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Use of Fuzzy Relations for Affordability Decisions in High Technology

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Abstract

In the aeronautics industry, effective assessment of propulsion technologies for future investment and applications is required to reflect the total cost impact in the trade-off between cost and performance. Application of fuzzy relational techniques will allow more effective predictions of cost, uncertainty, indefiniteness and confidence in cost solutions, and will also facilitate more realistic analysis of cost/benefits vs. performance trade-off than probability theory when a sufficiently large sample of technological data characterising the manufactured artifact is not yet available. In our study, a method of repertory grid techniques is used to elicit the initial information for affordability studies. Data thus obtained is represented as fuzzy relations. BK-Fuzzy Relational Products and Fast Fuzzy Relational Algorithms are used to further analyze the elicited knowledge about an artifact. In the case study discussed in the paper, the evaluated part is a Low Pressure Turbine Cover Plate. The attributes characterising two technological processes aplied to the part, namely, extrusion and forging form a fuzzy relational structure. From this structure, the attribute-to-attribute relation is extracted and further processed, thus extracting an ordering from the relation. This ordering forms a parametrized familty of graphical hierarchies or heterarchies (preorders) which are displayed as Hasse diagrams. Testing the fuzzy relational structure for various relational properties allows us to discover dependencies, hierarchies, similarities, and equivalences of the attributes characterizing technological processes and manufactured artifacts in their relationship to costs and performance. This technique of fuzzy reational analysis plays an important role in our NSF supported project that aims at establishing a methodology for technical/business decision making in the situations when it is to be based on incomplete, uncertain data.

1. Introduction: Integrating the Cost Factors into Early Stages of Engineering Design

Assuring and maintaining a competitive position in the aerospace market requires constant diligence in developing and implementing manufacturing technologies which produce significant efficiencies and cost containment, but more specifically must now include affordability as a major component in all design and business decision-making. The need to provide cost-effective propulsion technology is critical in today's globally competitive environment.

The ability to make sound decisions about the cost and affordability of new technologies is a key to achieving affordability goals of future propulsion systems. The "Summary Findings from the Aeronautics Materials and Manufacturing Technologies Activity", Propulsion System Technologies Task Group, National Center for Advanced Technologies (Bill Yee, Pratt & Whitney, chair), stressed at the AIAA 33rd Aerospace Sciences Meeting (January 11, 1995, Reno, NV) that

- material and manufacturing shortfalls include a "lack of understanding of cost/affordability" and
- industry needs to develop "more accurate modeling and analysis tools to effectively trade affordability versus performance".

Indeed, *early cost-driven decisions* are critical to low cost development and production. For example, eighty percent of the cost of a jet engine is built-in during the early conceptual design phase. For firms that are at the leading edge of high technology, it often involves determining the essential cost factors of a *design that has never before been manufactured.* This creates a formidable problem, forcing the companies to make technological and business decisions based on incomplete, uncertain information about the product that is to be designed and manufactured. Furthermore, because the information about the cost of alternatives is incomplete and uncertain, it is not provided to the designers. This leads to further escalation of costs of design.

Yet, despite this urgent need, a *systematic accounting of incomplete or conflicting information*, constraints, and consequences *has not been* adequately addressed during engineering design and planning in the propulsion industry. This impacts any affordable aeronautics propulsion system incorporating new technology or product development, the analysis of cost, and realistic and reliable cost/benefit analysis in the design phase of advanced jet propulsion systems.

This situation is, however, slowly changing. In aerospace for example, the integration of design and cost databases, to support efficient performance/cost trades and design-to-cost, is currently being supported by a

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number of agency-supported programs. National Science Foundation Manufacturing Technology Directorate states the following in the document entitled *NSF 95-11 - MOTI: 1995 Focus on Affordability of New Technology*

> *The evasive impact of costs across an enterprise suggests that better understanding of the financial links between manufacturing and other elements of a firm is needed.*
>
> *The engineering design and manufacturing activities must be fully integrated with the management, organizational, accounting and financial activities of the enterprise. Therefore, the focus of research supported by this program will be on the integration of management and organizational factors with the engineering aspects of technological innovation.*

Further, NSF in the same document stresses that *Engineering managers need to*

- *estimate the impact of new technologies on manufacturing and life-cycle costs,*
- *deal with the uncertainty in the estimates, and incorporate the estimates concurrently into engineering design.*

Research is needed

- *to identify all the technical, human and organizational contributions to costs,*
- *deal with the uncertainty in the estimates, and incorporate the estimates concurrently into engineering design.*

Application of fuzzy logic and fuzzy relational algorithms will allow more effective predictions of cost, uncertainty or confidence in cost solutions, and analysis of cost/benefits than probability theory.

