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UPSTREAM BLOCKING AND AIRFLOW OVER MOUNTAINS

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INTRODUCTION

The reality of upstream blocking in stratified flows has been recognized for many years. If a stratified flow with Brunt-Väisälä frequency N ($N^2 = -g/\rho\ d\rho/dz$, where ρ is the fluid density, g the acceleration due to gravity, and z the vertical coordinate) is set in motion with mean velocity U over a (two-dimensional) obstacle of height h, then naive energy arguments (and common sense) indicate that if Nh/U is sufficiently large, fluid near the ground would be blocked on the upstream side and not flow over the obstacle. Casual observations and "folklore" have long indicated that this phenomenon is common near mountain ranges in the atmosphere. However, the nature and mechanics of how it occurs have only recently become clear. It is now known that upstream blocking in large-Reynolds-number flows propagates as a wave phenomenon, generated by nonlinear effects over the topography. These waves may be linear or nonlinear depending on circumstances, and they propagate primarily as "columnar" motions, meaning that they permanently alter the density and horizontal velocity profiles as they pass through the fluid ahead of the obstacle. Blocking occurs when these changes reach sufficient amplitude. Since they alter the upstream conditions, the understanding of these upstream disturbances caused by the obstacle is a prerequisite for calculating the steady-state flow over an obstacle, regardless of the other details of the flow. These effects generally depend on the topography being approximately two-dimensional (2D) with sufficiently large height. They are common in geophysical situations such as fjords, estuaries, and in the atmosphere.

Since blocking is primarily a two-dimensional stratified phenomenon, in this review we exclude the effects of rotation and are concerned with

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topography that is at least nearly two dimensional. The literature on stratified flow over topography is quite large, but most of the earlier studies were focused on downstream phenomena such as lee waves and windstorms, rather than upstream effects. This article is primarily concerned with the latter effects, and downstream-flow properties are only discussed insofar as they relate to upstream phenomena. From this viewpoint, the study of the subject began with the pioneering work of Long (1954, 1955). Since then, the state of the subject has been reviewed by Long (1972) and, for laboratory experiments, by Baines & Davies (1980).

The character of the flow will depend on the mean density stratification of the fluid, and here there are two main considerations. Firstly, the stratification may take the form of a number of homogeneous layers, or the density may vary continuously with height. In the latter case a layered model can be used as an approximation, although many layers may be needed. Secondly, the fluid depth may be finite or infinite. In the finite-depth case the stratified fluid is bounded above by a rigid horizontal boundary or an infinitely deep homogeneous layer, so that all upward-propagating energy is reflected downward; the vertical spectrum of linear internal waves consists of discrete modes. In the infinite-depth case, wave energy may propagate upward out of the region of interest without any downward reflection. This may be achieved by a fluid that is effectively infinitely deep or that has a region which absorbs and dissipates internal wave energy above some sufficiently high level. The vertical spectrum of internal wave energy is continuous, with no downward energy propagation. The behavior of finite- and infinite-depth systems is quite different in general. In particular, finite-depth systems contain an additional parameter—the total depth of the stratified fluid. Furthermore, the linearized solutions (for flow over obstacles with small h) become singular for layered and finite-depth systems when the speed of an internal wave mode is zero relative to the topography, whereas this does not occur for infinite-depth systems without trapped modes.

Both finite- and infinite-depth continuously stratified systems may contain a critical layer, which in the present context implies a level in the flow where the (initial) mean velocity of the fluid is zero relative to the topography. Critical layers introduce considerable complications, and in order to focus on the essentials of upstream blocking we assume that they are initially absent in the flows discussed here. However, topographic disturbances may themselves produce *local* critical levels.

A simple criterion for upstream blocking can be obtained from the following energy argument, due to Sheppard (1956). One may liken stratified fluid approaching an obstacle to balls being rolled uphill. Relative to neutral stratification, an approaching fluid particle must overcome a potential-energy deficit due to the stratification if it is to surmount the barrier. For continuously stratified fluid, a fluid particle with velocity U will not have sufficient kinetic energy to reach a height h if

$$\tfrac{1}{2}\rho U^2 < g \int_0^h (h-z)\left(-\frac{d\rho}{dz}\right) dz.$$

In the particular case where U and the Brunt-Väisälä frequency N are constant with height, this relation gives the criterion for blocking as

$$\frac{Nh}{U} > 1.$$

Laboratory experiments with 3D axisymmetric and near-axisymmetric obstacles (Hunt & Snyder 1980, Snyder et al. 1985) show that this criterion agrees closely with observations taken on the centerline. However, this agreement must be regarded as almost a coincidence, since the theoretical derivation ignores the effects of neighboring fluid particles through the pressure term. For two-dimensional topography this energy argument is not consistent with observations, and the value of Nh/U required for blocking is closer to 2, as shown below.

The most common dimensionless number in this topic is the Froude (pronounced "Frood") number F. Unfortunately, this name is used for different quantities in different circumstances by different people. In the flow of homogeneous fluid with a free surface, F is defined to be $U/(gl)^{1/2}$, where l may be an obstacle length L, the fluid depth D, or (conceivably but rarely) an obstacle height h. So defined, F may represent any one of three parameters, and these have very different physical significance. The first $[U/(gL)^{1/2}]$ was used extensively by William Froude and relates to wave drag. The second has been commonly termed the Froude number since the work of Moritz Weber (Rouse & Ince 1957) and is the ratio of a fluid speed to a linear wave speed. The use of "Froude number" for both terms must be regarded as accepted terminology. For the case of continuous stratification with constant N, we have the corresponding parameters U/NL, U/ND and U/Nh. All three (plus their squares and reciprocals and suitable constant multiples) have been termed the "Froude number" by various authors. This proliferation of the term has caused unnecessary confusion because, again, these three parameters have very different physical significance: The first relates to internal wave drag, the second is the ratio of a fluid speed to a wave speed, and the third relates to nonlinear wave steepening and upstream blocking. By analogy with free-surface flows it may (regrettably) be regarded as accepted practice to term U/NL and U/ND Froude numbers, but there seems to be little sense[1] or justification for using the same appellation for U/Nh (although the present author is as guilty of this in the past as anyone else). I suggest that it is more appropriate to write this number as Nh/U, and we leave it nameless with no symbol in this article. A suitable name might be "Nhu."

In the following sections we consider the nature of the blocking phenomenon in systems of increasing complexity. We begin with a single homogeneous layer with a free surface, then proceed to multilayer systems, and finally discuss continuously stratified systems of finite and infinite depth. Most theoretical studies have assumed that the obstacle has a long horizontal length scale, so that the flow is mostly hydrostatic (apart from certain situations mentioned below); this provides a substantial simplification of the equations, and the flows calculated should at least be representative of the character of flow over shorter obstacles, because the essential nonlinearities are retained.

