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Use of Markov-encoded sequential information in numerical signal detection*

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Twelve Ss made binary decisions with feedback on numbers from one of two normal distributions with equal variances and unequal means. Sequences of distribution choices corresponded to first-order two-state Markov processes with probabilities of change of state of $p_1 = p_2 = .50$, $p_1 = p_2 = .75$, and $p_1 = p_2 = .25$. Performance was best (in d' terms) when $p_1 = p_2 \neq .50$. First-order sequential response dependencies tended to mirror the first-order stimulus dependencies. Violations of a fixed cutoff point decision rule were concentrated in the region of the average critical point, with a bandwidth of about $1/2\sigma$, in which violations were strikingly more frequent than would be expected if they had occurred randomly. These results imply that in this task Ss are using a criterion-band decision rule instead of a fixed cutoff point rule, and that they are basing decisions in the region of the criterion band on information extracted from the sequence of decisions presented to them. The average bandwidth is generally different from the optimum bandwidth used by an ideal O in combining the two sources of information.

The present study was designed to investigate whether and how people detect and use sequential information in making binary numerical detection decisions. This problem is important for at least two reasons. First, many investigators have found that Ss, in identification and detection tasks with sensory stimuli, typically exhibit response biases, often called "sequential effects," which have been interpreted as active searching for nonexistent sequential information in the sequence of stimuli presented to them for judgment (Ward & Loackhead, 1971). Typically, with many stimuli, the response to a particular stimulus is assimilated to the immediately preceding stimulus and response and contrasted with stimuli and responses from about 2-6 trials back in the sequence (Ward & Lockhead, 1971). When there is no information feedback, the assimilation seems to be extended further back in the sequence before the contrast appears (Ward & Lockhead, 1971). When there are only two stimuli, feedback also affects the form of the sequential effects, with contrast between the response to a particular stimulus and the immediately preceding stimulus being reported when there is no feedback (Parducci & Sandusky, 1965) and assimilation when feedback is given (Tanner, Rauk, & Atkinson, 1970). Also, some recent studies show that sequential patterns can, in fact, be discriminated from each other (e.g., Pollack, 1971), so it is plausible that Ss may detect and use sequential information if it is available.

Second, a recent paper by Kubovy, Rapoport, and Tversky (1971) indicated that in a detection task free from sensory components, Ss seem to use a fixed cutoff point decision rule of the type proposed for the ideal O in signal detection theory (SDT—Green & Swets, 1966). In their study, Kubovy et al (1971) had Ss decide from which of two overlapping normal distributions numerical stimuli were drawn. The Ss made several thousand judgments before experimental data were collected so that they would be familiar with the distributions from which the stimuli were drawn and learning effects would be minimized. In addition, the only pay received by the Ss was contingent on their performance. This was done to ensure that Ss would be motivated to use an optimum decision strategy. Kubovy et al (1971) found that, in general, a deterministic (fixed cutoff point) model of decision making fit the data rather well, much better than a generalized (probabilistic) micromatching model similar to that proposed by Lee and his associates (Lee, 1971). However, the test passed by the deterministic model was a statistical one, and, as Kubovy et al (1971) point out, that model is intolerant of any error. In an analysis of the violations of the deterministic model, they found that on average 5.87% of the decisions made by their Ss violated the model, and this after several thousand practice trials. In several conditions, for some Ss, there were over 15% violations.

Although the study by Kubovy et al (1971) seems to have been somewhat successful in controlling sensory, learning, and motivational components in the decision process, the presence of violations of the proposed model leaves doubt as to whether it wholly explains decision behavior in binary detection situations. It is possible that, similarly to binary detection situations in which sensory stimuli are used, Ss' decisions are subject to sequential response biases which are giving rise to the observed violations of the model. In the present study, therefore, sequential information was introduced into the situation, and analyses of performance, sequential response dependencies, and violations of the deterministic model were performed in an effort to expose the reasons for the violations of that model.

