Use of Hypolimnetic Oxygen Depletion Rate as a Trophic State Index for Lakes

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The feasibility of lake water quality management planning has been greatly increased over the past 10 years with the development of relatively simple, empirical methods for assessing eutrophication problems. These relate phosphorus loading, hydrology, and morphometry to such traditional trophic state indices as phosphorus concentration, chlorophyll-a concentration, and transparency. One of the difficulties associated with use of these methods is that water quality criteria, as related to beneficial use, do not generally correspond to subjective definitions of 'trophic state.' This paper attempts to improve upon existing methods by relating measures of phosphorus, chlorophyll-a, and/or transparency to hypolimnetic dissolved oxygen, which is of direct relevance to existing water quality standards, particularly for fisheries management. A modified version of Carlson's (1977) trophic state index summarizes relationships among summer, epilimnetic measurements of total phosphorus, chlorophyll-a, and transparency. On the basis of data from 30 lakes this index is shown to be highly correlated with areal hypolimnetic oxygen depletion rate when the apparent effects of mean depth are also taken into account ($R^2 = 0.91$). Tests of the empirical model on a separate data base of 86 lakes indicate that the approach can be used to predict oxygen status based upon lake morphometry and trophic index. The methodology provides a link between phosphorus mass balance models and existing water quality criteria for dissolved oxygen.

INTRODUCTION

Traditional strategies for classifying lakes with respect to eutrophication have relied primarily upon subjective assessments of one or more types of water quality or biological characteristics. Recently, the increased availability of data has made it possible to develop more objective criteria for ranking and classifying lakes at a regional level on the basis of observed lake conditions [*Shannon and Brezonik*, 1972; *U.S. Environmental Protection Agency*, (EPA), 1974; *Carlson*, 1977] or the factors governing them, such as nutrient loading, hydrology, and morphometry [*Dillon*, 1975; *Vollenweider*, 1976]. The development of these methods has greatly increased the feasibility of lake management planning.

The development of an objective basis for specifying standards or criteria with regard to lake water quality is generally lacking and can be considered a weak link in the planning process. This has arisen partially out of the fact that water quality concerns are related to beneficial use and do not always correspond with traditional trophic state criteria. While some states may have considered or be considering phosphorus standards, the nutrient, in itself, does not hinder water use. It is the indirect effects of the nutrient on such water quality aspects as transparency, odor, and dissolved oxygen that are of concern from a water use standpoint. Both theoretical developments and empirical evidence indicate that the effects of phosphorus supply on primary production and water quality vary with impoundment morphometric and hydrologic characteristics and depend upon supplies of other nutrients. Thus it may not be advisable to establish universal phosphorus standards. Standards should be based on those water quality responses which are of direct concern to water use. To do this, we need to develop methodology for predicting such effects.

The trophic state index discussed below provides a simple basis for describing and ranking impoundments based upon one or more types of measurements. By relating dissolved oxygen to other traditional trophic state indices (phosphorus, transparency, and chlorophyll-a) the system provides a link between the empirical models designed to predict eutrophication and existing water quality standards. As shown in Figure 1, the index can be used in combination with a phosphorus mass balance model [*Dillon*, 1975; *Vollenweider*, 1976] to predict and compare the effects of alternative phosphorus loadings on lake phosphorus, chlorophyll-a, transparency, and hypolimnetic dissolved oxygen levels. The data base, development, testing, and applications of the index are discussed in detail below.

DATA BASE

Because this study has been initiated for use in Connecticut, over half of the data base is derived from surveys of 24 Connecticut impoundments (17 natural and 7 artificial) by the Connecticut Agricultural Experiment Station [*Norvell and Frink*, 1975] and the Environmental Protection Agency's (EPA) National Eutrophication Survey [*EPA*, 1975]. These data have been screened to eliminate lakes or reservoirs with questionable data (as acknowledged by the respective authors) or with extremely eutrophic conditions (total P greater than 250 mg/m^3). To provide a more extensive data base for study of hypolimnetic oxygen depletion rates, survey data have also been used from 13 Canadian lakes [*Dillon and Rigler*, 1974; *Lasenby*, 1975] and from 8 lakes in the U.S. portion of the Organization for Economic Cooperation and Development (OECD) North American Project [*Rast and Lee*, 1978; *Seyb and Randolph*, 1977; W. Rast, personal communication, 1978].

Summer average transparencies and epilimnetic concentrations of chlorophyll-a and total phosphorus have been compiled for each impoundment. Because summer values were not available for the Canadian lakes, spring total P values have been substituted. Areal rates of dissolved oxygen depletion below the thermocline reflect conditions from the onset of spring stratification through August or until anaerobic conditions develop in the hypolimnion. Basic morphometric and hydrologic characteristics have also been compiled. A statistical summary of the data is given in Table 1.

