# UPPER LIMIT ON THE $\nu_{\tau}$ MASS FROM $\tau \rightarrow 3 h \nu_{\tau}$ DECAYS 

## The OPAL Collaboration


#### Abstract

A new limit on the $\nu_{\tau}$ mass has been determined using $\mathrm{Z}^{0} \rightarrow \tau^{+} \tau^{-}$events selected from $140 \mathrm{pb}^{-1}$ of data collected with the OPAL detector at LEP during the period 1990-1994. Using $Z^{0} \rightarrow \tau^{+} \tau^{-}$events in which both $\tau$-leptons decay to three charged particles, a novel likelihood analysis is applied to the 2-dimensional distribution of missing mass squared and missing energy, from which we obtain a limit of $M_{\nu_{\tau}}<$ 35.3 MeV at $95 \%$ confidence level. Combining this result with OPAL's previous published measurement from $\tau \rightarrow 5 \pi \nu_{\tau}$ decays, we obtain the new $95 \%$ confidence level limit of $M_{\nu_{\tau}}<29.9 \mathrm{MeV}$.


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## 1 Introduction

One of the most interesting current topics in particle physics and astrophysics is the possibility of non-zero neutrino mass. In particle physics, massive neutrinos are common features of appealing extensions of the minimal standard model, which assumes vanishing neutrino masses. The mass hierarchy between the charged and neutral leptons can be explained by the "see-saw" mechanism [1]. In this context, the $\tau$ neutrino is the best candidate to have an experimentally observable mass. In astrophysics, massive neutrinos are candidates for dark matter and may also serve as a solution for the observed deficit of solar neutrinos [2]. A heavy and unstable neutrino with mass in the MeV range, which could be directly explored by experiments at $\mathrm{e}^{+} \mathrm{e}^{-}$colliders, can have a host of interesting astrophysical and cosmological consequences [3]. The best $95 \%$ confidence level (C.L.) upper limit on the $\nu_{\tau}$ mass $M_{\nu_{\tau}}$ of 24 MeV has been determined by ALEPH [4], using a 2 -dimensional likelihood fit to the hadronic invariant mass and hadronic total energy of $25 \tau \rightarrow 5 \pi\left(\pi^{0}\right) \nu_{\tau}$ decays.

In this letter we present an upper limit on the $\nu_{\tau}$ mass obtained with a high statistics sample of $\tau \rightarrow 3 h \nu_{\tau}$ decays in 3 -prong versus 3 -prong (3-3) $\mathrm{Z}^{0} \rightarrow \tau^{+} \tau^{-}$events, where $h$ is either a charged $\pi$ or K meson. We employ a novel 2 -dimensional likelihood technique in the plane of the missing energy and missing mass variables, which allows the extraction of a limit on the $\nu_{\tau}$ mass using a statistical approach.

## 2 The OPAL detector

The OPAL detector is described in detail elsewhere [5]. Only the elements of the apparatus relevant to this analysis are described briefly here. The coordinate system is defined so that the $z$-axis follows the $\mathrm{e}^{-}$-beam direction and the $x-y$ plane is perpendicular to it with the $x$-axis lying horizontally in the plane of the LEP ring; $\theta$ and $\phi$ are the usual polar coordinates. Tracking of charged particles is performed by a central tracking detector, consisting of a silicon microvertex detector, a vertex chamber, a jet chamber and $z$-chambers. The central tracking detector is immersed in a 0.435 T solenoidal magnetic field. The momentum resolution obtained is approximately $\sigma_{p} / p=\sqrt{0.02^{2}+(0.0015 p)^{2}}$, where $p$ is in GeV . The silicon microvertex detector, which covers a polar angle range $|\cos \theta|<0.83$, provided both $\phi$ - and $z$-coordinates for data taken in 1993-1994, but $\phi$ coordinates only for 1991 and 1992. Only $\phi$-coordinate information is used in this analysis.

The barrel electromagnetic calorimeter is located outside the magnet coil, behind an average of 2.4 radiation lengths of material. It consists of 9440 lead-glass blocks of 24.6 radiation lengths pointing toward the beam interaction region, each covering approximately $40 \times 40 \mathrm{mrad}^{2}$. In the endcap there are 2264 lead-glass blocks of the same cross section but with axes parallel to the beam direction. The energy resolution is $\sigma_{E} / E=2.1 \%$ in the barrel and $\sigma_{E} / E=3.1 \%$ in the endcap for electrons with $E=45.6 \mathrm{GeV}$, determined from $\mathbf{e}^{+} \mathrm{e}^{-}$events.

## 3 Event selection

This analysis is based on a data sample of $140 \mathrm{pb}^{-1}$ collected by OPAL from 1990 to 1994. To suppress effects caused by the dependence of initial state radiation on the $\mathrm{e}^{+} \mathrm{e}^{-}$center-of-mass energy, $E_{\text {C.m. }}$, only data on the $\mathrm{Z}^{0}$ peak are used (i.e. beam energies between 45.5 GeV and 45.7 GeV ). The preselection of $\mathrm{Z}^{0} \rightarrow \tau^{+} \tau^{-}$events uses the same quality cuts for charged tracks and clusters of electromagnetic energy, cosmic ray rejection, $\tau$ finding algorithm, and $q \bar{q}$ background removal used in [6]. In this preselection, the $\tau$ decay products are identified as jets of particles lying within cones of half-angle $35^{\circ}$; these
are referred to as " $\tau$-jet cones" throughout this paper. A $\mathrm{Z}^{0} \rightarrow \tau^{+} \tau^{-}$candidate must contain exactly two $\tau$-jet cones. We use the event thrust axis to estimate the $\tau$ flight direction, and therefore in order to ensure good resolution of the thrust axis direction we require that each $\tau$-jet cone contains precisely three charged tracks. The event thrust axis calculated from the momenta of all good charged tracks lying within the detector acceptance must satisfy $\left|\cos \theta_{\text {thrust }}\right|<0.9$. The total charge of each $\tau$-jet cone must be $\pm 1$ and the total charge of the event must be zero.