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Table 1
d′ Measures of Decision Performance

S	Order	ALT(1)	RAN(2)	RUNS(3)
3	123	1.008	.807	1.428
5	132	.760	1.106	.900
10	123	1.787	1.101	1.349
12	132	.921	.511	1.492
1	213	1.114	1.054	1.526
4	231	.890	.800	1.147
8	213	1.223	.744	.965
11	231	1.232	.977	1.058
2	321	1.257	.899	1.483
7	312	.430	.605	.783
13	312	1.072	.551	1.227
14	321	1.155	.874	1.299
Mean		1.071	.836	1.221

METHOD

Sequential Information

If the sequence of stimuli in a binary detection task is viewed as a first-order two-state Markov process, then it can be seen that run length and frequency, and frequency of stimulus occurrence, can be manipulated by manipulating the probabilities of change of state in the transition matrix of the Markov process. When the matrix is symmetrical about the main diagonal, overall probabilities of occurrence of the two states will be equal. This is the case for all of the matrices used in this study.

Three conditions varying in the amount and type of sequential information present were created by choosing sequences of stimuli according to probabilities of change of state of .25, .50, and .75. A sequence with probabilities of change of state of $p_1 = p_2 = .50$ corresponds to the typical "random" sequence of stimuli, and there is no information present in the sequence as to which of the two stimuli is more likely on a given trial. However, when $p_1 = p_2 = .25$, there will be more and longer runs of the same state (stimulus) than in the "random" sequence, and on a given trial the probability that the stimulus is identical to that of the previous trial is .75. When $p_1 = p_2 = .75$, the situation is reversed, with the probability of an alternation from the stimulus of the preceding trial equal to .75. Thus, in the latter two conditions, probabilistic information relevant to the identity of the stimulus is present in the sequence of stimuli.

Stimuli

In the present study, each state of the Markov process discussed above corresponded to the selection of a number from a particular one of two overlapping normal distributions with standard deviations of $\sigma_1 = \sigma_2 = 100$ and means of $\mu_1 = 1{,}500$ and $\mu_2 = 1{,}600$. The task of the S, when presented with a number (integer) drawn at random from one of these two distributions, was to say from which distribution it was drawn by responding "1" or "2."

Procedure

Ss were run on a Datel remote terminal connected to the time-sharing facility (Call 360/OS) of the IBM 360/67 computer at Rutgers. A program written in the BASIC language controlled stimulus selection and presentation. The program decided what state of the Markov chain it was in on a given trial according to the appropriate probabilities of change of state for the particular condition being run, selected an integer at random from the indicated normal distribution, presented the number to the S, recorded his response, and provided feedback as to the correct response on that trial. Fourteen volunteer Ss (undergraduates at Rutgers University) were run in the study; the data of two of these were discarded without analysis when serious procedural violations were discovered. Each of the remaining 12 Ss had 100 practice and 400 experimental trials in each of the three conditions outlined above. Conditions were run in counterbalanced order in a repeated measures design; two Ss were run in each of the six possible orders of the three conditions. Ss had one condition per 1.5-h session and one session per day.

In an attempt to make the situation comparable to that of Kubovy et al (1971), Ss were given a sheet of paper on which were drawn two prototypical overlapping normal distributions together with a brief explanation about probability distributions and the expected frequency of occurrence of numbers at various places on the continuum. The means of the distributions were those actually used in the experiment, but the variances were not veridically displayed. It was hoped that this would at least partially compensate for the small number of practice trials allowed Ss. In addition, Ss also received monetary compensation contingent upon their performance for participation in the experiment. A symmetric payoff matrix was used, in which each correct answer was worth .7 cents and each incorrect answer was worth nothing. Ss stopped every 100 trials to calculate their payoffs for the preceding 100 trials. Ss made an average of $6 for the whole experiment, paid after all conditions were completed. It might be noted that about half of the Ss were relatively sophisticated with respect to probability theory, having completed a first course in statistics during the previous semester. The other half of the Ss were relatively naive. Degree of sophistication in general made no difference in the results to be reported.

