex fractional program [34].

Appendix B
Proof of Theorem 1

Let $\theta^* = \min_m f_m = f_{m'}$. Thus, $\forall m \neq m'$, $f_m \geq \theta^*$. Let $\widetilde{\mathcal{M}}$ denote the subset of MTs with $f_m > \theta^*$. From KKT conditions, we have at optimality

$$\mu_m^*\{\sum_{n\in\mathcal{N}}\sum_{s\in\mathcal{S}_n}(R_{nsm}^* - \lambda(\frac{P_{nsm}^*}{\rho} + Q_{nsm} + \zeta_{nsm}B_{nsm})) - \theta^*\} = 0, \quad \forall m. \tag{40}$$

Hence, for $m \in \widetilde{\mathcal{M}}$, $\mu_m^* = 0$, and we have two cases:

1) $\phi_m = 0$: From (20), $B_{nsm}^* = 0$ and thus by (26) $P_{nsm}^* = 0$, leading to $f_m = \theta_m = -\lambda\sum_n\sum_s Q_{nsm}$;
2) $\phi_m > 0$: From (26) and (27), we have

$$P_{nsm}^* = B_{nsm}[\frac{\phi_m}{\ln(2)\{\frac{\nu_m^*}{\rho} + \omega_{nsm}\}} - \frac{N_0}{h_{nsm}}]^+, \quad \forall n,s,m \tag{41}$$

$$\log_2(1 + \frac{P_{nsm}^*h_{nsm}}{N_0B_{nsm}^*}) - \frac{P_{nsm}^*h_{nsm}}{\ln(2)(N_0B_{nsm}^* + P_{nsm}^*h_{nsm})} = \frac{\zeta_{nsm}\nu_m^* + \beta_{ns}^*}{\phi_m}. \tag{42}$$

Using (41) and (42), f_m can be determined in this case.

For $\phi_m \geq 0$, if $\theta^* > f_m$, $f_m < f_{m'}$ $\forall m \in \widetilde{\mathcal{M}}$. However, $\forall m \neq m'$, f_m cannot be less than $f_{m'}$. Hence, at the optimal point, f_m $\forall m \in \mathcal{M}$ must have the same value, which is equal to θ^*. Otherwise, at optimal point, for m with $\theta^* \leq f_m$, $\mu_m^* = 0$.

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