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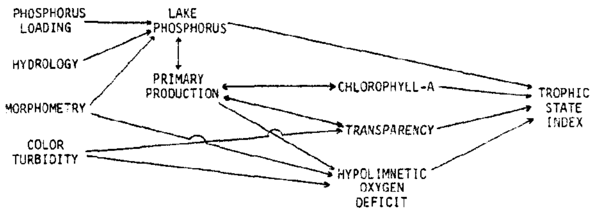

Fig. 1. Assumed causal pathways in the development of the trophic state index.

Chlorophyll-a, Phosphorus, and Transparency Indices

To provide a basis for comparisons with oxygen depletion rate, modified versions of the Trophic State Indices proposed by *Carlson* [1977] have been used to summarize relationships among chlorophyll-a, phosphorus, and transparency for this collection of lakes. Carlson based his index scale upon equivalent transparency values, which were suggested as relative indicators of biomass. Since transparency is partially influenced by factors which are independent of algal standing crop, the modified scale is based upon equivalent chlorophyll-a values rather than transparency. Certainly, chlorophyll-a is neither a direct nor a proportional measure of biomass, but it is probably less sensitive than transparency to such orthogonal factors as dissolved color and inorganic suspended solids. There are also a variety of methods for measuring chlorophyll-a, which may introduce variabilities in these types of studies. Modifications to Carlson's index scheme seemed appropriate for these lakes because of an apparent positive bias in the transparency index when compared with the chlorophyll-a or phosphorus indices.

The location and scale of the modified index are arbitrarily defined so that an index value of zero corresponds to a chlorophyll-a concentration of 0.25 mg/m^3 and concentration doubles for each increase of 10 index units. This is equivalent to the following expression:

$$I_B = 20.0 + 33.2 \log_{10} B \tag{1}$$

where

I_B chlorophyll-a index;
B chlorophyll-a concentration, mg/m^3.

The results of regression analyses relating chlorophyll-a to phosphorus and to transparency have been transformed to develop the following expressions for phosphorus- and transparency-based indices:

$$I_P = -15.6 + 46.1 \log_{10} P \tag{2}$$

TABLE 1. Statistical Summary of Data Base

Variable	Minimum	Median	Maximum
Mean depth, m	0.9	7.4	33.
Maximum depth, m	3.1	18.	76.
Hydraulic residence time, years	0.002	0.74	4.8
Summer total P, mg/m^3	4.1	18.	86.
Summer chlorophyll-a, mg/m^3	0.8	4.0	64.
Summer Secchi depth, m	0.8	4.0	8.7
Hypolimnetic oxygen depletion rate, g/m^2 day	0.06	0.31	1.73

$$I_T = 75.3 + 44.8 \log_{10} (1/Z_s - \alpha) \tag{3}$$

where

P total phosphorus concentration, mg/m^3;
Z_s Secchi depth, m;
α term representing nonalgal influence on transparency, m^{-1}.

Relationships among the three versions of the index are shown in Figures 2–4.

The slope of the chlorophyll-a/total phosphorus relationship for these lakes is 1.39, not significantly different from the values found by *Dillon and Rigler* [1974], *Jones and Bachman* [1976], and *Carlson* [1977]. The second term inside the parentheses of (3) represents a correction factor for the effects of nonalgal materials on the transparency measurement. On the basis of the analysis of limited data from Connecticut lakes the influence of dissolved color on α can be approximately represented by

$$\alpha \simeq 0.04 + 0.0025C \tag{4}$$

where C is the true color (Pt-Co units). This model essentially assumes that the Secchi depth is inversely related to the light extinction coefficient, which, in turn, is a linear function of dissolved color [*Meta Systems, Inc.*, 1978; *Rast and Lee*, 1978]. The size of the color correction term for the Connecticut lakes is small, since average color values are less than or equal to 25 Pt-Co units. The slope of the color dependence is not significantly different from the result derived by *Brezonik* [1978] on the basis of data from Florida lakes with color values up to 550 Pt-Co units. While an additional linear term in nonalgal suspended solids concentration seems appropriate for (4), the required data are not available.