RESULTS AND DISCUSSION

Performance

The d′ measure of performance was calculated in two ways. First, the overall percentages of hits and false alarms for the data of each S in each condition were used to calculate the traditional d′ measure. These d′s are displayed in Table 1. For the data, ANOVA shows no effect of order of running in the different conditions and no interaction between order and condition. However, the average d′s of the three conditions are significantly different ($F = 7.53$, $p < .005$). A Newman-Keuls test on the pairs of means indicates that the ALT and RUNS conditions do not differ at the .05 level, but that each is significantly different from the RAN condition.

A second set of d′s, in which any first-order sequential response dependencies were taken into account, was calculated for the same data (see Sandusky, 1971, for details of the analysis). ANOVA of these d′s yielded no significant differences over the three conditions, indicating that the differences in the ordinary d′ measures reported above were due to the presence of first-order sequential response dependencies in the data, and not to improved discriminability of the distributions.

These sequential dependencies were analyzed further by forming a S_{n-1} by R_n (Stimulus on Trial $N - 1$ by Response on Trial N) matrix for each S for each condition. Chi-square tests of significance on these matrices indicated that every S in the ALT condition had a significant tendency ($p < .05$ or better) to give the response different from the immediately previous correct response, every S in the RUNS condition had a significant tendency to repeat the immediately previous correct response, and 9 of 12 Ss showed no significant sequential response dependencies in the RAN condition. In the latter condition, 2 of the 12 showed a significant tendency to alternate and 1, a significant tendency to repeat the immediately previous correct response. Thus, the first-order sequential response dependencies tended to mirror the transition probabilities of the Markov chain of stimuli. This result is very similar to that found by Anderson (1960) for a two-choice learning situation, and in combination with the second d' analysis above indicates that the differences in the first d' analysis are due to Ss' tendency to respond in a fashion that utilizes available sequential information.

However, it is easily seen that the available sequential information was not used optimally. An S who combined the two sources of information (distributional and sequential) in an ideal way would use a criterion band decision rule (see Appendix), which would give a maximum (ordinary) d' for the ALT and RUNS conditions of about 1.454. In the RAN condition, the criterion band would have zero width, leading to a cutoff point decision rule and a d' of 1.0. Of course, if the simple cutoff point rule was used in the ALT and RUNS conditions, d' would be 1.0 over a large number of trials in those conditions also.

It can be seen in Table 1 that in no condition is the average d' very near the ideal maximum, although several of the individual entries do approach it. In addition, since the mean d's are significantly different from each other, it is unlikely that Ss are simply using a fixed cutoff point decision rule in this experiment, especially in the RAN and RUNS conditions, where the average d' is also significantly different from 1.0 ($t_{RAN} = -3.02$, $p < .02$; $t_{RUNS} = 3.34$, $p < .01$).

It seems reasonable to assert that use of available sequential information in the ALT and RUNS conditions is leading to a response advantage in those conditions. It is also possible that unsystematic attempts to use sequential information that is not there is causing lowered performance in the RAN condition. Since the sequential and stimulus information are uncorrelated, such behavior should often show up as violations of a fixed cutoff point decision rule. An analysis of these violations might indicate where and how this information is being used.

Violations of the Fixed Cutoff Point Decision Rule

A critical point (a stimulus value for which the number of violations of the fixed cutoff point decision rule is minimal—see Kubovy et al, 1971) was calculated for the data of each S in each condition. The critical point was treated as a hypothetical cutoff point, and each of the 14,400 decisions made in this experiment were checked for violation of a fixed cutoff point decision rule. A violation was recorded when a response of "1" was made to a stimulus value above the hypothetical cutoff point for the particular S and condition, and when a response of "2" was made to a stimulus value below the hypothetical cutoff point. There was no systematic variation in the calculated critical points. The average critical points were 1,554 for the ALT condition, 1,558 for the RAN condition, and 1,552 for the RUNS condition, all quite near the ideal cutoff point of 1,550 for this situation, indicating no significant bias in favor of one of the two responses.

