Use of a Spreadsheet Program in Electromagnetics

Ahmad Yamani and Abdelwahab Kharab

Abstract—**This paper describes how a spreadsheet program can assist electromagnetics education at the undergraduate level. Spreadsheet software has proven to be an important pedagogical tool for a variety of problems in electrical engineering education. Using the power of the 123 macros, a menu driven spreadsheet program is used to compute the solution of some electrostatic boundary value problems for an introductory-level electromagnetics course. With the on-screen numerical and visual feedback and the ease of entering data, students will be able to take a close look at the effects of varying the input data of the model problems.**

Index Terms—**Capacitance, electric field, electromagnetic, macro, spreadsheet.**

I. INTRODUCTION

ELECTROMAGNETISM is one of the most fundamental elements to be taught in many branches of science and engineering. It requires a strong mathematical background coupled with a three-dimensional geometrical imagination. As with many fundamental concepts, knowing the mathematical formulas is not enough for students to have a "feeling" for their applications. The way in which spreadsheets can help in generating the insight of many fundamental concepts in science and engineering using macro programs is now becoming widely recognized. Though spreadsheets have been used most broadly in the financial arena, they have found their way into many applications in science and engineering. They have been recently used as an educational tool in many engineering models. In [3], Kharab used a spreadsheet program for the numerical solution of a first-order differential equation using the Runge–Kutta–Fehlberg method. In [1], Finely used a spreadsheet for the numerical solution of Poisson's equation in a pn junction diode. In [5], Rao used it for analyzing logical networks, and in [2], Hagler used spreadsheets to approximate the solution of hyperbolic and elliptic partial differential equations using a finite difference method.

The advantage of using a spreadsheet, rather than conventional computer languages, is that the programing is streamlined and less time is needed to enter the necessary code. In addition, the spreadsheet program itself provides for screen graphics, easy-data manipulation, on-screen numerical and visual feedback, and fast calculations. This paper shows the use of a spreadsheet program to solve some electrostatic boundary value problems. The objective is to provide students with better alternative tools which can be used to enhance their understanding of concepts that are otherwise abstract for an average student. The 1-2-3 macro program is found to be very helpful in viewing the effects of varying the input data.

II. PROBLEM FORMULATION

In electrostatics, the determination of the electrical field $\vec{E}$ is generally performed through the use of either Coulomb's law or Gaussian law when the volume charge distribution ρ_v or the electrical potential V is known throughout the region. In most practical situations, however, these quantities are only specified at some surfaces, and it is desired to find the electrical field and the electrical potential V throughout the volume. Such problems are usually referred to as boundary-value problems and are solved using Poisson's or Laplace equations, which can easily be derived from Gauss's law as follows:

$$\nabla \cdot \left(\varepsilon \vec{E} \right) = \rho_v \qquad (1)$$

and

$$\vec{E} = -\nabla V \qquad (2)$$

where ε is the permitivity of the medium.

Substituting (1) into (2) yields

$$-\nabla \cdot (\varepsilon \nabla V) = \rho_v \qquad (3)$$

which is known as Poisson's equation.

For homogeneous medium, (ε constant) (3) becomes

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}. \qquad (4)$$

In Cartesian coordinate system, (4) takes the form

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\varepsilon}. \qquad (5)$$

Various methods have been developed [see [4] for solving (5)]. The simplest method, known as the direct integration method, which is applicable to only one-dimensional field problems, that is $V(x)$, will be used here to present several examples in different coordinate systems. Unlike the Cartesian coordinate system, interchanging the independent variables in a cylindrical coordinate system will result in different solutions. For this, only one case is considered in the Cartesian coordinate system; whereas, two cases (i.e., $V(\rho)$ and $V(\phi)$) are considered in cylindrical coordinate systems.

III. SPREADSHEET IMPLEMENTATION

The 1-2-3 macro shown in Fig. 1 computes the distribution of the voltage, electrical field, electric flux, and charge densities.