SINGLE LAYER

We consider the flow of a single layer over a long (slowly varying) obstacle, so that the flow is mostly in hydrostatic balance. We also note that the equations governing hydrostatic flow of a single layer are the same as those for hydrostatic flow of a two-layer system with an infinitely deep inert upper layer, if g is replaced by $g' = g(\rho_1 - \rho_2)/\rho_1$.

This single-layer system provides examples of the two main types of nonlinear disturbances produced by topography in finite-depth flows. The first of these is the *hydraulic jump*, which is the end result of a steepening process due to nonlinear advection. For many purposes these jumps may be regarded as traveling discontinuities that do not change their shape or properties with time; their detailed structure will depend on a balance between nonlinear steepening and a combination of linear dispersion, dissipation, and wave breaking. The second type of disturbance is the *rarefaction*—a term borrowed from gasdynamics, but here the word implies that the *disturbance* is being rarefied, rather than the fluid density. This type occurs when the trailing part of the disturbance travels more slowly than the leading part (conversely to the hydraulic-jump case), so that nonlinear advection causes the disturbances to become progressively more stretched out as time passes. Both of these types of disturbance are important for stratified flows over obstacles in general.

The effects of two-dimensional topography on a single layer have been

[1] This point will be discussed in more detail in the monograph "Topographic effects in stratified flows" by the author.

investigated by Long (1954, 1970, 1972—theory and experiments) and independently by Houghton & Kasahara (1968—theory and numerical experiments). Their results have been summarized in a unified form in Baines & Davies (1980). If a fluid layer of depth d_0 is impulsively set into motion with velocity u_0 in the presence of an obstacle of maximum height h, the resulting flow may be characterized by two dimensionless parameters—a Froude number $F_0 = u_0/(gd_0)^{1/2}$ and $H = h/d_0$. From the equations of momentum and mass conservation, one may infer that the final steady state depends on F_0 and H, as shown in Figure 1. The equations for the various curves are

$$\mathrm{A'B'+A'E'}: H = 1 - \frac{3}{2}F_0^{2/3} + \frac{1}{2}F_0^2,$$

$$\mathrm{A'F'}: H = \frac{8(F_0^2+1)^{3/2}+1}{16F_0^2} - \frac{1}{4} - \frac{3}{2}F_0,$$

$$\mathrm{B'C'}: F_0 = (H-1)\left(\frac{1+H}{2H}\right)^{1/2}.$$

To the left of curve F′A′B′, where the flow is either supercritical ($F_0 > 1$) or subcritical ($F_0 < 1$), the flow upstream and downstream is the same as the initial undisturbed flow (apart from transients), and the flow over the obstacle is given by the Bernoulli equation. To the right of B′C′ the obstacle height is sufficiently large to completely block the flow. When the flow is partially or totally blocked, a hydraulic jump propagates upstream to infinity, reducing the incident mass flux and altering the upstream fluid

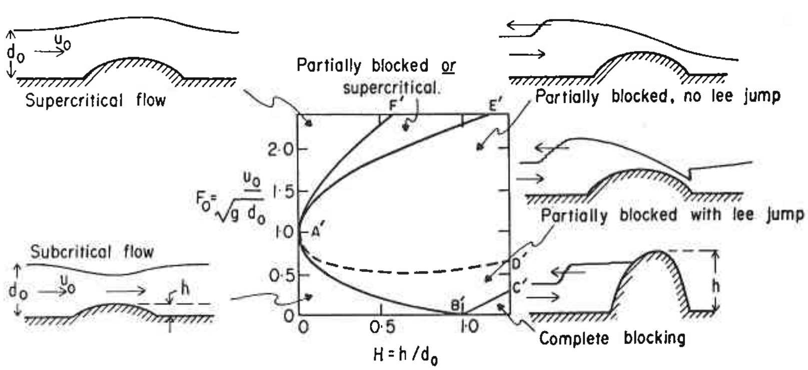

Figure 1 Hydrostatic single-layer flow over an obstacle: The flow regimes are obtained after an impulsive start from rest for various values of F_0 and H, where h is the maximum obstacle height and u_0 is the speed of the obstacle relative to the initial undisturbed stream, which has depth d_0.

velocity u and layer thickness d. An equation relating jump speed to the change in conditions across the jump may be derived and used to obtain the properties of the overall flow. In the partially blocked case, flow over the obstacle crest is controlled by the local condition $F = u/(gd)^{1/2} = 1$. On the downstream side, a hydraulic jump may be attached to the obstacle (below A′D′) or swept downstream (above A′D′): farther downstream, a rarefaction (simple wave) disturbance connects the flow to the original undisturbed state. In the region E′A′F′ the flow may be either partially blocked or supercritical, depending on the initial conditions, so that a hysteresis phenomenon exists in this system. The existence of these double equilibria has been verified numerically by Pratt (1983) and experimentally by Baines (1984).

When two long obstacles are present in two-dimensional flow, the hydrostatic long-wave model may not be applicable. If the steady-state flow for a single obstacle is everywhere subcritical or supercritical, the steady-state flow pattern for each of two long obstacles of the same height will be the same as that for a single obstacle. However, if upstream blocking occurs, the long-wave theory may yield no sensible answer; in these cases nonlinear wave trains are observed in the region between the obstacles (Pratt 1984). Various flow regimes obtained experimentally for a range of heights of two obstacles are shown in Figure 2. Apart from possible wave breaking, the observed flows were all completely steady. This phenomenon may be interpreted, at least in part, with the theory of Benjamin & Lighthill (1954). We define the mass flux Q, energy R, and momentum flux S of a uniform stream of velocity u_1 and depth d_1, taking density as unity, by $Q = u_1 d_1$, $R = \frac{1}{2}u_1^2 + gd_1$, $S = u_1^2 d_1 + \frac{1}{2}gd_1^2$. Then if R_c and S_c denote the values of R and S for a *critical* stream ($F = u_1/\sqrt{gd_1} = 1$) of given volume flux Q, the possible values of R and S for steady flows on this stream are given in Figure 3. The upper boundary of the cusp represents subcritical uniform stream flows ($F < 1$), the lower boundary represents supercritical uniform flows ($F > 1$) and solitary waves, and the region in between represents

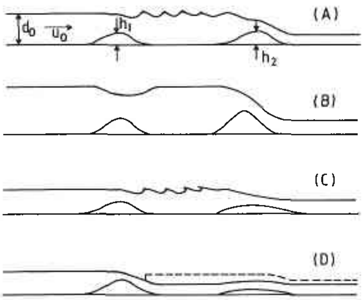

Figure 2 Sketches of experimentally found flow regimes for a single layer over two obstacles (from Pratt 1984). (*a*) $h_1 \approx h_2$; laminar lee waves between obstacles. (*b*) $h_1 < h_2$; long-wave subcritical flow between obstacles. (*c*) $h_1 > h_2$; breaking lee waves. (*d*) $h_1 \gg h_2$; long-wave supercritical flow between obstacles (solid line) or containing hydraulic jump (dashed line).