Table 2
Frequency of Violation of the Simple Deterministic Model

S	Order	ALT(1)	RAN(2)	RUNS(3)	Mean
3	123	11	51	132	
5	132	56	32	34	
10	123	99	49	84	60.8
12	132	25	82	75	
1	213	33	13	76	
4	231	37	78	46	
8	213	9	52	29	44.5
11	231	89	20	52	
2	321	102	48	98	
7	312	135	72	43	
13	312	99	110	20	73.8
14	321	37	37	85	
Mean		61.0	53.7	64.5	

There were 2,150 violations (14.9%) in the 14,400 trials checked. This is more than found by Kubovy et al (1971) for a similar situation. However, they introduced no sequential information, and the introduction of such information in the present study might have led to more violations of the model. Therefore, only the violations found in the data of the RAN condition for the Ss who had that condition first were considered, this being the best comparison to Kubovy et al (1971). Of the 1,600 decisions in that condition, 10.3% were violations of the simple model. This is closer to the 5.87% reported by Kubovy et al (1971), but still larger. It must be presumed that, although a good deal of the larger number of violations in the present study is due to the introduction of sequential information, at least some of the difference may be due to less experience with the distributions used.

Table 2 displays the number of violations of the cutoff point decision rule for each S and each condition. Although the average number of violations in each condition mirrors the average performance data, the differences are not statistically significant. The only significant effect in the data of Table 2 is the effect of order of presentation of conditions ($F = 4.59$, $p < .05$). Ss who ran in the RUNS condition first produced the most violations overall; Ss who had the RAN condition first produced the least. This is evidence that the

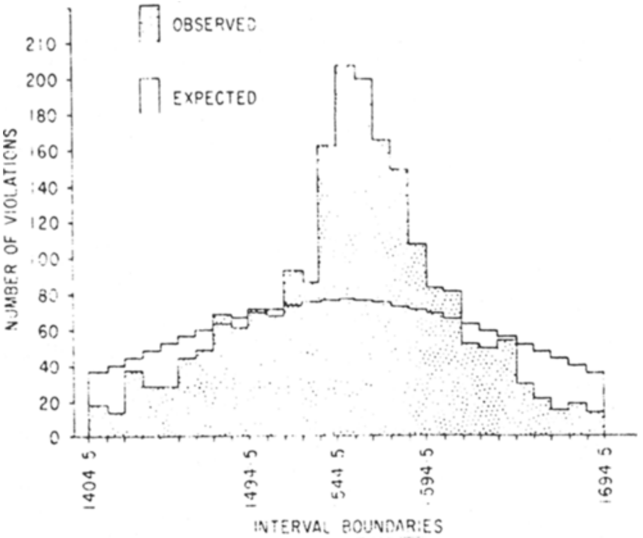

Fig. 1. Observed and expected frequency distributions of the simple deterministic model for stimulus values between 1,404.5 and 1,694.5. The data are collapsed over Ss and conditions.

detection of sequential information and its use in appropriate situations is conducive to more attempts to use it even when it is inappropriate. It can also be seen, by comparison of Tables 1 and 2, that for the Ss whose performance indicates use of sequential information to improve performance, the overall number of violations of the cutoff point decision rule increases over sessions. This is the opposite of what would be happening if continuing learning of the stimulus distributions were producing closer correspondence of the Ss' decision behavior to the simple decision rule. On the other hand, some Ss do seem to show fewer violations in their later sessions, and, indeed, they are generally those Ss who did not seem to make use of the sequential information to increase performance in the ALT and RUNS conditions. These Ss could be said to be learning the distributions, although, as will be pointed out, they did not differ from the others as to where their violations occurred, and their data showed similar sequential response dependencies.

