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Upper Bound of the Lightest Higgs Boson Mass in the Minimal Supersymmetric Standard Model

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In the minimal supersymmetric standard model, it is shown that a radiative correction of the top and stop loops gives a finite, but non-negligible contribution to Higgs scalar masses if $m_t \simeq 150$–250 GeV. The upper limit to the lightest-scalar mass becomes 70–190 GeV in the range of heavy top quark.

The mechanism of electroweak symmetry breaking is one of the most important issues in the present particle physics. In the standard electroweak model a fundamental Higgs doublet is introduced to cause the spontaneous symmetry breaking. Supersymmetry (SUSY), eliminating all quadratic divergences, may provide a better theoretical basis to describe a fundamental Higgs boson with a relatively small mass to a high energy cutoff scale, say the Planck scale for example.[1)]

In the minimal SUSY extension of the standard electroweak model the Higgs sector consists of two chiral superfields of Higgs doublets Φ_{H_1} and Φ_{H_2} with opposite hypercharges. They are required to give masses for all quarks and leptons and to guarantee the absence of the gauge anomaly. Five physical Higgs bosons among them survive the gauge symmetry breaking, namely, there appear two neutral scalars φ_a and φ_b, a neutral pseudoscalar χ and a pair of charged scalars $\chi^{\pm}$ as physical particles. It has been, furthermore, shown by many authors[2)] that there is at least one neutral scalar boson lighter than $Z^0 (m_\varphi < m_{Z^0})$ in the minimal SUSY model. This has strongly motivated many recent analyses of Higgs boson production at LEP energies.[3)]

In this paper, however, we stress that a radiative correction gives a significant contribution to the Higgs mass term if the top quark is suficiently heavy as $m_t \simeq 150$—250 GeV. Therefore, the presence of the Higgs scalar lighter than m_{Z^0} is not an inevitable prediction of the minimal SUSY standard model.

Let us discuss the Higgs sector in the minimal SUSY model. With general soft-breaking terms[4)] of SUSY the tree-level Higgs potential is given by

$$V=\frac{g^2}{8}(\bar{H}_1\tau_a H_1+\bar{H}_2\tau_a H_2)^2+\frac{g'^2}{8}(\bar{H}_1 H_1-\bar{H}_2 H_2)^2$$
$$+m_1{}^2\bar{H}_1 H_1+m_2{}^2\bar{H}_2 H_2-m_3{}^2(H_1 H_2+\bar{H}_1\bar{H}_2)\,. \tag{1}$$

The parameters g and g' are gauge coupling constants of $SU(2)$ and $U(1)$, respectively.

If the model is coupled to a hidden sector of the minimal broken supergravity, the induced soft-breaking terms at the Planck scale satisfy

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$$m_1{}^2 = m_2{}^2 = \mu^2 + m_{3/2}^2 ,$$
$$m_3{}^2 = -(A-1)\mu m_{3/2} , \qquad (2)$$

where the mass parameters μ and $m_{3/2}$ are a SUSY symmetric Higgs mass and gravitino mass, respectively and A is a dimensionless constant determined by the potential of the hidden sector.[5] Renormalization effects change these parameters in the range between the Planck and the electroweak scale.[5] We may calculate the soft-breaking masses at low energies in terms of the original parameters at the Planck scale. For the present purpose, however, we do not need to specify the origin of the SUSY soft-breaking terms.

The electroweak symmetry breaking caused by

$$\langle H_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} / \sqrt{2} ,$$
$$\langle H_2 \rangle = \begin{pmatrix} v_2 \\ 0 \end{pmatrix} / \sqrt{2} \qquad (3)$$

yields the following tree-level mass formulae of the physical Higgs scalars,

$$m_{\chi^0}^2 = m_1{}^2 + m_2{}^2 ,$$
$$m_{\chi^\pm}^2 = m_{\chi^0}^2 + m_{W^\pm}^2 ,$$
$$m_{a,b}^2 = \frac{1}{2}\{ m_{\chi^0}^2 + m_{Z^0}^2 \pm \sqrt{(m_{\chi^0}^2 + m_{Z^0}^2)^2 - 4 m_{\chi^0}^2 m_{Z^0}^2 \cos^2 2\theta} \} . \qquad (4)$$