Because neither suspended solids nor color data were available for many of these lakes, an average α value of 0.08 m^{-1} has been used in deriving (3). Some adjustment in this value would obviously be necessary for lakes with high color, high nonalgal turbidity, or, on the other hand, high clarity (transparency greater than 12 m). Because of dependence on α the Secchi depth measurement is not a very reliable measure of chlorophyll-a in less productive lakes. Generally, deviations from the chlorophyll-a/transparency relationship may be in-

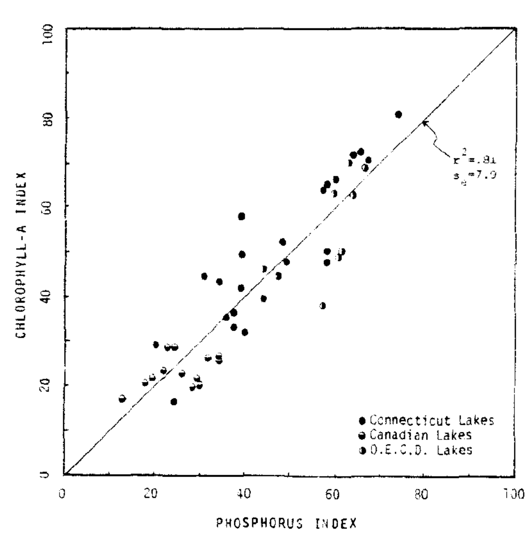

Fig. 2. Chlorophyll-a index versus phosphorus index.

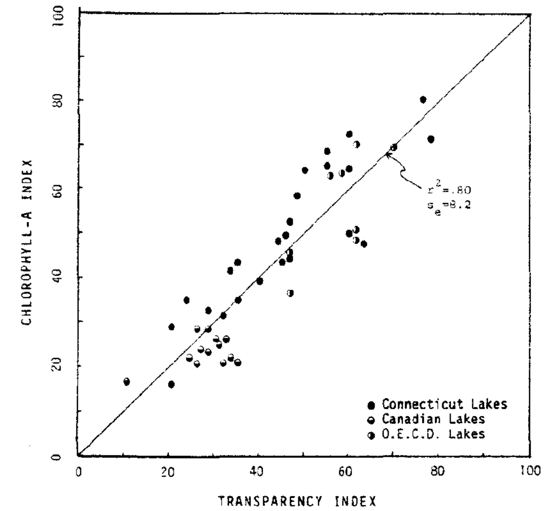

Fig. 3. Chlorophyll-a index versus transparency index.

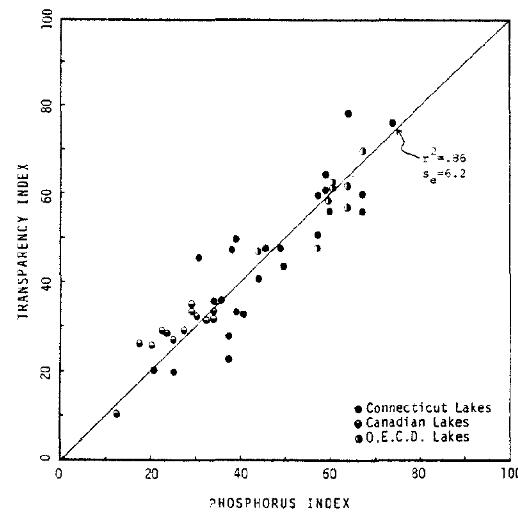

Fig. 4. Transparency index versus phosphorus index.

dicative of the differences in the relative magnitudes of the algal versus nonalgal components of the light extinction coefficient.

Observed differences among the various versions of the index can be interpreted as combinations of model error and measurement error. The former is attributed to the effects of factors which are not accounted for in these simple relationships. These would include, for instance, the effects of nitrogen limitation or high mineral turbidity. Measurement error includes the usual errors associated with analytical procedures for phosphorus and chlorophyll-a and with field measurements of Secchi depth. Another type of measurement error, probably more important in this case, is actually a form of statistical sampling error resulting from the use of discrete samples in time and space to estimate average conditions. To quantify this type of error, the individual observations made in a given lake could be used to estimate a standard error of the mean for each variable on the basis of the corresponding standard deviation, number of samples, and any observed serial correlation. This could be converted, in turn, to an estimate of the standard error of the corresponding index. An error analysis of this type could be used to assess the adequacy of sampling frequency in a monitoring program designed to gather data for assessing lake conditions and interpreting lake response using the index system. Currently lacking, however, are the assessments of within-lake spatial and temporal variabilities needed to apply this approach to this collection of lakes.

If, on the basis of a comparison of the various indices, one is willing to accept that a given lake conforms to the index scheme, a useful aggregate estimate of the index can be obtained by using a weighted averaging procedure. The simplest averaging rule would be the following:

$$\bar{I} = (I_B + I_P + I_T)/3 \qquad (5)$$

Averaging provides a means of reducing the effects of individual sampling and measurement errors and thus developing a more robust estimate of the index, provided that more than one type of measurement is available. The mean index calculated according to (5) explains 91% of the variance of the individual index values with a residual standard error of 4.9 index units. A somewhat more sophisticated approach would weight each component of the index in inverse proportion to its measurement variance, which could be estimated using the approach described above. This weighting procedure would provide a mean index estimate with minimum variance but would require more data and analysis to implement than (5).