The objectives of our project are (1) to provide the propulsion industry with the ability to assess and evaluate the cost of new technologies, and (2) to design and manage for affordability of leading-edge technologies, and (3) to improve technical decisions (choices) based on incomplete, uncertain data. The use of fuzzy attributes for cost models and affordability applications addresses the problem of the limited (or unavailable) data in the situation of materials with limited characterization data, processes with limited empirical data, and manufacturing processes with little or no manufacturing base. The use of verbal descriptors parallels the knowledge-gathering exercise and will provide the ability to assign different levels of accuracy, precision, or certainty to each element, such as cost, material input, or processing condition.

Transactions on Information and Communications Technologies vol 16, © 1996 WIT Press, www.witpress.com, ISSN 1743-3517

In the context of this project, knowledge acquisition has to be based on the careful analysis of existing technical, human and organizational subsystems of the total production system as well as on the ranges of feasible modifications, innovations and new technologies. Exploratory knowledge elicitation has to be performed first, in order to acquire the understanding of the relative qualitative relationships in all these subsystems and for all their substantial interactions prior to quantizing these. We need to elicit not only the values of fuzzy membership function but also the relevant linguistic labels and their implied meaning before introducing any quantization. In the following sections, we describe the method of knowledge acquisition which will be applied to this project.

2. Engineering Scenario: A mechanical component to be evaluated

A *Low Pressure Turbine (LPT) Cover Plate* is to be manufactured, using new material, namely, gamma titanium. Prior to any production characterization, the part is to be costed out, using the expert knowledge concerning manufacturing processes and available cost estimation that is available for other small gamma titanium parts.

There are five main processing steps that will be involved in manufacturing the Low Pressure Turbine Cover Plate:

1. Initial ingot production
2. Extrusion
3. Forging
4. Heat treat
5. Machining

Two of the steps, initial ingot production and heat treating, are well-known processes, with little variability, for the LPT cover plate material. There may initially be more than one production scenario, as currently the cover plate may be produced via a process that calls for one heat treatment or via a process calling for more than one heat treat step. However, each of the heat treat scenarios can be defined and the heat treatments are well-known.

The extrusion step is currently being costed out. There has been difficulty in justifying the costs since few extrusions of the cover plate material have been carried out and cost data is limited.

The forging process is better known than the extrusion. However, there are some unknowns: the size is bigger than any similar components produced previously, and there is no production knowledge available for the component.

The machining step currently is the most uncertain, with the most incomplete knowledge. Some small parts are being machined and data collected under those conditions.

In our study, we are using this part and the above five steps as major cost drivers. Current plans call for the same part to be used for a probabilistic cost study being conducted under a concurrent program based on statistical exploratory data analysis [7],[6],[11],[13].

3. Application of Knowledge Engineering Methods

3.1. Exploratory Knowledge Elicitation Using Relational Fuzzy Methods of Knowledge Extraction and Representation

As there is no historical base for this component, and any preliminary physical characterisation studies of the enginnering processes involved may also be costly, we proceed with exploratory elicitation of knowledge possessed by experts, engineers and other specialist staff that has direct or indirect relevance to the manufacturing problem. In order to proceed successfully with this pursuit, exploratory knowledge elicitation methods [8] have to be combined with fuzzy relational representation of elicited knowledge [10].

In this context, the use of fuzzy attributes for cost models and affordability applications makes it possible to address the problem of the limited (or unavailable) data in the situation of materials with limited characterization data, processes with limited empirical data, and manufacturing processes with little or no manufacturing base. The use of verbal descriptors parallels the knowledge-gathering exercise and will provide the ability to assign different levels of accuracy, precision, or certainty to each element, such as cost, material input, or processing condition.