To remove two-photon background events, the acollinearity angle between the $\tau$-jet cones must be less than $15^{\circ}$, where the directions of the jets are given by the momentum sums of the tracks and electromagnetic clusters. Residual two-photon background events are rejected by exploiting the low visible energies and very low net transverse momenta typical of $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-} X$ events. An event is rejected if the sum of visible energies of the jets (taken for each jet as the larger value of the sum of track momenta and of the sum of electromagnetic cluster energies) is less than $1 \%$ of $E_{\text {C.M. }}$. Further, the event is rejected if the sum of visible energy is less than $20 \%$ of $E_{\text {C.M. }}$ and the missing transverse momenta, calculated separately for charged tracks and for electromagnetic clusters, are both less than 2 GeV .

Events with tracks originating from photon conversions are eliminated with an invariant mass cut of $M_{\mathrm{e}^{+}} \mathrm{e}^{-}<0.05 \mathrm{GeV}$, assuming electron masses for oppositely-charged tracks in any $\tau$-jet cone. Tracks from $\mathrm{K}^{0} \rightarrow \pi^{+} \pi^{-}$decays are eliminated with an invariant mass cut of $0.473 \mathrm{GeV}<M_{\pi^{+} \pi^{-}}<0.523 \mathrm{GeV}$, assuming $\pi$ masses for the oppositely charged tracks in any $\tau$-jet cone. A total of 1056 events in the data are rejected by the $\gamma$ conversion removal cut alone, which is consistent with the expected number $1089 \pm 25$ from the $\mathrm{Z}^{0} \rightarrow \tau^{+} \tau^{-}$Monte Carlo (KORALZ4.0 [7]) and $8.2 \pm 5.8$ events from the $\mathrm{q} \overline{\mathrm{q}}$ Monte Carlo (JETSET7.4 [8][9]). The Monte Carlo events were passed through a full simulation of the OPAL detector [10]. The $\mathrm{K}^{0}$ rejection removes 34 events in the data after all other cuts, compared with $29 \pm 4$ events expected from $\mathrm{Z}^{0} \rightarrow \tau^{+} \tau^{-}$and zero events from $\mathrm{q} \overline{\mathrm{q}}$. A total of $23543-3 \mathrm{Z}^{0} \rightarrow \tau^{+} \tau^{-}$events is selected after the above cuts. The estimates for the fractions of the different $\tau$-decays in this preselected $3-3 \mathrm{Z}^{0} \rightarrow \tau^{+} \tau^{-}$event sample are listed in Table 1. The expected number of $q \bar{q}$ background events in the preselected sample is $45.1 \pm 13.8((1.91 \pm 0.58) \%)$ from the Monte Carlo.

In each event, at least one $\tau$-jet cone is required to satisfy the following requirements, aimed at selecting $\tau \rightarrow 3 h \nu_{\tau}$ decays. To ensure well measured track momenta, the $3 \pi$ energy reconstructed from the three charged tracks, $E_{3 \pi}$, assuming the $\pi$ mass for each track in a $\tau$-jet cone, must satisfy $0.2<E_{3 \pi} / E_{\text {beam }}<1.2$, where $E_{\text {beam }}$ is the beam energy. To suppress poorly measured tracks, the $3 \pi$ mass reconstructed from the three charged track momenta $M_{3 \pi}=\sqrt{E_{3 \pi}{ }^{2}-\left|\vec{p}_{3 \pi}\right|^{2}}$ must satisfy $0.5 \mathrm{GeV}<M_{3 \pi}<2.0 \mathrm{GeV}$. To suppress decays in which $\pi^{0}$ s are present, we require the following:

1. the sum of electromagnetic energy in the $\tau$-jet cone must be less than $60 \%$ of the sum of momenta of charged tracks in the $\tau$-jet cone, $\Sigma p$. This cut is sensitive to cases where the $\pi^{0}$ makes a relatively large contribution to the energy in the $\tau$-jet cone.
2. the sum of electromagnetic energies in the $\tau$-jet cone not associated with any charged track must be less than $40 \%$ of $\Sigma p$. This cut is designed to reject $\tau$-decays with an electromagnetic cluster from a $\pi^{0}$ which is well separated from any charged $\pi$ tracks.
3. to suppress cases where a $\pi^{0}$ hits the same lead-glass block as a charged $\pi$ track, the electromagnetic energy associated with each charged track is required to be less