The computed mean index values for these lakes can be compared with subjective trophic state classifications derived from the original data sources. Figure 5 depicts the stratification of the various trophic states along the axis defined by mean index values (equation (5)). The separations among the oligotrophic, mesotrophic, and eutrophic states appear to be well defined, with transition zones approximately located between 25 and 30 and between 45 and 50 index units. The eutrophic/highly eutrophic boundary is less distinct and has been arbitrarily set between 65 and 70 index units. These results provide some perspective as to the significance of the index in relation to traditional classification schemes.

OXYGEN INDEX

The rate of dissolved oxygen depletion below the thermocline has been suggested as an indictor of the rate of primary production in the surface waters of stratified lakes. This depletion has commonly been represented on an areal basis as the rate of change of the hypolimnetic oxygen deficit (ΔHOD, g/m^2 day) [*Mortimer,* 1941]. Oxygen depletion rates were avail-

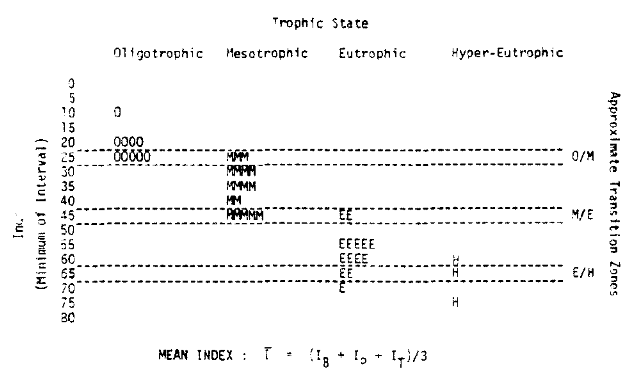

Fig. 5. Stratification of mean trophic indices across lake trophic states.

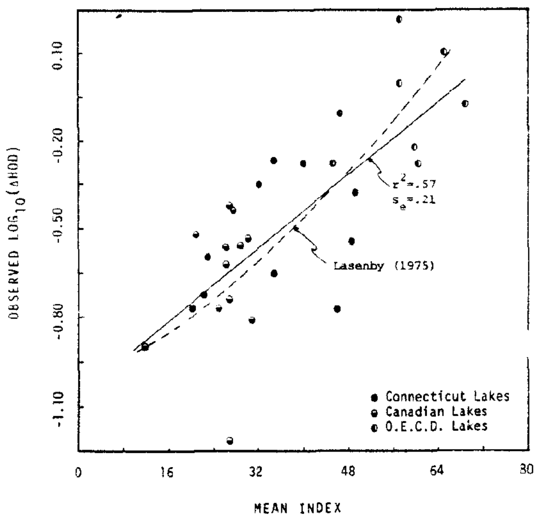

Fig. 6. Oxygen depletion rate versus mean trophic index.

able for 30 of the lakes studied above. A regression analysis summarizes the relationship between oxygen depletion rate and trophic index:

$$\log_{10}(\Delta HOD) = -1.06 + 0.016I \quad r^2 = 0.57 \quad s_e = 0.21 \quad (6)$$

where ΔHOD is the hypolimnetic oxygen depletion rate (in grams per square meter day). The relationship is shown in Figure 6 in comparison with predictions from the equation derived by *Lasenby* [1975] on the basis of data from 21 natural lakes:

$$\log_{10}(\Delta HOD) = 0.35 - 1.35 \log_{10} Z_s \quad (7)$$

To develop an expression of Lasenby's model which is analogous to (6), Secchi depths have been transformed to corresponding index values using (3) and assuming an average α value of 0.08 m^{-1}. While about half of the data used by Lasenby is also included in this analysis (Canadian lakes), Figure 6 shows that there is essentially no difference in the predictive abilities of the above two equations when they are compared in the context of the data examined here.

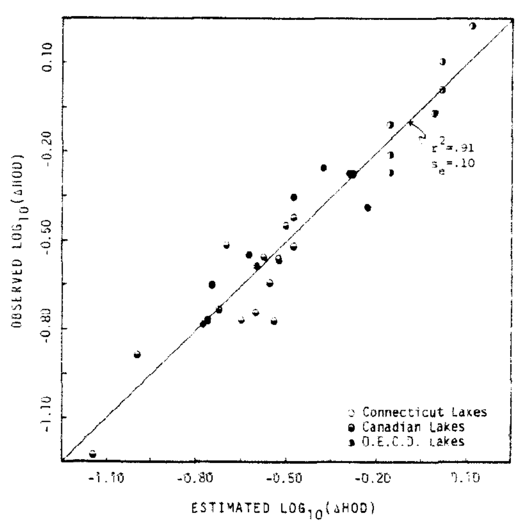

Fig. 7. Observed versus estimated oxygen depletion rate.