3.2. Knowledge Elicitation

Knowledge Elicitation is extraction of expert's declarative and procedural knowledge. The *knowledge elicitation* process is concerned with identification and formation of domain concepts and structural domain knowledge. Various methods can be used to abstract internal and causal knowledge possessed by experts.

In our study, we use a method of *Repertory Grid* techniques to elicit the initial information for affordability studies.

3.2.1. Repertory Grid (RG)

The *Repertory Grid* is a tool for eliciting a relation between some entities. It refers to three major components: *objects*, *attributes* and *respondents*. **Objects** consist of a name and a physical/functional description of items or elements, or things. **Atributes** (properties, or discriminants) characterise objects in some way. For example, in Urban Studies models [12], objects are the places in their urban environment, where the inhabitants live, go, eat, etc. and attributes are some psychological constructs attached to places. In a medical system, objects may be diseases and atributes may be signs or symptoms [2]. In a propulsion system, its components may be objects; factors which impact the cost in each process may be represented as attributes. Attributes are made up of two opposing concepts called poles; the emphasized, positive or favored pole is alleged to be the primary pole. **Respondents** (or participants) are, for example, the set of individual inhabitants of the urban area whose opinion is elicited by the repertory grid, medical doctors assessing patients or technologists involved in a high technology design.

An individual respondent is presented with a list of objects(elements) and a list of atributes (e.g. constructs or properties) in a repertory grid form. Then, the person is asked to apply those properties to the objects using a graded scale expressing the degree to which the relation between objects and atributes holds(semantic hedges). Such degree describes not only the level of emphasis placed on a pole but also which pole is being stressed. Thus, the objects are evaluated and an opinion is formed.

Pr.		3	2	1	0	-1	-2	-3	
1	Capable Analytical modeling								Limited Analytical modeling
2	Large Processing windows								Small Processing windows
3	Low Temperature								High Temperature
4	Good Lubricity								Poor Lubricity
5	Air Furnace atmosphere								Vacuum Furnace atmosphere
6	Good Process control								Limited Process control
7	Available Tooling								New Tooling
8	Flat Die shape								Shaped Die shape
9	Long Die life								Short Die life

Table 1: The Repertory Grid in Extrusion Process

The aim of the techniques is to capture not only technological but also human factors involved in the technological process. Table 1 shows a part of the repertory grid in extrusion process of propulsion system. Table 2 shows a summary of the rating sheet. Responses of 2 respondents describing 2 different objects are summarized. Each column displays the values of a specific rating sheet. How this information which is acquired from engineers will be processed further by the relational algorithms is discussed in the subsequent sections.

Transactions on Information and Communications Technologies vol 16, © 1996 WIT Press, www.witpress.com, ISSN 1743-3517

	respondent-1		respondent-2	
	P_1	P_2	P_1	P_2
C_1	n/a	n/a	0	0
C_2	+1	+1	-1	-1
C_3	+2	+1	0	+1
C_4	0	-1	-1	-1
C_5	+1	+1	-1	-1
C_6	+1	0	-1	-1
C_7	+2	+2	-1	-1
C_8	0	+2	-1	-1
C_9	+1	+1	-1	-1

Table 2: The Summary Table of a Repertory Grid

3.3. Structure Formation

Given a set of attributes(or objects), we seek to identify thier relationships implicit on data, pertinent to a particular perspective of our concerns. It falls upon fuzzy relational theory to approximate the connection of attributes of objects by extracting them out of the knowledge of how the respondents apply the attributes to the objects. The attributes and their connection form a relational structure.

Structure formation consists of two major steps: formation of an attribute-attribute relation from the summary table of a repertory grid and extraction of an ordering from the relation, forming a graphical hierarchy. Before we describe the steps of structure formation, we briefly explain some basic technicalities concerning Fuzzy Relational Products.

3.4. A Brief Overview of BK-Products

Relational representation of computational structures and simulation models makes it possible to perform all the computations and decision making in a uniform way [9], by means of *special relational compositions* called triangle and square products first introduced by Bandler and Kohout in 1977, and referred to as the BK-products in the literature. Theory and applications have made substantial progress since then. See the survey in [9] with a list of 50 selected references on the theory and applications

Mathematical definitions. Where R is a relation from X to Y, and S a relation from Y to Z, a *product relation* $R * S$ is a relation from X to Z, determined by R and S. There are several types of product used to produce product-relations [4], [9],[1].