Here $\tan\theta \equiv v_2/v_1$. From Eq. (4) it is easily proved that there exists a scalar boson φ_b lighter than $Z^0 (m_b{}^2 \leq m_{Z^0}^2 \cos^2 2\theta)$. The radiative correction due to gauge interactions may change the mass formulae in Eq. (4), but the mass shift is negligible since the gauge coupling constants $\alpha_2 \equiv g^2/4\pi$ and $\alpha_1 \equiv g'^2/4\pi$ are very small. In fact, the radiative correction from the gauge interactions has been calculated in Ref. 6), where it is shown that the upper bound to the lightest Higgs mass cannot exceed 95 GeV. Therefore the presence of the light scalar boson φ_b seems a quite general conclusion in the minimal SUSY model.

We are now at the point to show that the one-loop diagrams of the top quark multiplets induce a finite, non-negligible contribution to the potential (1) if the top-quark mass is fairly large as $m_t \simeq 150$—250 GeV.*) The Yukawa coupling h_t of top quark is given by

$$h_t = \sqrt{2} m_t / v_2 \qquad (5)$$

and for the range $m_t \simeq 150$—250 GeV h_t becomes $O(1)$. Here the Yukawa coupling h_t is defined in the superpotential as

$$G = h_t \Phi_{\bar{t}_R} \Phi_{q_L^3} \Phi_{H_2} , \qquad (6)$$

*) It has been pointed out in Ref. 7) that large radiative corrections to the tree-level mass formulae (4) may arise in the case of heavy top quark. However, the upper bound of the lightest Higgs boson mass has not been derived in Ref 7), since the mass itself has divergences in their formalism.

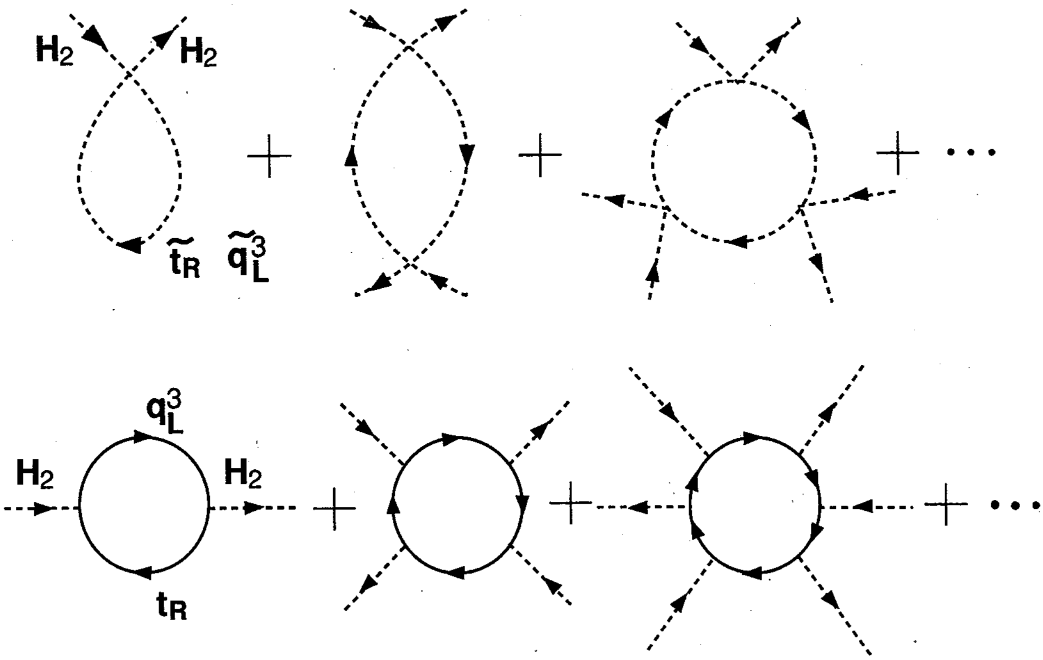

Fig. 1. One-loop Feynman diagrams contributing to the effective potential $V(H_2, \bar{H}_2)$. The dotted lines in the loops represent scalar quarks $\tilde{t}_R$ and $\tilde{q}_L{}^3$ and the solid line quarks t_R and $q_L{}^3$.

where $\Phi_{\bar{t}_R}$ and $\Phi_{q_L^3}$ are chiral superfields of the right-handed top quark and the left-handed quark doublet in the third generation, respectively.