| Mode | Initial (\%) [11] | Preselected (\%) | $3 h \nu_{\tau}$ candidates (\%) |
| :---: | ---: | ---: | ---: |
| $\tau \rightarrow 3 \pi \nu_{\tau}$ | 8.89 | $54.53 \pm \mathbf{0 . 4 7}$ | $76.99 \pm 0.54$ |
| $\tau \rightarrow \mathrm{KK} \pi \nu_{\tau}$ | 0.17 | $1.16 \pm \mathbf{0 . 1 0}$ | $1.77 \pm 0.17$ |
| $\tau \rightarrow \mathrm{~K} \pi \pi \nu_{\tau}$ | 0.20 | $1.53 \pm \mathbf{0 . 1 2}$ | $2.22 \pm 0.19$ |
| $\tau \rightarrow 3 \pi \pi^{0} \nu_{\tau}$ | 5.01 | $27.39 \pm \mathbf{0 . 4 2}$ | $14.67 \pm \mathbf{0 . 4 6}$ |
| $\tau \rightarrow 3 \pi 2 \pi^{0} \nu_{\tau}$ | 0.42 | $1.96 \pm \mathbf{0 . 1 3}$ | $0.40 \pm \mathbf{0 . 0 8}$ |
| $\tau \rightarrow 3 \pi 3 \pi^{0} \nu_{\tau}$ | 0.06 | $0.25 \pm 0.05$ | $0.02 \pm 0.02$ |
| non 3-prong $\tau$-decays | 85.26 | $12.31 \pm \mathbf{0 . 3 1}$ | $3.93 \pm \mathbf{0 . 2 5}$ |

Table 1: Decay fractions in the initial, preselected, and finally selected $\tau \rightarrow 3 h \nu_{\tau}$ candidates, in $\tau$ MC samples.
than $80 \%$ of the charged track momentum.
After the above requirements, $2694 \tau \rightarrow 3 h \nu_{\tau}$ candidates are selected. From Monte Carlo simulation the selection efficiency for $\tau \rightarrow 3 h \nu_{\tau}$ decays is ( $67.9 \pm 0.6$ ) \% within the preselected 3-3 topology events. The estimates for the fractions of the different $\tau$-decays in the selected $\tau \rightarrow 3 h \nu_{\tau}$ candidates are summarized in Table 1. The effect of small contributions from $\tau \rightarrow \mathrm{KK} \pi \nu_{\tau}$ and $\tau \rightarrow \mathrm{K} \pi \pi \nu_{\tau}$ decays is incorporated into the analysis for the $\tau \rightarrow 3 h \nu_{\tau}$ signal as described below. The background fraction from $q \bar{q}$ events is expected to be $(0.46 \pm 0.26) \%$.

## 4 Method

We determine a limit on $M_{\nu_{\tau}}$ from the distribution of the missing energy normalized by the beam energy $\omega=E_{\text {miss }} / E_{\text {beam }}=\left(E_{\text {beam }}-E_{3 \pi}\right) / E_{\text {beam }}$ and the missing mass squared $\eta=M_{\text {miss }}^{2}=E_{\text {miss }}^{2}-p_{\text {miss }}^{2}$ for the $\tau \rightarrow 3 h \nu_{\tau}$ candidates. The missing momentum vector $\vec{p}_{\text {miss }}=\hat{p}_{\tau} \sqrt{E_{\text {beam }}^{2}-M_{\tau}^{2}}-\vec{p}_{3 \pi}$ is computed for each three-track decay, where $\vec{p}_{3 \pi}$ is the sum of the momentum vectors of the three charged tracks, and the $\tau$ flight direction, $\hat{p}_{\tau}$, is estimated using the event thrust axis direction, $\hat{p}_{\text {thrust }}$. The vector $\hat{p}_{\text {thrust }}$ is calculated from the sum of the three-track momentum vectors $\vec{p}_{3 \pi}$ and $\vec{p}_{3 \pi}^{\prime}$ for the two $\tau$-decays: $\hat{p}_{\text {thrust }}=\left(\vec{p}_{3 \pi}-\vec{p}_{3 \pi}^{\prime}\right) /\left|\vec{p}_{3 \pi}-\vec{p}_{3 \pi}^{\prime}\right|$. The use of the variable $M_{\text {miss }}^{2}$ is a new feature of this analysis. A non-zero $M_{\nu_{\tau}}^{2}$ contributes directly to $M_{\text {miss }}^{2}$. The $M_{\text {miss }}^{2}$ peak becomes broadened because of the assumption that the $\tau$ flight direction is the thrust axis, as can be seen from the expression $M_{\text {miss }}^{2}=M_{\nu_{\tau}}^{2}+2 p_{\tau} p_{3 \pi}\left(\cos \Psi_{\text {thrust }}-\cos \Psi_{\tau}\right)$, where $\Psi_{\text {thrust }}$ and $\Psi_{\tau}$ are the event thrust axis and the true $\tau$ flight direction angles with respect to the $\vec{p}_{3 \pi}$ decay. The event thrust axis direction typically differs from the true $\tau$ flight direction by 7 mrad in $3-3 \mathrm{Z}^{0} \rightarrow \tau^{+} \tau^{-}$events due to the recoil of $\nu_{\tau}$. This is the dominant contribution to the observed width of $M_{\text {miss }}^{2}$ and can be predicted by the Monte Carlo for a given input value for $M_{\nu_{\tau}}$. The observed $\eta$ and $\omega$ distributions from the selected $\tau \rightarrow 3 h \nu_{\tau}$ data and Monte Carlo samples are shown in Figure 1. The agreement is reasonable: it should be noted that the analysis method does not rely on the Monte Carlo to model these distributions; instead the resolution function is determined from data, as explained below.