	IA	IB	IC	ID	IE	IF	IG	IH
1	{HOME}							
2	/mr/rmcA~IA1~							
3	{BLANK A4..L100}							
4	{BLANK C A5..CP80}~							
5	{WINDOWSOFF}							
6	/rmcMENU1~IA20 IC22~							
7	/rmcMENU2~IA25 IG27~							
8	/rmcN~I2~							
9	/rmcFLAG~HV1~							
10	/rmcVOLT~IA30~							
11	/rmcELEC~IB30~							
12	/rmcFLUX~IC30~							
13	/rmcDENSITY~ID30~							
14	/rmcCHARGE~IE30~							
15	/rmcCAPAC~IF30~							
16	/rmcENERGY~IG30~							
17	{WINDOWSON}							
18	{MENUCALL MENU1}							
19								
20	CARTESIAN	CYLINDRICAL 1	CYLINDRICAL 2					
21	Cartesian Coordinate	Cylindrical Coordinate changes with Rho	Cylindrical Coordinate changes with Phi					
22	{LET FLAG.1}	{LET FLAG.2}	{LET FLAG.3}					
23	{MENUCALL MENU2}	{MENUCALL MENU2}	{MENUCALL MENU2}					
24								
25	VOLT	ELEC	FLUX	DENSITY	CHARGE	CAPAC	ENERGY	QUIT
26	Voltage	Electrical Field	Electric Flux density	Electric charge density	Total charge	Capacitance	Energy stored	QUIT
27	{VOLT}	{ELEC}	{FLUX}	{DENSITY}	{CHARGE}	{CAPAC}	{ENERGY}	{HOME}
28	{BRANCH IA23}	{BRANCH IA23}	{BRANCH IA23}	{BRANCH IA23}	{BRANCH IA23}	{BRANCH IA23}	{BRANCH IA23}	{QUIT}~
29								
30	{IF FLAG=2}{BRANCH IA39}~	{IF FLAG=2}{BRANCH IB39}~	{IF FLAG=2}{BRANCH IC36}~	{IF FLAG=2}{BRANCH ID35}~	{IF FLAG=2}{BRANCH IE35}~	{IF FLAG=2}{BRANCH IF35}~	{IF FLAG=2}{BRANCH IG35}~	
31	{IF FLAG=3}{BRANCH IA46}~	{IF FLAG=3}{BRANCH IB48}~	{IF FLAG=3}{BRANCH IC40}~	{IF FLAG=3}{BRANCH ID38}~	{IF FLAG=3}{BRANCH IE38}~	{IF FLAG=3}{BRANCH IF38}~	{IF FLAG=3}{BRANCH IG38}~	
32	{GOTO}I4~^x~	{GOTO}I4~^x~	{GOTO}I4~^D~	{GOTO}I4~^Rhos~	{GOTO}I4~^Q~	{GOTO}G4~^C~	{GOTO}H4~^W~	