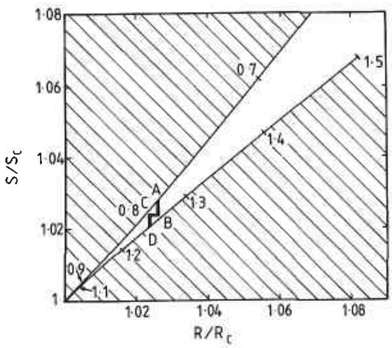

Figure 3 The energy density (R)–momentum flux (S) diagram for possible steady states of a single layer with given mass flux Q. For critical flow, we have $R = R_c$, $S = S_c$. Numbers on the cusp denote values of F (from Benjamin & Lighthill 1954).

flow with cnoidal wave trains. Flow over an obstacle causes a decrease in S equal to the (inviscid) drag force; hence, in passing over an obstacle, the point on the diagram representing the stream flow moves downward from the upper branch of the cusp. For a single obstacle it may reach the lower branch, but, with a second obstacle, in the cases of interest it will only traverse part of this gap (AB in Figure 3), giving a cnoidal wave train downstream of the first obstacle. If these waves are large enough to break, then a decrease in R will result (BC), and then a further decrease in S (CD) at the second obstacle. The details of these changes in S may be dependent on the spacing between the obstacles and their shape; the phenomenon needs further study. If several obstacles are present in the flow, we may expect a succession of such zigzags in the R-S plane, so that R and S decrease toward their minimum values R_c and S_c and the downstream flow tends toward criticality. On the other hand, if any one obstacle blocks the flow, it is blocked everywhere.

For three-dimensional (3D) topography (for example, a 3D barrier in a channel) the flow will be totally blocked if and only if the barrier is higher than the 2D blocking height (given in Figure 1) continuously across the channel. Also, if the channel is narrow relative to the longitudinal length scale, the flow may be controlled by a critical condition that depends on the topographic height profile at the "minimum gap" (rather than a single height); we discuss this point in a broader framework below.

TWO OR MORE LAYERS

The upstream effects of two-layer flow have been investigated numerically by Houghton & Isaacson (1970) and experimentally by Long (1954, 1974) and the author (Baines 1984). The last paper gives a comprehensive description of the various flow types that occur with two immiscible fluids when the flow is commenced from a state of rest, so that the velocities of the two layers are initially equal. The experiments have been carried out with moderately long obstacles (with length comparable to the depth), and the observations have been satisfactorily compared with results from a hydraulic two-layer model (using mass and momentum equations for each layer). The observed upstream disturbances may take one of three forms, as follows. (*a*) A hydraulic jump (Figure 4*a*), similar in character to those observed in single-layer flows. The jump is undular at small amplitudes; at large amplitudes the interface becomes turbulent at and on the lee side of the crests due to Kelvin-Helmholtz instability. (*b*) A limiting bore plus a rarefaction (Figure 4*b*). As the amplitude of a bore and the downstream lower-layer depth are increased, the effect of the upper-layer thickness becomes more important; the speed of the bore tends to a maximum value at a particular amplitude, and the energy loss across the bore decreases to zero at (or very near) this same point. This bore of maximum amplitude and zero dissipation is termed a "limiting bore," and it consists of a monotonic increase in the lower-layer depth, which still propagates without changing shape. If the downstream lower-layer depth is forced to increase further, this must result in a rarefaction that propagates more slowly than the bore. (*c*) A pure rarefaction (Figure 4*c*). If the lower-layer depth is initially greater than or equal to a value that is approximately half the total depth (and depends on ρ_2/ρ_1), an increase in the lower-layer depth is propagated as a rarefaction only.

When hydraulic jumps are present, the hydraulic model requires a relationship between the jump speed and the conditions upstream and downstream of it. In order to obtain this relationship for multilayered

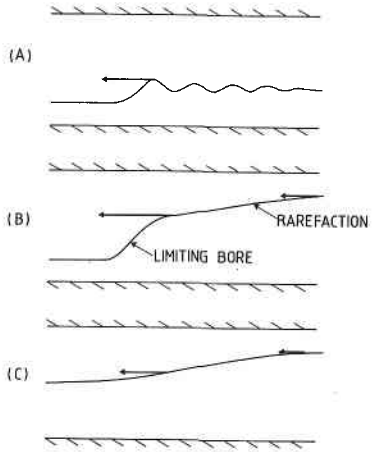

Figure 4 Examples of the types of non-linear disturbances in two-layer flow. (*a*) A hydraulic jump, which propagates at constant speed without changing shape. (*b*) A limiting bore and rarefaction; the limiting bore is a bore of maximum amplitude that propagates at constant speed without changing shape, and for the following rarefaction, the leading part propagates faster than the trailing part. (*c*) A pure rarefaction. The arrows represent relative propagation speeds of the interface height.

flows, an assumption about the flow within the jump is required. One assumption that meets the requirements is that the flow within the jump is hydrostatic, and this is equivalent to the assumptions used by Yih & Guha (1955), Houghton & Isaacson (1970), Long (1970, 1974), Su (1976), and Baines (1984). However, it is obviously not strictly correct, and Chu & Baddour (1977) and Wood & Simpson (1984) have suggested that for two-layer systems, it may be replaced by an assumption of conservation of energy in the contracting layer in the jump. In cases where the two criteria have been compared with observations (Wood & Simpson 1984, Baines 1984), the difference between them is small and the comparisons are inconclusive, and hence the question of the most appropriate assumption is still open.

We now consider the results for flow between rigid upper and lower boundaries with $(\rho_1-\rho_2)/\rho_1 \ll 1$, starting from a state of rest. The resulting flow may be specified by three parameters F_0, H, and r, where

$$F_0 = \frac{u_0}{c_0}, \qquad c_0^2 = \frac{g(\rho_1-\rho_2)}{\dfrac{\rho_1}{d_{10}}+\dfrac{\rho_2}{d_{20}}}, \qquad H = \frac{h}{D}, \qquad r = \frac{d_{10}}{D},$$

where u_0 is the initial fluid velocity relative to the topography, ρ_1, d_{10} and ρ_2, d_{20} denote the density and initial thickness of the lower and upper layers, respectively, h is the maximum height of the obstacle, and the total depth $D = d_{10}+d_{20}$. Figure 5 shows the model results in terms of F_0, H for $r = 0.1, 0.5$. For $r = 0.1$ the diagram is very similar to Figure 1 for a single layer when $F_0 \lesssim 1.4$. However, when $F_0 > 1.4$ the upstream disturbance may be sufficiently large for the flow to become critical immediately upstream of the obstacle (the dashed line in Figure 5*a*); this marks an upper limit to the magnitude of the upstream disturbance, which does not increase further if H is increased. On part of this curve the upstream bore has reached its maximum amplitude, and a small-amplitude rarefaction follows it. Flow states with upstream bores in the two-state (hysteresis) region may not be realizable experimentally because of interfacial friction (Baines 1984). For $r = 0.5$, on the other hand, no upstream jumps occur, and the only upstream disturbances are of the rarefaction type. As r increases from 0.1 to 0.5, the F_0-H diagram evolves continuously from Figure 5*a* to Figure 5*b*.