Transactions on Information and Communications Technologies vol 16, © 1996 WIT Press, www.witpress.com, ISSN 1743-3517

PRODUCT TYPE	SET-BASED DEFINITION	MANY-VALUED LOGIC FORMULA
Circle product:	$x(R \circ S)z \Leftrightarrow xR$ intersects Sz	$(R \circ S)_{ik} = \bigvee_j (R_{ij} \bigwedge S_{jk})$
Triangle Subproduct:	$x(R \lhd S)z \Leftrightarrow xR \subseteq Sz$	$(R \lhd S)_{ik} = \bigwedge_j (R_{ij} \rightarrow S_{jk})$
Triangle Superproduct:	$x(R \rhd S)z \Leftrightarrow xR \supseteq Sz$	$(R \rhd S)_{ik} = \bigwedge_j (R_{ij} \leftarrow S_{jk})$
Square product:	$x(R \square S)z \Leftrightarrow xR = Sz$	$(R \square S)_{ik} = \bigwedge_j (R_{ij} \equiv S_{jk})$

By choosing appropriate many-valued logic (MVL) operations for the logic connectives in the above definitions of products, the Boolean (crisp) case extends to a wider variety of many-valued logic based (fuzzy) relational systems [1],[3],[9].

BK-relational product can be used to compare relational structures. Thus, if R is any relation (perhaps itself a product of other relations) from X to Y $\mathcal{R}(X \rightsquigarrow Y)$ and R^T its *transpose*, then the product $R * R^T \in \mathcal{R}(X \rightsquigarrow X)$ (where $* \in \{\circ, \lhd, \rhd, \square\}$) might exhibit some relational properties that reveal important characteristics of the source of information from which they were generated. Relational properties, such as reflexivity, symmetry, and transitivity, and classes such as tolerances, equivalences and partial orders are well known for crisp relations.

Closures and interiors of relations [5],[4] play an important role in design of fast fuzzy relational algorithms used in our approach. The idea of *comparison of a relation with its closure* and *comparison of a relation with its interior* leads to design and to validity proofs of fast fuzzy relational algorithms (FFRA) that can test various local properties and also automatically discover the cases when the tested properties hold not only locally, but also globally. Algorithms for transitive closures, local preorder closures, and preorder closures have been embodied in successful software tools TRISYS and TRIMOD [5].

3.5. Using Fuzzy Relational Triangle Subproduct

The values in the repertory grids are normalized over the values [0, 1] in the many-valued binary relation. The normalized grid yields the fuzzy relational matrix, say R, to be further processed.

Let R be a relation from X to Y where X is the set of *Properties* and Y is the set of *Objects*. Then, R_{xy} is the degree to which a respondent assigns property x to object y. On the other hand, R^T is a relation Y to X where $R^T_{yx} = R_{xy}$ is the degree to which a respondent assigns to object y property x. By composing the normalized matrix R with its transpose R^T, the fuzzy relational triangle subproduct $R \lhd R^T$ yields a property-property relation over objects by applying a fuzzy implication operator. $(R \lhd R^T)_{x_i x_j}$ gives the degree to which property x_i implies property x_j based on how a respondent applied both properties to the objects. The fuzzy relational triangle subproduct $(R \lhd R^T)$ may be computed in two ways: by taking either the minimum value or the mean value over all objects. For example,

$$(R \lhd R^T)_{x_ix_k} = \begin{cases} min_j(R_{x_ix_j} \rightarrow R^T_{x_jx_k}) & \forall j \in J : harsh\ criteria \\ mean_j(R_{x_ix_j} \rightarrow R^T_{x_jx_k}) & \forall j \in J : mean\ criteria \end{cases}$$

Different kinds of fuzzy implication operators can be chosen for the computation of fuzzy relational triangle subproduct such as S#, S, G43, Łukasiewicz, Kleene-Dienes-Łukasiewicz, Kleene-Dienes, Early Zadeh, Willmott and other operators [1].