33	{D}0~{D}+A5+0$2/$N~	{D}0~{D}+A5+0$2/$N~	{D}-A$2*F$2/(D$2*36*@PI)~	{D}+A$2*F$2/(36*@PI*D2)~	{D}+A$2*F$2/(D2*36*@PI)~	{D}+A2*^F2*E2/(72*@PI*D2)~	{D}+A2*^F2*E2/(72*@PI*D2)~	
34	/c~ {D N-1}~	/c~ {D N-1}~	/c~ {D N}~	{RETURN}	{RETURN}	{RETURN}	{RETURN}	
35	{UP 2}{R}^V~	{UP 2}{R}^V~	{GOTO}C4~^E~	{GOTO}E4~^Rhos~	{GOTO}E4~^Q~	{GOTO}G4~^C~	{GOTO}H4~^W~	
36	{D}+A$2/0$2*(L)~	{D}+A$2/0$2~	{D}+A$2*F$2/(36*@PI*A5'@ln(C$2/B$2))~	{D}+A2*F2/(B2*36*@PI*@ln(C2/B2))~	{D}+A2*F2/(18*@ln(C2/B2))~	{D}+F2/(18*@ln(C2/B2))~	{D}+F2/(36*@ln(C2/B2))*A2^2~	
37	/c~ {D N}~	/c~ {D N}~	/c~ {D N}~	{RETURN}	{RETURN}	{RETURN}	{RETURN}	
38	{BRANCH IA52}~	{BRANCH IB45}~	{GOTO}I4~^Rho~	{GOTO}E4~^Rhos~	{GOTO}G4~^Q~	{GOTO}H4~^C~	{GOTO}H4~^W~	
39	{GOTO}I4~^Rho~	{GOTO}A4~^Rho~	{RETURN}	{D}+A$2*F$2/(36*@PI*H$2*C5)~	{D}+F2*A2*E2*@ln(100*G2)/(36*@PI*H2*G2)~	{D}+F$2*E2*@ln(100*G2)/(72*@PI*H2*G2)/A2^2~	{D}+F$2*E2*@ln(100*G2)/(72*@PI*H2*G2)*A2^2~	
40	{D}+B$2~{D}+A5+(C$2-B$2)/$N~	{D}+B$2~{D}+A5+(C$2-B$2)/$N~	{GOTO}I4~^D~	/c~ {D N}~	{RETURN}	{RETURN}		
41	/c~ {D N-1}~	/c~ {D N-1}~	/c~ {D N}~	{RETURN}				
42	{UP 2}{R}^V~	{UP 2}{R}^E~	{UP 2}{R}^E~					
43	{D}+A$2*@ln(C$2/(L))/@ln(C$2/B$2)~	{D}+A$2/A5*@ln(C$2/B$2)~	{D}+A$2*F$2/(36*@PI*H$2*C5)~					
44	/c~ {D N}~	/c~ {D N}~	/c~ {D N}~					
45	{BRANCH IA52}~	{GOTO}A5~/gr~tx {D N}~	{RETURN}					
46	{GOTO}A4~^Phi~	A(R 2) {D N}~vq						
47	{D}0~{D}+A5+H$2/$N~	{RETURN}						
48	/c~ {D N-1}~	{GOTO}C4~^Rho~						
49	{D}+0.01~+C5+(G$2-.01)/$N~	{D}+0.01~+C5+(G$2-.01)/$N~						
50	{D}+A$2/H$2*(L)~	/c~ {D N-1}~						
51	{UP 2}{R}^E~	{UP 2}{R}^E~						
52	/c~ {D N}~	{D}+A$2/(H$2*(L))~						
53	{GOTO}A5~	/c~ {D N}~						
54	/gr~tx {D N}~vq	{GOTO}C5~/gr~tx {D N}~						
55	A(R) {D N}~vq	A(R) {D N}~vq						
56	{RETURN}	{RETURN}						
57								