Mathematical analyses of the nonlinear region near resonance ($F_0 \sim 1$) have recently been carried out by Grimshaw & Smyth (1986) and by W. K. Melville & K. R. Helfrich (private communication). These studies enable the fluid response for fairly long obstacles with small H to be calculated as the solution of a forced Korteweg–de Vries (KdV) equation;

an extended KdV (EKdV) equation incorporating cubic nonlinearities is required to model two-layer effects, such as limiting bores. An example of the results obtained from the KdV equation is shown in Figure 6. Results are in qualitative agreement with laboratory observations for small r, and Melville & Helfrich obtained reasonable detailed quantitative agreement with the EKdV equation for larger r in some cases.

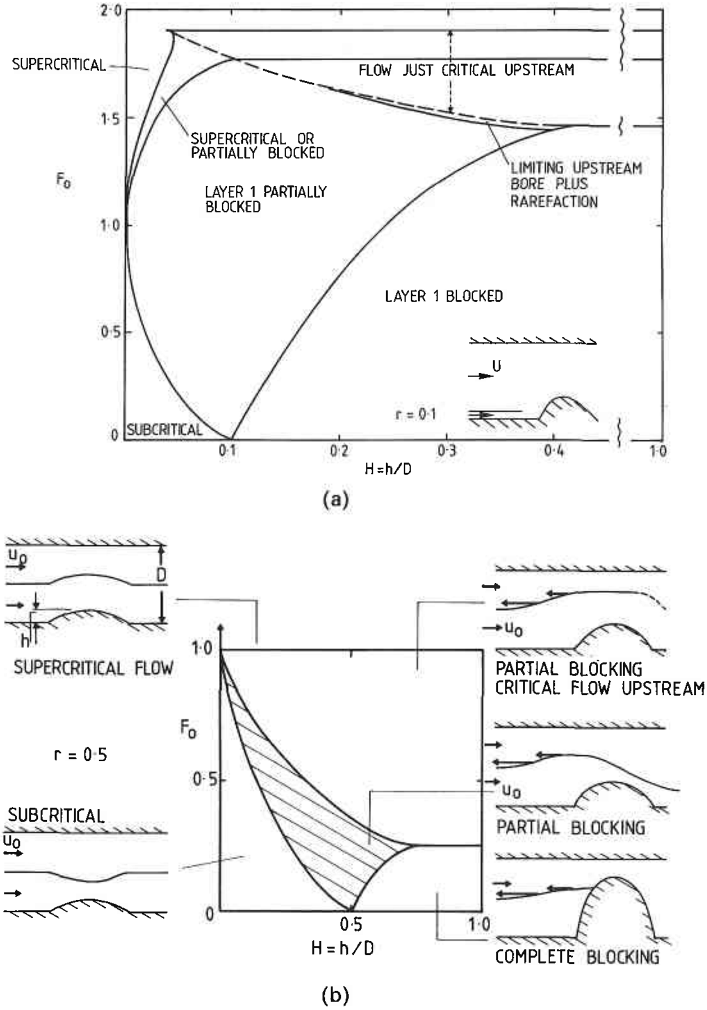

Figure 5 Flow-regime diagrams in terms of F_0, H for two-layer flows: (*a*) $r = 0.1$; upstream disturbances are mostly hydraulic jumps (cf Figure 1). (*b*) $r = 0.5$; upstream disturbances are all rarefactions.

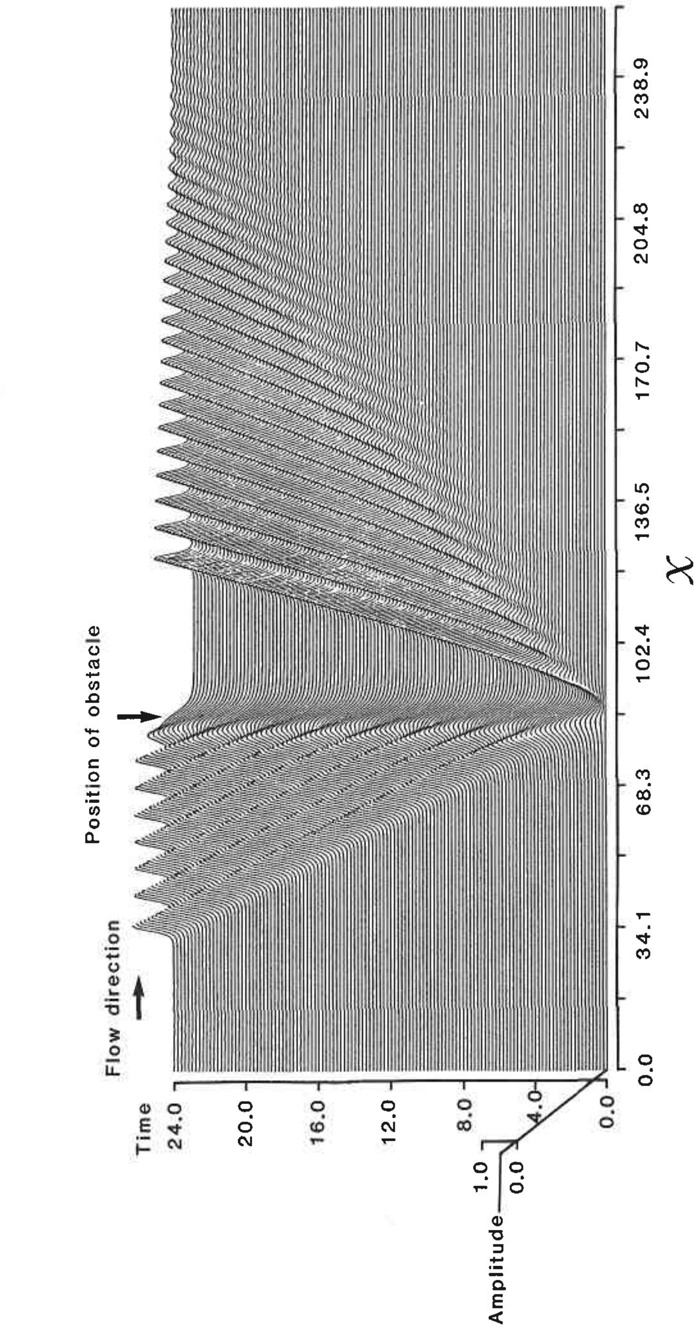

Figure 6 An example of a solution of the forced Korteweg–de Vries equation at resonance, showing an upstream (undamped) undular bore that parallels the two-layer experiments with a thin lower layer ($r \ll 1$). For larger r, cubic nonlinearities must be included to obtain agreement with observations (from Grimshaw & Smyth 1986).