Mortimer [1941] suggested that a characteristic upper ΔHOD limit for oligotrophic lakes is about 0.25 g/m² day, while eutrophic lakes have values generally greater than 0.55 g/m² day. According to (6), Mortimer's criteria correspond to mean index values of 29 and 50, respectively. These values are in reasonable agreement with the trophic state transition zones depicted in Figure 5.

Lasenby excluded shallow lakes from his analysis on the basis of the 20-m maximum depth criterion suggested by *Hutchinson* [1957]. Shallow lakes are also of concern from a water quality management point of view, however, and, when stratified, can be more susceptible to hypolimnetic dissolved oxygen problems than deep lakes, which generally have greater supplies of dissolved oxygen per unit area at the onset of stratification. For these reasons, shallow lakes are included in this analysis. To test for morphometric effects, the residuals from (6) have been examined against mean depth. A positive depth dependence is apparent in lakes with mean depths less than about 10 m, while a flat or slightly negative relationship is indicated in deeper lakes. This dependence is described in more detail below.

Equation (6) has been modified to account for morphometric effects using a multiple regression model of the following form:

$$\log_{10}(\Delta HOD) = a_0 + a_1 I + a_2 \log_{10} Z + a_3(\log_{10} Z)^2 \quad (8)$$

where Z is the mean depth (in meters) and a_0, a_1, a_2, and a_3 are the empirical coefficients. As shown in Figure 7, a model of this type explains 91% of the variance in the data with a standard error of 0.10. Optimal coefficients and corresponding standard errors are as follows:

$$a_0 = -3.58 \qquad a_1 = 0.0204 \pm 0.0013$$
$$a_2 = 4.55 \pm 0.52 \qquad a_3 = -2.04 \pm 0.25$$

An additional characterization of the relationship is given by the correlation matrix of regression coefficients in Table 2. Note that the correlation between a_2 and a_3 is high. This suggests that the exact shape of the curvilinear depth dependence is quite uncertain. The a_1 coefficient, however, is only weakly correlated with the other two. This indicates that reasonable separation of the trophic state and depth influences on the oxygen depletion rate has been achieved.

The apparent nonlinear dependence of oxygen depletion rate on depth is depicted in Figure 8. The effects of the trophic index have been removed according to (8). The positive depth dependence in shallow lakes may reflect the combined effects of (1) variations in epilimnion thickness as a function of mean depth, (2) weak or intermittent stratification, (3) hypolimnetic photosynthesis, and/or (4) initial (partially allochthonous) concentrations of oxygen demanding substances in the hypolimnion at the onset of stratification. As indicated by the dashed line in Figure 8, additional data are needed to establish the depth response in deep lakes more clearly. The slightly negative depth dependence, if significant, might be due to lower hypolimnetic temperatures in deeper lakes or to

TABLE 2. Correlation Matrix of Regression Coefficients

	a_1	a_2	a_3
a_1	1.0		
a_2	0.27	1.0	
a_3	−0.23	−0.99	1.0

greater exchange rates between the hypolimnion and epilimnion [*Blanton,* 1973]. The latter would cause more rapid thermocline erosion and possibly greater oxygen transfer into the hypolimnion during the stratified period. Future work should examine some of these possibilities with the aid of theoretical models of the type discussed above or elsewhere [*Imboden,* 1974]. Specific data on epilimnion and hypolimnion depths would be required for such an effort.

The residuals from (8) are not significantly related to other lake morphometric or hydrologic characteristics, including maximum depth, hydraulic residence time, or surface overflow rate. The equation overpredicts depletion rate by an average of about 10% in lakes with maximum depth/mean depth ratios less than 2.2. Six lakes in the data set are in this category; the effect is not considered strong enough to warrant additional complication of the model but is worthy of additional analysis using a refined data base.

Solving (8) for I yields an expression for a trophic state index based upon hypolimnetic oxygen depletion rate and lake mean depth:

$$I_0 = 175 + 49 \log_{10} \Delta HOD - 223 \log_{10} Z + 100(\log_{10} Z)^2 \quad (9)$$

where I_O is the oxygen-based trophic state index. The equivalence between this version and that based upon chlorophyll-a, phosphorus, and transparency measurements (equation (5)) is demonstrated in Figure 9. The oxygen-based index I_O explains 89% of the variance in $\bar{I}$ with a residual standard error of 5.0 index units.

USE IN PREDICTING OXYGEN STATUS

Areal oxygen depletion rate, mean hypolimnion depth, and oxygen concentration at spring turnover are three important factors determining summer oxygen status. On the basis of a mass balance the following statistic seems to be a rational means of representing the combined influence of these factors,

$$T_{DO} = O_i Z_H / \Delta HOD \quad (10)$$

where

- T_{DO} effective number of days of oxygen supply present in the hypolimnion at spring turnover, days;
- O_i oxygen concentration at spring turnover, g/m^3;
- Z_H mean hypolimnion depth, m.