Fig. 1.   Macro program.

In addition, the total charge induced on the conductor surface, the capacitance, and the energy stored in the region of interest are calculated in both Cartesian and cylindrical coordinate systems for the following model problems.

Cartesian coordinates: Parallel plate capacitors (see Fig. 2). Here we have

Formula	Quantity	unit
$V(x) = \frac{V_0}{d} x$	Voltage	V
$\vec{E} = -\frac{V_0}{d} \vec{a}_n$	Electric field	V/m
$\vec{D} = -\frac{\varepsilon_r V_0}{36\pi d} \vec{a}_n$	Electrical flux density	nc/m^2
$\rho_s = \frac{\varepsilon_r V_0}{36\pi d}$	Electrical charge density	nc/m^2
$Q = \frac{\varepsilon_r V_0}{36\pi d} S$	Total charge	nc
$C = \frac{\varepsilon_r S}{36\pi d}$	Capacitance	nF
$W = \frac{\varepsilon_r S V_0^2}{72\pi d}$	Energy stored	nJ

where x varies from zero to d, V_0 is the potential difference between the two plates, $\vec{a}_n$ is a outward vector normal to the conductor plate, and S is the total area of the conductor.

Cylindrical coordinates:

a) Voltage changes with ρ: Coaxial capacitors (see Fig. 3).

Formula	Quantity	unit
$V(\rho) = \frac{V_0 \ln(b/\rho)}{\ln(b/a)}$	Voltage	V
$\vec{E}(\rho) = \frac{V_0}{\rho \ln(b/a)} \vec{a}_\rho$	Electric field	V/m
$\vec{D}(\rho) = \frac{\varepsilon_r V_0}{36\pi \rho \ln(b/a)} \vec{a}_\rho$	Electrical flux density	nc/m^2
$\rho_{s in} = \frac{\varepsilon_r V_0}{36\pi a \ln(b/a)}$	Electrical charge density	nc/m^2
$Q = \frac{\varepsilon_r V_0}{18 \ln(b/a)}$	Total charge / unit length	nc/m
$C = \frac{\varepsilon_r}{18 \ln(b/a)}$	Capacitance / unit length	nF/m
$W = \frac{\varepsilon_r V_0^2}{36 \ln(b/a)}$	Energy stored / unit length	nJ/m

where ρ varies from a to b and $\vec{a}_\rho$ is a unit vector in radial direction.

b) Voltage changes with ϕ: Radial capacitor (see Fig. 4).

Formula	Quantity	unit
$V(\phi) = \frac{V_0}{\alpha} \phi$	Voltage	V
$\vec{E}(\rho) = -\frac{V_0}{\alpha \rho}$	Electric field	V/m
$\vec{D}(\rho) = \frac{\varepsilon_r V_0}{36\pi \alpha \rho}$	Electrical flux density	nc/m^2
$\rho_s(\rho) = \frac{\varepsilon_r V_0}{36\pi \alpha \rho}$	Electrical charge density	nc/m^2
$Q = \frac{\varepsilon_r V_0 S}{36\pi \alpha L} \ln(100L)$	Total charge	nc
$C = \frac{\varepsilon_r S}{36\pi \alpha L} \ln(100L)$	Capacitance	nF
$W = \frac{\varepsilon_r S V_0^2}{72\pi \alpha L} \ln(100L)$	Energy stored	nJ

where ϕ varies from zero to α and L is the radial length of the conductor.

This program is particularly useful for classroom presentation in an introductory course in electromagnetics. With a LitePro projector, for example, and a laptop PC, students will be able to experience the solution of the model problems described above for different sets of data. The program can be also useful for laboratory sessions. It is menu-driven, user-friendly, and inter-
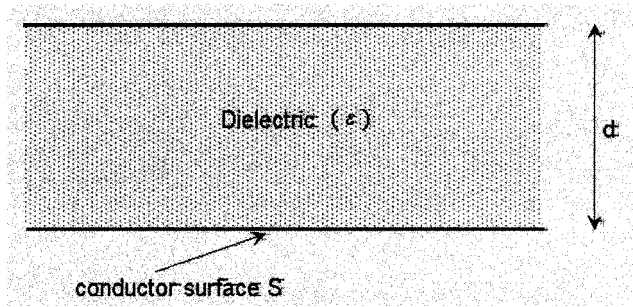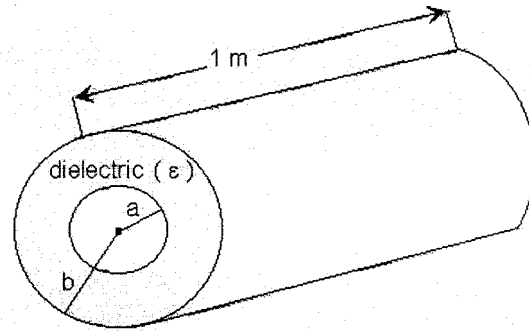

Fig. 2. Parallel plate capacitor geometry.
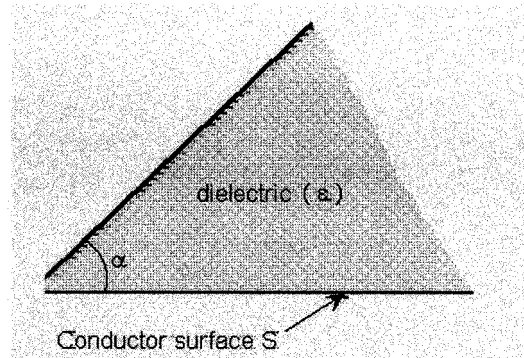

Fig. 3. Coaxial capacitor.