The hydraulic model may be extended to systems with more than two layers (and hence with more than one internal mode) by the following procedure (P. G. Baines, submitted for publication, 1986). It may be shown (Benton 1954, Lee & Su 1977) that at the crest of an obstacle, either the horizontal gradients of all interfaces must vanish or else *one* mode must be critical there (i.e. its propagation speed relative to the topography must be zero). If one particular mode is critical at the crest and the obstacle height is increased by a small amount, the flow may adjust to retain this critical condition by sending a small-amplitude columnar disturbance upstream that has the structure of the critical mode. This disturbance will have the character of a jump if $dc/da > 0$ and a rarefaction if $dc/da < 0$, where c is the linear wave speed propagating against the upstream flow and a is the amplitude of the *preceding* columnar disturbances. By these means, it is possible to construct the F_0-H diagram for any number of layers, although the procedure becomes more difficult as H increases and the upstream disturbances become more complex. In particular, criteria for blocking of the lowest layer may be obtained. Figure 7 shows the F_0-H diagram for three layers between rigid boundaries, originally of equal thickness and with equal density increments. Up to the point where blocking of the lowest layer begins, the upstream disturbances are pure rarefactions. Treatment of the flow with a blocked layer present is

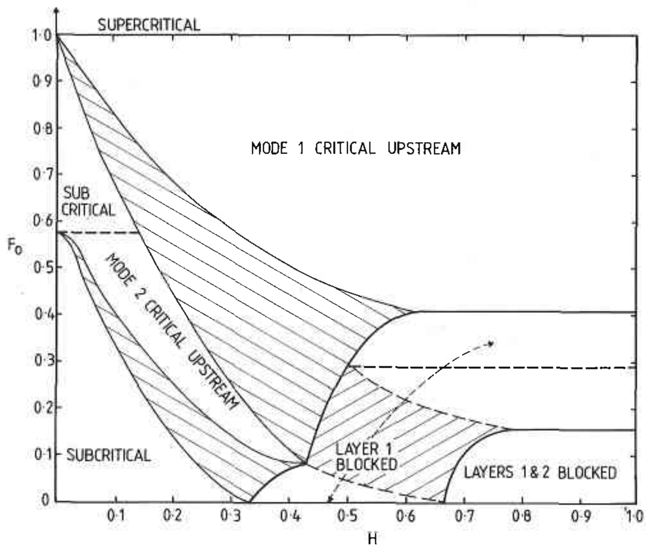

Figure 7 F_0-H regime diagram for three-layer flow, with layer velocities and thicknesses initially equal. In the shaded regions the upstream disturbance of the appropriate mode increases in amplitude as H increases.

more complicated, and the details are given in P. G. Baines & F. Guest (submitted for publication, 1986).

For hydraulic flows (long obstacles) governed by a critical condition at $dh/dx = 0$, in many cases (such as the one- and two-layer systems described above) the upstream flow may be determined independently of the downstream flow, although this will not be true in general. Downstream flows are more complicated and are less well understood. Smith (1976) observed lee waves behind short obstacles in subcritical two-layer flows and found that the wave amplitudes were substantially larger than those predicted by linear theory. For long obstacles in two-layer flow with $r \ll 0.5$, when upstream bores are present the lee-side flow may contain a stationary jump or the jump may be swept downstream. When the flow is critical just upstream, it must be supercritical over the obstacle and then adjust to downstream conditions by a sudden descent of the interface on the lee side to another supercritical state through what is sometimes called a "hydraulic drop" (Baines 1984). These flows may be affected by lee-side flow separation, which is conspicuously present in some cases when the upstream flow is supercritical. Lawrence (1985) has made a detailed study of downstream flow features using miscible fluids and a large flume that permits the flow to exist in a steady state for long periods. In particular, the nature of mixing processes and their dependence on various features of flows with a downstream hydraulic jump have been explored; mixing was observed to be due primarily to Kelvin-Helmholtz billows in the region of maximum shear upstream of the hydraulic jump, rather than to processes within the jump itself. Armi (1986) has made an experimental study of two-layer flow through horizontal contractions.

There do not appear to have been any relevant studies of layered flows with three-dimensional barriers, or with two or more barriers.

CONTINUOUSLY STRATIFIED FLOW—FINITE DEPTH

One might expect that the upstream phenomena present in layered flows would have their analogues in continuously stratified fluid. This is in fact the case, although the subject has developed quite independently. Virtually all reported studies of upstream effects in continuously stratified fluids have used approximately uniform stratification, i.e. fluid with constant Brunt-Väisälä frequency N. In a fluid of depth D, mean velocity U, and obstacle height h, we have the dimensionless parameters

$$F_0 = \frac{\pi U}{ND}, \qquad H = \frac{h}{D}, \qquad \frac{Nh}{U} = \frac{\pi H}{F_0}.$$

Here F_0 (sometimes written as $1/K$) is a Froude number based on the lowest internal wave mode. Linear theory (with small h) does not predict steady upstream disturbances unless $F_0 < 1$ and the topography is semi-infinite [or effectively semi-infinite (Wong & Kao 1970)]. In this case, upstream columnar motions of $O(h)$ are obtained as linear "transients"; these are not in fact transient, because the obstacle has no downstream end, and so they constitute a steady upstream disturbance. For obstacles of finite length, weakly nonlinear theories by Benjamin (1970) (single layer), Keady (1971) (two layer) and McIntyre (1972) (constant N) predict an $O(h^2)$ columnar motion upstream of and related to the downstream lee-wave train; for the constant-N case, these effects are numerically very small and have been looked for experimentally without success (Baines 1977). Solutions to the linear equations, in fact, become singular when the phase (and group) velocity of long waves for some internal mode is zero relative to the topography. For constant N this implies $F_0 = 1/n$, where n is an integer. This resonance causes nonlinear terms to become significant over the obstacle, even for small H, and it is this process that causes the steady-state upstream disturbances. For stratification with constant N the nonlinear steepening effects are extremely small, so that upstream disturbances propagate as linear waves (though their generation over the obstacle is nonlinear), even for moderate amplitudes (provided that the background state is not significantly altered). Consequently, only modes that are subcritical ($c_n = ND/n\pi > U$) can propagate upstream.

The first observations of upstream effects in continuously stratified fluids were made by Long (1955), who observed upstream jets and blocking close to the obstacle when $F_0 < 1$ and Nh/U was sufficiently large. Wei et al. (1975) noticed that these upstream disturbances propagated far upstream as unattenuated columnar linear modes; the obstacles used in their experiments were steep sided, and Wei et al. regarded these upstream effects as consequences of lee-side separation and a turbulent wake. The present author (Baines 1977, 1979a,b) observed these columnar modes and upstream blocking for smooth streamlined obstacles and described their properties for various values of F_0 and H. For small Nh/U, linear lee-wave theory describes the steady-state flow quite well, except near the points of resonance ($1/F_0 \lesssim n$) (Baines 1979a). For $1/n+1 < F_0 < 1/n$, as H increases, a critical value is reached beyond which steady upstream columnar motions of mode n are observed, and this height is zero for $F_0 = 1/n$.