Perhaps a more interesting question concerns *where* the violations of the simple decision rule are occurring. Several authors, in investigating sequential effects found in sensory identification and detection situations, have suggested that attempted use of sequential information occurs mainly in areas of the stimulus continuum where the S is highly uncertain as to the appropriate response (Parducci & Sandusky, 1965; Sandusky, 1971; Ward & Lockhead, 1971). In fact, Parducci and Sandusky (1965) proposed that Ss used a criterion-band decision rule in their position-detection task. As would the ideal combiner of stimulus and sequential information, Ss seemed to base decisions on stimulus information for stimuli outside the criterion band, and at least partly on "response biases" associated with the previous sequence of stimuli for stimuli within the band. If Ss are using a similar variant of the ideal criterion-band decision rule in the present situation, then there should be an area of the stimulus range in which violations of a cutoff point decision rule are more frequent than in other areas; this area would correspond to the criterion band. A reasonable place to look for this band would be around the ideal critical point as calculated from the simple model, since in this region the likelihood ratio between the two distributions is closest to 1.0, implying maximum stimulus uncertainty. Of course, the ideal criterion band is centered about the ideal critical point and, in the RAN condition, is identical to it.

In order to investigate the above speculations, the interval between 1,404.5 and 1,694.5 on the stimulus continuum (a range of about 3σ) was divided into 29 equal intervals, and the number of violations that occurred to stimulus values within each interval (and the regions on each end) was compiled separately for each S and each condition. The overall result of this analysis is displayed in the observed frequency distribution of violations in Fig. 1. The expected distribution of violations is calculated on the assumption that if violations occurred at random over the range of stimulus values, the probability of a violation in a particular interval would be equal to the probability of a stimulus occurrence in that interval. To get the expected frequencies of violations, these probabilities were simply multiplied by the overall frequency of violations (2,150). The distributions displayed in Fig. 1 do not give quite a veridical picture, since there were a number of violations observed and expected beyond the region of intensive analysis. In the region below 1,404.5, 49 violations were observed, while 208.98 were expected there. In the roughly corresponding region above 1,694.5, 39 violations were observed, while 213.28 would have been expected if violations occurred at random over the range of stimuli. The figures match the relation in the tails of the two distributions in Fig. 1; in general, fewer violations than expected occurred there. However, in the middle of the range of stimulus values in Fig. 1, centered roughly about the average critical point of 1,555, there is a region in which the occurrence of violations of the simple deterministic model is far more frequent than expected. Because of these differences, the observed distribution of violations is significantly different from the expected distribution (χ^2 = 709.25, df = 31). The shape of the observed distribution is identical for each S in each condition, although, of course, more variable. Especially interesting in this connection is the observed distribution for the RAN condition for those Ss who had that condition first, and thus had not been exposed to sequential information at the time they made their judgments in this condition. The shape of that distribution is identical to the overall distribution, and it, too, is statistically significantly different from the expected distribution of violations (χ^2 = 180.1, df = 31).

These data are evidence for the use of a criterion-band decision rule by the Ss in this experiment for all conditions, regardless of the presence or absence of sequential information. We may even speculate about the width of the criterion band from observing the width of the sharp peak in the distribution of observed violations in Fig. 1. This peak is roughly five intervals (or 50 numbers) wide; with $\sigma_1 = \sigma_2 = 100$, the average criterion bandwidth in the present situation is about ½σ. This is quite different from the optimum bandwidth for the present situation of 220 units, or 2.2σ for the ALT and RUNS conditions and zero for the RAN condition (see Appendix), and of course may not be the bandwidth over all judgment situations. In fact, since bandwidth can be thought of as related to the relative weighting given sequential information, it may vary in a systematic way as the experimental parameters are changed. For example, when bandwidths are estimated separately for the three conditions of the present experiment, the widest bandwidth is found in the RUNS condition (about .6σ), the next widest in the RAN condition (about .5σ), and the narrowest in the ALT condition (about .4σ). This is exactly the order of conditions when the distance of the average d′ from the d′ predicted by a fixed cutoff point decision rule is considered. It could be speculated that sequential information is most salient, and thus most heavily weighted, when long runs of one stimulus or the other occur. This was also found by Anderson (1960) for two-choice learning situations. The fact that the d′ in the ALT condition is not significantly different from 1.0 may be due to relatively light use of sequential information in that condition.