Fig. 4. Top view of a radial capacitor.

active. Little knowledge is required to use this program. The way the program has been written is as follows:

The program includes seven subroutines named VOLT, ELEC, FLUX, DENSITY, CHARGE, CAPAC, and ENERGY that compute the distribution of the voltage, the electrical field, the electric flux density, the electric charge density, the total charge induced on the conductor surface, the capacitance, and the energy stored in the region of interest respectively. The commands that create these subroutines reside in cells IA10...IA16. Fig. 5 shows the ranges where each subroutine is contained in the worksheet.

At the start of the macro, a main menu is activated with the command residing in cell IA18. It asks the user to choose the coordinate system (Cartesian or cylindrical) in which the model problem is to be solved. Once a coordinate system is selected, the user will then be prompted by a submenu as shown in Fig. 6. The commands that call the submenu are contained in cells IA23, IB23, and IC23.
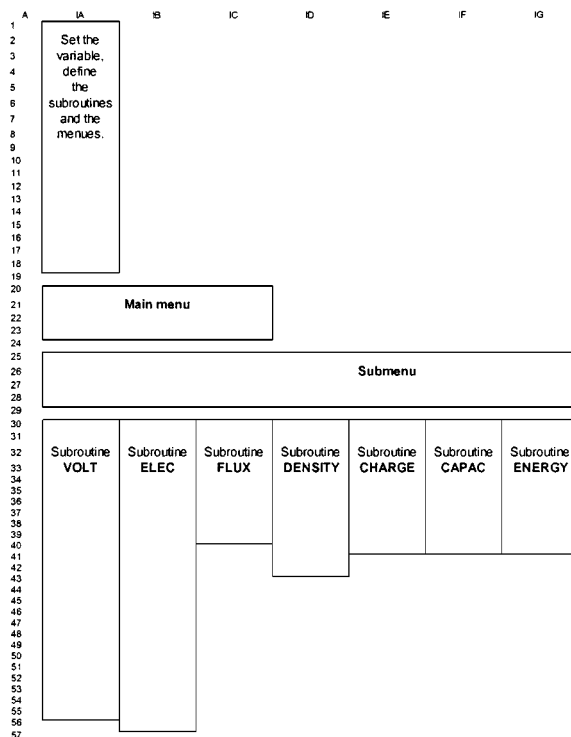
Fig. 5.   Setup of the 1-2-3 macro.

A. *How to Enter the 1-2-3 Macro into a Worksheet*

The 1-2-3 macro shown in Fig. 1 should be stored within a Lotus 1-2-3 worksheet. In writing the 1-2-3 macro, the authors used Columns IA, IB,…,IH away from the range of output. When entering the macro into a 1-2-3 worksheet, one should make sure that it is written exactly as shown in Fig. 1, that is, starting at cell IA1 and finishing at cell IH28.

B. *How to Run the 1-2-3 Macro*

Once the 1-2-3 macro is entered into a 1-2-3 worksheet, name it \A by moving the cursor to cell IA1 and then using the command $/rnc\backslash A^\sim IA1^\sim$. Enter the data as shown in Fig. 7 and then press the combination key Alt-A to invoke the macro. For example, to enter the value of V_0, move the cursor to cell A2 and type 100.

The Lotus 1-2-3 offers a useful interactive capability which is ideal in studying the effects on the solution of the model problem when input data are changed, the so-called "What-if" analysis. Once the program is run using one of the coordinate systems, the spreadsheet recalculates all dependent variables in realtime upon the a change of any of the input data.