The distribution of data in the 2 -dimensional $(\eta, \omega)$ plane is compared to that predicted by Monte Carlo samples in which the $\nu_{\tau}$ is assigned various masses and the detector resolution and acceptance effects are included. We perform a cone-by-cone maximum likelihood fit to extract the best value for $M_{\nu_{\tau}}$. The likelihood for each selected $\tau \rightarrow 3 h \nu_{\tau}$
candidate $i$ takes the form:

$$
\mathcal{L}_{i}\left(M_{\nu_{\tau}}, \eta_{i}, \omega_{i}\right)=\frac{\int \mathrm{d} \eta \int \mathrm{~d} \omega \mathcal{P}\left(M_{\nu_{\tau}}, \eta, \omega\right) \mathcal{D}\left(\eta-\eta_{\mathrm{i}}, \omega-\omega_{\mathrm{i}}, \sigma_{\eta_{\mathrm{i}}}, \sigma_{\omega_{\mathrm{i}}}, \rho_{\mathrm{i}}\right)}{\int \mathrm{d} \eta \int \mathrm{~d} \omega\left\{\mathcal{P}\left(M_{\nu_{\tau}}, \eta, \omega\right) \int \mathrm{d} \eta^{\prime} \int \mathrm{d} \omega^{\prime} \mathcal{D}\left(\eta-\eta^{\prime}, \omega-\omega^{\prime}, \sigma_{\eta_{\mathrm{i}}}, \sigma_{\omega_{\mathrm{i}}}, \rho_{\mathrm{i}}\right)\right\}},
$$

where $\mathcal{P}$ is the physics function, $\mathcal{D}$ is the detector resolution function, $\sigma_{\eta}, \sigma_{\omega}$, and $\rho$ are the errors and correlation coefficient for $\eta$ and $\omega$. These last three variables are obtained from track parameters and error matrices of the $i$-th $3 \pi$ decay candidate, assuming all charged decay products are pions. The physics function is defined for the region $-1 \mathrm{GeV}^{2}<\eta<$ $1 \mathrm{GeV}^{2}$ and $0<\omega<1$. The cut $\left|\eta_{i}\right|<1 \mathrm{GeV}^{2}$ is also applied to measured candidates. A total of $2514 \tau \rightarrow 3 h \nu_{\tau}$ candidates are left after this cut.

The physics function consists of a term for $\mathrm{Z}^{0} \rightarrow \tau^{+} \tau^{-}$events and one for non- $\tau$ background events. We consider $\mathrm{q} \overline{\mathrm{q}}$ events to be the only non- $\tau$ background, and their distribution is independent of the true $M_{\nu_{\tau}}$. The physics function can be written as:

$$
\mathcal{P}\left(M_{\nu_{\tau}}, \eta, \omega\right)=\left(1-f_{\mathrm{q} \overline{\mathrm{q}}}\right) \mathcal{P}_{\tau \tau}\left(M_{\nu_{\tau}}, \eta, \omega\right)+f_{\mathrm{q} \overline{\mathrm{q}}} \mathcal{P}_{\mathrm{q} \overline{\mathrm{q}}}(\eta, \omega),
$$

where $f_{\mathrm{q} \overline{\mathrm{q}}}$ is the fraction of $\mathrm{q} \overline{\mathrm{q}}$ events in the selected sample. A JETSET7.4 event sample with full detector simulation was used to obtain $\mathcal{P}_{\mathrm{q} \overline{\mathrm{q}}}(\eta, \omega)$; the full simulation was needed in order to determine the acceptance, but the true particle momenta were used to define $\eta$ and $\omega$, since in the fit these are convolved with the detector resolution function $\mathcal{D}$. To include the effect of the correlations between the two $\tau$-jet cones through the thrust axis direction, the 2 -dimensional physics function $\mathcal{P}_{\tau \tau}$ has separate contributions from $3 h$ and $3 h \mathrm{n} \pi^{0}(\mathrm{n} \geq 1)$ decay modes in each $\tau$-jet cone. The function $\mathcal{P}_{\tau \tau}$ for a candidate $\tau \rightarrow X \nu_{\tau}$ decay is given by:

$$
\mathcal{P}_{\tau \tau}\left(M_{\nu_{\tau}}, \eta, \omega\right)=\sum_{X, X^{\prime}} f_{X} f_{X^{\prime}} \mathcal{T}_{X, X^{\prime}}\left(M_{\nu_{\tau}}, \eta, \omega\right) \epsilon_{X}(\eta, \omega)+\mathcal{P}_{n o n 3}(\eta, \omega)
$$