The above expression is exact only for the case of linear oxygen deficit development [*Lasenby,* 1975] and constant (average) hypolimnion depth. In such a case, comparing T_{DO} with the length of the stratified period should provide an indication of whether the hypolimnion is likely to become anaerobic before fall overturn. Because of the possible effects of nonlinearities in deficit development and variations in hypolimnion depth, testing of (10) as a predictor of oxygen status is required.

An independent set of data has been compiled to provide a basis for testing the T_{DO} statistic as a predictor of oxygen status. *Reckhow* [1977] used data primarily from the EPA's *National Eutrophication Survey* (NES) [*EPA,* 1975] and *Snodgrass* [1974] to classify a collection of northern temperate lakes and reservoirs according to stratification tendency and hypolimnetic oxygen status. The latter was indicated by one of four categories: (1) oxic, (2) possibly oxic, but uncertain, (3) possibly anoxic, but uncertain, and (4) anoxic. Categories 2 and 3 contained impoundments for which there were insufficient data to make reliable assessments of oxygen status during the late summer or early fall months. For the purposes of this analysis these two categories have been combined to form one 'uncertain' class. For each impoundment, Reckhow tabulated mean depth and median, summer total phosphorus measurements, which permit estimation of the trophic index and oxygen depletion rate according to (2) and (8), respectively.

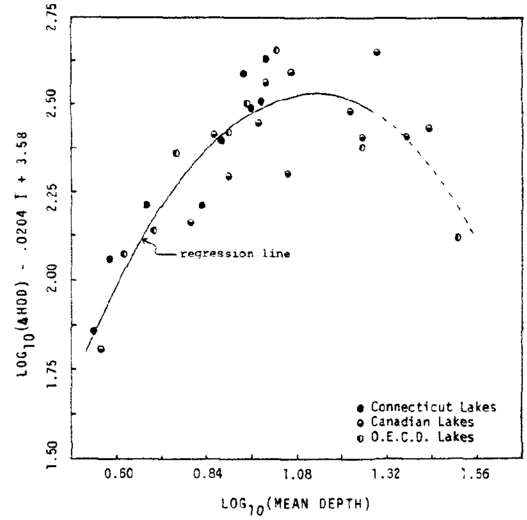

Fig. 8. Apparent effect of mean depth on oxygen depletion rate.

Oxygen concentrations at spring overturn are required for estimating T_{DO} values according to (10). In the absence of specific data for each lake, an average O_i value of 12 g/m^3 has been used [*Norvell and Frink,* 1975]. As discussed by *Hutchinson* [1957], this type of assumption may introduce errors in large, deep lakes with appreciable winter oxygen deficits or in lakes with long ice cover periods which heat rapidly and late in the spring.

A relationship developed by *Snodgrass* [1974] has been used to estimate average thermocline depth:

$$Z_T = 1.6Z^{.57} \quad (11)$$

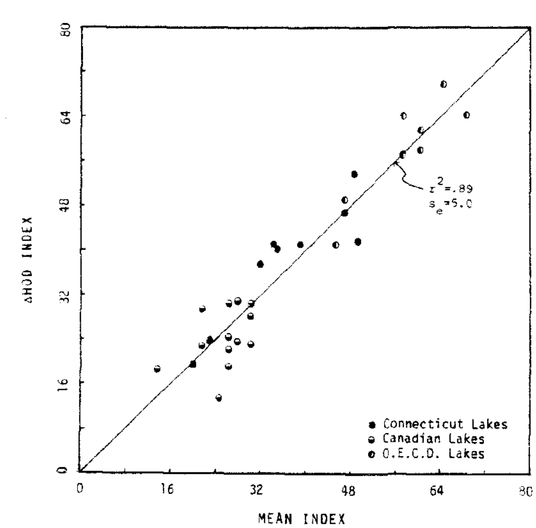

Fig. 9. Oxygen depletion rate index versus mean of indices based upon chlorophyll-a, phosphorus, and transparency.

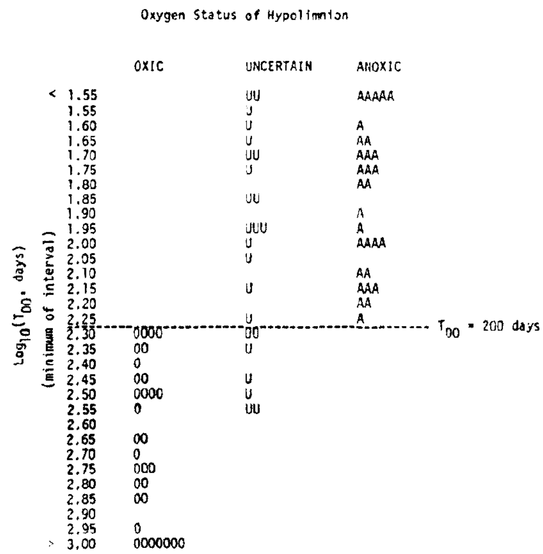

Fig. 10. Stratification of estimated T_{DO} values across groups defined by observed hypolimnetic oxygen status.