IV. EXAMPLES

A. *Example 1*

The two plates of a parallel-plate capacitor are separated by a distance $d = 10$ mm and maintained at potentials 0 and 10 V. Assuming negligible fringing effect at the edges, the students should determine the potential at any point between the plates, the surface charge densities on the plates, the electric field intensity at any point between the plates, and the capacitance given that the space between the plates is filled with a medium having
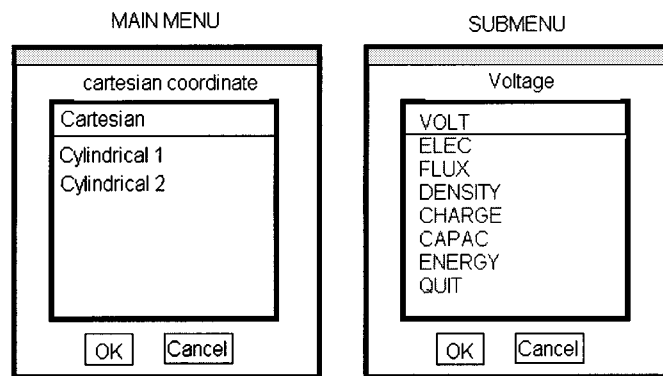

Fig. 6.   Menus activated by the spreadsheet program.

a dielectric constant $\varepsilon_r = 2.5$ and that each plate has a surface $S = 2$ cm^2.

The input data V_0, d, S, ε_r and N (number of grid points) should be entered in cells A2, D2, E2, F2 and I2, respectively (see Fig. 8), and then the 1-2-3 macro should be invoked to get the output shown in Fig. 8. Note that V is given in column B, $\vec{E}$ in column C, ρ_s in F5 and C in G5.

B. *Example 2*

Two conducting plates of size 1×5 m are inclined at 45° to each other with a negligible insulating gap separating them and are maintained at a potential difference of $V_o = 50$ V. Assuming that the medium between the plates has $\varepsilon_r = 1.5$. The students should determine the potential and electric field at any point between the plates, the charge per plate, the capacitance of these plates, and the energy stored in the capacitor by taking the radial distance 5 m.

The input data V_0, S, ε_r, L, α, and N should be entered in cells A2, E2, F2, G2, H2, and I2, respectively (see Fig. 9), and then the 1-2-3 macro should be invoked to get the output shown in Fig. 9. It is noted that V is given in column B and $\vec{E}$ in column D, Q in G5, C in H5, and W in I5.

C. *Example 3*

A cylindrical capacitor has an inner and outer radius of 5 mm and 5 mm, respectively. If $V(\rho = 5$ mm$) = 100$ V, and $V(\rho = 15$ mm$) = 0$ V, calculation of the potential and the electric field at any point between the conductors, the charge density $\rho_{\sin}$ on each conductor and the capacitor per unit length of this capacitor can be accomplished.

The input data V_0, a, b, ε_r and N (number of grid points) can be entered in cells A2, B2, C2, F2, and I2, respectively (see Fig. 10), and then the 1-2-3 macro can be used to get the output shown in Fig. 10. Note that V is given in column B and $\vec{E}$ in column C, $\rho_{\sin}$ in F5 and C in G5.

V. CONCLUSION

A spreadsheet program has been developed to solve some electrostatic boundary value problems. The program is menu-driven, user friendly, and interactive. It can be run by pressing a single key, and data can be entered very easily. It allows the user to carry out a series of numerical experiments using different sets of data and to view the effect automatically in graphical

	A	B	C	D	E	F	G	H	I
1	V0	a	b	d	S	EPSr	L	α	N
2	50	5	15	2	2	1.5	5	45	
3									

Fig. 7. Input data.