3.6. Extracting an order

Fast Fuzzy Relational algorithms [5] are used to test relational properties and extract equivalences, preorders and other mutual dependencies between objects or properties.

3.6.1. Local Preorder Closure

It is essential for a relation to have the properties of reflexity, transitivity , antisymmmetry to be an ordering. We compute local preorder closure of a relation, say R, which is the least inclusive relation which has the property of reflexity and transitivity, and includes R. And, we compute the degree of approximation of the original relation to the closure. When the degree is sufficiently high, the closure is taken as the the relation used in further analysis.

3.6.2. Alpha(α)-cuts

By taking proper α-cuts, crisp structures are extracted from fuzzy relations. These α-cuts are used for additional analysis and interpretation in a variety of crisp perspectives. So an α-cut may be viewed as a filtering process and crisp examination of a fuzzy relation. A value at the α-cut(R_α) is chosen either as 1 if the degree of the fuzzy relation is greater than or equal to the α-value, or 0 if it is less than the α-value.

As the α-value descends, each relation contains its predecessors with perhaps additions. At the α-cut, the strength of implication between two properties is at least α-strong. α-values may be chosen at four different levels: height(max), half-upper ((max + mean)/2), mean, half-lower((mean + min)/2) over all elements of R. Every α-cut possesses the relational properties of preorderness (reflexivity & transitivity).

3.6.3. Hasse Diagram(HD)

The order relationships of properties are represented in the use of HD and equivalence classes. The graphical representation of the implicit hierarchy on the crisp matrix is the HD which shows ordering. The properties hierarchy gives us insight into the opinions of a respondent's view of her/his modeling system (see Fig.1 in section 4.1 below).

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3.7. From Abstract Relations to Conceptual Meaning of Fuzzy Relational Structures

To have abstract relations is not enough. Each relations must possess a clearly defined meaning giving it a concrete practical linguistic interpretation within the domain of its application. This interpretation also determines the linguistic meaning of the composed relation computed by the relational product. In order to further clarify this point, let us look at easily understandable example taken from the health care domain. Many examples could be quoted from the other fields, but the medical ones are most easily understandable without extra technical description.

An illustrative example of the semantics application of relational products.

If R is the relation between *patients* and *individual signs*, and S a relation between *signs* and *diseases, impairments or body malfunctions*, then $R * S$ will be a relation between *patients and diseases* etc. The conceptual interpretation of each distinct logical type (e.g. the triangular square product types) of these product-relations has a **distinct medical meaning**:
$x(R \circ S)z$: x has at least one sign of impairment z.
$x(R \lhd S)z$: x's signs are among those which characterize z.
$x(R \rhd S)z$: x's signs include all those which characterize z.
$x(R \square S)z$: x's signs are exactly those of impairment z.

Similarly, taking the semantics of the relations from an engineering domain, the products may acquire the following meaning. If R is a relation between the set X of subsystems or parts of some enginnering system and the set Y of the observable or measurable signs of functions out of range; and S is a relation between Y and Z (the set of possible faults), the composed relation $R * S$ represents the diagnostic relation showing the detected faults of individual subsystems with semantics analogous to that given above for the square and triangle products.

4. A Case Study: Low Pressure Turbine (LPT) Cover Plate

In this section we present and discuss the results of the relational computations that used the data elicited by the repertory grids shown above in Figure 1. These grids were made for evaluation of manufacturing processes of LPT Gamma-Titanium Cover Plate.

4.1. Semantics of the Relationships of the Relational Model of LPT Cover Plate

Our model relation M to be used in the sequel $M \in \mathcal{R}(Y \times (O \times G) \times P)$, relates the following three lists (sets) of entities:

- the set Y of process attributes;
- The Cartesian product $O \times G$ of the set O of observers, assessors or measuring sensors; with the set G of process identifiers;
- the set of Parts or Components.

Applying the usual selection and projection operators the ternary relation is decomposed into a family of 2-ary relations in $\mathcal{R}(Y \times (O \times G))$, indexed by the set P. For the relation used in our example, see Table 2. The processes P_1 and P_2 are extrusion and forging, respectively. The relation R from the set of process attributes Y to the set $(O \times G))$ is composed with its transpose R^T by means of *triangle subproduct*(see the definition in section 3.4. above) and the local preorder closure computed by the TRYSIS system. The result of this computation is a relation from the process attributes Y to Y. This relation shows the dependencies of process attributes represented as a preorder relation.