where Z_T is the thermocline depth (in meters). Assuming regular morphometry, mean hypolimnion depths have been estimated from the following:

$$Z_H = Z(1 - Z_T/Z_M) \tag{12}$$

where Z_M is the maximum depth (in meters). This relationship holds in cases where lake volume development can be approximated as a single-term power function of total depth. The above assumptions concerning spring oxygen levels and hypolimnion depths have been made only for the purposes of model testing with available data. Specific measurements of these quantities should be used in model applications.

Some initial screening of Reckhow's data has been done to eliminate lakes and reservoirs which do not conform with the morphometric limits of the data base used to develop (8). Modifications made with reference to EPA/NES reports [*EPA*, 1975] include (1) reclassification of Lake Pepin, Minnesota, as unstratified and (2) elimination of Cannonsville Reservoir, New York, which was classified as 'oxic,' but sampled by the EPA/NES only in May. In addition, a mean depth of 21 m has been assumed for Hallwillersee [*Imboden*, 1974]. To supplement the data base for deep, eutrophic lakes, data from Lake Washington in 1964 have also been included [*Rast and Lee*, 1978]. With these modifications, data from a total of 86 impoundments classified by Reckhow as 'stratified' or 'weakly stratified' have been used for model testing.

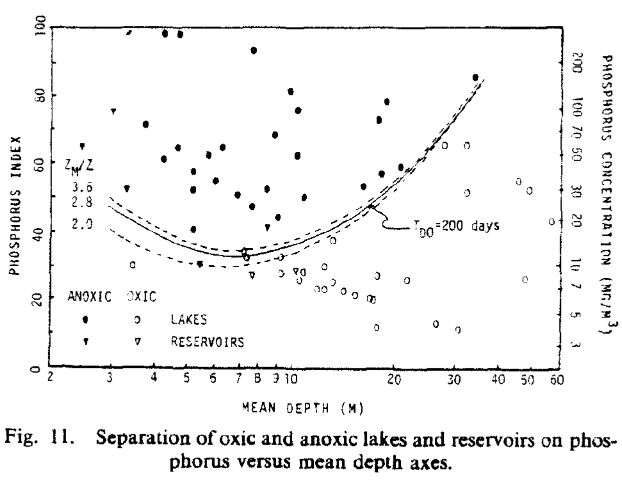

Fig. 11. Separation of oxic and anoxic lakes and reservoirs on phosphorus versus mean depth axes.

Figure 10 depicts the stratification of estimated $\log_{10}(T_{DO})$ values across groups defined by hypolimnetic oxygen status. While the uncertain group is not distinguished from the others, the oxic and 'anoxic' groups are clearly separated at a T_{DO} value of about 200 days. Anoxic conditions are estimated to develop if there are less than 200 days of oxygen supply in the hypolimnion at the onset of stratification. This criterion could be modified in individual cases on the basis of knowledge of the stratification period and/or spring oxygen concentrations. Assuming an average date of April 1 for the onset of stratification, a T_{DO} value of 200 days provides oxygen supply through October 17, which is typical of turnover dates in the northern temperate zone. Despite the variabilities which were not accounted for in this test (stratification tendencies, possible nonlinear deficit development, irregular morphometries, thermocline migration, initial oxygen concentrations at spring turnover, and sampling dates for detecting oxygen status), Reckhow's data appear to be consistent with the models developed above using a different data base.

Some additional insights into the relative importance of algal production and impoundment morphometry in controlling oxygen status are derived from Figure 11, a plot of the phosphorus-based trophic state index against mean depth. Lakes and reservoirs classified by Reckhow as oxic or anoxic are distinguished by different symbols. The curves represent the solutions of (8), (10), (11), and (12) for a T_{DO} value of 200 days and for three typical values of Z_M/Z. Figure 11 shows the ability of these functions to discriminate between oxic and anoxic lakes in two dimensions. Estimated T_{DO} values are less sensitive to Z_M/Z ratios in deeper lakes. The parabolic relationship indicates that oxygen status is most sensitive to phosphorus level in lakes with mean depths between 5 and 10 m. Below this range, areal oxygen depletion rates decrease with decreasing depth, as demonstrated in Figure 8. Above this range, areal depletion rates are relatively insensitive to depth, and an increasing supply of oxygen is available in the hypolimnion per unit area at the onset of stratification. This gives deeper lakes greater tolerance to algal production (or phosphorus) from a dissolved oxygen aspect. *Reckhow* [1978] also noted the importance of mean depth as a factor determining the response of lake oxygen status to phosphorus loading on the basis of a discriminant analysis.