Although the criterion-band model of decision behavior has been found useful in explaining data in both "sensory" and "nonsensory" judgment situations, there may be differences in Ss' decision behavior within the criterion band depending on type and availability of extradistributional information. For example, a recent study by Sandusky (1971) found better performance in a no-feedback sensory detection situation when the probability of change of state was high (ALT) than when it was low (RUNS). Sandusky's analysis indicated that this relative response advantage was due to a tendency of Ss over all conditions to alternate responses more often than stimuli would alternate in a random sequence. In the no-feedback experiment, sequential information, even if present, is not available to the Ss with any certainty, and Ss may attempt to use their general "knowledge" about sequences to mediate their uncertainty in the region of the criterion band. Several authors have reported that most people think that random sequences have fewer and shorter runs than they, in fact, do have (Baken, 1960; Chapanis, 1963; Jenkins & Cunningham, 1949). This would explain the tendency to alternate responses in no-feedback experiments.

Just the opposite tendency seems to be operating in some sensory detection experiments where feedback is provided (e.g., Tanner, Rauk, & Atkinson, 1970). Sandusky (1971) suggests that repetition of the immediately previous correct response is the rule within the criterion band in experiments of this type. This suggestion is based on experiments in which the probability of occurrence of stimuli was manipulated (asymmetric transition matrices in Markov chains of stimuli). This manipulation could lead to perception of an overall lower than .5 probability of change of state when one stimulus was more probable, and could give rise to a tendency to repeat the immediately previous correct response. An earlier study by Friedman and Carterette (1964), in which several varieties of transition matrices were used to generate Markov chains of auditory stimuli for detection in noise with feedback, lends credence to this view. Performance results in that study were similar to those in the present study, and analyses of sequential response dependencies in the data indicated that Ss' decision behavior seemed to track the sequential contingencies of the various transition matrices used, including the asymmetric ones.

Thus, it seems that in many types of judgment situations, Ss attempt to combine two sources of information, using "general" or specific sequential information to mediate uncertainty in areas where stimulus information is ambiguous. They do not, however, seem to combine the two sources optimally, indicating either a general fixed response strategy or a lack of ability to discriminate accurately the relative usefulness of the two types of information. Which of these is the case is an interesting question for further investigation.

APPENDIX

This appendix contains a brief discussion of the performance of the ideal O in combining the two sources of information used in the present study, distributional information and sequential information. In what follows, the ordinary simplifying assumptions of SDT are made. In addition, only symmetric payoff matrices and symmetric transition matrices of first-order two-state Markov chains of stimulus presentations with feedback are considered. Changes in these conditions change the analysis somewhat and require somewhat more general statements. However, the general ideas presented remain valid with such changes.

In the simple SDT situation, the ideal O is assumed to know or to calculate some monotonic function of the likelihood ratio

$$\ell(x) = \frac{P(x \mid 2)}{P(x \mid 1)},$$

where the value x is given rise to by a stimulus which could have come from either of two probability distributions. In the present case, x is given directly as a number, and it could have been chosen from either of two normal distributions (labeled 1 and 2) with $\sigma_1 = \sigma_2$ and $\mu_1 < \mu_2$. A simple cutoff point decision rule is used, so that the ideal O

$$\text{responds} \begin{Bmatrix}1\\2\end{Bmatrix} \text{ when } \ell(x) \begin{Bmatrix}<\\>\end{Bmatrix} \beta = \ell(x_c),$$

where x_c is the cutoff point or criterion. In essence, the likelihood ratio $\ell(x)$ is a measure of the relative discriminability of the two distributions from one another—the farther from 1.0 is $\ell(x)$, the more discriminable are the distributions. Ideally, the response chosen on every trial would be the one which asserted that the distribution most likely to have given rise to a particular x did, in fact, do so.