where $f_{X}$ is is the decay fraction of $\tau$ into 3 -prong decays in the sample of $\tau \rightarrow 3 h \nu_{\tau}$ candidates and $f_{X^{\prime}}$ is the corresponding fraction in the opposite $\tau$-jet cone ( $X, X^{\prime}=3 h$ or $\left.3 h \mathrm{n} \pi^{0}\right), \epsilon_{X}(\eta, \omega)$ is the normalized efficiency function ( $\iint \mathrm{d} \eta \mathrm{d} \omega \epsilon_{\mathrm{X}}=1$ ) for the $\tau \rightarrow X \nu_{\tau}$ decay, $\mathcal{T}_{X, X^{\prime}}\left(M_{\nu_{\tau}}, \eta, \omega\right)$ is the theoretical distribution for $\tau \rightarrow X \nu_{\tau}$ decays with $\tau \rightarrow X^{\prime} \nu_{\tau}$ in the opposite $\tau$-jet cone, and $\mathcal{P}_{\text {non } 3}(\eta, \omega)$ is the contribution to the physics function from events with background sources other than 3 -prong $\tau$ decays in either $\tau$-jet cone. Some 1-prong decays, such as $\tau \rightarrow \rho \nu_{\tau}$ and $\tau \rightarrow \mathrm{K}^{*} \nu_{\tau}$, survive the $3 \pi$ selection through photon conversions. The contribution from these backgrounds is expected to have negligible sensitivity to $M_{\nu_{\tau}}$ because of large fluctuations on the plane of $(\eta, \omega)$ and the small fraction of these events in the final sample. The functions $\mathcal{T}_{X, X^{\prime}}\left(M_{\nu_{\tau}}, \eta, \omega\right)$ are obtained from the theoretical decay distributions on the ( $\eta, \omega$ ) plane taking account of the bias from the thrust axis direction, using $X, X^{\prime}$ events without detector simulation generated with KORALZ4.0 in steps of 10 MeV for $M_{\nu_{\tau}}$ and including multiple initial and final state photon radiation. The functions $\epsilon_{X}(\eta, \omega), \mathcal{P}_{\text {non }}(\eta, \omega)$, are estimated from KORALZ4.0 event samples with full detector simulation. The dependence of $\epsilon_{X}(\eta, \omega)$ on $\omega$ for $\tau \rightarrow 3 h \nu_{\tau}$ decays is relatively small, while the $\tau \rightarrow 3 h \mathrm{n} \pi^{0} \nu_{\tau}$ decays with large $\omega$ are suppressed by the $\pi^{0}$ rejection cuts. In Figures 2 a)-d), contour plots of reference histograms of the physics functions are shown. The region of greatest sensitivity lies around the peak in $\eta$ and near the end-point of $\omega=0$. From these plots one can also observe the good separation on the $(\eta, \omega)$ plane between the contributions from $3 h$ and $3 h \mathrm{n} \pi^{0}$ decays. Furthermore, the fit for $M_{\nu_{\tau}}$ to the 2 -dimensional distribution can substantially reduce the systematic uncertainties in the measured $M_{\nu_{\tau}}$ arising from the momentum scale, resolution, and beam
energy, because of the different sensitivities of the two correlated variables $(\eta, \omega)$ to these effects.

The detector resolution function is given by the 2 -dimensional Gaussian distribution: $\mathcal{D}\left(\eta-\eta_{i}, \omega-\omega_{i}, \sigma_{\eta_{i}}, \sigma_{\omega_{i}}, \rho_{i}\right)$. The experimental resolutions strongly depend on the properties of specific events (e.g. presence of $z$-chamber or silicon microvertex detector hits). In order to take this into account, the resolutions $\sigma_{\eta_{i}}$ and $\sigma_{\omega_{i}}$ and the correlation coefficient $\rho_{i}$ are individually reconstructed for each of the selected $\tau \rightarrow 3 h \nu_{\tau}$ candidates using the track parameters and their respective error matrices. These resolutions depend on the track parameters in both $\tau$-jet cones, because of the dependence of $\eta_{i}$ on the thrust axis direction. We emphasize again that the resolution used to estimate the likelihood is taken from data, not Monte Carlo. The distributions of $\sigma_{\eta_{i}}$ and $\sigma_{\omega_{i}}$ have peaks at 0.03 $\mathrm{GeV}^{2}$ and 0.01 , respectively, which are lower than their average values, $0.10 \mathrm{GeV}^{2}$ and 0.02 .

Each of the track error matrix elements is corrected by a scaling factor determined from $Z^{0} \rightarrow \mu^{+} \mu^{-}$events after correcting for the effects of initial state radiation and the center-of-mass energy spread. The systematic uncertainty on the scaling factors is estimated to be $4 \%$ using $\mathrm{D}^{*} \rightarrow \mathrm{~K} \pi \pi$ decays as described below. To improve the resolution of $\eta$, the three tracks are constrained to a reconstructed secondary vertex if one can be formed with $\chi^{2}$-probability $>1 \%$. Only $14 \%$ of the selected $\tau \rightarrow 3 h \nu_{\tau}$ decays do not meet this requirement. The $\eta$ resolution is, on average, $0.09 \mathrm{GeV}^{2}$ for the $\tau \rightarrow 3 h \nu_{\tau}$ decays with a good vertex and $0.18 \mathrm{GeV}^{2}$ for the decays without a good vertex and therefore the events without a good vertex carry little statistical weight in the fit.