A sample result displaying dependences and equivalences of process attributes is shown in Figure 1. These figures show the Hasse Diagram (HD) structures displaying the preorders of process factors computed by the fuzzy relational triangle subproduct over processes.

The implication operators used to compute the Hasse diagrams displayed in Figure 1 of this paper are indicated directly in the figures. The formulas defining these implication operators used are as follows: Lukasiewicz implication operator $a \xrightarrow{5} b = min(1, 1 - a + b)$; Kleene-Dienes implication operator $a \xrightarrow{6} b = max(1 - a, b)$; and S* (Heyting-Gödel) implication operator $a \xrightarrow{3} b = 1$ if $a \leq b$, b otherwise. Each implication operator represents an additional hypothesis on the properties of data. The sub-structures of Hasse Diagrams that coincide although computed by different implication oprtators are more robust and invariant with respect the properties of the family of implication operators involved.

From the figure we can see that the Hasse diagrams are parametrized by α-values and the type of fuzzy implication operator used for computation of triangle subproducts. By applying various evaluation criteria, the most relevant cuts and desirable implication operators are selected from those provided by Hasse Diagram structures. In Figure 1-a, at the α-cuts of the height, half-upper and mean, HD structures of factors show the identical structures for different implication operators. This indicates that the dependences are stable for a range of α-cuts between the *height* and *mean*. We can see that in the displayed HD structures, some factors form equivalent classes. For example in Figure 1-a, the process characteristics *C2* (Large Processing Windows), *C5* (Air Furnace Atmosphere) and *C9* (Long Die Life) are in the same equivalence class. This indicates that these have equivalent (or closely similar) effect in characterising the LPT cover plate. As the chosen value of α-cut decreases, the equivalence classes become larger. It can be seen from both figures 1-b and 1-d, however, that the parameter