The analysis of Grimshaw & Smyth (1986) generalizes the forced Korteweg–de Vries equation for two-layer flow near resonance to arbitrary finite-depth flows near resonance ($F_0 \sim 1/n$); the coefficients are dependent on the mean velocity profile and stratification. Comparisons between this model and experiments with continuous stratification have yet to be made.

As Nh/U increases, the upstream disturbances in the laboratory experiments are observed to increase in amplitude until upstream blocking occurs. If $F_0 < 0.5$ this occurs for $Nh/U \gtrsim 2$, for obstacles of witch of Agnesi shape. The onset of upstream blocking is manifested as a layer of fluid of finite thickness (typically $\sim \frac{1}{2}h$) coming to rest, rather than as a stationary thin layer near the ground that then thickens vertically.

All the experiments just described were carried out by towing obstacles along tanks of finite length filled with stratified fluid. The columnar modes produced at the obstacle will reflect from the upstream end of the tank (McEwan & Baines 1974), but the observations were made before these returned to influence the observed field of flow significantly. Snyder et al. (1985) reported a series of observations of the density field upstream of two-dimensional obstacles (as well as other shapes), and they attributed upstream blocking to a "squashing" phenomenon. For their experiments, reflection from the upstream end (and in some cases, also the downstream end) was significant, so that the term "squashing" is applicable to their results. However, contrary to their suggestion, it is *not* applicable to the above-cited experiments that simulate a tank of infinite length (albeit for a finite time). Snyder et al. also pointed out that the most slowly moving upstream modes $(n, n-1, \ldots)$ have significant amplitudes, so that the flow may take a long time to reach steady state at a fixed distance upstream. This is quite consistent with the flow-field observations of Baines (1979a,b), for example, who reported steady (or nearly steady) flow in the immediate vicinity of the obstacle near the end of the observing period.

A hydraulic model of the type described in the previous section, but one with 64 layers, has been developed to model flow over long obstacles with continuous stratification when H is not small (P. G. Baines & F. Guest, submitted for publication, 1986). Results are given in Figure 8, up to the point of blocking of the lowest layer, for $F_0 > 0.3$. In this parameter range, only modes $n = 1, 2$, and 3 may become subcritical (and hence propagate) upstream. A 64-layer model is a good approximation to continuous stratification when the upstream disturbances are small, but this is not necessarily the case when they are large, particularly near $F_0 = 1/n$; as slow-moving layers become thicker, their discreteness becomes significant. Some similarity between Figure 8 and the two- and three-layer calculations (Figures 5*b* and 6, respectively) is evident. The speeds of the upstream disturbances vary little with amplitude and are treated as rarefactions. These results have not yet been tested experimentally. This is partly because laboratory experiments with hydrostatic stratified flow are difficult because they require obstacles whose lengths are much greater than the fluid depth. Experiments described in Baines (1979a) for moderately short obstacles (length/depth $\sim$ 1.5) give the curve for the onset of upstream disturbances shown (lightly) dashed in Figure 8; this implies that shorter, steeper obstacles may generate upstream disturbances for smaller h than longer obstacles. No results from fully numerical models for these finite-depth systems have yet been published.

We next consider the flow in a channel of width W past a two-dimensional transverse barrier with a small gap at one end of width w. This models a two-dimensional ridge with gaps of width $2w$ spaced periodically along the ridge at intervals of $2W$. If $w/W \ll 1$ we may expect the gap(s) to have negligible effect on the upstream motion. Experiments with this geometry have been reported for a particular obstacle (a short witch of Agnesi) by Baines (1979b) and Weil et al. (1981) for a range of gap sizes. If blocked

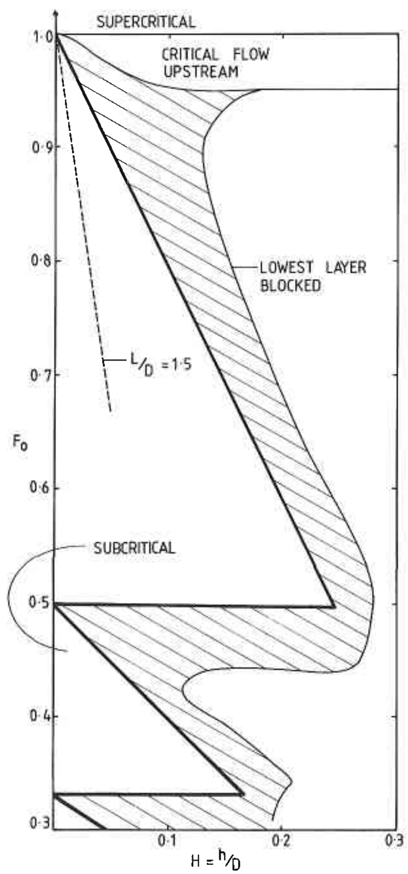

Figure 8 F_0-H diagram for the hydrostatic 64-layer model, approximating uniform stratification, up to the point of blocking for $F_0 > 0.3$. The "critical flow upstream" region ($0.95 < F_0 < 1$) is an artifact of discrete layering. In the shaded region, upstream disturbances increase in amplitude with increasing H. The dashed line denotes the observed onset of steady upstream disturbances in uniform stratification for flow with obstacle-length/depth $\cong$ 1.5 [i.e. nonhydrostatic flow (Baines 1979a, Figure 6*a*)] (from P. G. Baines & F. Guest, submitted for publication, 1986).

fluid is present upstream in the 2D case ($w = 0$), for $w/W \ll 1$ the "blocked" fluid will slowly converge on the gap and flow through it, but its upstream depth will only be affected slightly. If the gap is made wider, the depth of this nearly blocked fluid decreases as a result of increased leakage through the gap. The depth of the nearly blocked layer, z_s, a height that separates fluid flowing horizontally around the barrier from fluid above flowing over it, is quite sharply defined, as shown in Figure 9. For $w/W = 0.125$ this depth is given approximately in terms of Nh/U by

$$z_s/h = 1 - 2U/Nh.$$

If w/W is large enough there may be no permanent upstream disturbances at all. For long 3D obstacles (hydrostatic flow), in order to have upstream disturbances it is necessary for the flow to become critical at the minimum cross section, and the largest value of w/W for which this occurs will mark the change from flow that is 2D-like to 3D-like.

CONTINUOUSLY STRATIFIED FLOW—INFINITE DEPTH

This last case is the one of greatest relevance to the atmosphere. The upper radiation condition implies that there is no downward-propagating energy at the upper region of the fluid, so that (initially at least) a discrete spectrum of vertical modes does not exist; the spectrum of vertical wave numbers is continuous. Nevertheless, purely horizontally propagating linear internal waves are possible, provided that they have infinitely long horizontal wavelength. Furthermore, propagation of these waves in the upstream direction is possible for vertical wave numbers $n < N/U$, with wave speeds (both phase and group velocities) $c = N/n - U$ in the upstream direction (see, for example, Lighthill 1978, Section 4.12). The question is, Under what circumstances are they produced, given the absence of the resonance mechanism with discrete modes? For this system the important parameter is Nh/U, with the length and shape of the obstacle having only secondary significance.