## 5 Results

Using the 2-dimensional likelihood technique described in Section 4, we estimate the total likelihood values for the 2514 selected $\tau \rightarrow 3 h \nu_{\tau}$ candidates as the assumed true $M_{\nu_{\tau}}$ is changed from 0 MeV to 150 MeV in 10 MeV steps. The results of these total likelihood values favour $M_{\nu_{\tau}}=0 \mathrm{MeV}$ as shown in Figure 3 a). To obtain an upper limit on $M_{\nu_{\tau}}$, we considered the following three different approaches:

1. We fit the likelihood distribution as a function of $M_{\nu_{\tau}}$ using a third order polynomial multiplied by a Gaussian (Figure 3 a)). By integrating the function over the physical region, we obtain an upper limit at $95 \%$ C.L., $M_{\nu_{\tau}}<32.1 \mathrm{MeV}$.
2. We fit the likelihood distribution as a function of $M_{\nu_{\tau}}^{2}$ using a Gaussian, which has the advantage that the likelihood parameterization can be extrapolated into the unphysical region. The peak of the Gaussian lies at negative $M_{\nu_{\tau}}^{2}$, but is statistically consistent with the assumption of zero mass. The $95 \%$ C.L. limit on $M_{\nu_{\tau}}$ is $M_{\nu_{\tau}}<$ 33.6 MeV , constraining $M_{\nu_{\tau}}^{2}>0 \mathrm{MeV}^{2}$.
3. We form the distribution of $\Delta \ln \mathcal{L}=\ln \mathcal{L}(0)-\ln \mathcal{L}\left(M_{\nu_{\tau}}\right)$ and obtain a $95 \%$ C.L. limit by noting the value of $M_{\nu_{\tau}}$ at which $\Delta \ln \mathcal{L}=1.92$. The $95 \%$ C.L. limit is $M_{\nu_{\tau}}<31.7 \mathrm{MeV}$.
For our quoted limit, we use the first approach and consider the others as consistency checks. The second approach verifies that the limit is sensitive neither to the assumption of uniform a priori probability in $M_{\nu_{\tau}}$ or $M_{\nu_{\tau}}^{2}$, nor to the details of the fit parameterization used. The third approach finds that value of $M_{\nu_{\tau}}$ that, if it were the true value, would result in a probability of $95 \%$ to measure a smaller difference between $\ln \mathcal{L}\left(M_{\nu_{\tau}}\right)$ and the observed maximum of $\ln \mathcal{L}$, if the experiment were to be repeated many times. Although having a different meaning, this value lies close to the results of the other two approaches. This raw result must be corrected for the following systematic effects:

- Beam energy: The absolute beam energy, used to calculate the $\tau$ momentum in the determination of $\eta$, is varied by its uncertainty of 6.0 MeV , as determined by the working group on LEP energy [12].
$-\tau$ mass: The value of the $\tau$ mass $M_{\tau}=1777.02 \mathrm{MeV}$, used to calculate the $\tau$ momentum $p_{\tau}$ in the determination of $\eta$, is varied by its present total error of $0.30 \mathrm{MeV}[13]$.
$-M_{a_{1}}$ and $\Gamma_{a_{1}}$ : For the reference histograms of the physics functions and the Monte Carlo events with full-detector simulation, $\mathrm{M}_{\mathrm{a}_{1}}$ and $\Gamma_{\mathrm{a}_{1}}$ are set to 1.251 GeV and 0.599 GeV respectively. These values are obtained from fits to ARGUS data for the $a_{1}$ spectral function assuming the same theoretical model for the partial decay rate of $\mathrm{a}_{1}[15]$ as is implemented in KORALZ4.0. They are also consistent with the recent OPAL measurement [14]. The errors in the determination of $\mathrm{M}_{\mathrm{a}_{1}}$ and $\Gamma_{\mathrm{a}_{1}}, 13 \mathrm{MeV}$ and 44 MeV , which are also taken from [15], are used as systematic uncertainties for $M_{a_{1}}$ and $\Gamma_{a_{1}}$ in the Monte Carlo. To evaluate the uncertainty due to $M_{a_{1}}$, we generated another set of reference histograms using an input $\mathrm{M}_{\mathrm{a}_{1}}$ shifted by 13 MeV and observed the limit variation from this shift. A similar technique was used in order to estimate the systematic uncertainty arising from the error on $\Gamma_{\mathbf{a}_{1}}$.
- Decay fractions in the final sample: The absolute uncertainties for the final fractions of $3 h n \pi^{0}$, non 3 -prong, and $q \bar{q}$ are estimated to be $1.5 \%, 0.3 \%$, and $0.4 \%$, respectively, as follows. For the $3 h n \pi^{0}$ component, a control sample enriched in such decays was used to assess the consistency between data and Monte Carlo for the $3 h \mathrm{n} \pi^{0}$ rejection cuts. The uncertainty in the $3 h \mathrm{n} \pi^{0}$ branching ratio [11] was also taken into account. For the non 3-prong decays, the difference between the photon conversion rates in data and Monte Carlo, and the Monte Carlo statistical error, were taken into account. The uncertainty in the $q \bar{q}$ background was estimated using a control sample enriched in $q \bar{q}$ events.
- Normalized efficiency function: The uncertainty in the slope of the $\omega$-dependence of the efficiency function is conservatively estimated to be $\pm 0.20 \omega$. Changing the slope by this amount has negligible influence on the $95 \%$ C.L. limit.
- Resolutions: From cross checks with a $\mathrm{D}^{*} \rightarrow \mathrm{~K} \pi \pi$ sample, we find that the width of the $M_{\mathrm{D}^{*}} / \sigma\left(M_{\mathrm{D}^{*}}\right)$ distribution from $\mathrm{D}^{*}$ decays is consistent with unity within the statistical error of $4 \%$ after the background subtraction. An uncertainty of $4 \%$ is taken.
- Momentum, $\theta$, and $\phi$ offset: The absolute momentum uncertainty is estimated to be less than $0.25 \%$ from the $\mu^{-}$and $\mu^{+}$momentum distribution in $Z^{0} \rightarrow \mu^{+} \mu^{-}$ events, which is essentially due to calibration effects. The offset is expected to be smaller for low momentum tracks. Furthermore, the measurements of the $\mathrm{D}^{0}$ and the $\mathrm{D}^{+}$mass are sensitive to such systematics, particularly multi-track effects. We have measured the mass of $\mathrm{D}^{0}$ and $\mathrm{D}^{+}$from $\mathrm{D}^{0} \rightarrow \mathrm{~K}^{-} \pi^{+}$and $\mathrm{D}^{+} \rightarrow \mathrm{K}^{-} \pi^{+} \pi^{+}$decays and the discrepancies from the world average values $[16]$ are $(0.016 \pm 0.054) \%$ and $(0.145 \pm 0.064) \%$, respectively. The sums in quadrature of the discrepancies and the errors, $0.056 \%$ and $0.158 \%$, can be converted into corresponding measured momentum uncertainties of $0.07 \%$ and $0.17 \%$, which are lower than our estimate of $0.25 \%$ for the momentum offset. Possible offsets of $\theta$ and $\phi$ are also estimated by observing the acollinearity angle in $Z^{0} \rightarrow \mu^{+} \mu^{-}$events. The uncertainties are 0.2 mrad for both angles. The variations in the limits for $M_{\nu_{\tau}}$ are investigated as momenta, $\theta$, and $\phi$ angle of tracks are changed, independently, by amounts corresponding to the above offsets in the process of reconstruction of $M_{\text {miss }}^{2}, E_{\text {miss }}$, and their covariances.