The one-loop Feynman diagrams are given in Fig. 1. By the straightforward calculation the one-loop correction to the effective potential is obtained as

$$V_{\rm eff}^{(1)}=3\left(\frac{1}{4\pi}\right)^2\left\{(m^2+h_t{}^2\bar{H}_2H_2)^2\left(\log\frac{m^2+h_t{}^2\bar{H}_2H_2}{\Lambda^2}-\frac{1}{2}\right)\right.$$
$$\left.-h_t{}^4(\bar{H}_2H_2)^2\left(\log\frac{h_t{}^2\bar{H}_2H_2}{\Lambda^2}-\frac{1}{2}\right)\right\}. \tag{7}$$

Here we have assumed the SUSY breaking mass m^2 common to all scalar partners of t_R and $q_L{}^3$, for simplicity. Notice that the logarithmic divergence appears only in the Higgs mass term. For $m^2 \gg h_t{}^2\langle\bar{H}_2H_2\rangle(=m_t{}^2)$, the effective potential in Eq. (7) may be expanded as

$$V_{\rm eff}^{(1)}\simeq 3\left(\frac{1}{4\pi}\right)^2\left\{-2m^2\log\frac{\Lambda^2}{m^2}h_t{}^2\bar{H}_2H_2\right.$$
$$\left.+\frac{3}{2}h_t{}^4(\bar{H}_2H_2)^2-\log\frac{h_t{}^2\bar{H}_2H_2}{m^2+h_t{}^2\bar{H}_2H_2}h_t{}^4(\bar{H}_2H_2)^2\right\}. \tag{8}$$

The divergent mass term in Eq. (8) can be absorbed by the renormalization of $m_2{}^2$ in Eq. (1). We must restrict our analysis in the case $h_t{}^2/4\pi \ll 1$ so that the one-loop approximation is reliable.

With having the finite correction (8) to the tree-level potential (1), we find masses for the physical scalar particles,*)

$$m_{\chi^0}^2 = m_1{}^2 + m_2{}^2 + \delta' \sin^2\theta \cdot v^2 ,$$
$$m_{\chi^\pm}^2 = m_{\chi^0}^2 + m_{W^\pm}^2 . \tag{9}$$

The mass matrix for the two scalar bosons φ_a and φ_b is given by

$$(\varphi_a\ \varphi_b)\begin{bmatrix} m_{\chi^0}^2 + m_{Z^0}^2 \sin^2 2\theta & -m_{Z^0}^2 \sin 2\theta \cos 2\theta \\ +\frac{1}{2}\delta \sin^2 2\theta \cdot v^2 & +\delta \sin^2\theta \sin 2\theta \cdot v^2 \\ -m_{Z^0}^2 \sin 2\theta \cos 2\theta & m_{Z^0}^2 \cos^2 2\theta \\ +\delta \sin^2\theta \sin 2\theta \cdot v^2 & +2\delta \sin^4\theta \cdot v^2 \end{bmatrix} \begin{pmatrix} \varphi_a \\ \varphi_b \end{pmatrix}, \tag{10}$$

where

$$\delta \simeq 3\left(\log \frac{m^2 + m_t{}^2}{m_t{}^2}\right)\left(\frac{h_t{}^2}{4\pi}\right)^2 ,$$
$$\delta' \simeq 3\left(1 + \log \frac{m^2 + m_t{}^2}{m_t{}^2}\right)\left(\frac{h_t{}^2}{4\pi}\right)^2 \tag{11}$$

and $v = \sqrt{v_1{}^2 + v_2{}^2} = 250$ GeV.**) It is easy to see that one of the eigenvalues is always smaller than $m_{Z^0}^2 \cos^2 2\theta + 2\delta \sin^4\theta \cdot v^2$ and hence, using Eq. (5), the mass of the lightest scalar boson φ_{light} is bounded as

$$m_{\text{light}} \le \sqrt{m_{Z^0}^2 \cos^2 2\theta + \frac{6}{(2\pi)^2}\left(\log \frac{m^2 + m_t{}^2}{m_t{}^2}\right)\frac{m_t{}^4}{v^2}} . \tag{12}$$