	A	B	C	D	E	F	G	H	I
1	V0	a	b	d	S	EPSr	L	α	N
2	10			0.01	0.0002	2.5			10
3									
4	x	V	E	D	Rhos	Q	C	W	
5	0	0	-1000		22.10485		0.000442		
6	0.001	1	-1000						
7	0.002	2	-1000						
8	0.003	3	-1000						
9	0.004	4	-1000						
10	0.005	5	-1000						
11	0.006	6	-1000						
12	0.007	7	-1000						
13	0.008	8	-1000						
14	0.009	9	-1000						
15	0.01	10	-1000						
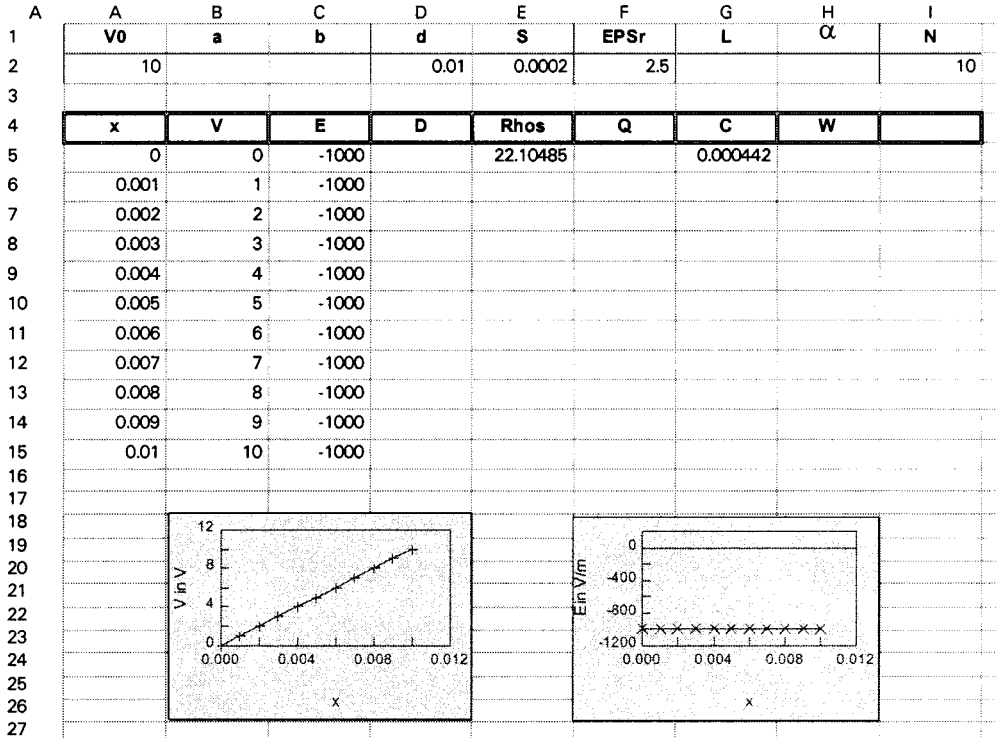
Fig. 8. Example 1 macro output.