| Source | uncertainty | limit variation $(\mathrm{MeV})$ |
| :--- | ---: | :---: |
| $E_{\text {beam }}$ | 6 MeV | 0.1 |
| $M_{\tau}$ | 0.30 MeV | $<0.1$ |
| $\mathrm{M}_{\mathrm{a}_{1}}$ | 13 MeV | 0.8 |
| $\Gamma_{\mathrm{a}_{1}}$ | 44 MeV | 1.2 |
| $f_{3 h n \pi^{0}}$ | $1.5 \%$ | 0.6 |
| $f_{\text {non } 3}$ | $0.3 \%$ | 0.1 |
| $f_{\mathrm{q} \overline{\mathrm{q}}}$ | $0.4 \%$ | $<0.1$ |
| $\epsilon_{3 h n \pi^{0}}(\eta, \omega)$ | $20 \%$ | $<0.1$ |
| resolution | $4 \%$ | 1.2 |
| momentum offset | $0.25 \%$ | 2.2 |
| $\theta$ offset | 0.2 mrad | 0.1 |
| $\phi$ offset | 0.2 mrad | 0.1 |
| $M_{\text {miss }}^{2}$ cuts |  | 1.2 |
| total |  | 3.2 |

Table 2: List of systematic errors. The variation of the $95 \%$ C.L. $M_{\nu_{\tau}}$ limit is shown for each systematic source.

- Cuts in $M_{\text {miss }}^{2}$ : In the distribution of $M_{\text {miss }}^{2}$, the tail in the region of $-0.6 \mathrm{GeV}^{2}<$ $M_{\text {miss }}^{2}<-0.4 \mathrm{GeV}^{2}$ is not modelled well by Monte Carlo (See Figure 1 a) ). A tighter cut of $-0.4 \mathrm{GeV}^{2}<M_{\text {miss }}^{2}<1.0 \mathrm{GeV}^{2}$ was used and the number of candidates decreased by 186 . The limit increases by 3.8 MeV if the 186 candidates are removed. From $\tau$ Monte Carlo events with the same statistics, the tighter cut removes 111 candidates and the variation in the limit is 3.5 MeV . We take 1.2 MeV as an uncertainty from the tail by subtracting 3.5 MeV and 3.8 MeV in quadrature.
The variations in the $95 \%$ C.L. $M_{\nu_{\tau}}$ limit for each systematic source are listed in Table 2. Assuming that the above systematic errors are independent of each other, the total systematic error in the $M_{\nu_{\tau}}$ measurement is estimated from adding them in quadrature. The result of 3.2 MeV is added linearly to the raw upper limit to form the corrected value:

$$
M_{\nu_{\tau}}<35.3 \mathrm{MeV}, \quad 95 \% \text { C.L. }
$$

## 6 Cross checks

The sensitivity of the fit to a massive $\nu_{\tau}$ is investigated with dedicated samples of Monte Carlo events with full detector simulation in which $M_{\nu_{\tau}}$ is set to 50 and 100 MeV and about $3500 \tau$-decays remain after selection cuts. The results of the likelihood fits are $47_{-23}^{+18} \mathrm{MeV}$ and $104_{-14}^{+11} \mathrm{MeV}$, respectively. With about 7000 decays the results are $55_{-16}^{+12} \mathrm{MeV}$ and $108_{-7}^{+6} \mathrm{MeV}$, respectively.