Numerical studies of upstream effects in this system with N and U initially uniform have been reported by Pierrehumbert (1984) and Pierrehumbert & Wyman (1985), and laboratory studies have been described by Baines & Hoinka (1985). Earlier numerical studies of similar systems have concentrated on downstream effects, although some upstream disturbances are visible in the results of Peltier & Clark (1979). Pierrehumbert & Wyman employed a Boussinesq hydrostatic model with a terrain-following coordinate system and a sponge layer at the top to absorb wave energy. With obstacles of Gaussian shape and an

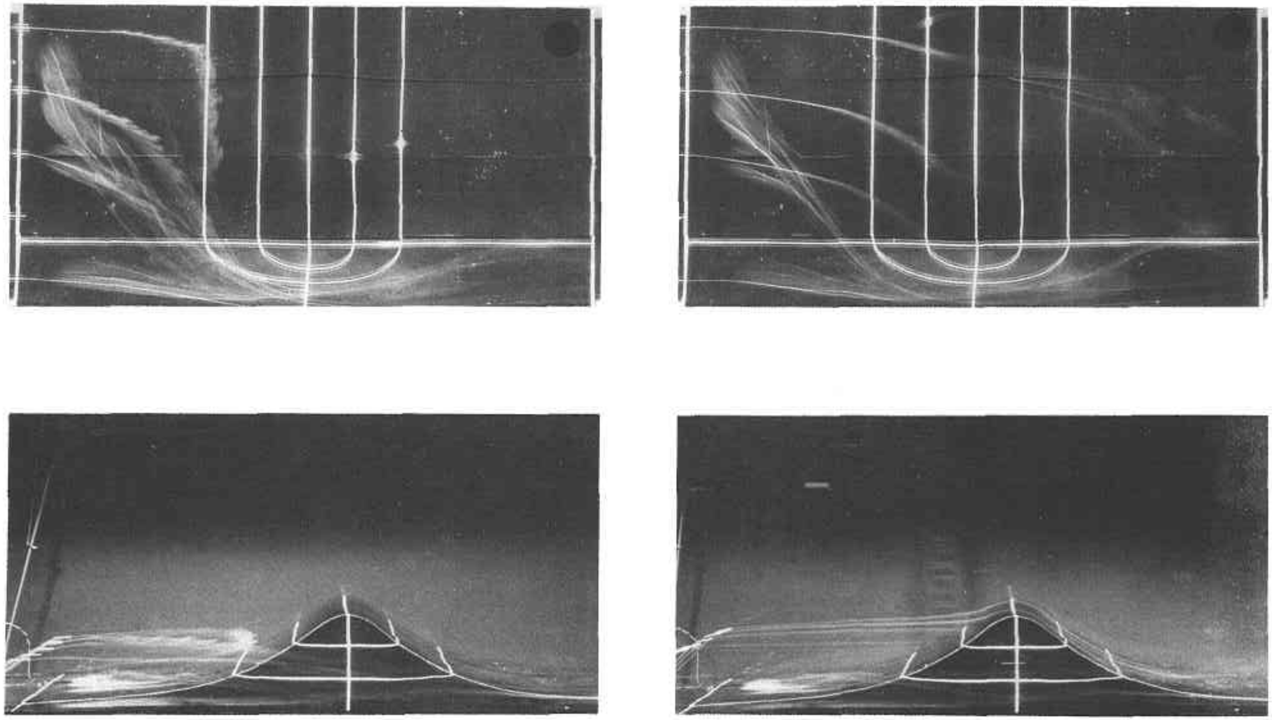

Figure 9 Plan (*upper*) and side (*lower*) views of uniformly stratified flow incident from the left on a barrier with a gap at one end with $Nh/U = 5.9$. The flow field is the same in both cases, but the dye is released from an upstream dye rake at a slightly higher level (9 mm higher, with a total obstacle height of 6.26 cm) in the right-hand frames. Flow in the left-hand frames passes around the obstacle, whereas flow in the right-hand frames passes over it, which demonstrates the abrupt change in flow character with height (from Baines 1979b).

impulsive start to the flow, they found that the steady-state flow was well described by the Long's model solution [a solution that extends steady-state linear theory to finite amplitude when N/U is constant (Long 1955, Lilly & Klemp 1979)] up to the point of overturning ($Nh/U < 0.75$). For $Nh/U > 0.75$, columnar upstream disturbances of *finite* amplitude were generated, and these increased in amplitude with Nh/U. Upstream blocking occurred near the obstacle for $Nh/U > 1.5$ (Gaussian shape) and $Nh/U > 1.75$ (witch of Agnesi shape), but upstream propagation of this blocked fluid was not observed until $Nh/U \gtrsim 2$. Figure 10 shows the time evolution of the flow field computed by Pierrehumbert & Wyman for $Nh/U = 2.0$.

Baines & Hoinka (1985) carried out towing experiments in a stratified tank similar to those described earlier, but with the difference that a radiation condition at the top of the working fluid was simulated with a novel geometrical arrangement. Experiments were carried out up to the point where the flow in the vicinity of the obstacle appeared to be steady and before this flow could be significantly affected by wave motion reflected from the upstream end of the tank. Five different obstacle shapes were used, and a broad range of Nh/U values were covered for each one. The obstacles were not long enough for the flow to be hydrostatic. Near-steady-state flow fields are shown in Figure 11 for the witch of Agnesi. The principal results were as follows. (*a*) For $0 < Nh/U < 0.5$ (± 0.2) no steady upstream effects were observed, and the steady-state flow was generally consistent with linear theory and Long's model solutions. (*b*) For $Nh/U > 0.5$ (± 0.2) steady upstream columnar disturbances were observed, with amplitude increasing from zero as Nh/U increased above 0.5. As the "error bars" indicate, this lower limit was only determined approximately because of the presence of upstream transients and the smallness of the signal. However, it seemed to be independent of obstacle shape and was not dependent on overturning in the lee-wave field, which was not observed until $Nh/U \gtrsim 1.5$. Upstream blocking was observed when Nh/U reached a value in the range 1.3 to 2.2, with the actual value depending on the obstacle shape, but for symmetric obstacles the value was approximately 2. As may be seen in Figure 11, reduced velocities and blocking at low levels upstream are accompanied by increased velocities above the level of the obstacle, and this velocity profile oscillates with decreasing amplitude as the height increases. The density gradient is very small in the slow-moving or blocked fluid, and it is correspondingly large in the overlying jet region; as Nh/U increases, this region becomes more like an interface that can support horizontally propagating waves, as shown on the lee side in the last two frames of Figure 11. For $Nh/U > 1.5$ a stagnant region (or "wave-induced critical level")