Discussion

The relationships identified and tested above provide a link between phosphorus management and dissolved oxygen criteria for lakes. In combination with a phosphorus mass balance model they can be used to assess the impacts of phosphorus loadings on hypolimnetic dissolved oxygen. Applications should be restricted to thermally stratified, phosphorus-limited, natural lakes in the northern temperate zone with mean depths between 3 and 33 m. These relationships are not applicable to lakes with high levels of allochthonous suspended solids, which may restrict light penetration, reduce phosphorus availability, and exert oxygen demand [*Walker and Kühner*, 1978].

While Figure 11 indicates that the seven reservoirs with available data are correctly classified by the model, it should be noted that the data base used to develop the oxygen depletion rate model consists exclusively of natural lakes. The dissolved oxygen dynamics of reservoirs may be significantly different from those of natural lakes owing to differences in morphometry and hydrodynamics. In addition, allochthonous sources of oxygen demand may be more important in reservoirs than in natural lakes. For these reasons, additional analysis needs to be done to determine whether modifications are necessary for reservoirs.

In reducing the Connecticut lake data provided by *Norvell and Frink* [1975], only those sampling dates with phosphorus, transparency, and chlorophyll-a measurements have been included, resulting in an average of only 2.1 sampling dates at one station per lake. This provides a benchmark for assessing data adequacy in applications and demonstrates the feasibility of using the approach with limited amounts of information, as is often the case in lake management situations. While this level of information may be adequate for preliminary analyses or for screening lake problems at a regional level, more intensive monitoring efforts are required to provide a basis for management strategy design and implementation.

The magnitudes of potential errors should be considered in applying these relationships. Table 3 compares the standard errors involved in estimating any of the four versions of the trophic state index on the basis of any one of the remaining three. These standard errors range from 5.0 to 7.5 index units. By comparison, the standard errors of phosphorus mass balance models typically used to predict lake phosphorus concentrations from phosphorus loadings, hydrologic factors, and morphometric factors are on the order of 0.2, on a base 10 logarithmic scale [*Reckhow*, 1977; *Walker*, 1977]. On the basis of (2) a standard error of 0.2 in the logarithm of lake total phosphorus concentration corresponds to a standard error of 9.2 in estimating the phosphorus-based trophic state index from the above external factors. On the basis of a comparison of error variance there appears to be more uncertainty involved in predicting lake phosphorus concentrations from external factors than in summarizing relationships among within-lake measures of trophic state (phosphorus, chlorophyll-a, transparency, and oxygen depletion rate). As discussed above, these errors result from combinations of model and data errors. The relatively poor performance of phosphorus mass balance models may be due, in part, to the relatively poor quality of phosphorus loading estimates, which are often more difficult to quantify than within-lake conditions owing to the temporal and spatial variability of flows and phosphorus concentrations in tributary streams and point sources.

The feasibility of improving upon these types of relationships depends strongly upon the quality of the data used in model development and testing. The data base used above has been derived from a number of studies employing different spatial and temporal sampling frequencies, analytical methods, and data reduction methods. Uniformity in these aspects would permit more direct comparison of lake characteristics. This may lead to the development of models which are more theoretically based and better understood from an error analysis point of view than those presented above.

TABLE 3. Standard Error Matrix

Symbol	Index Version	Equation	I_P	I_B	I_T	$\bar{I}$*
I_P	phosphorus	(2)	···	···	···	···
I_B	chlorophyll-a	(1)	7.5	···	···	···
I_T	transparency	(3)	5.7	6.8	···	···
I_O	oxygen depletion	(9)	6.4	6.9	6.6	5.0

Based upon data from 30 lakes.
* Defined in (5).

CONCLUSIONS

Using a simple, empirical framework, this paper has demonstrated the feasibility of relating measures of epilimnetic algal standing crop to hypolimnetic oxygen depletion. The influence of lake morphometry on these relationships has been identified and quantified. An empirical model for estimating areal hypolimnetic oxygen depletion rate as a function of summer, epilimnetic phosphorus concentration and mean depth has been shown to give reasonable predictions of late summer hypolimnetic oxygen status when tested on an independent data base. These relationships have been summarized in the form of an index which can be computed from chlorophyll-a, transparency, phosphorus, and/or oxygen depletion rate measurements.

In applications to lake management the index provides a link between phosphorus mass balance models and existing water quality criteria for dissolved oxygen. Rational phosphorus criteria for maintaining acceptable hypolimnetic oxygen concentrations depend upon lake morphometry. This dependence should be taken into account in establishing phosphrous standards and in ranking lakes with respect to potential dissolved oxygen problems. More work is needed to explain these relationships on a theoretical basis, to identify and separate error components, and to test their validity in reservoirs and in deep and/or colored lakes.

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