We have also fitted the data for $M_{\nu_{\tau}}$ in the 1-dimensional $\omega$ distributions using the same likelihood scheme presented in Section 4 but simplified. For this fit using the observed $\omega$ distribution, only $394 \tau \rightarrow 3 h \nu_{\tau}$ decays in the region of $-0.1<\omega_{i}<0.1$ are used, and so the fractions $f_{X}$ are reestimated. The result of the likelihood fit is consistent with $M_{\nu_{\tau}}=0 \mathrm{MeV}$ and the $95 \%$ C.L. limit is obtained at 86 MeV . Furthermore, a higher statistics check is done using $2579 \tau \rightarrow 3 h \nu_{\tau}$ decays in $1-3 \mathrm{Z}^{0} \rightarrow \tau^{+} \tau^{-}$events. Again we observe consistency with $M_{\nu_{\tau}}=0 \mathrm{MeV}$ and the $95 \%$ C.L. limit of 59 MeV . The uncertainty in the measured momentum offset of $0.25 \%$, as described in Section 6, corresponds to a
systematic shift of the likelihood peak by 70 MeV in this 1-dimensional analysis. This kind of systematic variation is substantially reduced in the 2 -dimensional likelihood technique.

The framework of the fitting scheme, scaling factors, and systematics of detector response are checked by measuring the $\tau$ lifetime using the vertex separation of $3-3$ $\mathrm{Z}^{0} \rightarrow \tau^{+} \tau^{-}$events. The measured value is $292.4 \pm 8.3 \mathrm{fs}$, where the error is statistical only, in agreement with the most recent OPAL published value of $289.2 \pm 2.1 \mathrm{fs}[17]$ and the world average $295.6 \pm 3.1$ fs [18].

## 7 Conclusions

We obtain an upper limit of $M_{\nu_{\tau}}<35.3 \mathrm{MeV}$ at the $95 \%$ C.L. from $2514 \tau \rightarrow 3 h \nu_{\tau}$ candidates in the $3-3$ topology of $\mathrm{Z}^{0} \rightarrow \tau^{+} \tau^{-}$through a comparison of the 2-dimensional distribution of missing mass squared and missing energy with theoretical predictions as a function of $M_{\nu_{\tau}}$.

The likelihood curve for this analysis combined with the published OPAL measurement from $\tau \rightarrow 5 \pi \nu_{\tau}$ decays [19], obtained by multiplying raw likelihoods from both measurements at the same $M_{\nu_{\tau}}$ points, is shown in Figure 3 b ). From these new likelihood values as a function of $M_{\nu_{\tau}}$, we find a combined upper limit for $M_{\nu_{\tau}}$ without correcting for the above systematic uncertainties of $M_{\nu_{\tau}}<26.7 \mathrm{MeV}$ at the $95 \%$ C.L. After adding the total systematic variation in this analysis to the combined raw upper limit, the final combined limit on $M_{\nu_{\tau}}$ is found to be:

$$
M_{\nu_{\tau}}<29.9 \mathrm{MeV}, \quad 95 \% \text { C.L. }
$$

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Figure 1: Distributions of a) reconstructed $\eta=M_{\text {miss }}^{2}$ in $\mathrm{GeV}^{2}$ and b) reconstructed $\omega=E_{\text {miss }} / E_{\text {beam }}$ after all selection cuts except the cut $\left|M_{\text {miss }}^{2}\right|<1 \mathrm{GeV}^{2}$ are imposed for $\tau \rightarrow 3 h \nu_{\tau}$ candidates in the data and Monte Carlo $\tau$ and $\mathrm{q} \overline{\mathrm{q}}$ events. Data are represented as points with error bars. Monte Carlo expectations at $M_{\nu_{\tau}}=10 \mathrm{MeV}$ are shown as the solid histogram. The $3 h \mathrm{n} \pi^{0}$ and non 3 -prong decays are shown as the shaded histogram. The $q \bar{q}$ background expectation is shown as the hatched histogram. The cut $\left|M_{\text {miss }}^{2}\right|<1 \mathrm{GeV}^{2}$ is shown as dashed lines on the $\eta=M_{\text {miss }}^{2}$ plots. Monte Carlo predictions are normalized to the number of preselected $3-3 \mathrm{Z}^{0} \rightarrow \tau^{+} \boldsymbol{\tau}^{-}$events.


Figure 2: Contour plots of reference histograms of the physics functions. They show ten equidistant levels of probability densities of the physics functions; a) $\mathcal{P}_{3 h-3 h}$, b) $\mathcal{P}_{3 h-3 h n \pi^{0}}$ c) $\mathcal{P}_{3 h n \pi^{0}-3 \mathrm{~h}}$ d) $\mathcal{P}_{3 h n \pi^{0}-3 h n \pi^{0}}$ for $M_{\nu_{\tau}}=0 \mathrm{MeV}$, where the first channel indicates the decay in the measurement cone and the second represents the decay in the opposite cone.

OPAL



Figure 3: Total likelihood values versus $M_{\nu_{\tau}}$ for a) this 2-dimensional analysis using $\tau \rightarrow 3 h \nu_{\tau}$ and b) combined with the OPAL $\tau \rightarrow 5 \pi \nu_{\tau}$ analysis. Note that the $95 \%$ C.L. limits on $M_{\nu_{\tau}}$ shown have not been corrected for the effect of systematic uncertainties.