In Fig. 2 we show the maximal values of the lightest scalar mass for various m_t. Here we have taken the SUSY breaking scale $m = 1$ TeV. We see that the upper bound of m_{light} varies from 70 to 190 GeV in the region of $m_t = 150$—250 GeV. It is not unreasonable that the radiative correction becomes larger than the tree-level mass, since the Yukawa coupling h_t reponsible to the correction is not present in the tree-level potential (1), which contains only the gauge coupling constants g^2 and g'^2.

Several comments are in order. There is another soft-breaking term of SUSY,

$$\mathcal{L}_{\text{SUSY}} = m_4\, \tilde{\bar{t}}_R \tilde{q}_L{}^3 H_2 + \text{h.c.}, \tag{13}$$

where $\tilde{\bar{t}}_R$ and $\tilde{q}_L{}^3$ denote the scalar components of the chiral superfields $\Phi_{\bar{t}_R}$ and $\Phi_{q_L{}^3}$.

*) The physical mass is defined as a pole of the scalar propagator. The inverse propagator is $p^2 - m_{\text{tree}}^2 + \Sigma(p^2)$, where $\Sigma(p^2)$ is the higher order correction of the self-energy. The second derivative $V_{(1)}''$ of the one-loop potential corresponds to the one-loop self-energy at zero momentum $\Sigma(0)$ and m_{tree}^2 is V_{tree}''. Thus, the pole appears at $p^2 = V_{\text{total}}'' + \Sigma(m_{\text{tree}}^2) - \Sigma(0)$ at one-loop level. However, the difference $\Sigma(m_{\text{tree}}^2) - \Sigma(0)$ associated with the wave function renormalization is negligible in the present analysis, since we are interested in a mass correction of the same order of the tree-level mass m_{tree}.

**) Notice that the logarithmic term in Eq. (8) gives rise to a mass term in the broken phase.

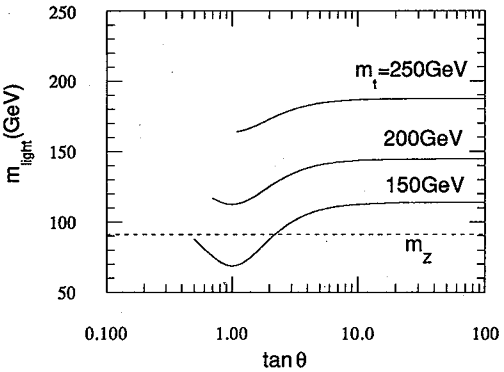

Fig. 2. The upper limit to the mass of the lightest Higgs boson is given in various top-quark masses m_t. We restrict our analysis to the case where the Yukawa coupling $h_t^2/4\pi$ does not exceed 0.3.

This breaking term also contributes to the mass correction.*) However we find that this always gives a negative contribution to δ in the one-loop approximation, and hence our main conclusion in this paper concerning the upper limit to the φ_{light} mass is unchanged.

In our analysis we have assumed the same mass m for all scalar partners of t_R and $q_L{}^3$ for simplicity. If we consider the model coupled to the minimal supergravity, this is true only at the Planck scale. The renormalization effects may change these masses at the Fermi scale. Since the main renormalization effect coming from the QCD interaction is common for the masses of $\tilde{t}_R$ and $\tilde{q}_L{}^3$, we expect the mass difference to be small compared to m itself. The detailed analysis including the trilinear-coupling term, the differences of scalar quark masses, etc. will be given elsewhere.[8)]

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Note added in proof: In Ref. 8) we find that Eq. (13) gives a positive contribution to the Higgs mass through the interference with SUSY interactions.

*) This trilinear-coupling term induces, in the broken phase of the electroweak symmetry, a mixing between scalar partners of $\bar{t}_R$ and $q_L{}^3$. Thus, m_4 is limited such that it never causes an instability of the QCD symmetric vacuum.