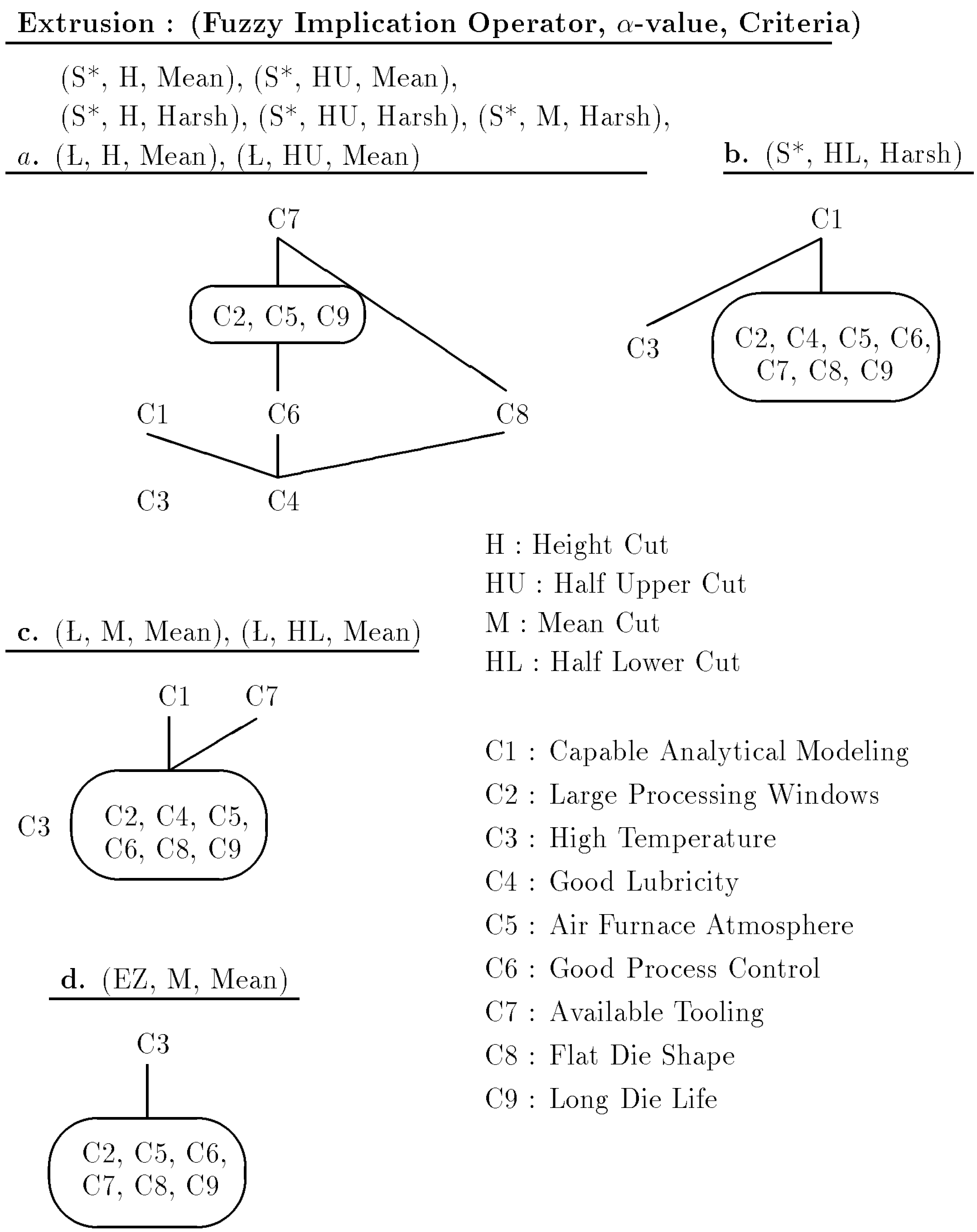

Figure 1: Hasse Diagrams of α-cuts with various α values

C3 (high/low temperature) is highly dissimilar to other attributes in its dynamic effect. This is also an important finding, allowing for qualitative differentiation of participating process attributes.

4.2. Other Alternatives of LPT Data Analysis

A number of other assessments and evaluative schemes can be formulated, showing inter-process dependences, inter-observer dependences, etc. Because the purpose of this paper is to show basic techniques of relational analysis as applied to manufacturing processes, not to provide a detailed analysis of engineering of LPT cover plate, further details of these computations are not presented in this paper.

By and large, the following three scenarios for using a repertory grid on LPT parts have addressed useful analysis goals:

SCENARIO 1:
1 object (LPT cover plate) and a group of respondents (engineers) where each of the respondents would assess the object independently in a selected process. The aim of this is to find the dependences between process characteristics as well as the inter respondent consistency.

SCENARIO 2:
1 respondent (engineer) and a collection of objects (different LPT parts). Here, the aim is to detect characteristic similarities and differences between objects.

SCENARIO 3:
1 respondent, 1 object, several situations or processes in which the object may appear. In this scenario, the primary goal is to detect similarities and dependences between process attributes of different processes.

SCENARIO X:
Any possible combination of the three above.

5. Conclusion

The relational methods combining linguistic labels with BK-products give a natural conceptual framework for knowledge representation and inference from imprecise, incomplete, or not totally reliable information in a consistent manner. Using the above described knowledge acquisition method, we examine the interrelationships of the attributes (properties of objects) which impact the cost of processes and components comprising a technological artifact, e.g., part of a propulsion system. Such structures show us the dependencies, hierarchies, similarities, and equivalences of the attributes characterizing development and manufacturing costs and help establish a methodology for technical/business decisions based on incomplete, uncertain data often encountered in the aeronautics industry.

The most important advantage of fast fuzzy relational algorithms that are used for extracting the structural relationships between the vague parameters characterizing engineering parts and manufacturing processes is that these do not impose any a priori assumptions on the presence or absence of order, symmetry or other properties. This is unlike well known path analysis used in sociology or economics, which assumes the presence of partial ordering. The well known statistical correlational coefficients on the other hand assume the symmetry property of the data. If these properties are not present in the data, the mentioned path analysis and correlational analysis break down and present spurious results. This is unlike our relational methods that work also under the weaker conditions of absence of some of these properties.

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