However, in the present situation, there are two likelihood ratios available on each trial. First, a (redefined) ratio calculated from the stimulus probability distributions,

$$\ell(x) = \frac{P(x \mid i)}{P(x \mid j)} \text{ for } P(x \mid i) > P(x \mid j),\ i,j = 1,2,$$

and second,

$$\ell'(x) = \begin{cases} \dfrac{P(s_n = s_{n-1})}{P(s_n \neq s_{n-1})} \\[2ex] \dfrac{P(s_n \neq s_{n-1})}{P(s_n = s_{n-1})} \end{cases}$$

$$\text{where } P(s_n = s_{n-1}) \begin{Bmatrix}>\\<\end{Bmatrix} P(s_n \neq s_{n-1}),$$

which is the same for all x and is calculated from the probabilities of the (symmetrical) transition matrix of the Markov chain of stimulus presentations.

The ideal O, confronted with $\ell(x)$ and $\ell'(x)$, would use whichever one gave the best discriminability (i.e., value farthest from 1.0). For $\ell'(x) = k$ and $\ell(x)$ calculated for x from two normal distributions with equal variance and unequal means, there would be a region of the stimulus continuum in which $\ell'(x) > \ell(x)$, and regions to either side of the points where $\ell'(x) = \ell(x)$ [x_{cl} and x_{cu}], in which $\ell(x) > \ell'(x)$. The best decision rule in this case would be

$$\text{respond} \begin{cases} 1 & \text{for } x \leqslant x_{cl} \\ 2 & \text{for } x \geqslant x_{cu} \\ R_n & \text{for } x_{cl} < x < x_{cu} \end{cases}$$

where

$$R_n = \begin{Bmatrix} s_{n-1} \\ \sim s_{n-1} \end{Bmatrix} \text{ if } P(s_n = s_{n-1}) \begin{Bmatrix}>\\<\end{Bmatrix} P(s_n \neq s_{n-1}).$$

Thus, the region $x_{cl} < x < x_{cu}$ is a criterion *band*, outside of which responses are determined by distributional information and inside of which sequential information is used. Of course, where the probabilities in the transition matrix of the Markov chain of stimulus presentations are all .5 (the case of "random" stimulus presentation order), $\ell'(x)$ would equal 1.0, and the criterion band would be of zero width and would correspond to the cutoff point in SDT.

For the present situation, the performance of the ideal O can be calculated quite easily. Since $P_1 = P_2 = .5$ in the RAN condition, the criterion band has zero width and the ideal cutoff point is 1.550. Maximum d′ here is 1.0 over a large number of trials. For the ALT and RUNS conditions, $\ell'(x) = .75/.25 = 3.0$. The criterion band extends from $x_{cl} = 1.440$ to $x_{cu} = 1.660$ [at these points $\ell(x) = 3.0$], a width of 220 numbers, or 2.2σ. Maximum d′ in these conditions over a large number of trials is about 1.454 if the ideal decision rule described above is used. This is calculated by weighting expected P(HIT) and P(FA) for x inside and outside of the criterion band by the relative proportion of observations occurring overall in these regions (w_i and w_o), i.e.,

$$P(HIT)_T = w_i P(HIT)_i + w_o P(HIT)_o,$$

$$P(FA)_T = w_i P(FA)_i + w_o P(FA)_o;$$

$$w_i + w_o = 1.0.$$

The d′ is then calculated from the composite hit and false alarm rates.

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