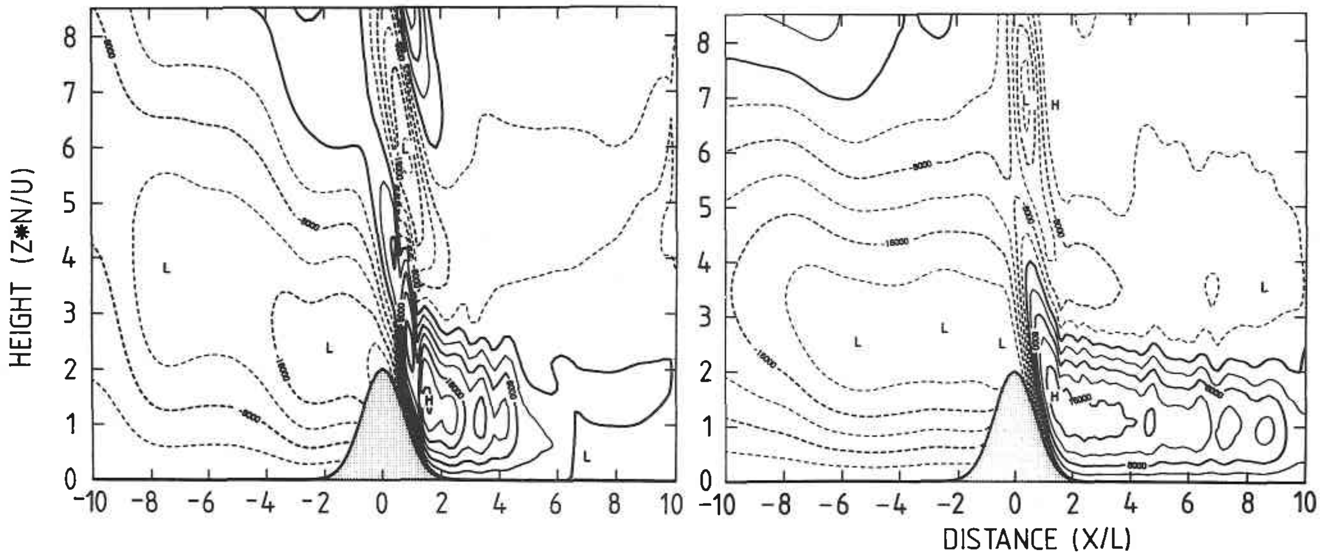

Figure 10 Time development of the computed stream-function perturbation (units of $m^2 s^{-1}$) for hydrostatic flow with $Nh/U = 2.0$ (from Pierrehumbert & Wyman 1985). Left frame: $t = 7.2\ L/U$; right frame: $t = 14.4\ L/U$, where t denotes time from the impulsive commencement of motion.

exists above the jet over the lee side of the obstacle, and as Nh/U increases, the wave field at upper levels becomes less apparent. The flows then appear to be qualitatively similar to the finite-depth flows for the same Nh/U, provided $F_0 \ll 1$. The behavior shown in Figure 9 for 3D topography should also occur in the infinite-depth case.

There is, as yet, no mechanistic model that can explain and describe the upstream motions for this infinite-depth case. Unlike finite-depth systems, upstream effects are not observed unless Nh/U is sufficiently large, and the value at which this occurs is different in the numerical and laboratory

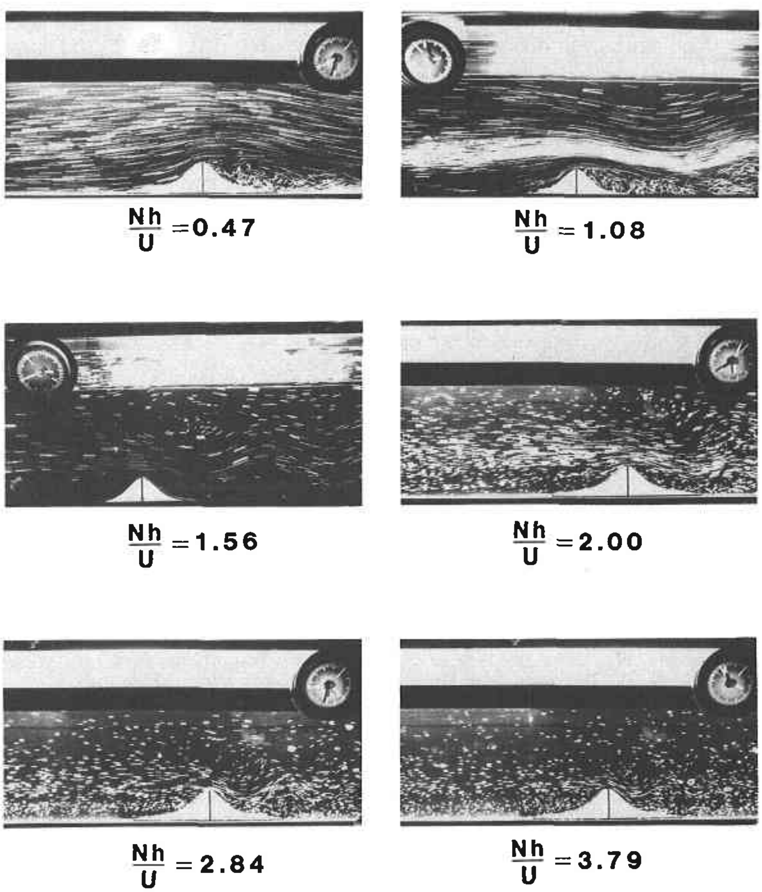

Figure 11 Near-steady-state streamlines for the witch of Agnesi for a range of Nh/U values. Flow is from left to right. Note the upstream blocking in the last three frames (from Baines & Hoinka 1985).

experiments. If, as the laboratory observations suggest, upstream motions may appear without lee-side overturning, then this result implies possible hysteresis in the system because the Long's model solutions are valid steady-state solutions up to the point of overturning. Recent non-hydrostatic computations by J. T. Bacmeister & R. T. Pierrehumbert (private communication) have investigated various start-up conditions, and the results suggest some steady upstream motion for $Nh/U > 0.5$ for a gradual commencement of motion, but the results are complicated by a slow approach to steady state.

Finally, two further aspects deserve mention, although space limitations preclude detailed discussion. Firstly, for application to the atmosphere, where time scales of more than a few hours are important, the Earth's rotation must be considered. This has been discussed for finite Nh/U by Pierrehumbert & Wyman (1985). Upstream effects are restricted to a distance of order Nh/f, where f is the Coriolis parameter. Secondly, the question of stagnant fluid versus sweeping out of periodic valleys in 2D stratified flow across the valleys has been studied by Bell & Thompson (1980) for finite-depth systems and by P. Manins & F. Kimura (private communication) for infinite-depth systems. (Both studies employed numerical and laboratory models.) Bell & Thompson found that blocking in the valleys occurred for $Nh/U \gtrsim 0.8$. Manins & Kimura observed that blocking in valleys was related to wave breaking and obtained a similar criterion, although the flow fields were different in many respects from those described by Bell & Thompson.

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