	A	B	C	D	E	F	G	H	I
1	V0	a	b	d	S	EPSr	L	α	N
2	50				5	1.5	5	45	10
3				E					
4	Phi	V	Rho	E	D	Rhos	Q	C	W
5	0	0	0.001	-1111.11			0.091582	0.001832	2.28955
6	4.5	5	0.5	-2.22222					
7	9	10	0.999	-1.11222					
8	13.5	15	1.498	-0.74173					
9	18	20	1.997	-0.55639					
10	22.5	25	2.496	-0.44516					
11	27	30	2.995	-0.37099					
12	31.5	35	3.494	-0.31801					
13	36	40	3.993	-0.27826					
14	40.5	45	4.492	-0.24735					
15	45	50	4.991	-0.22262					
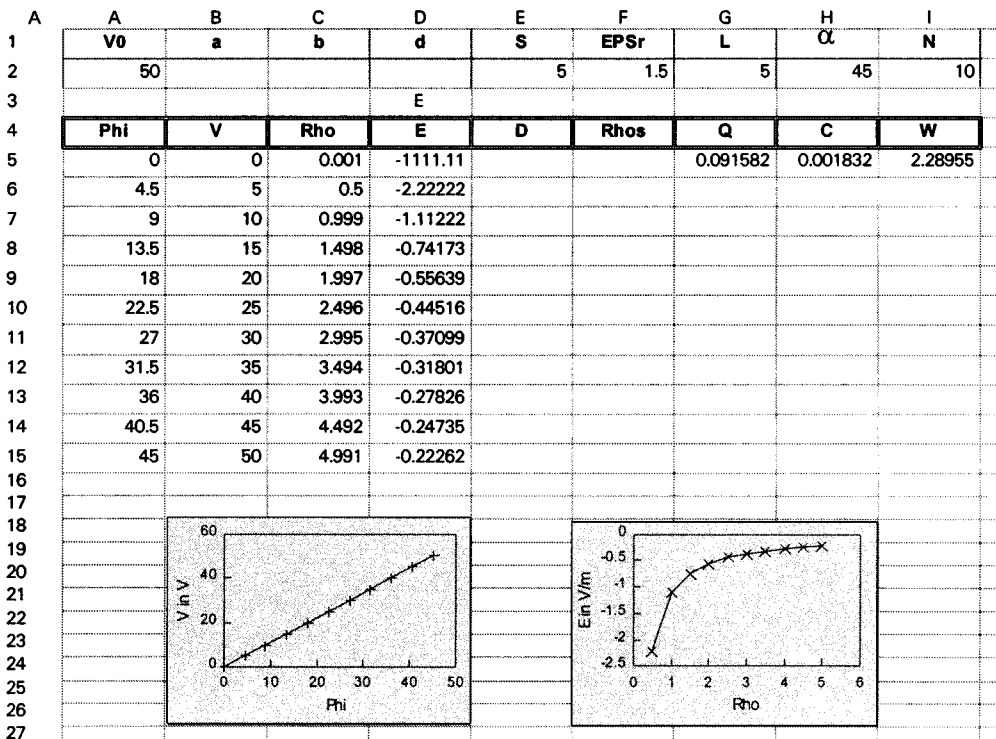
Fig. 9. Example 2 macro output.

V0	a	b	d	S	EPSr	L	α	N
100	0.005	0.015			1.5			10

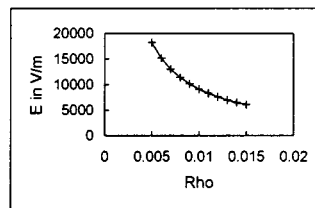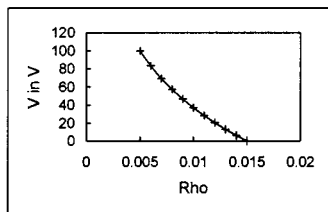| ρ	V	E	D	Rhos	Q	C	W	
0.005	100	18204.785		241.44845		0.0758533		
0.006	83.404377	15170.654						
0.007	69.372977	13003.418						
0.008	57.218426	11377.99						
0.009	46.497352	10113.769						
0.01	36.907025	9102.3923						
0.011	28.231518	8274.9021						
0.012	20.311401	7585.3269						
0.013	13.0256	7001.8402						
0.014	6.2800018	6501.7088						
0.015	4.042E-14	6068.2615						

Fig. 10.   Example 3 macro output.

form. These features are difficult to obtain by using a structured programming language such as FORTRAN. Classroom and lab use of this program has resulted in favorable comments by the students. The examples used here demonstrate that spreadsheet programs have proven to be an important pedagogical tool for a variety of problems in engineering education.

Note: The spreadsheet program can be downloaded from the second author's web site at *www.kfupm.edu.sa/math/awkharab*

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Ahmad Yamani, Photograph and biography not available at the time of publication.

Abdelwahab Kharab, Photograph and biography not available at the time of publication.